K Academies &c

THE PHILOSOPHICAL TRANSACTIONS

From the Year MDCC.

(Where Mr. *LOWTHORP* ends)

To the Year MDCCXX.

ABRIDG'D,

AND

Diſpos'd under General HEADS.

In Two VOLUMES.

By BENJ. MOTTE.

LONDON:

inted for R. WILKIN, R. ROBINSON, S. BALLARD,
W. and J. INNYS, and J. OSBORN. MDCCXXI.

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MVSEVM BRITANNICVM

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VOL. I. Containing

Part I. The MATHEMATICAL Papers.
Part II. The ANATOMICAL and MEDICAL Papers.

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MVSEVM
BRITAN
NICVM

To the MOST NOBLE,

JAMES,

DUKE and BARON of

CHANDOS,

MARQUESS and EARL of

CAERNARVAN;

Viſcount WILTON of *Stukeley-Caſtle* in the County of *Hereford*,

AND

Lord Lieutenant and *Cuſtos Rotulorum* of the Counties of *Hereford* and *Radnor*.

THESE MATHEMATICAL and MEDICAL Papers, Abridg'd and Diſpos'd under General Heads, are, with all Gratitude and Humility, dedicated by

HIS GRACE'S

Obedient, Devoted,

Humble Servant,

BENJ. MOTTE.

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THE PREFACE.

AFTER *the Reader has been inform'd, that what now lies before him, is only a Continuation of* Mr. LOWTHORP's *Abridgment from the Period of his Work to the preſent Time; and that I have throughout copy'd after his Pattern, ſo far as my inferiour Judgment will give me leave: it will be unneceſſary to ſpend many Words as to the Method here purſu'd; and ſufficient to refer to his Preface for a ſuccinct and particular Account of it. At the ſame time I look upon it as a Duty incumbent on me ingenuouſly and with Gratitude to confeſs, that whatever Advantages may accrue to my ſelf or the Publick by the preſent Performance, are owing to the ingenious Hint of that judicious Gentleman; and to the Applauſe and Encouragement which his uſeful Undertaking was moſt juſtly receiv'd with.*

IT *may not however be improper briefly to take notice of the following Particulars.*

1. THE *Anatomical Papers, which are by* Mr. LOWTHORP *made the Third Part, are here plac'd in the Second: which was*

done only for the Convenience of Binding, and to divide the Two Volumes equally. This Reaſon, I preſume, will be ſufficient to juſtify ſo minute an Alteration.

2. I *have omitted ſuch Experiments of the late Mr.* Hawksbee, *as he has himſelf collected and publiſh'd in his* Phyſico-Mechanical Experiments *printed in* 1709. *ſince this Book, in all probability, makes a part of the Library of any Gentleman whoſe Curioſity and Genius inclines him to Studies of that kind.*

3. SEVERAL *Catalogues of Plants, Minerals, Foſſils, Inſects,* &c. *publiſh'd by the late inquiſitive Naturaliſt Mr.* James Petiver, *are alſo omitted; by reaſon the moſt material Particulars have been by himſelf inſerted in the ſeveral Parts of his* Gazophylacium Naturæ & Artis: *and beſides, that thoſe Catalogues are of Bulk ſufficient to make a handſome Volume of themſelves, if ſuch a Deſign could meet with ſuitable Encouragement.*

4. THE *Tracts relating to the Diſputes between the Mathematicians of our own Country and ſome Gentlemen abroad, are above the Capacity of the Majority of Readers; and were intended rather for an honeſt Repreſentation of matter of Fact to Poſterity, than for the immediate Inſtruction of the Studious in thoſe Sciences. For which reaſon I have contented my ſelf with referring to the Places in the* Tranſactions, *where thoſe Tracts are to be found, for the Service of ſuch as are deſirous to be acquainted with the Merits of that Controverſy.*

IT *may perhaps be expected that I ſhould ſay ſomething in this place to excuſe what Imperfections may occur in the Courſe of this*

Work; and take notice of a Competition wherewith I am threatned. As to the firſt of theſe, I have already confeſt myſelf indebted to a more able Hand for the Method, and to the Labours of many eminent Men for the Materials, whereof it is compos'd. After this, he muſt be maſter of very little Skill and Judgment, who with ſuch Aſſiſtances is not able to furniſh out a tolerable Performance; and I may without vanity hope, the Reader will find no Errors or Defects, but what a good natured Man may excuſe, on account of the Length and Tediousneſs of the Work, and the number of Papers, and variety of Subjects whereof it conſiſts.

As *to the Competition there is reaſon to expect, it is a Misfortune which can in no wiſe be imputed to me. Before I had any Apprehenſion of a Deſign of that nature, I was engaged too far to recede with any Conveniency. I notwithſtanding endeavour'd an Accommodation, provided it could be concluded upon Terms which implied an Equality; but this readineſs of mine to comply was always fruſtrated by an imagin'd Superiority of Learning, or Intereſt, or ſomething elſe on the other ſide. How reaſonable theſe Preſumptions were, muſt be ſubmitted to the Judgment of the Publick. In the mean time, it would be low and impertinent in this place to deſcend to the further Particulars of an Affair, which has relation only to two private Perſons.*

I *ſhall conclude this Preface with offering one Conſideration to the Thoughts of ſuch Gentlemen as deſign to communicate their Labours to the World: which is the Convenience that would enſue, if in every Book that is publiſh'd, a Memorial was inſerted of the Day* *and Year when it firſt appear'd in Print, and this Date continu'd in ſucceeding Impreſſions. It would give an additional Pleaſure to a future Reader, to obſerve by what Steps and Advances Learning was improv'd; and would enable Poſterity to do Juſtice to the true Author of a fine Thought or uſeful Diſcovery; which is no more than neceſſary in an Age where Plagiary is ſo common a Vice. Other Reaſons might be given to confirm this Caution, but it would be tedious to inſiſt at large on what I only mention as an Hint, in my opinion, worthy Obſervation; and not improper to be prefixt to a Book which treats of Miſcellaneous Subjects.*

Sept. 28. 1721.

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ERRATA.

PArt II. p. 13. *firſt Note in Marg. read* n. 337. p. 281. p. 70. *in Marg. for* Fig. 9. *r.* Fig. 15. p. 101. *l.* 33. *r. Figures.*

THE
Philoſophical Tranſactions
From the Year 1700. to 1720.
Abridg'd and Methodically Digeſted

PART I.
The *Mathematical* Papers.

CHAP. I.
Geometry, Algebra, Logarithmotechny, Arithmetick.

SIT *DE* Axis Tranſverſus Ellipſeos, *AO* Axis alter, & C centrum Sectionis. Sit *P* punctum quodvis in circumferentia ejus; *PQ* Tangens curvæ ad *P*, occurrens Axi tranſverſo ad *Q*; puncta *SF* Foci; *CP*, *CK* Semidiametri conjugatæ; *PH* Semilatus rectum ad diametrum *PC*: *PG* normalis ad Tangentem, cui occurrat *HG*, perpendicularis ipſi *PCH*, in puncto *G*: ut fiat *PG* radius Curvaturæ Ellipſeos in puncto *P*: ſint etiam *ST*, *CR*, *FV* perpendiculares in Tangentem *PQ* demiſſæ: Jungatur *SO*, & demittatur in Axem normalis *PL*. His poſitis, dico quod,

I. *Some Properties of the Conick Sections, by Mr.* Abr. de Moivre, *n.* 352. *p.* 622. *Plate* I. *Fig.* 1.

I. *Rectangulum ſub diſtantiis ab utroque Ellipſeos Foco, ſive* SP × SF *æquale eſt quadrato Semidiametri* CK.

Demonſtratio.

$PSq = PCq + CSq - 2CS \times CL$ per 13. II. *Elem.*
$PFq = PCq + CSq + 2CS \times CL$ per 12. II. *Elem.*
Unde $PSq + PFq = 2PCq + 2CSq$.
Jam $PS + PF = DE = 2CD$, ac propterea
$PSq + PFq + 2PS \times PF = 4CDq$.

Quare tranſponendo, $2PS \times PF = 4CDq - 2PCq - 2CSq$.
Ac Dimidiando $PS \times PF = 2CDq - PCq - CSq$.
Eſt autem CS *quad.* $= CD$ *quad.* $- CO$ *quad.* atque adeo,
$PS \times PF = CDq + COq - PCq$.
Sed $CDq + COq = PCq + CKq$. *per* 12. VII. *Conic. Apollonii.*
Quare $PS \times PF = CKq$. *Q. E. D.*

II. *Diſtantia à Foco* SP *eſt ad perpendicularem in Tangentem demiſſam, ut Semidiameter Conjugata* CK *ad Semiaxem minorem* CO.

Demonſtratio. Ob ſimilia Triangula SPT, FPV, erit $PS : PF :: ST : FV$; ac componendo $PS + PF$ erit ad $ST + FV$, & earundem dimidia CD ad CR, ut PS ad ST. Unde $CD \times CK$ erit ad $CR \times CK$ ut PS ad ST. Sed $CR \times CK$ æquale eſt rectangulo ſub Semiaxibus CD in CO, per 31. VII. *Conic.* Proinde PS eſt ad ST ut CD in CK ad $CD \times CO$, ſive ut CK ad CO. Ac pari argumento demonſtrabitur PF eſſe ad FV in eadem ratione. *Q. E. D.*

III. *In eadem etiam eſt ratione Semiaxis Tranſverſus* CD *ad normalem è centro* C *ad Tangentem demiſſam, ſive ad* CR.

Etenim cum rectangulum $CR \times CK$ æquale ſit rectangulo $CD \times CO$, uti jam dictum eſt, erit ἀνάλογον CD ad CR ut CK ad CO. *Q. E. D.*

IV. *Semidiameter quævis* PC *eſt ad diſtantiam puncti* P *à foco* S, *ſive ad* SP, *ut diſtantia ab altero Foco* FP *ad dimidium lateris recti ad Verticem* P *pertinentis, ſive ad* PH.

Hoc autem manifeſtum eſt ob *Prop.* I. cum nempe quadratum ex CK æquale ſit rectangulo ſub $SP \times PF$.

V. *Rectangulum Semiaxium* CD × CO *eſt ad quadratum ſemidiametri conjugatæ* CK, *ut* CK *ad Radium Curvaturæ in puncto* P, *ſive ad* PG.

Sunt enim Triangula PCR, PGH inter ſe ſimilia, unde CR eſt ad PC, ut ſemilatus rectum PH ad PG: hoc eſt, per præmiſſam Proprietatem III, $\frac{CD \times CO}{CK} = CR$ eſt ad PC ut $\frac{CK^2}{PC} = PH$ ad $\frac{CK^3}{CD \times CO} = PG$. proinde ἀνάλογον $CD \times CO : CK^2 :: CK : PG$. *Q. E. D.*

Theorema Generale I. *Vis centripeta ad idem punctum* S *tendens, in Curvis omnibus, eſt ſemper proportionalis Quantitati* $\frac{SP}{PG \times ST^3}$.

Hoc Theorema ante plures annos à me inveſtigatum & cum amicis communicatum, propriis demonſtrationibus firmavere Geometræ Clariſſimi D. *J. Bernoullius* in *Act. Lipſiæ*; D. *J. Keillius* in harum *Tranſact.* * N. 317. & D. *Jac. Hermannus* in *Phoronomiâ* ſua pag. 70. quos vide.

* *v. infrà,* *Tract.* XX.

Scribendo autem CK^3 pro PG, per *Propr.* V; & $\frac{SP}{CK}$ juxta *Propr.* II. pro ST, (ob datas ſcilicet CD, CO) erit Vis centripeta tendens ad focum Ellipſeos S, ſemper ut $\frac{SP \times CK^3}{CK^3 \times SP^3}$, hoc eſt ut $\frac{SP}{SP^3}$ vel $\frac{1}{SP^2}$, nempe reciproce ut quadratum ex SP. Unde patet quod ſi Sectio fuerit Ellipſis motu corporis deſcripta, erit Vis Centripeta ut quadratum diſtantiæ a centro Virium reciprocè. Ex his Proprietatibus conſequuntur Corollaria nonnulla notatu non indigna.

Corol. 1. *Velocitas Corporis in Ellipſi revolventis, ad punctum quodlibet* P, *eſt ad Velocitatem revolventis in circulo ad eandem diſtantiam* SP *à centro Virium, in ſubdupla ratione diſtantiæ ab altero foco* PF, *ad Semiaxem tranſverſum Sectionis, ſive ut media proportionalis inter* PF *&* CD *ad* CD.

Eſt enim velocitas revolventis in Ellipſi ad diſtantiam SP, ad Velocitatem revolventis in Circulo vel Ellipſi ad diſtantiam Semiaxis CD vel SO, ut CO ad ST; hoc eſt per *Propr.* II. ut $\sqrt{PF}$ ad $\sqrt{SP}$. Velocitas autem revolventis in Circulo ad diſtantiam CD eſt ad velocitatem revolventis in Circulo ad diſtantiam SP, ut $\sqrt{SP}$ ad $\sqrt{CD}$. Ex æquo igitur, Velocitas revolventis in Ellipſi ad diſtantiam SP, eſt ad Velocitatem revolventis in Circulo ad eandem diſtantiam ut $\sqrt{PF}$ ad $\sqrt{CD}$.

Coroll. 2. *Ex datis Velocitate in Ellipſi, poſitione Tangentis, & centro Virium ſeu Foco, facile eſt determinare Focum alterum.*

Sit enim Velocitas Data R; ea autem Velocitas qua deſcriberetur Circulus ad datam a centro diſtantiam SP ſit Q; ac per *Coroll.* præcedens, R eſt ad Q ut $\sqrt{PF}$ ad $\sqrt{CD}$, adeoque QQ eſt ad RR ut CD ad PF, & $2QQ - RR$ erit ad RR ut SP ad PF: Datur autem SP; data eſt igitur PF magnitudine. Datur etiam poſitione, ob angulum VPF angulo SPT æqualem. Datur igitur punctum F alter Focorum. Quo invento pronum eſt Sectionem deſcribere.

Si vero $\frac{1}{2}RR$ majus fuerit quadrato ex Q, $2QQ - RR$ fit quantitas Negativa, & loco Ellipſeos Trajectoria deſcribenda in Hyperbolam tranſit. Eritque $RR - 2QQ$ ad RR ut SP ad PF diſtantiam alterius Foci, ad alterum Tangentis latus ponendam, ut habeatur Focus F. Proprietates autem omnes quas in Ellipſi demonſtravimus, mutatis mutandis, etiam Hyperbolæ competunt.

Plate 1. *Fig.* 2.

Quod ſi acciderit QQ æquale eſſe dimidio quadrati ex R; evaneſcente quantitate $2QQ - RR = 0$, quarta proportionalis PF fit infinita: proinde Trajectoria deſcribenda Parabolica eſt, Foco ſcilicet altero in infinitum abeunte. Axis autem Trajectoriæ poſitione datur; eſt enim ipſi PF parallelus, exiſtente ſcilicet angulo FPV angulo dato SPT æquali.

Coroll. 3. *Velocitas revolventis in data Sectione Conica ad diſtantiam* SP *eſt ad Velocitatem ejuſdem ad diſtantiam aliam* SX, *ut media proportionalis inter* FP *&* SX *ad mediam proportionalem inter* SP *&* FX.

Velocitas enim in P eſt ut $\sqrt{\frac{FP}{SP}}$ (per *propr.* II.) & per eandem, Velocitas in X eſt ut $\sqrt{\frac{FX}{SX}}$. Unde manifeſta eſt propoſitio.

Coroll. 4. *Ratio etiam Velocitatum duorum Corporum in eodem Syſtemate, ſed in datis Coniſectionibus diverſis, revolventium, datis utriuſque à communi Orbium Foco diſtantiis, ope Corollarii* 1^{mi}. *ſtatim obtinebitur.*

Cum enim Velocitas corporis in P ſit ad Velocitatem in Circulo ad eandem diſtantiam SP, ut $\sqrt{PF}$ ad $\sqrt{CD}$; & in alia ſuppoſita Coniſectione, cujus Semiaxis cd & Foci S, f, ad diſtantiam Sp, Velocitates illæ ſint ut $\sqrt{pf}$ ad $\sqrt{cd}$: Velocitas autem revolventis in Circulo ad diſtantiam SP ſit ad Velocitatem in Circulo ad diſtantiam Sp ut $\sqrt{Sp}$ ad $\sqrt{SP}$; Compoſitis rationibus, erit Velocitas in P ad Velocitatem in p, ut $\sqrt{PF \times cd \times Sp}$ ad $\sqrt{pf \times CD \times SP}$. Quod ſi Sectio illa altera fuerit Parabola, erunt cd, pf infinitæ, ſed in ratione 1 ad 2; proinde ratio Velocitatum erit ut $\sqrt{PF \times Sp}$ ad $\sqrt{2CD \times SP}$.

Coroll. 5. *Quod ſi in Hyperbola punctum* P *abeat in infinitum, ex præcedentibus manifeſtum eſt, Velocitatem ultimam ac minimam, qua cum corpus in æternum aſcenderet, æqualem eſſe ei qua, ad diſtantiam* CD *Semiaxi tranſverſo æqualem, Circulum deſcriberet.*

Coroll. 6. *Ex data diſtantia à Foco, datur quoque Poſitio Tangentis, ſive angulus* SPT, *ſub diſtantia* SP *&* *Tangente* PT *contentus.*

Eſt enim (per *propr.* II.) PS ad ST ut CK ad CO ſive ut $\sqrt{SP \times PF}$ ad CO, atque ita Radius ad Sinum anguli SPT. At in Ellipſibus Circulis affinibus præſtaret angulum PST, ejuſdem complementum ad quadrantem, inquirere: Hujus autem Sinus eſt ad Radium ut $\sqrt{SP \times PF - COq}$ ad $\sqrt{SP \times PF}$.

Coroll. 7. *Atque hinc conſequuntur Velocitates quibuſcum diſtantiæ* SP *creſcunt vel decreſcunt.*

Nam cum, ex Corollario præcedente, $\sqrt{SP \times PF}$ ſit ad $\sqrt{SP \times PF - COq}$ ut Radius ad ſinum anguli PST, ac in eadem ſit ratione Velocitas Corporis in P ad Velocitatem momenti ipſius SP; Velocitas autem illa in P ſit (per *propr.* II.) ut $\sqrt{\frac{PF}{SP}}$; eliſis ſuperfluis, erit $\sqrt{\frac{SP \times PF - COq}{SP}}$ Velocitati, qua creſcit vel decreſcit diſtantia SP, ſemper proportionalis.

Theorema Generale II. *In omni Trajectoria Curvilinea Velocitates angulares circa centrum Virium ſunt reciprocè proportionales quadratis diſtantiarum à centro.*

Nam ob Sectorum minimorum Areas æquales, arcus angulis minimis ſubtenſi ſive Baſes, ſunt reciproce ut Radii: Anguli autem minimi quibus Baſes æquales ſubtenduntur ſunt etiam reciproce ut Radii. Proinde anguli Sectorum minimorum Area æqualium, ſunt inter ſe reciproce in dupla ratione Radiorum, ſive ut quadrata diſtantiarum.

Coroll. 8. *Hinc Velocitates angulares revolventium in diverſis Ellipſibus datis comparantur inter ſe.*

Velocitates enim angulares quibuſcum ad diſtantias Semiaxibus Tranſverſis æquales circuli deſcriberentur, ſunt reciproce in ratione ſeſquialtera Axium, ſive ut $\frac{1}{CD\sqrt{CD}}$. Velocitates autem angulares has medias habent Corpora revolventia, cum quadrata diſtantiarum æquantur rectangulis ſub ſemiaxibus Ellipſeôn. Ideo (per *Theor.* II.) erit SPq ad

$CD \times CO$ ut $\frac{1}{CD\sqrt{CD}}$ ad $\frac{CO}{SPq \times \sqrt{CD}}$; quæ quidem Quantitas eſt ut Velocitas anguli ad centrum S, motu rectæ SP, tempore quam minimo dato, deſcripti.

Coroll. 9. *Velocitas angularis qua circumgyratur Tangens* PT, *ſive recta in Tangentem perpendicularis* ST, *eſt ad Velocitatem angularem recta* SP, *ut Semiaxis tranſverſus* CD *ad diſtantiam ab altero foco* PF.

Demonſtratio. Sint puncta P, p, quamproxima inter ſe, ductiſque SP, Sp, ſint PT, pt, duæ Tangentes, ad quas demittantur normales ST, St; iiſque parallelæ ducantur radii Curvaturæ PG, pG coeuntes in G: ac deſcribatur, centro S & radio SP, arcus minimus PE occurrens ipſi Sp in E. Manifeſtum eſt angulum PGp æqualem eſſe angulo TSt, ſive angulari Velocitati normalis ST. Eſt autem angulus PSp angularis velocitas rectæ Sp; quare angulus PGp eſt ad angulum PSp ut angularis velocitas ipſius ST ad angularem velocitatem rectæ SP; hoc eſt, ut $\frac{Pp}{PG}$ ad $\frac{PE}{PS}$. Sed $Pp : PE :: SP : ST :: CK : CO$ (per *propr.* II.) Hæ igitur Velocitates ſunt ut $\frac{CK}{PG}$ ad $\frac{CO}{PS}$. Pro PG ſcribe $\frac{CK^3}{CD \times CO}$ (per *propr.* V.) ac $\frac{CK}{PG}$ fiet $\frac{CD \times CO}{CKq} = \frac{CD \times CO}{PS \times PF}$. Hinc $\frac{CD \times CO}{PS \times PF}$ erit ad $\frac{CO}{PS}$ ſive, deletis ſuperfluis, CD ad PF, ut angulus TSt ad angulum PSp, ſive Velocitas angularis Tangentis ad angularem Velocitatem diſtantiæ SP: proinde Velocitas, qua circumgyratur Tangens, ſemper proportionalis eſt quantitati $\frac{CO \times \sqrt{CD}}{PF \times SPq}$.

Plate I. Fig. 3.

II. Sit A Area Curvæ cujus Abſciſſa x, & ordinatim Applicata $x^{m}\sqrt{dx - xx}$. Sit B Area Curvæ cujus Abſciſſa eadem cum priori, ſed ordinatim Applicata $x^{m-n}\sqrt{dx - xx}$; ponatur $\sqrt{dx - xx} = y$. Erit Area $A =$

A way to ſquare certain kinds of Curves, or to reduce them to more ſimple Ones. by the ſame. n. 278. p. 1113.

$$d^{1} B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} \text{ in } \frac{2m-5}{2m-2} \text{ \&c.} = \text{P}.$$

$$- \frac{1}{m+2} x^{m-1} y^3 = - \text{Q}.$$

$$= \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} x^{m-2} y^3 = - \text{R}.$$

$$= \frac{dd}{m} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} x^{m-3} y^3 = - \text{S}.$$

$$= \frac{d^3}{m-1} \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-1}{2m+2} \text{ in } \frac{2m-3}{2m} x^{m-4} y^3 = - \text{T}.$$

&c.

Ubi notandum 1° quod n ſupponitur numerus integer & affirmativus; 2° Quod Quantitas $d^n B$ in ſerie per P deſignata, multiplicari debet in tot terminos quot ſunt unitates in n; 3° quod tot ſequentes ſeries per $-Q-R,-S,-T$ &c. deſignatæ ſumi debeant, quot ſunt unitates in n; quod ut Exemplo uno vel altero clarius fiat, dico quod ſi $n=1$, tunc A

$$= d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3 \text{ \& ſi } n=2,$$

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} \text{ in } \frac{2m-2}{2m+2} - \frac{1}{m+2} x^{m-1} y^3$$

$$- \frac{d}{m+1} \text{ in } \frac{2m+1}{2m+4} \text{ in } x^{m-2} y^3$$

4° quod ſi y ponatur $= \sqrt{dx-xx}$ tunc A erit $= Q-R+S-T$ &c. $\pm P$.

Corollarium. Si m ponatur æqualis termino cuivis ſequentis Seriei

$$-\frac{1}{2},\ \frac{1}{2},\ \frac{3}{2},\ \frac{5}{2},\ \frac{7}{2},\ \frac{9}{2} \text{ \&c.}$$

quadratura Curvæ cujus ordinatim Applicata $x^m \sqrt{dx-xx}$, aut $x^m \sqrt{dx+xx}$ finita evadit & exhibetur per ſeriem noſtram; quod ut Exemplo illuſtretur, Inquirenda ſit Area Curvæ cujus ordinatim Applicata $x^{-\frac{1}{2}} \sqrt{dx-xx}$; fingatur Curvam hanc comparari cum Curva cujus ordinatim Applicata $x^{-\frac{3}{2}} \sqrt{dx-xx}$, quoniam hoc in caſu $n=1$, ideo

$$A = d^n B \text{ in } \frac{2m+1}{2m+4} - \frac{1}{m+2} x^{m-1} y^3$$

$$\text{ſed } m = -\frac{1}{2}, \text{ ergo } 2m+1=0, \text{ ideoq;}$$

$$A = -\frac{1}{m+2} x^{m-1} y^3 = -\frac{2y^3}{3\sqrt{x^3}}.$$

Hic Obſervatu dignum eſt quod Area ſic reperta interdum data quantitate deficit a vera Area, aut eandem data quantitate excedit; quo autem exceſſus iſte aut defectus innoteſcat, ſupponatur Area reperta augeri minuive data quantitate q, tuncque poſita $x=0$, ſupponatur Area aucta minutave æqualis nihilo, ſic in præſenti caſu q reperietur $= \frac{2}{3} d\sqrt{d}$, adeoq;

$$A = \frac{2}{3} d\sqrt{d}\ \frac{2y^3}{3\sqrt{x^3}}.$$

Corollarium 2. Si n ponatur æqualis termino cuivis ſequentis ſeriei 3, 4, 5, 6, 7, &c. Quadratura Curvæ cujus ordinatim applicata $x^{-n} \sqrt{dx-xx}$ aut $x^{-n} \sqrt{dx+xx}$, finita evadit, & exhibetur per ſeriem noſtram;

ſtram; Inquirenda ſit Area Curvæ cujus ordinatim applicata $x^{-3}\sqrt{dx-xx}$, finge eam comparari cum Area Circuli, quæ vocetur A; erit $m=0$, $n=3$, adeoq; $A=P-Q-R-S$. Sed cum quantitas $2m$ infinite parva ſeu potius nulla, in Denominatore termini tertii per quem $d^n B$ multiplicatur, extet; Quantitas deſignata per P infinita eſt; atque ob eandem cauſam, Quantitas deſignata per $-S$ infinita evadit, adeoque Quantitates A, $-Q$, $-R$ evaneſcunt: Igitur $P=S$, diviſaque æquatione per $\frac{2m+1}{2m+4}$ in $\frac{2m-1}{2m+2}$ fit

$$d^n B \text{ in } \frac{2m-3}{2m} = \frac{dd}{m} x^{m-3} y^3 \text{ ſeu } d B \text{ in } \frac{2m-3}{2}$$

$$= dd\, x^{m-3} y^3 \text{: ſcriptiſque 0 \& 3 pro } m \text{ \& } n \text{, prodibit}$$

$$d B \text{ in } -\frac{3}{2} = \frac{y^3}{x^3}, \text{ ſeu } B = -\frac{2y^3}{3x^3}.$$

Corollarium 3. Si m ponatur æqualis termino cuivis ſequentis ſeriei, -2, -1, 0, 1, 2, 3, 4, 5, *&c.* quadratura Curvæ cujus ordinata $x^m\sqrt{dx-xx}$, pendet a quadratura Circuli: Area vero Curvæ cujus ordinata $x^m\sqrt{dx+xx}$, pendet a quadratura Hyperbolæ, & relatio iſtius Curvæ cum Circulo aut Hyperbola exhibetur per Seriem noſtram in terminis finitis.

Corollarium 4. Si m exponatur per alium quemvis terminum differentem ab iis quas ſupra memoravimus, Curva cujus ordinata $x^m\sqrt{dx-xx}$ aut $x^m\sqrt{dx+xx}$, neque quadratur exacte, nec ab Hyperbola aut Circulo pendet, ſed ad Curvam ſimpliciorem reducitur per ſeriem noſtram.

Theorema II.

Sit A Area Curvæ cujus Abſciſſa x & ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$. Sit B area Curvæ cujus Abſciſſa eadem cum priori ſed ordinatim applicata $\frac{x^{m-n}}{\sqrt{dx-xx}}$ ponatur $\sqrt{dx-xx}=y$. Erit $A=$

$$d^n B \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} \text{ in } \frac{2m-7}{2m-6} \text{ \&c.} = P.$$

$$\text{I} \quad -\frac{1}{m} x^{m-1} y = -Q.$$

$$\text{II} \quad \frac{d}{m-1} \text{ in } \frac{2m-1}{2m} x^{m-2} y = -R.$$

$$\text{III} \quad \frac{dd}{m-2} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} x^{m-3} y = -S.$$

$$\text{IV} \quad \frac{d^3}{m-3} \text{ in } \frac{2m-1}{2m} \text{ in } \frac{2m-3}{2m-2} \text{ in } \frac{2m-5}{2m-4} x^{m-4} y = -T, \text{ \&c.}$$

Obſervationes ad primum Theorema, hic & in ſequentibus locum habent.

Corollarium 1. Si m ponatur æqualis Termino cuivis ſequentis ſeriei, $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$, &c. quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$ aut $\frac{x^m}{\sqrt{dx+xx}}$ finita evadit, & exhibetur per hanc ſeriem.

Corollarium 2. Si n ponatur æqualis Termino cuivis ſequentis ſeriei 1, 2, 3, 4, 5, 6, 7, &c. Curva omnis cujus ordinatim applicata $\frac{x^{-n}}{\sqrt{dx-xx}}$ aut $\frac{x^{-n}}{\sqrt{dx+xx}}$ quadratur per hanc ſeriem in terminis finitis.

Corollarium 3. Si m exponatur per terminum quemlibet ſequentis ſeriei, 0, 1, 2, 3, 4, 5, 6, 7, &c. Curva cujus ordinatim applicata $\frac{x^m}{\sqrt{dx-xx}}$ pendet a Quadratura Circuli. Curva vero cujus ordinatim applicata $\frac{x^m}{\sqrt{dx+xx}}$ a quadratura Hyperbolæ. Etenim ſi Centro C, Diametro
Plate 1. Fig. A. $AB=d$ deſcribatur Circulus AEB, ac ſumatur $AD=x$; erecto DE normaliter, junge CE. Sector AEC per $\frac{1}{8}dd$ diviſus æqualis eſt Areæ Curvæ cujus Ordinata $\frac{x^0}{\sqrt{dx-xx}}$ Eodem modo, ſi Centro C, Tranſverſo
Fig. B. axi $AB=d$, deſcribatur æquilatera Hyperbola AE, ſumatur $AD=x$, erigatur DE ad angulos rectos, jungatur CE; ſector ACE per $\frac{1}{8}dd$ diviſus æqualis eſt Areæ Curvæ cujus ordinata $\frac{x^0}{\sqrt{dx+xx}}$.

Corollarium 4. Si m ponatur æqualis Termino cuivis, qui non in limitationes præcedentes cadat, Curva cujus ordinata $\frac{x^m}{\sqrt{dx+xx}}$ neque quadratur exacte, nec a Circulo aut Hyperbola pendet, ſed ad Curvam ſimpliciorem reducitur.

Theorema 3.

Sit A Area Curvæ cujus Abſciſſa x, ordinatim applicata $x^m\sqrt{rr-xx}$, ſit B area Curvæ cujus Abſciſſa itidem x, ordinatim applicata $x^{m-2n}\sqrt{rr-xx}$, ponatur $\sqrt{rr-xx}=y$.

Erit $A =$

$$r^{2n} B \text{ in } \frac{m-1}{m+2} \text{ in } \frac{m-3}{m} \text{ in } \frac{m-5}{m-2} \text{ in } \frac{m-7}{m-4} \text{ \&c.} = P.$$

$$- \frac{1}{m+2} x^{m-1} y^3 = -Q.$$

$$- \frac{rr}{m} \text{ in } \frac{m-1}{m+2} x^{m-3} y^3 = -R.$$

$$- \frac{r^4}{m-2} \text{ in } \frac{m-2}{m+2} \text{ in } \frac{m-3}{m} x^{m-5} y^3 = -S, \text{ \&c.}$$

Corollarium 1. Si *m* exponatur per terminum quemvis ſequentis ſeriei 1, 3, 5, 7, 9, *&c.* Quadratura Curvæ cujus ordinata $x^m \sqrt{rr-xx}$ aut $x^m \sqrt{rr+xx}$ finita evadit, & exhibetur per hoc Theorema.

Corollarium 2. Si *n* exponatur per terminum quemvis ſequentis ſeriei 2, 3, 4, 5, 6, *&c.* Curva cujus ordinata $x^{-2n} \sqrt{rr-xx}$, aut $x^{-2n} \sqrt{rr+xx}$ quadratur exacte per hoc Theorema.

Corollarium 3. Si *m* exponatur per Terminum quemvis ſequentis ſeriei −2, 0, 2, 4, 6, 8, *&c.* Quadratura Curvæ cujus ordinata $x^m \sqrt{rr-xx}$, pendet a Circulo. Quadratura vero Curvæ cujus ordinata $x^m \sqrt{rr+xx}$, pendet ab Hyperbola.

Corollarium 4. Si *m* exponatur per Terminum quemvis differentem ab illis quos ſupra memoravimus, Curva cujus ordinata $x^m \sqrt{rr-xx}$, aut $x^m \sqrt{rr+xx}$, neque exacte quadratur, nec a Circulo aut Hyperbola pendet, ſed ad ſimpliciorem Curvam reducitur.

Theorema IV.

Sit *A* Area Curvæ cujus abſciſſa *x*, ordinatim applicata $\frac{x^m}{\sqrt{rr-xx}}$, Sit *B* Area Curvæ cujus Abſciſſa itidem *x*, Ordinatim applicata $\frac{x^{m-2n}}{\sqrt{rr-xx}}$.

Erit $A =$

$$r^{2n} B \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} \text{ in } \frac{m-7}{m-6} \text{ \&c.} = P.$$

$$- \frac{1}{m} x^{m-1} y = -Q.$$

$$- \frac{rr}{m-2} \text{ in } \frac{m-1}{m} x^{m-3} y = -R.$$

$$- \frac{r^4}{m-4} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} x^{m-5} y = -S.$$

$$- \frac{r^6}{m-6} \text{ in } \frac{m-1}{m} \text{ in } \frac{m-3}{m-2} \text{ in } \frac{m-5}{m-4} x^{m-7} y = -T, \text{ \&c.}$$

Corollarium 1. Si *m* exponatur per terminum quemvis ſequentis ſeriei 1, 3, 5, 7, 9, &c. Quadratura Curvæ cujus ordinata $\frac{x^m}{\sqrt{rr-xx}}$ aut $\frac{x^m}{\sqrt{rr+xx}}$, per hoc Theorema habetur in finitis Terminis.

Corollarium 2. Si *n* exponatur per terminum quemlibet ſequentis ſeriei 1, 2, 3, 4, 5, 6, &c. Curva cujus ordinatim applicata $\frac{x^{-2n}}{\sqrt{rr-xx}}$ aut $\frac{x^{-2n}}{\sqrt{rr+xx}}$ exacte quadratur per hoc Theorema.

Corollarium 3. Si *m* exponatur per terminum quemvis ſequentis ſeriei 0, 2, 4, 6, 8, 10, &c. Quadratura Curvæ, cujus ordinatim applicata $\frac{x}{\sqrt{rr-xx}}$ pendet a quadratura Circuli. Etenim ſi Centro C radio Fig. A. $CA = r$ deſcribatur Circulus *AEG*, ſumatur $CD = x$, erigatur *DE* normalis ad *CD*, Jungatur *CE*: Sector *CAE* per $\frac{1}{2} rr$ diviſus æqualis eſt Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{\sqrt{rr-xx}}$. Eodem modo ſi Fig. B. Centro C, Tranſverſo ſemiaxi $CA = r$, deſcribatur æquilatera Hyperbola *EAM*, ducatur *CF* ad *AC* perpendicularis $= x$, ducatur *FE* axi parallela donec occurrat Hyperbolæ in *E*, jungatur *CE*: ſector Hyperbolicus *ACE* per $\frac{1}{2} rr$ diviſus æqualis eſt Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{\sqrt{rr+xx}}$.

Corollarium 4. Si *m* exponatur per terminum quemlibet a præcedentibus differentem, Curva cujus ordinata $\frac{x^m}{\sqrt{rr-xx}}$ aut $\frac{x^m}{\sqrt{rr+xx}}$ neque quadratur exacte, nec a Circulo aut Hyperbola pendet, ſed ad Curvam ſimpliciorem reducitur.

Theorema V.

Sit *A* Area Curvæ cujus abſciſſa *x*, ordinatim applicata $\frac{x^m}{d-x}$; ſit *B* Area Curvæ cujus abſciſſa itidem *x*, ejuſq; ordinatim applicata $\frac{x^{m-n}}{d-x}$.

Erit Area

$$A = d^n B - \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} - \frac{ddx^{m-2}}{m-2} \text{ \&c.}$$

Sit ordinatim applicata $\frac{x^m}{d+x}$, tunc Area erit =

$$A = \frac{x^m}{m} - \frac{dx^{m-1}}{m-1} + \frac{ddx^{m-2}}{m-2} \text{ \&c.} \pm d^n B.$$

Corollarium. Si m exponatur per terminum quemlibet ſequentis ſeriei, 0, 1, 2, 3, 4, 5, 6, &c. Quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{d-x}$, aut $\frac{x^m}{d+x}$ pendet a quadratura Hyperbolæ; Etenim ductis DE, EF ad angulos rectos, ſumatur $EG = d$, ducatur GH normalis ad EF & ipſi EG æqualis. Intra Aſymptotos DE, EF deſcribatur Hyperbola per H tranſiens, quo facto ſumatur $GK = x$ verſus E pro primo caſu, at verſus F pro ſecundo; ducatur ordinatim applicata KL: Area $HGKL$ per dd diviſa æqualis eſt Areæ Curvæ cujus ordinatim applicata $\frac{x^0}{d-x}$ aut $\frac{x^0}{d+x}$. Hinc Solidum generatum a portione Ciſſoidis, dum circa Diametrum circuli genitoris revolvit, in finitis terminis exhibetur, data Hyperbolæ Quadratura. Vide Fig. C.

Theorema VI.

Sit A Area Curvæ cujus abſciſſa x, ordinatim applicata $\frac{x^m}{rr+xx}$; Sit B Area Curvæ cujus abſciſſa itidem x, ordinatim applicata $\frac{x^{m-2n}}{rr+xx}$,

Erit Area $A = \frac{x^{m-1}}{m-1} - \frac{rr\,x^{m-3}}{m-3} + \frac{r^4\,x^{m-5}}{m-5}$ &c. $\mp r^{2n} B$.

Corollarium. Si m exponatur per terminum quemlibet ſequentis ſeriei 0, 2, 4, 6, 8, &c. Quadratura Curvæ cujus ordinatim applicata $\frac{x^m}{rr+xx}$ pendet a rectificatione circularis Arcus. Etenim ſi centro C radio $CA = r$ deſcribatur Circulus AEG, ducatur Tangens $AK = x$ jungatur CK peripheriæ occurrens in E, arcus AE per rr diviſus æqualis eſt Areæ curvæ cujus ordinata $\frac{x^0}{rr+xx}$. Fig. B.

Corollarium generale ad hæc ſex Theoremata.

Curva omnis mechanica cujus quadratura pendet ab aliqua e Curvis numero infinitis, cujus ordinatæ formas ſequentes adipiſci poſſunt $x^m\sqrt{dx \pm xx}$, $\frac{x^m}{\sqrt{dx \pm xx}}$, $x^m\sqrt{rr \pm xx}$, $\frac{x^m}{\sqrt{rr \pm xx}}$, $\frac{x^m}{d \pm x}$, $\frac{x^m}{rr+xx}$, per ſeries has quadrari poteſt. Hoc Exemplo unico indicare ſatis erit.

Poſito quod Cubus Arcus Circularis Sinui verſo correſpondentis fiat Ordinata Curvæ, cujus Abſciſſa ſit idem Sinus verſus. Inquirenda eſt Area iſtius Curvæ.

Sit Abſciſſa x, arcus circularis v, fluxio Areæ ſit $v^3\dot{x}$.

Sit Area $v^3 x - q$. Igitur $v^3 \dot{x} + 3 v^2 \dot{v} x - \dot{q} = v^3 \dot{x}$, unde $\dot{q} = 3 v^2 \dot{v} x$; ſed $\dot{v} = \frac{d\dot{x}}{2\sqrt{dx-xx}}$, igitur $\dot{q} = \frac{3 d v^2 x \dot{x}}{2\sqrt{dx-xx}}$ ſed per *Theorema* II. $\frac{x\dot{x}}{\sqrt{dx-xx}} = \frac{d\dot{x}}{2\sqrt{dx-xx}} - \dot{y} = \dot{v} - \dot{y}$, adeoq; $\dot{q} = \frac{3}{2} d v^2 \dot{v} - \frac{3}{2} d v^2 \dot{y}$, igitur $q = \frac{1}{2} d v^3 - \frac{3}{2} d v^2 \dot{y}$.

Ergo ad hoc perventum eſt ut fluentem quantitatem inveniamus cujus fluxio eſt $\frac{3}{2} d v^2 \dot{y}$.

Sit hæc quantitas $\frac{3}{2} d v^2 y - r$.

Igitur $\frac{3}{2} d v^2 \dot{y} + 3 d v \dot{v} y - \dot{r} = \frac{3}{2} d v^2 \dot{y}$.

Adeoque $\dot{r} = 3 d v \dot{v} y = \frac{3}{2} d d v \dot{x}$. Sit $r = \frac{3}{2} d d v x - s$.

Igitur $\frac{3}{2} d d v \dot{x} = \frac{3}{2} d d v \dot{x} + \frac{3}{2} d d x \dot{v} - \dot{s}$.

Adeoque $\dot{s} = \frac{3}{2} d d x \dot{v} = \frac{3 d^3 x \dot{x}}{4\sqrt{dx-xx}} = \frac{3}{4} d^3 \dot{v} - \frac{3}{4} d^3 \dot{y}$, per 2um *Theorema*.

Igitur $s = \frac{3}{4} d^3 v - \frac{3}{4} d^3 y$. adeoque area quæſita $= v^3 x - \frac{1}{2} d v^3 + \frac{3}{2} d v^2 y - \frac{3}{2} d d v x + \frac{3}{4} d^3 v. - \frac{3}{4} d^3 y$.

Quoniam autem Solida ex rotatione Curvarum genita, Superficies ab eadem rotatione genitæ, Longitudines Curvarum, & Centra Gravitatis horum omnium a Quadratura Curvarum pendent, hæc ſi a Curvis ſupradictis pendent facillime computantur.

Of deducing the Tangents of Curves, from the Theory of Maxima *and* Minima, *by Mr.* H. Ditton. *n.* 284. *p.* 1333.

Plate I. *Fig.* 4.

III. Sit Curva AG, cujus vertex A, axis AK, ordinatim applicata FD centrumque (ſiquod habet) punctum K. Sumpto puncto L in Axe ſit $AL = n$, $AD = x$, $FD = y$, $FL = z$; quarum quantitatum, tres poſteriores ſunt fluentes, prior vero n permanens ac ſtabilis, hæc enim una eademque prioribus variis ſemper reſpondet. Ex Triangulo Rectangulo FDL, hanc habemus Æquationem, $zz = yy + nn - 2nx + xx$; determinandoque z ad extremum, oritur $2 y \dot{y} - 2 n \dot{x} + 2 x \dot{x} = 0$; unde interpretando $2 y \dot{y}$ ſecundum propriam Curvæ naturam, relinquetur quantitas n expoſita in terminis etiam Curvæ propriis.

Cum vero z hoc modo ad valorem extremum determinatam habeamus; hoc eſt, linea FL omnium quæ a puncto L ad Curvam duci poſſunt vel maxima vel minima ſit, indeque ad Curvam in puncto F normalis; ipſam DL eſſe ſubnormalem patet, ex qua ſubtangens nullo negotio eruitur.

In exemplum producatur primo Parabola Apolloniana, quam curvam hic delineatam eſſe ſupponemus. Habemus ergo $2 y \dot{y} = r \dot{x}$ (poſito Parametro $= r$) unde $r \dot{x} - 2 n \dot{x} + 2 x \dot{x} = 0$, & $n = \frac{r}{2} + x$, ergoque DL ſubnormalis $= \frac{1}{2} r$. (Cujus Theorematis ſenſus hic eſt, *viz.* Si ultra terminum D abſciſſæ AD, deſignetur DL ſemiparametro æqualis, atque a puncto L producatur LF recta ad punctum F; recta ſic ducta Parabolæ in puncto F normalis erit, & omnium quæ a puncto L ad Curvam duci poſſunt minima. Dico minimam; alicui enim curvæ naturam ac indolem ſcienti, apparet Maximam eſſe non poſſe (id quod in ſequentibus notatum velim) ſed neceſſario eſt vel maxima vel minima, ideoque poſterior.) Hæcque pars prior eſt *Theor.* 5. *Lib.* 7. Conicor. Præclariſſimi *de La Hire.*

Ducatur ordinata EB, junganturque puncta EL; fit intercepta $BD = f$, unde $AB = x - f$, & $BL = \frac{r}{2} + f$. Jam $LE^q = \frac{rr}{4} + rx + ff$, & $FL^q = \frac{rr}{4} + rx + ff$, & $FL^q = \frac{rr}{4} + rx$, ergo $LE^q - FL^q = BD^q$; quæ pars poſterior eſt Theorem. 5. ejuſdem Lib. Conicor.

Quo propius punctum F, in quo curvam normalis ſecat, puncto A ſive vertici admovetur; eo propius etiam punctum L eidem venit. Ergo quando F cum A coincidit, & ſic evaneſcit ordinata FD, tunc ipſa Minima jacet in Axe AK, & ſemiparametri quantitatem adæquabit. Hoc eſt in illo caſu $n = \frac{1}{2} r$ tantum; in nihilum abeunte x abſciſſa ad ordinatam evaneſcentem pertinente. Si ergo $AL = n = \frac{1}{2} r$, ſumpto puncto D inter A & L, fiat $AD = x$; tum oritur $FL^q = \frac{rr}{4} + xx$, ergo $FL^q - AL^q = xx$, hoc eſt $FL^q - AL^q = AD^q$ ſemper. Ejuſdemque tenoris eſt Theor. 2. Lib. 7. Conicor. *de La Hire.*

Secundo ſit curva quædam ordinis Parabolici ſuperioris, cujus æquatio $r^{p-q} x^q = y^p$.

Tum $yy = r^{\frac{2p-2q}{p}} x^{\frac{2q}{p}}$, adeoque

$$2y\dot{y} = \frac{2q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} \dot{x};$$

ſubſtituendoque hunc valorem loco $2y\dot{y}$ in æquatione generali determinante x ad extremum, habemus inde

$$n = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}} + x;$$

& propterea ſubnormalis

$$DL = \frac{q}{p} r^{\frac{2p-2q}{p}} x^{\frac{2q-p}{p}}.$$

Hoc vero ſingulis hiſce curvis facillime applicatur, ſi indices p & q ſecundum uniuſcujuſque naturam ac genium debito modo exponantur.

Supponetur tertio Curvam eſſe Ellipſin cujus $\frac{1}{2}$ Axis Major AK; ex cujus etiam æquatione conſequitur $2y\dot{y} = r\dot{x} - \frac{2rx\dot{x}}{q}$. Unde provenit $r\dot{x} - \frac{2rx\dot{x}}{q} - 2n\dot{x} + 2x\dot{x} = 0$, & $n = \frac{r}{2} + x - \frac{rx}{q}$, ac propterea $\frac{r}{2} - \frac{rx}{q}$ ſubnormali DL æqualis. Si vero ellipſeos loco ſubſtitue-etur Circulus, æquationem eodem modo tractando, inveniemus $DL = r - x$, poſito r Circuli Radio æquali.

Sed ad Ellipſin revertendum, cujus alia proprietas ex hoc fonte deducenda eſt, prout in Parabola factum. Sit $BD = f$, unde $AB = x - f$. Habemus $LE^q \; (= LB^q + EB^q) = \frac{rr}{4} - \frac{rrx}{q} + \frac{rrxx}{qq} + ff + rx - \frac{rxx}{q} - \frac{rff}{q}$: & $FL^q \; (= FD^q + LD^q) = rx - \frac{rxx}{q} + \frac{rr}{4} - \frac{rrx}{q} + \frac{rrxx}{qq}$: Ergo, $LE^q - LF^q = ff - \frac{rff}{q}$; Hoc vero eſt Theor. 6. Lib. 7. Conic. *de La Hire.*

Poſtulat enim Geometra ille ſublimis, ut ſit $q : r :: \frac{q}{2} - x : LD$, cujus valor eſt $\frac{r}{2} - \frac{rx}{q}$ prout ſupra inventum; ideoque quarta proportionalis eſt tribus ante poſitis: Hoc vero ei conceſſo, LF eſſe minimam omnium rectarum quæ a puncto L ad Ellipſin duci poſſunt evidenter demonſtrat. Præterea quoniam eſt $q : q - r :: f : f - \frac{fr}{q}$, Ergo $\Box\, ff - \frac{rff}{q}$ ſive $f \times f - \frac{fr}{q}$; idem eſt quod rectangulum apud *D. De La Hire* exemplar vocatum: Hoc vero exemplar ſecundum ejus definitionem, eſt Rectangulum ſimile Rectangulo, differentiam inter Quadratum Axis Tranſverſi & Figuram conſtituenti (hoc eſt Rectangulo $qq - qr$) & præterea ad Rectam BD ſive f applicatum. Et quod Rectangulum $ff - \frac{rff}{q}$ omnes haſce conditiones poſſideat, luce Meridiana clarius eſt.

Notetur, ex valore quantitatis n ſupra invento, plane conſequi $n \sqsupset \frac{r}{2}$. Nam $n = \frac{r}{2} + x - \frac{rx}{q}$, ergo $qn + rx = \frac{qr}{2} + qx$, ſed (propter $q \sqsupset r$) $qx \sqsupset rx$, ergo, $qn \sqsupset \frac{qr}{2}$ & $n \sqsupset \frac{r}{2}$.

Quando (ut in Parabola modo obſervatum) punctum FA in verticem incidit, ipſa Minima in Axe deſignatur; & propter evaneſcentem x, habemus $n = \frac{r}{2}$: Aſſumptoque quovis puncto D inter A & L, ſi $AD =$ alicui x, comparando emergit $FL^q - AL^q = xx - \frac{rxx}{q}$; quod ipſum eſt Theor. 3. Lib. 7. Conic. D. La Hire. Quoniam enim eſt $q : q - r :: x : x - \frac{rx}{q}$, patet $\Box\, xx - \frac{rxx}{q}$ eſſe exemplar, ſed applicatum ad abſciſſam x; & præterea hoc eſſe menſuram adequatam defectus quadrati Minimæ a quadrato cujuſvis rectæ alterius, ab eodem puncto ad curvam protenſæ; hæcque demonſtrat ille loco citato.

Theoremata vero ad Axem minorem ſive conjugatum ellipſeos ſpectantia (hactenus enim majore ſive Tranſverſo uſi fuimus) eodem plane modo determinantur. Sit $AK \; \frac{1}{2}$ Axis Minoris $= \frac{c}{2}$, Parameter $= R$;

punctum L jam ultra centrum, ad alteras partes GK collocari ſupponitur. Operando ut prius, invenietur AL ſive $n = \frac{R}{2} + x - \frac{Rx}{c}$, & ſubnormalis $DL = \frac{R}{2} - \frac{Rx}{c}$; Hoc eſt $c : R :: \frac{c}{2} - x : \frac{R}{2} - \frac{Rx}{c}$, adeoque ducta FL omnium quæ a puncto L ad ellipſin duci poſſunt Maxima, & $LF^q - LE^q = \frac{Rff}{c} - ff =$ Rectangulo exemplar ad BD (ſive f) applicato. Quod vero hoc ſit exemplar, patet, eſt enim $c : R - c :: f : \frac{Rf}{c} - f$, adeoque ex definitione, $\frac{Rf}{c} - f \times f =$ Exemplari. Hoc vero Theor. eſt 7. Lib. 7. Conicor. De La Hire.

Iterum; Puncto F cum A coincidente; propter evaneſcentem x evaneſcentis tunc temporis ordinatæ, relinquitur $n = \frac{R}{2}$, & AL omnium quæ a puncto L ad Ellipſin duci poſſunt Maxima, & $AL^q - FL^q = \frac{Rxx}{c} - xx =$ Exemplari ad AD ſive x applicato; eodemque modo ſonat Theor. 4. Lib. prædicti Conicorum.

Obſervandum vero ad caſum præcedentem (quod prius ergo notari debuit) ubi invenimus $n = \frac{R}{2} + x - \frac{Rx}{c}$, quod $n \sqsubset \frac{R}{2}$; nam $cn + Rx = \frac{R}{2}c + cx$, & propter $R \sqsupset c$, adeoque $Rx \sqsupset cx$, relinquetur $cn \sqsubset \frac{Rc}{2}$, & $n \sqsubset \frac{R}{2}$.

Jam vero ut res in Ellipſi peracta eſt, ſic eodem prorſus modo in Hyperbola peragenda foret, Minimæque in hac curva lineæ determinandæ: ſed talis inter haſce curvas connectio, tam faciliſque ab una ad alteram tranſitus, ut vel Tyronibus ipſis labor inanis videatur. Nil aliud reſtat, v. gr. ad ſubnormalem determinandam, quam ut ſignum $-$ in $+$ mutetur. Nam cum in Hyperbola ſit $2y\dot{y} = r\dot{x} + \frac{2rx\dot{x}}{q}$, & $n = \frac{r}{2} + x + \frac{rx}{q}$ (ex æquatione generali) manet $DL = \frac{r}{2} + \frac{rx}{q}$.

Concipietur Quarto Curvam MSN (in altera Fig. parte delin.) Eſſe unam ex Hyperboloidibus, cujus Aſymptoti AK, KH, rectamque SR ad Aſymptoton KH ordinatam, SR ſit $= y$, $SP = z$, $KR = x$, $KP = n$, quæ hic neceſſario minor erit quam x, ut conſideranti patet. Æquatio curvæ propria eſt $y^p x^q = r^q s^p$ cujus loco (propter r & s quantitates determinatas) ſcribi poſſit $y^p = x^{-q}$, adeoque $y^2 = x^{-\frac{2q}{p}}$, & $2y\dot{y} = -\frac{2q}{p}\dot{x}x^{+\frac{2q-p}{p}}$; hinc cum $zz = yy + xx - 2nx + nn$, pro extremo habemus $2y\dot{y}+2x\dot{x}-2n\dot{x}=0$, hoc eſt $-\frac{2q}{p}\dot{x}x^{-\frac{2q-p}{p}}+2x\dot{x}=2n\dot{x}$, & $n=x-\frac{q}{p}x^{-\frac{2q-p}{p}}$ adeoque ſubnormalis PR $(=x-n)$ $=\frac{q}{p}x^{-\frac{2q-p}{p}}$.

Curvam jam AFG (ultimo loco) Cycloiden primariam concipiamus; ſitque r Radius, c Arcus & y ordinata Circuli genitoris, cujus Diameter per AK repreſentatur, centrumque inter L & K poſitum. Tum vocata FD cycloidis ordinata a, cæteriſque ut prius; curvæ æquatio eſt $aa=yy+2cy+cc$, adeoque zz $(=aa+nn-2nx+xx)=yy+2cy+cc+nn-2nx+xx$, & ($z$ ad extremum determinata) $2y\dot{y}+2c\dot{y}+2y\dot{c}+2c\dot{c}-2n\dot{x}+2x\dot{x}=0$. Eſt vero $\dot{y}=\frac{r\dot{x}-x\dot{x}}{y}$ & $\dot{c}=\frac{r\dot{x}}{y}$, ergo hos valores ſubſtituendo, ac æquationem debite reducendo; habemus $2r-2x+\frac{2rc-2xc}{y}+2r+\frac{2cr}{y}=2n-2x$; ac propterea $2r-x+\frac{2rc-xc}{y}=n-x=DL$ ſubnormali.

Coroll. 1. Exempla hactenus oblata percurrenti, in ſingulis patebit, quod $2y\dot{y}-2n\dot{x}+2x\dot{x}=0$, poſito nempe loco n in hac æquatione, valore ejus ſecundum curvæ naturam. In Hyperboloidibus ergo ex. gr. $\frac{2q}{p}\dot{x}x^{-\frac{2q-p}{p}}-2x\dot{x}+\frac{2q}{p}x\dot{x}^{-\frac{2q-p}{p}}+2x\dot{x}=0$, quod (ipſo oculo judice) manifeſtum eſt; & ſic in aliis (ſine ulla demonſtratione) veritas facile perſpicietur.

Coroll. 2. Ex ſubnormalium inventione, curvarum ordinatas Maximas & Minimas facile determinabimus. Hacque in re dico, ſi ſubnormalis (pro aliquo curvæ puncto) nihilo ponatur æqualis, habemus ordinatam iſtius curvæ ad extremum determinatam; & quidem maximam ſi ad partes curvæ concavas, minimam vero ſi ad convexas applicari intelligatur. Ex. gr. in Circulo (poſita ſubnormali $=l$) eſt $l=r-x$; ſit $r-x=0$; ergo $r=x$, ac inde $y=r$, hoc eſt applicata maxima Radio æqualis. Similiter in Ellipſi, $l=\frac{r}{2}-\frac{rx}{q}$; ſit $\frac{r}{2}-\frac{rx}{q}=0$, tum $rq=2rx$, ac $x=\frac{q}{2}$, ergo $yy=\frac{rq}{4}=$ 4tæ parti Figuræ (uti vocant) ſive ſemiaxis conjugati quadrato, adeoque maxima $y=$ iſti ſemiaxi. Nec Methodo diſſimili cum aliis curvis operandum foret; inveniatur ſubnormalis ex æquatione data, eaque nihilo æquali poſita, ordinatam curvæ maximam vel minimam determinatam habebimus; priorem ad partem curvæ verſus axem concavam, poſteriorem ad convexam.

P. S. Æque facile hac methodo determinatur Tangens ad partes curvæ convexas operando, ac ad partes concavas uti prius. Sit enim AC Tangens verticalis inque ea ad libitum ſumpto puncto C, ſit $AC = n$, $CO = z$ (quo etiam charactere omnes lineæ, a puncto C ad curvam convexam AEG ductæ, inſigniantur) ergo ducta MO ſemper ad AC perpendiculari, erit $CM = n - y$, & cum $OM = x$, erit $zz = nn - 2ny + yy + xx$, adeoque (pro extremo ipſius z valore) $2y\dot{y} + 2x\dot{x} - 2n\dot{y} = 0$. In qua æquatione ſi exponatur $2x\dot{x}$ ſecundum curvæ naturam, lineam CZ (quæ hoc loco ſubnormalis vicem ſubibit) determinatam dabimus. Res clarior eſt quam quæ exemplis illuſtrantibus indigeat; quæ-quæ jamjam dicta ſunt facile hoc opus excuſabunt.

Secundo, Sicut Methodo priore, Curvarum Tangentes invenimus, ipſas lineas LE vel CO a puncto dato vel in Axe vel in Tangente verticali ſumpto productas, ad extremum determinando; ſic etiam conſiderando lineas QE, &c. a puncto in Axe dato ultra verticem productas, idem (idque Univerſaliter) perficere poſſumus. Omnes enim lineæ QE valoris fluentis ſunt ac perpetuo mutabilis, ſola vero Tangens QF (poſito quod QF curvam tangat) ſtabilis eſt ac ad unicum valorem determinata. Hoc ergo loco, τῷ extremi Hypotheſi non innitemur, ſed quantitatem permanentem tantum ſpeculabimur. Aſſumantur duo puncta QL, indeque ad idem curvæ punctum E duæ ſemper lineæ ducantur LE, QE. Inter punctum F contactus ac verticem, angulus QEL ſemper erit obtuſus, ad alteras vero partes puncti F acutus erit, ſuppoſito (quod prius monitum) QF curvam tangere, ac FL ei ad angulos rectos inſiſtere. Sit $QA = p$. $AL = n$. $AB = x$. $BE = y$. $QE = z$. VE (intercepta inter punctum E & V ubi cadit QV perpendicularis ab Q in LE productam) $= v$. Jam propter Triangulum obtuſangulum QE habemus hanc æquationem $zz = p^2 + 2pn - y^2 - x^2 + 2nx - \overline{y^2 + n^2 - 2nx + xx}|^{\frac{1}{2}} \times 2v$; ſive loco $\overline{y^2 + n^2 - 2nx + xx}|^{\frac{1}{2}}$ ſcribendo f eſt $zz = p^2 + 2pn - y^2 - x^2 + 2nx - 2fv$, ideoque $2z\dot{z} = 2y\dot{y} - 2x\dot{x} + 2n\dot{x} - 2f\dot{v} - 2v\dot{f}$. Si z jam fiat quantitas ſtabilis, quo in Caſu QE cum QF tangente coincidet, erit tum $-2y\dot{y} - 2x\dot{x} + 2n\dot{x} = 0$ (rectangulo $2fv$ ejuſque adeo fluxione penitus evaneſcente.) Hæc vero eſt ipſa æquatio Generalis Methodo ſuperiori determinata, quæque uti videmus non minus facile ac naturaliter ex hoc ſuppoſito quantitatis ſtabilis principio, quam ex illo extremi deducitur.

IV. Cum ex data relatione inter z & y, quæritur $\int: z\,dy$, tum Methodus noſtra in determinandis Figurarum Quadraturis, tum Methodus Calculi differentialis inverſa poſtulant, ut z per y & datas exprimatur, quod tamen fieri non poteſt, quando æquatio relationem illam definiens ultra

A Specimen of a general method of determining the Quadratures of Figures, by Mr. Jo. Craig. n. 284. p. 1345 ultra Cubicam, vel Biquadraticam aſcendit. Porro quamvis Regula innoteſceret generalis inveniendi Radices æquationum cujuſcunque gradus: huic tamen Methodo inverſæ prorſus foret inutilis: Radix enim z ſurdis tam complicatis involveretur, ut nulla arte (hactenus cognita) a differentiali ad integrale regreſſus dari poſſet. Ob has rationes, alias vias, & conatu non prorſus irrito rem ſum aggreſſus.

Sit $z^m + ay^n = bz^e y^r$ æquatio exprimens relationem inter Ordinatam z & abſciſſam y; in qua Exponentes m, n, e, r, denotant quoſlibet Numeros, Integros vel Fractos, Affirmativos vel Negativos. Ponatur $r - n = c$. Erit

$$\text{AREA} = \frac{m}{m+n} zy +$$

$$\frac{mc+ne}{m\times\overline{m+n}\times\overline{c+1}+n\times\overline{m+n}\times\overline{e+1}} \times \frac{b}{a} z^{e+1} y^{c+1} +$$

$$\frac{\overline{m-e}\times\overline{c+1}+r\times\overline{e+1}}{m\times\overline{2c+1}+n\times\overline{2e+1}} \times \frac{bB}{a} z^{2e+1} y^{2c+1} +$$

$$\frac{\overline{m-e}\times\overline{2c+1}+r\times\overline{2e+1}}{m\times\overline{3c+1}+n\times\overline{3e+1}} \times \frac{bC}{a} z^{3e+1} y^{3c+1} +$$

$$\frac{\overline{m-e}\times\overline{3c+1}+r\times\overline{3e+1}}{m\times\overline{4e+1}+n\times\overline{4e+1}} \times \frac{bD}{a} z^{4e+1} y^{4c+1} +$$

$$\frac{\overline{m-e}\times\overline{4e+1}+r\times\overline{4e+1}}{m\times\overline{5c+1}+n\times\overline{5e+1}} \times \frac{bE}{a} z^{5e+1} y^{5c+1} + \&c.$$

De hac Serie hæc ſunt notanda: (1.) Quod literæ majuſculæ B, C, D, &c. deſignent coefficientes terminorum ipſis immediate præcedentium: (2.) Quod exhibeat Quadraturas omnium Figurarum Quadrabilium, quarum Curvæ per æquationem trium terminorum definiuntur: (3.) Et quod ſemper ſint Quadrabiles, quando $\frac{m\times r-r}{mn-rm-en}$ eſt numerus integer & affirmativus, quem vocemus l. (4.) Speciatim $l+1$ dat numerum Terminorum (ab initio ſumptorum) Seriei Aream quæſitam conſtituentium: (5.) Quod ſi ponatur $e = 0$, mutabitur hæc Series in Celebre Theorema Newtonianum pro Binomio communi; & proinde hoc Theorema eſt hujus Seriei caſus ſpecialis & ſimplex: (6.) Cum ſit Applicatio hujus Seriei ad Figuram particularem, hæ regulæ ſunt obſervandæ. 1. Reducatur æquatio Curvam datam definiens ad formam generalem, & ex comparatione particularis cum generali invenientur coefficientes a, b; ut & exponentes m, n, e, r. Secunda, Si exponentes ſic determinati non faciant l numerum integrum & affirmativum, (juxta conditionem in Not. 3. aſſignatam,) tum alius terminus æquationis particularis a quantitate z liberetur; & ſi nunc exponentibus denuo determinatis non competat illa Quadrabilitatis conditio, tum reliquus terminus a quantitate z libe-

z liberetur: Nam nullo labore quilibet ex tribus terminis æquationem datam conſtituentibus a quantitate z liberari poteſt. Tertia, Si æquationi per Regulam præcedentem tractatæ non conveniat prædicta Quadrabilitatis conditio; tum per Seriem quæratur Areæ complementum $\int: ydz:$ quo cognito ſtatim habetur Area quæſita; nam, ut omnibus notum, $zy - \int: ydz = \int: zdy$. Et ut ſine confuſione Complementum per Seriem obtineatur; in æquatione data Curvam particularem definiente pro z ſcribatur Y, & pro y ſcribatur Z: Factaque hac mutatione Ordinatæ in Abſciſſam, & Abſciſſæ in Ordinatam, tractetur æquatio juxta præcepta regulæ ſecundæ; donec illi conveniat Quadrabilitatis conditio, vel eandem ipſi non poſſe convenire pateat.

Exemplum 1. Sit $z^3 + y^3 = bzy$. Quia hic $m = 3$, $n = 3$, $e = 1$, $r = 1$, $a = 1$; ideo $l = 1$, adeoque $l + 1 = 2$. Et proinde (juxta Not. 4.) duo primi Seriei termini dant Aream $= \frac{1}{2}zy - \frac{1}{6}bz^2y^{-1}$.

Exemp. 2. Sit $z^7 + ay^3 = bzy^2$, ubi $m = 7$, $n = 3$, $e = 1$, $r = 2$; qui faciunt $l = 2$; ideo (juxta Not. 4.) tres primi Seriei termini dabunt quæſitam

$$\text{AREAM} = \frac{7}{10}zy - \frac{b}{15a}z^2 - \frac{2b^2}{15a^2}z^3y^{-1}.$$

Exemp. 3. Sit $z^3 + ky^5 = bz^{-2}y^{11}$; ubi $m = 3$, $n = 5$, $e = -2$, $r = 11$; at quia hi non faciunt l numerum integrum & affirmativum; ideo (per Regulam ſecundam) libero terminum $bz^{-2}y^{11}$ a quantitate z; & ſic æquatio fit $z^5 - by^{11} = -kz^2y^5$; ubi $a = -b$, $b = -k$; & $m = 5$, $n = 11$, $e = 2$, $r = e$; qui faciunt $l = 1$: Unde

$$\text{AREA} = \frac{5}{16}zy - \frac{k}{16b}z^3y^{-5}.$$

Exemp. 4. Sit $z^2 - by^2 = -kz^2y^2$; ubi $m = 2$, $n = 2$, $e = 2$, $r = 2$; qui non faciunt l numerum integrum & affirmativum; ideo libero terminum $-kz^2y^2$ a quantitate z; & tum $z^0 + ky^2 = bz^{-2}y^2$; ubi $a = k$, $b = b$; & $m = 0$, $n = 2$, $e = -2$, $r = 2$, qui faciunt $l = 1$, ideo

$$\text{AREA} = \frac{b}{k}z^{-1}y$$

Exemp. 5. Sit $z^2 - \frac{4g^2}{b}y^6 = -\frac{g}{b}z^2y^4$; ubi $m = 2$, $n = 6$, $e = 2$, $r = 4$; qui non faciunt l numerum integrum & affirmativum; idemque contingit liberato (a quantitate z) utrolibet ex reliquis: Ideo juxta regulam Tertiam quæro Complementum; quare (ut jam præmonui) pono $z = Y$; $y = Z$; unde æquatio data erit

$Y^2 - \frac{4g^2}{h}Z^6 = -\frac{g}{b}Z^4Y^2$; quæ (juxta Reg. 1.) reducta ad formam generalem erit hujus modi $Z^6 - \frac{b}{4g^2}Y^2 = \frac{1}{4g}Z^4Y^2$

ubi $m=6$, $n=2$, $e=4$, $r=2$; qui non faciunt l numerum integrum & affirmativum; ideo (juxta Reg. 2.) libero terminum ultimum a Z; unde $Z^2-\frac{1}{4g}Y^2=\frac{b}{4g^2}Z^{-4}Y^2$; ubi $m=2$, $n=2$, $e=-4$, $r=2$; unde $l=r$; & $a=-\frac{1}{4g}$, $b=\frac{b}{4g^2}$; Unde Areæ quæſitæ complementum eſt $=\frac{1}{2}ZY-\frac{b}{2g}Z^{-3}Y$ ſeu $\frac{1}{2}zy-\frac{b}{2g}zy^{-3}$; Ergo etiam Area quæſita $\int:zdy=\frac{1}{2}zy+\frac{b}{2g}zy^{-3}$.

Sit $z^m+ay^n=bz^{2e}y^{2c+n}+fz^ey^{c+n}$ æquatio exprimens Relationem inter Ordinatam z & Abſciſſam y. Erit

$$\text{AREA}=Azy+Bz^{e+1}y^{c+1}+Cz^{2e+1}y^{2c+1}+$$
$$Dz^{3e+1}y^{3c+1}+Ez^{4e+1}y^{4c+1}+$$
$$Fz^{5e+1}y^{5c+1},\ \&c.$$

Ubi (poſitis $2c+n=r$, $c+n=s$) $A=\frac{n}{m+n}$;

$$B=\frac{\overline{m-e+s}\times A+e-m}{m\times\overline{c+1}+n\times\overline{e+1}}\times\frac{f}{a}.$$

$$C=\frac{\overline{m-2e+r}\times bA+\overline{\overline{m-e}\times\overline{c+1}+r\times\overline{e+1}}\times fB+2eb-mb}{ma\times\overline{2c+1}+na\times\overline{2e+1}}$$

$$D=\frac{\overline{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}\times bB+\overline{\overline{m-e}\times\overline{2c+1}+s\times\overline{2e+1}}\times fC}{ma\times\overline{3c+1}+na\times\overline{3e+1}}$$

$$E=\frac{\overline{\overline{m-2e}\times\overline{2c+1}+r\times\overline{2e+1}}\times bC+\overline{\overline{m-e}\times\overline{3c+1}+s\times\overline{3e+1}}\times fD}{ma\times\overline{4c+1}+na\times\overline{4e+1}}$$

$$F=\frac{\overline{\overline{m-2e}\times\overline{3c+1}+r\times\overline{3e+1}}\times bD+\overline{\overline{m-e}\times\overline{4c+1}+s\times\overline{4e+1}}\times fF}{ma\times\overline{5c+1}+na\times\overline{5e+1}}$$

De hac Serie (cujus progreſſio primo fere intuitu eſt manifeſta) hæc ſunt notanda. (1.) Quod figuræ (quarum Curvæ prædicta æquatione definiuntur) ſunt Quadrabiles, quando Numeri exponentiales m, n, e, c; & coefficientes a, b, f habent relationes modo aſſignandas; ſcil: quando $\frac{\overline{2c+m}\times\overline{n-2e}}{-cm-en}$ eſt numerus integer & affirmativus, quem vocemus l, & (cum l eſt major quam 2) quando Coefficientium relatio eſt hæc.

$m-2$

$$\frac{\overline{m-2e}\times\overline{lc-c+1}+r\times\overline{le-e+1}}{\overline{e-m}\times\overline{lc+1}-s\times\overline{le+1}}\times\frac{bU}{f}=$$

$$\frac{\overline{m-2e}\times\overline{lc-2c+1}+r\times\overline{le-2e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{bP}{a}+$$

$$\frac{\overline{m-e}\times\overline{lc+c+1}+r\times\overline{le-e+1}}{m\times\overline{lc+1}+n\times\overline{le+1}}\times\frac{fU}{a}.$$

Ubi U & P denotant Coefficientes Terminorum duorum, qui immediate præcedunt ultimo Areæ quæſitæ Termino; ſcil: U eſt coefficiens termini ad Ultimum propioris, P coefficiens termini ab ultimo remotioris: ut ſi $Fz^{5e+1}y^{5c+1}$ eſſet ultimus Areæ quæſitæ terminus, tum U denotaret E, & P denotaret D. (2.) Ultimus ille Areæ quæſitæ terminus ex valore numeri l cognoſcitur; nam hic etiam $l+1$ dat numerum terminorum (ab initio ſumptorum) Seriei, qui Aream quæſitam conſtituunt. (3.) Si fuerit $l=1$, tum coefficientium relatio debet eſſe hæc

$$\frac{\overline{2e-m}\times\overline{1-A}+rA}{\overline{e-m}\times\overline{c+1}-s\times\overline{e+1}}\times\frac{b}{f}=\frac{\overline{e-m}\times\overline{1-A}+sA}{m\times\overline{c+1}+n\times\overline{e+1}}\times\frac{f}{a};$$

Si $l=2$; relatio debet eſſe hæc

$$\frac{\overline{m-2e}\times\overline{c+1}+r\times\overline{e+1}}{\overline{e-m}\times\overline{2c+1}-s\times\overline{2e+1}}\times\frac{bB}{f}=\frac{\overline{2e-m}\times\overline{1-A}+rA}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{b}{a}+$$

$$\frac{\overline{m-e}\times\overline{c+1}+s\times\overline{e+1}}{m\times\overline{2c+1}+n\times\overline{2e+1}}\times\frac{fB}{a}.$$

Sit $z^m=ay^n+bz^ey^{c+n}+fz^{2e}y^{2c+n}+gz^{3e}y^{3c+n}+hz^{4e}y^{4c+n}+$ &c. æquatio exprimens relationem inter ordinatam z & abſciſſam y; & conſtans terminis quotcunq; Erit

Area $=Azy+Bz^{e+1}y^{c+1}+Cz^{2e+1}y^{2e+1}+Dz^{3e+1}y^{3c+1}+Ez^{4e+1}y^{4e+1}+$ &c.

Calculo perfacili inveniuntur A, B, C, D, E, &c. ut & Quadrabilitatis conditiones, & quot termini ſeriei Aream quæſitam conſtituant. Creſcit quidem numerus harum conditionum pro multitudine terminorum, ex quibus conſtat æquatio relationem inter z & y definiens. Et ſpeciatim ſi illa terminorum multitudo vocetur N; tum $N-2$ eſt numerus conditionum Quadrabilitatis; quarum una Exponentium m, n, e, c relationem reſpicit, eſtq; hæc; ut $\frac{Nc-2c+2e-Ne+m+n}{-cm-en}$ eſt numerus (quem vocemus l) Integer & affirmativus. Reliquæ vero conditiones coefficientium a, b, f, g, h, &c. reſpiciunt. Ac deniq; $l+1$ dat numerum terminorum (ab initio ſumptorum) ſeriei, qui Aream quæſitam conſtituunt.

Corol. Ex hac Serie generali deduci poteſt Series, quæ exhibeat Quadraturas Figurarum, quarum Curvæ definiuntur per æquationem conſtantem terminis quibuſvis, qui æquationem Sectionis tertiæ generalem conſtituunt. Nam ad hanc obtinendam opus tantum eſt Seriem computare pro æquatione conſtante tot terminis (ab initio ſumptis) æquationis generalis, quot includent Terminos æquatio Curvas definiens. Tum ex valoribus quantitatum *A*, *B*, *C*, *D*, *&c.* Eliminentur illæ coefficientes *b*, *f*, *g*, *&c.* quæ ad æquationem propoſitam non ſpectant; reliquæ dabunt aream quæſitam. Exemplo res patebit.

Sit $z^m = ay^n + bz^e y^{c+n} + gz^{3e} y^{3c+n}$ Æquatio exprimens relationem inter Z & Y. Jam quia $z^m = ay^n + bz^e y^{c+n} + fz^{2e} y^{2c+n} + gz^{3e} y^{3c+n}$, eſt illa pars æquationis quæ (ſumptis terminis in ordine a principio) includit æquationem datam; quam deinceps (brevitatis cauſa) æquationem completam vocabo; ideo Figurarum (quarum Curvæ definiuntur per æquationem completam.)

Areæ $= Azy + Bz^{e+1} y^{c+1} + Cz^{2e+1} y^{2c+1} + Dz^{3e+1} y^{3c+1} + Ez^{4e+1} y^{4c+1} + Fz^{5e+1} y^{5c+1} +$ *&c.* & *a*, *b*, *f*, *g* ingredientur valores quantitatum *B*, *C*, *D*, *E*, *F &c.* Si ergo in his valoribus ponatur ubique $f = 0$ (quia $fz^{2e} y^{2c+n}$ æquationem datam non ingreditur) habebis valores quantitatum *A*, *B*, *C*, *D*, *E*, *&c.* qui in Serie ſubſtituti dabunt Areas quæſitas. Et Calculo inito inveni

$$A = \frac{m}{m+n}. \quad B = \frac{e - \overline{m - c - n \times A} + m - e}{m \times \overline{c+1} + n \times \overline{e+1}} \times \frac{b}{a}.$$

$$C = \frac{\overline{c+n} \times \overline{e+1} + \overline{m-e} \times \overline{c+1}}{m \times \overline{2c+1} + n \times \overline{2e+1}} \times \frac{bB}{a}.$$

$$D = \frac{\overline{\overline{m-3e} \times \overline{1-A} + \overline{3c+n} \times -A \times g}}{ma \times \overline{3c+1}}$$

$$\frac{+\overline{\overline{m-e} \times \overline{2c+1} + \overline{c+n} \times \overline{2e+1}} \times -bC}{+na \times \overline{3e+1}}.$$

$$E = \frac{\overline{\overline{m-3e} \times \overline{c+1} + \overline{3c+n} \times \overline{e+1}} \times}{ma \times \overline{4c+1}}$$

$$\frac{-gB + \overline{\overline{m-e} \times \overline{3c+1} + \overline{c+n} \times \overline{3e+1}} \times -bD}{+na \times \overline{4e+1}}.$$

$$F = \frac{\overline{\overline{m-3e} \times \overline{2c+1} + \overline{3c+n} \times \overline{2e+1}} +}{+ma \times \overline{5c+1}}$$

$$\frac{-gC + \overline{\overline{m+e} \times \overline{4c+1} + \overline{c+n} \times \overline{4e+1}} \times -bE}{+na \times \overline{5e+1}}.$$

$$G = \frac{\overline{\overline{m-3e}\times\overline{3c+13}+\overline{c+n}\times\overline{3e+1}}\,.\times}{+ma\times\overline{6c+1}}$$

$$\frac{-gD+\overline{\overline{m+e}\times\overline{5c+1}+\overline{c+n}\times\overline{5e+1}}\times -bF}{+na\times\overline{6e+1}}.$$

Ex his patet progreſſio reliquorum in infinitum. Et ſic habetur Series exhibens Quadraturas omnium Figurarum, quarum Curvæ definiuntur per hanc æquationem quatuor terminorum $z^n = ay^n + bz^e y^{c+n} + gz^{3e} y^{3c+n}$. Et notandum quod conditiones Quadrabilitatis & numerus terminorum Seriei, Aream quamlibet quæſitam conſtituentium, eadem ſunt cum conditionibus Quadrabilitatis, & numero Terminorum, quæ conveniunt Figuris, quarum Curvæ per æquationes completas definiuntur.

Corol. Præter has duas ſeries in § 2 & 4 propter Figuras quatuor terminorum, poſſunt eodem modo infinitæ aliæ ſeries computari pro cæteris caſibus Figurarum quatuor terminorum. Quod etiam intelligendum eſt de omnibus aliis Figuris, quarum Curvæ per æquationes quotlibet terminorum numero conſtantes definiuntur.

Non jam vacat ipſam Methodum minutiatim deſcribere per quam ad hujuſmodi Series pervenio; brevem tamen ejus rationem exponere forte non ingratum erit. Aſſumo Seriem ex z pariter ac y compoſitam, ſc. $Azy + Bz^p y^q + Cz^g y^h + Dz^l y^k + \&c. = \int : z\,dy$. Cujus ſinguli termini (præter primum) habeant Exponentes incognitos. Tum æquationem inſtituo inter duos valores quantitatis dz, quorum alter ex hac ſerie, alter ex æquatione relationem inter z & y definiente per Methodum Calculi Differentialis directam facile invenitur. Ex terminis hujus æquationis rite reductæ primo determino exponentes incognitos p, q, g, h, l, k &c. Et dein coefficientes A, B, C, &c. Et ſi plures ſint comparationes, quam quæ determinandis his coefficientibus ſufficiunt, tum ex reliquis deduco Quadrabilitatis conditiones. Si recta ineatur via, Calculus eſt longe facillimus; multaſque habeo Regulas huc ſpectantes quas alias forſan tradam; ut & uſum hujus Methodi in inveniendis Quadraturis irrationalibus finitis, quando rationales non dantur: res enim omnino in poteſtate eſt.

V. Having found the Conſtruction of a Curve, from whence (beſides its own *Quadrature* and *Rectification*) the *Quadrature* of the *Hyperbola* is deriv'd, I thought the following Account might not be unacceptable.

Conſtruction and Properties of a Quadratrix *to the* Hyperbola, *by Mr. - -* Perks. *n.* 306. *p.* 2253

Let AB, CD, be two ſtraight Rulars joyned at B, and there making a right Angle. (Their length according to the largeneſs of the Figure you will deſcribe.) EF is another Rular ſomewhat longer than AB. Near the one end E, let a little *Truckle-wheel* (repreſented edge-wiſe by gh, and made of a thin Plate of Braſs or Iron) be faſtned to the Rular by a Pin (i) thorow its Center, ſo that the Wheel may turn about upon the Pin (i) tight to the Rular without joggling.

Fig. 5. *Plate* I.

On the under ſide of this Rular (the ſide from the Eye in the Scheme) let there be pinn'd or glewed a little piece of Wood (in the form of a Quadrant, the part which is ſeen being mark'd kl) whoſe edge (or limb) kl, is an arch of a Circle of Center (i) and *Radius* ib (the ſame with the little Wheel.) The deſign of this piece of Wood is, that in the ſeveral Poſitions of the Rular EF, the circular Limb kl always touching and ſliding by the edge of the Rular AB, the Center of the Wheel may be always in a line (im) parallel to the Rular AB.

In the Rular CD make $MB = ib$ or ik, and at M faſten a little Pin, and another to the Rular EF near the Wheel, as at p. To theſe two Pins let be faſten'd the two ends of a String MR, ſo that its whole length (from Pin to Pin) $+ pi$, be equal to the intended Axis of the Curve TW.

The Inſtrument being thus prepar'd, let a ſtrong Rular SO, be faſtned (or held faſt) upon the Paper or Plain that the Curve is to be drawn upon. Lay the Rular EF from M towards A, and parallel to AB, ſo that the String lye all ſtraight along the edge of the Rular EF, from M to p, the point Sk of the Quadrantal piece of Wood reſting upon the edge of the Rular AB. Then with a ſmall Pin at M, keeping the String cloſe to the edge of the Rular EF, and with your other hand upon the end E, keeping the Wheel tight to the Paper or Plain, move the Pin, String and Rular EF from M towards O, the Rular CD ſliding along by the faſtned Rular SO in a right line, the Wheel gh will by its motion deſcribe the deſired Curve TV.

Note, The Semi-diameter of the little Wheel muſt be about the Sum of the thickneſſes of the two Rulars EF and AB, that it may touch the Paper. Alſo it will be convenient that its edge be thin, and a little rough, that it may not ſlide flat-ways, and that it may leave a viſible impreſſion.

From this Conſtruction the following Properties are demonſtrable.

I. It is evident from the Conſtruction, that the *Sum of the Tangent and Subtangent*, is every where equal to the ſame given Line $= MR + Ri$ $= TW$.) for the String (firſt ſtraight at TW, afterwards making an Angle at R) being every where the ſame, the Line Ri (or $Rp + pi$) is always the Tangent, and the Remainder RM the Subtangent; the Contact of the Wheel with the Plain, being the point of the Curve to which they belong.

II. It hence follows, that any aſſignable part of the Curve is *Rectifiable*, or equal to any aſſignable ſtraight Line. Let FAE be a part of the Curve, its Vertex F. HDd is the Line deſcribed by the motion of the Pin R, and may be ſhewn to be Aſymptote to the Curve. FH a perpendicular to HD. Let A be a given point in the Curve, AD the Tangent, and BD the Subtangent to the ſame point A. Let a be another point in the Curve infinitely near to A, to which let ad be the Tangent, and bd the Subtangent. Draw AG, ag perpendicular to FH; and AB, ab, perpendicular to HD. By the Conſtruction $AD + DB =$

$ad + db$. Let $a\delta$ be made equal to aD, and draw $D\delta$. Then becauſe $ad + bd = AD + DB$, ſubtract bD and aD (or $a\delta$) from both Sums (Equals from Equals) there remains $\delta d + dD = Aa + Bb$ (or Ca). AaC, $Dd\delta$ are like Triangles (or differing infinitely little from ſuch) therefore $Ca\,(Bb) : Aa :: \delta d : Dd$, and compounding $Bb + Aa : Aa :: \delta d + Dd : Dd$. Alternating $Bb + Aa : \delta d + Dd :: Aa : Dd$. But $Bb + Aa = \delta d + Dd$ (as is ſhewn above) therefore $Aa = Dd$. Aa is the fluxional Particle of the Curve FA, and Dd is the fluxional Particle of the Line HD: Theſe Fluxions or Augments, being equal, their flowing quantities beginning together, are themſelves therefore equal, *viz.* $FA = HD$.

Let $FG = x$. $GA\,(= HB) = y$. $AD = t$. $BD = S$. So is the Curve $FA = HD = y + S$: that is, *the Curve from the Vertex to any given point therein, is equal to the Sum of its Ordinate, and Subtangent to the ſame point which is its ſecond Property.*

III. The next Property (and whereupon I call it the *Hyperbolic Quadratrix*) is this, let FAE be a part of the Curve, (*&c.* as before) $FIKH$ is a Square upon the Line FH. ‸ IL is an Equilater Hyperbola whoſe Vertex is I, its Aſymptotes HO, HR, its Ax $HI\mu$. From a given point L in the Hyperbola, (below its Vertex I) draw LA parallel to the Aſymptote RH, interſecting the Diagonal IH in M, FH in G, and touching the Quadratrix in A. I ſay, that the Hyperbolic Area ILM is equal to a Rectangle, whoſe ſides are the *Ordinate* GA, and twice FH, the Ax to the Quadratrix, that is, Trilin. $ILM = 2FH \times GA$.

Fig. 6. Plate I.

Let $FH = a$, $FG = x$, $GA = y$. Becauſe of the Hyperbola $GL \times GH\,(LS) = FHq$, therefore $GL = \frac{FHq}{GH}$; and $LM = \frac{FHq}{GH} - GH\,(MG)$ that is, $LM = \frac{aa}{a - x} - a + x = \frac{2ax - xx}{a - x}$, and conſequently the fluxion of the Area $ILM = \frac{2ax - xx}{a - x}\dot{x}$.

In the Rectangle triangle ADB, $AB = a - x$, $BD = S$, $AD = t = a - S$; then is $ADq = ABq + BDq$: or $aa - 2aS + SS = aa - 2ax + xx + SS$, which being reduced, gives $S = \frac{2ax - xx}{2a}$.

Let la be a right line ſuppoſed infinitely near and parallel to LA, and interſecting AB in C. Becauſe of like triangles ACa, ABD; $AB : BD :: AC : Ca$ that is $a - x : S\,\left(= \frac{2ax - xx}{2a}\right) :: \dot{x} : \dot{y}$. therefore $\dot{y} = \frac{2ax - xx}{2aa - 2ax}\dot{x}$. Multiply each by $2a$, and 'tis $2a\dot{y} = \frac{2ax - xx}{a - x}\dot{x}$. The *Flowing quantity* of $2a\dot{y}$ is $2ay$ and the *flowing quantity* of $\frac{2ax - xx}{ax}\dot{x}$ is the Hyperbolic Area ILM (as is ſhewn before.) Theſe two Area's beginning together at F and I, and having every where equal *Fluxions*, or Augments, are therefore themſelves every where equal.

IV. The Quadrature of the Trilinear Figure *ILM* being thus found, any other Area bounded with the Curve Line *IL*, and any other Right Lines, is alſo given.

IV. Suppoſing the ſame things as in the precedent Propoſition, I ſay, that the Area of the Quadratrix *FabHF* is equal to half the ſquare of *Fg*, wanting the Cube of *Fg* divided by ſix *FH*, or $FabHF = \frac{xx}{2} - \frac{xxx}{6a}$. The Fluxion of this Area is the Rectangle $CabB = \overline{a-x} \times \dot{y} = \overline{a-x} \times \frac{2ax - xx}{2aa - 2ax}\dot{x} = x\dot{x} - \frac{xx}{2a}\dot{x}$. The flowing quantity of $x\dot{x}$ is $\frac{1}{2}xx$: And the flowing quantity of $-\frac{xx}{2a}\dot{x}$ is $-\frac{xxx}{6a}$ [as is eaſily ſhewn by bringing back theſe flowing quantities to their reſpective Fluxions.] And hence alſo it follows, that the whole Area continued on infinitely towards *E*, *is one third of the Square FIKH*; or $\frac{1}{3}aa$. For ſuppoſing $x = a$ the Area above becomes $\frac{aa}{2} - \frac{aa}{6} = \frac{aa}{3}$.

V. Suppoſing ſtill the ſame things, I ſay that the Solid made by the converſion of the Area *FabHF* about the Line *Hb* as an Axis, is equal to a Cylinder whoſe Radius is $FH = a$, and height equal to $\frac{xx}{2a} - \frac{x^3}{2aa} + \frac{x^4}{8a^3}$. And the whole Solid made by converſion of the whole Figure infinitely continued, is equal to an eighth part of a Cylinder, whoſe Radius and Height are each equal to *FH* or *a*.

Let $\frac{P}{D}$ expreſs the Proportion of the Periphery and Diameter of a Circle. Then is $\frac{P}{D}\,ab$ quad. the Area of a Circle whoſe Radius is *ab*. And becauſe $Ca = \dot{y} = \frac{-\frac{xx}{2a}}{a-x}\dot{x}$ the fluxion of the Solid is $\frac{P}{D}\,\overline{ab}^2 \times \frac{x - \frac{xx}{2a}}{a-x}\dot{x}$, or $\frac{P}{D}\,\overline{a-x}^2 \times \frac{x - \frac{xx}{2a}}{a-x}\dot{x} = \frac{P}{D}\,\overline{ax - \frac{3}{2}xx + \frac{x^3}{2a}} \times \dot{x}$: whoſe flowing quantity is $\frac{P}{D} - \frac{axx - xxx}{2} + \frac{x^4}{8a}$. Which Solid being divided by $\frac{P}{D}\,aa$ (the Area of a Circle whoſe Radius is *a*) gives

$\frac{xx}{2a} - \frac{xxx}{2aa} + \frac{x^4}{8aa}$ for the height of a Cylinder on the ſaid circular Baſe, and equal to the Solid made by converſion of the Area *FabHF* about the Line *Hb* as an Axis. When $x = a$ (that is when the whole Figure is turn'd about its Aſymptote) the height $\frac{xx}{2a} - \frac{x^3}{2aa} + \frac{x^4}{8a^3}$ becomes $\frac{1}{8}a$.

VI. The Curve ſurface of the Solid generated by the Converſion of the Figure *FabHF* about *HB*, is equal to the Curve ſurface of a Cylinder, whoſe Radius is *a*, and height equal to $\frac{x}{2} - \frac{xx}{4a} + \frac{xxx}{12aa}$. And the whole Curve Surface of the Solid infinitely continued, is equal to *one third part of the Curve Surface of a Cylinder whoſe Radius and Height are equal to* F H *or* a. Which may be demonſtrated after the manner of the precedent Propoſition.

VII. The Radius of the Curvature of any Particle of the Quadratrix is $\frac{tt}{a-x}$. and this found Geometrically. *FAE* is the *Quadratrix*, *HD* the Aſymptote, *AD* the Tangent, *BD* the Subtangent to a given point *A*. Make $BV = AD$. Upon *V* raiſe the perpendicular *VW*, from *A* Fig. 7. Plate I.
draw *AW* perpendicular to the Tangent *AD*, till it meet *VW* in *W*. So is *AW* the Radius of the Curvature at *A*.

VIII. This Curve may be continued on infinitely above the point *F* (but by a different and more operoſe way of Conſtruction) whoſe Properties will be theſe, 1. The *Difference* of its Tangent and Subtangent (taking the Subtangent in the Line *HS*) will be always equal to the ſame given Line *FH* or *a*. That is, as $t + s = a$, below *F*, ſo $t - s = a$ above *F*. 2. As below *F* the Curve Line is equal to the *Sum* of its Ordinate and Subtangent, ſo above, it is equal to their *Difference* or $S - y$. 3. As below *F*, $2ay = ILM$, ſo above, $2ay = I\lambda\mu$. All which (and its other Properties) may be demonſtrated as the Precedent, *mutatis mutandis.*

IX. With a little variation in the precedent Conſtruction may the *Logarithmick Curve* be conſtructed, which is alſo a *Quadratrix* to the Hyperbola. Omitting the String *MRP*, let the diſtance *MR* be equal to the *Subtangent* of the intended Logarithmick Curve (which, as 'tis known, is invariable.) Stick a Pin at *R* in the Ruler *CD*, to which apply the Ruler *EF*, ſo that the edge of the little Quadrant *kl*, reſting upon the Ruler *AB*, the diſtance *Mi* be equal to *MR*. Then keeping the Ruler *EF* tight to the Pin *R* and Ruler *AB*, ſlide the Ruler *CD* along in a ſtraight Line (by the Ruler or Line *SO*.) So will the Wheel *gh* deſcribe a part of the Logarithmick Curve *TV*, whoſe *Subtangent* is every where *MR*.

X. Let FAE repreſent the *Logarithmick Curve,* whoſe Subtangent is equal to FH. $LI\lambda$ is an Equilater Hyperbola (*&c.* as before § III.) Let $FG = x$, $Ga = y$. $FH (= BD) = a$. $GH (= LS) = a - x$. $AC = \dot{x}$, $Ca = \dot{y}$. Then $AC : Ca :: AB : BD$, that is $\dot{x} : \dot{y} :: a - x : a :: a : \frac{aa}{a-x}$, therefore $a\dot{y} = \frac{aa}{a-x}\dot{x}$. The *Flowing quantity* of $a\dot{y}$ is ay; and the *Flowing quantity* of $\frac{aa}{a-x}\dot{x}$ is the Hyperbolick Area $FILG$ (for by the nature of the Hyperbola $GL = \frac{aa}{a-x}$) therefore is the Hyperbolick Area $FILG$ equal to ay, a Rectangle whoſe ſides are the Subtangent ($BD = FH$) and Ordinate GA (as here accounted) of a Logarithmick Curve.

Of the Length of Curve Lines. by Mr. Jo. Craig. *n.* 314. p. 64.

VI. Lemma. *Duorum Quadratorum ſummam in alia duo Quadrata dividere.*

Sint dz^2, ds^2, duo Quadrata data, quorum ſumma $dz^2 + ds^2$ dividenda eſt in alia duo Quadrata dx^2, dy^2; ſintque m & n duo quilibet numeri ad arbitrium ſumendi. Jam ex conditione Problematis eſt $dx^2 + dy^2 = dz^2 + ds^2$, unde (ut ex Diophanto conſtat) erit $dx = \frac{mm - nn \times dz + 2mnds}{mm + nn}$, $dy = \frac{nn - mm \times ds + 2mndz}{mm + nn}$. *Q. E. I.*

Problema. *Curvas innumeras invenire, quæ ſint ejuſdem Longitudinis cum Curva quavis propoſita, ſive Algebraica ſive Tranſcendente.*

Deſignent z, s Coordinatas Curvæ propoſitæ; & x, y Coordinatas Curvæ quæſitæ, quæ ejuſdem ſit longitudinis cum propoſita; Unde ex Curvarum Elementis $dx^2 + dy^2 = dz^2 + ds^2$, Ideoque per Lemma præcedens

$$dx = \frac{mm - nn \times dz + 2mnds}{mm + nn},$$

$$dy = \frac{nn - mm \times ds + 2mndz}{mm + nn};$$

Quarum integrales ſunt

$$x = \frac{\overline{m^2 - n^2} \times z + 2mns}{mm + nn},$$

$$y = \frac{\overline{n^2 - m^2} \times s + 2mnz}{mm + nn}.$$

Et ſic innoteſcunt Coordinatæ x, y unius ex Curvis quæſitis; ſimiliter ex hac una invenietur ſecunda, ex ſecunda tertia, & ſic porro innumeræ invenientur. *Q. E. I.*

Deſcription and Quadrature of a Curve of the Third Order, com. by Mr. de Moivre, n. 345. p. 329. *Fig.* 8.

VII. I have look'd a little farther into that Curve which fell lately under my conſideration. It is not the *Foliate* as I did at firſt imagine, but I believe it ought not to make a *Species* diſtinct from it. *AEB* is the Curve I thus deſcribe. Let *AB* and *BK* be perpendicular to each other. From the point *A* draw *AR* cutting *BK* in *R*, and make *RE* = *BR*, the point *E* belongs to the Curve. Draw *BC* making an Angle of 45 *grad.* with *AB*, this Line *BC* touches the Curve in *B*; from the point *E* draw *ED* perpendicular to *BC*, and calling *BD*, x; *DE*, y; *AB*, a; and making $\sqrt{8aa} = n$, the Equation belonging to that Curve is $x^3 + xxy + xyy + y^3 = nxy$ or $\frac{x^4 - y^4}{x - y} = nxy$. Taking *BG* = *AB*, and drawing *GP* perpendicular to *BG*, *PG* is an *Aſymptote*. In the *Foliate* the Equation is $x^3 + y^3 = \frac{1}{2} nxy$, in which the two Terms $xxy + xyy$ of the former Equation are wanting; and its *Aſymptote* is diſtant from *B* by $\frac{1}{3}$ *BA*. Again draw *EF* perpendicular to *AB*: let *BF* be called z, and *FE*, v; the Equation belonging to the Curve *AEB* is $vv = \frac{azz - z^3}{a + z}$. In the *Foliate* the Equation is $vv = \frac{azz - z^3}{a + 3z}$. From theſe two laſt Equations it ſeems that theſe Curves differ no more from one another than the *Circle* from the *Ellipſis*.

The Quadrature of the Curve here deſcribed hath ſomething of Simplicity with which I was well pleaſed. With the Radius *BA* and Center *B* deſcribe a Circle *AKG*, let the Square *HPST* circumſcribe it, ſo that *HP* be parallel to *AG*· prolong *FE* till it meet the Circumference of the Circle in *M*, and through *M* draw *LMQ* parallel to *HP*. The Area *BFE* is equal to the Area *KHLM*, comprehended by *KH*, *HL*, *LM* and the Arc *KM*. And the Area *Bfe* is equal to the Area *KmLH* or *KMPQ*. Therefore if *BF* and *Bf* are equal, the two Areas *BFE*, *Bfe* taken together are equal to the Rectangle *HQ*, and therefore the whole Space comprehended by *BEAXBeYGZ* (ſuppoſing *Y* and *Z* to be at an infinite Diſtance) is equal to the circumſcribed Square *HS*.

‘ *N.B.* This Quadrature is eaſily demonſtrated from the Equation: for by it $a + z : a - z :: zz : vv$, that is *AF* : *EF* :: *MF* : *FB*, and ſo *φF* the *Fluxion* of *AF* to *Ll* the *Fluxion* of *MF*. Hence the *Areola EFφe* will be always equal to the *Areola MLlμ*, and therefore the Area *AEF* always equal to the Area *MAL*.

‘ Hence it appears that this Curve requires the Quadrature of the Circle to ſquare it; whereas the *Foliate* is exactly quadrable, the whole *Leaf* thereof being but one third of the Square of *AB*, which in this is above three ſevenths of the ſame. Again in our Curve; the greateſt Breadth is when the Point *F* divides the Line *AB* in extreme and mean Proportion, whereas in the *Foliate* it is when *AB* is triple in power to

"*BF*. And the greateſt *EF* or Ordinate in the *Foliate* is to that of our "Curve nearly as 3 to 4, or exactly as $\sqrt{\frac{2}{3}\sqrt{\frac{1}{3}}-\frac{1}{3}}$ to $\sqrt{5\sqrt{\frac{5}{4}}-5\frac{1}{2}}$.

But ſtill theſe Differences are not enough to make them two diſtinct Species, they being both defin'd by a like Equation, if the Aſymptote *SGP* be taken for the Diameter. And they are both comprehended under the 40*th* kind of Curves of the 3*d* Order, as they ſtand enumerated by Sir *Iſaac Newton* in his incomparable Treatiſe on that Subject.

The Conſtruction and Meaſure of Curves, by Mr. Maclaurin, *n.* 356. *p.* 803.

VIII. § 1. Sint *L* & *l* puncta quamproxima in Curva *BlL*, ſit *lo* arcus centro *S* deſcriptus perpendicularis in *SL*; & erit *Ll* ut momentum Curvæ & *Lo* momentum Radii *SL*. Ac ſi detur ratio *Ll* ad *Lo*, vel ad *lo* in diſtantia *SL*, dabitur æquatio Curvæ ad centrum *S*. Sint *LP*, *lp* Tangentes Curvæ in punctis *L* & *l*, in quas ex *S* demittantur normales *SP*, *Sp* iis occurrentes in punctis *P* & *p*; ſimiliter in omnes Curvæ Tangentes demittantur perpendiculares ex dato puncto *S*, & conſtruetur Curva tranſiens per omnes Tangentium & perpendiculorum interſectiones. Hujus triangulum elementare *Pnp* ſimile erit triangulo *Lol*, quæ proinde dabitur ex data Curva *BlL*. Quippe ob æquales *Snp*, *PnL*, & rectos *Spn*, *SPL* æquiangula erunt triangula *Spn*, *PnL*, & proinde *Pn* : *pn* :: *Ln* : *Sn* :: *Lo* : *lo*, adeoque ob angulos *Pnp*, *SnL*, *Lol* æquales, erunt triangula *Pnp*, *SnL*, *Lol* ſimilia. Cum igitur eadem ſit ratio *Ll* ad *lo* quæ *Pp* ad *pn*, & *SL* ad *SP*, manifeſtum eſt data ratione *Ll* ad *lo*, & recta *SL*, dari rationem *Pp* ad *pn* & rectam *SP*, adeoque Curvam *DPp*. Eadem ratione ex *DP* conſtrui poteſt Tertia, & ex ea dein Quarta, & progrediendo prodibit ſeries Curvarum infinita, quæ omnes ex uno dato innoteſcunt. Quod ſi erigantur *LN* & *ln* perpendiculares in radios *SL*, *Sl*, ſibi mutuo occurrentes in *n*; & per omnia ſimiliter definita perpendicularium concurſuum puncta deſcribatur Curva *EN*: ea ipſa erit Curva ex qua deduci poteſt *BL*, eadem ratione qua conſtruximus *DP* ex *BL*. Ex *EN* ſimiliter conſtrui poteſt alia Curva, atque ex hac quoque parte Series infinita Curvarum conſtrui poterit.

Fig. 9.

§ 2. Curvarum vero hac ratione conſideratarum ſimpliciſſimæ ſunt quarum *Ll* eſt ad *Lo* in ratione poteſtatis alicujus Radii, ita ut, ſi *a* ſit data quantitas, *r* denotet Radium Curvæ, *n* numerum quemcunque, ſit *Ll* ad *lo* ut a^n ad r^n æquatio earum generalis. Omnes vero hæ Apſidem habent cum $r=a$, quoniam in eo caſu $Ll=lo$. Ut inveſtigem æquationem Curvæ *DP*, cum in *BL* eſt ut *Ll* ad *lo* ita a^n ad r^n, ita *r* ad $SP=\frac{r^{n+1}}{a^n}$, ita $a^{\frac{n}{n+1}}\times SP^{\frac{1}{n+1}}$ ad *SP*, ita $a^{\frac{n}{n+1}}$ ad $SP^{\frac{n}{n+1}}$, ita *Pp* ad *pn*. Proinde ſi ; repreſentet momentum Curvæ, $\dot{y}$ arcum circularem radio deſcriptum a centro *S*, & *r* radium correſpondentem, quæcunque ſit Curva cujus Æquatio inveſtigatur, erit Æquatio Curvæ *BL*, ; : $\dot{y}$ :: a^n : r^n; Æquatio vero Curvæ *DP*, ; : $\dot{y}$ :: $a^{\frac{n}{n+1}}$: $r^{\frac{n}{n+}}$. Angulus autem *PSp* erit ad Angulum LSl ut $\frac{pn}{SP}$ ad $\frac{lo}{SL}$, ſive ut $\frac{Pn}{SP}$ ad $\frac{Lo}{SL}$, vel (ſi SP dicatur x, & SL, r) ut $\frac{\dot{x}}{x}$ ad $\frac{\dot{r}}{r}$, hoc eſt, (ob $x = \frac{r^{n+1}}{a^n}$) ut $\frac{n+1}{r}\dot{r}$ ad $\frac{\dot{r}}{r}$, ſive ut $n+1$ ad 1. Hinc BSP eſt ad BSL ut $n+1$ ad 1; unde facilius abſque Tangentium ope duci poteſt Curva BP. Si ſumatur angulus BSP ad BSL in ratione $n+1$ ad 1, & in SP demittatur perpendicularis ex L, erit occurſus perpendiculi cum SP, in Curva BP prius Tangentium ope deſcripta. Fig. 10.

§ 3. Oſtendimus quo pacto ex una ſeries Curvarum infinita deducitur; quo vero pacto ſingularum longitudines ex illius & unius alterius longitudinibus datis innoteſcant pergo demonſtrare. Cum angulus $SPp = SLl$, atque LSl ſit ad PSp ut 1 ad $n+1$, erit Ll ad Pp ut SL ad $\overline{n+1}\,SP$, ſive (ob $SL : SP :: Ll : lo$) ut Ll ad $\overline{n+1}\,lo$, ac proinde $Pp = \overline{n+1}\,lo$: ſed $lo = ln - on = ln - LN + Nn$; ergo $Pp = \overline{n+1} \times \overline{ln - LN + Nn}$. Sed $ln - LN$ eſt momentum rectæ LN normalis in SL, Pp momentum Curvæ BP, & Nn momentum Curvæ BN. Cumque BP, BN, BL ſimul evaneſcant in B, erunt in ratione momentorum, adeoque $BP = \overline{n+1} \times \overline{BN + \text{vel} - LN}$. Unde Curva BP eſt ad ſummam vel differentiam Curvæ penultimæ in Serie ejuſque Tangentis ab intermedia interceptæ, ut $n+1$ ad 1; ſive, ſi m ſit Index æquationis Curvæ BP (quoniam $m = \frac{n}{n+1}$) ut 1 ad $1-m$. Fig. 10.

Hinc 1^{mo} in ſerie Curvarum infinita ſupra deſcripta, ſi dentur Longitudines duarum proximarum, dabuntur longitudines omnium; quippe menſura cujuſvis pendet a menſura penultimæ ſemper in ſerie, & proinde unum par omnibus menſurandis ſufficiet: Si una Curva ſit rectis commenſurabilis vel incommenſurabilis, erit integræ ſeriei pars dimidia rectis commenſurabilis vel incommenſurabilis. Hinc 2^{do}. Licet Curvæ BP & BN eſſent rectis incommenſurabiles, differentia tamen Curvæ BP ab $n+1$ Curvæ BN eſſet æqualis aſſignabili rectæ. 3^{tio}. Si Curva tranſit per S, recta LN evaneſcente in S, erit $BPS = \frac{BNS}{1-m}$.

§ 4. Curvarum de quibus egimus, quarum nimirum $\dot{s} : \dot{y} :: a^n : r^n$, maxime inſignis eſt Circulus, exiſtente S in circumferentia, cujus æquatio eſt $\dot{s} : \dot{y} :: a : r$, ut ex ſimilitudine triangulorum Lol, BLS manifeſtum eſt, adeoque $n = 1$; & proinde $m = \frac{n}{n+1} = \frac{1}{2}$, & æquatio Curvæ BP erit $\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$, quæ ipſa eſt æquatio *Epicycloidis* revolutione Circuli ſuper baſim ſibi æqualem revolventis deſcripti, ad punctum ubi punctum deſcribens tangit baſim, quæ D^{no} *Paſchal* dicitur *la Limaçon de M. Roberval*, quamque *M. De la Hire* conſiderat ut *Conchoidem* Baſis Circularis, in Actis Academiæ Pariſienſis Anni 1708. Perpendiculares omnes LN, ln concurrunt in puncto B, adeoque $BN = 0$; unde $BP =$ Fig. 11.

$\frac{BN+NL}{1-m} = 2BL$: Hinc Curva tota $BPS = 2BS$, ac longitudo *Epicycloidis* ſemper dupla eſt chordæ arcus in circulo correſpondentis. 2do. Ex *Epicycloide* deſcribatur Curva $B\Pi S$, eadem ratione qua *Epicycloidem* ex Circulo deſcripſimus: In hoc caſu $n = \frac{1}{2}$, & $m = \frac{n}{n+1} = \frac{\frac{1}{2}}{\frac{1}{2}+1} = \frac{1}{3}$, ac proinde æquatio Curvæ $B\Pi S$ erit $\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$. Longitudo Curvæ erit $\frac{BL+LP}{1-m} = \frac{3}{2}\overline{BL+LP} = \frac{3}{2}\overline{BL+LG}$, & proinde $B\Pi$ eſt ſeſquiplus ſummæ Arcus circularis ejuſque Sinus recti. Quod ſi ſumatur $CD = BD$, & radio SD centro S deſcribatur Circulus occurrens rectæ SP in H, & ſit HK perpendicularis in BS; quoniam $DH = \frac{3}{2}BL$, erit $B\Pi = DH + HK$. Hinc arcus $B\Pi$ neque ſunt rectis neque arcubus circularibus commenſurabiles, differentia tamen arcuum $B\Pi$ & DH eſt recta HK. In puncto S evaneſcit LG, adeoque $B\Pi S = \frac{3}{2}BLS$, unde tota Curva eſt ſeſcupla ſemicirculi. Nulla vero pars hujus Curvæ aſſignabilis commenſurari poteſt toti, nec integra Curva in data quavis ratione ſecabilis eſt, ita ut portiones rationem aſſignabilem habeant ad ſe mutuo aut ad totam. Si hæc curva in data aliqua ratione Geometrice ſecari poſſet, conſtaret Quadratura Circuli, nam ſi *e. gr.* eſſet $B\Pi$ ad $B\Pi S$ ut 1 ad m, & BL ad BLS ut 1 ad n, eſſet $B\Pi = \frac{B\Pi S}{m} = \frac{3BLS}{2m} = \frac{3nBL}{2m} = \frac{3}{2}\overline{BL+LG}$, unde eſſet $BL = \frac{mLG}{n-m}$ & $BLS = \frac{nm}{n-m}LG$. 3^{tio} Ex $B\Pi S$ conſtruatur explicata methodo Curva BR, & quoniam $n = \frac{1}{3}$ erit $m = \frac{n}{n+1} = \frac{1}{4}$, atque æquatio Curvæ BR erit $\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$. Hinc longitudo Curvæ fiet $\frac{4}{3}\overline{2BL+P\Pi}$, totalis vero Longitudo Curvæ $BRS = \frac{8}{3}$ diametri SB. Si harum Curvarum Conſtructiones continuentur, prodibit hujuſmodi ſeries Æquationum quæ facile producitur ad libitum.

Æquatio Circuli 1. $\dot{s} : \dot{y} :: a : r$
Epicycloidis 2. $\dot{s} : \dot{y} :: a^{\frac{1}{2}} : r^{\frac{1}{2}}$
Secundi 3. $\dot{s} : \dot{y} :: a^{\frac{1}{3}} : r^{\frac{1}{3}}$
Tertii 4. $\dot{s} : \dot{y} :: a^{\frac{1}{4}} : r^{\frac{1}{4}}$
Cujuſvis n. $\dot{s} : \dot{y} :: a^{\frac{1}{n}} : r^{\frac{1}{n}}$, &c.

Obſervare licet in genere, omnes quarum Indicum denominatores ſunt Numeri pares, perfectæ rectificationis eſſe capaces; cumque quævis ſit ad penultimam ut 1 ad $1-m$, perpendenti manifeſtum erit Curvæ cujuſvis longitudinem fore $= \frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m} \times \frac{1-6m}{1-7m}$, &c. $\times SB$

continuando ſeriem donec ad nihilum reducatur Fractio. Quod ſi Indicis denominator ſit Numerus impar, Curvæ erunt perfectæ rectificationis incapaces, & earum arcus quicunque erunt ſibi mutuo, ipſis totis, rectis quibuſvis & arcubus Circularibus incommenſurabiles: exprimi vero poſſunt omnes arcubus circularibus & rectis: At Curvæ cujuſvis totalis Longitudo erit ad Semicirculum ut $\frac{1}{1-m} \times \frac{1-2m}{1-3m} \times \frac{1-4m}{1-5m}$, &c. ad unitatem. Denique ſi Areola a Corpore in harum quavis revolvente ſumatur conſtans, hoc eſt ſi $r\dot{y} = 1$, ſubtenſa anguli contactus, cui ſemper (ob datum data area tempus) proportionalis eſt *Vis Centripeta* tendens ad S, erit reciproce ut poteſtas diſtantiæ cujus Index eſt $2m + 3$; atque hoc eſt non contemnendum harum Curvarum privilegium, quod in iis omnibus Vis centripeta tendens ad S ſit ut aliqua reciproca diſtantiæ dignitas, quæ ſimpliciſſima eſt, & utiliſſima, in Naturæ indagine, Virium Centripetarum lex.

§ 5. Curvarum quarum $\dot{s} : \dot{y} :: a^n : r^n$ proxime conſideranda venit (quæ Curva quidem improprie dicitur) ipſa Linea recta, exiſtente S extra rectam. In hac linea, ob ſimilia triangula Ppn, PBS erit (ſi $BS = a$ & $SP = r$) $\dot{s} : \dot{y} :: r : a$. Ex linea recta methodo directa nihil niſi punctum Fig. 12.
B conſtrui poteſt, Methodo vero inverſa, perpendicularium nimirum PL, pl concurſu, conſtrui poteſt Curva, cujus Index (ſi m ſit Index Curvæ BP) æqualis erit $\frac{m}{1-m}$; nam ſi Index Curvæ BL ſit n, erit $m = \frac{n}{n+1}$, ac proinde $n = \frac{m}{1-m}$. Unde in hoc caſu, cum $m = -1$ erit $n = \frac{-1}{2}$, & æquatio Curvæ BL erit $\dot{s} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$, quæ æquatio eſt Parabolæ ad Focum. Ex hac conſtrue aliam, conſtituendo angulum $LSN = LSB$ & erigendo LN normalem in SL occurrentem ipſi SN in N. Quoniam vero $m = \frac{-1}{2}$ erit $n = \frac{-1}{3}$, & æquatio Curvæ $\dot{s} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$ & $BP = \frac{BN - LN}{1-m} = \frac{1}{2}\overline{BN - LN}$, ergo $BN = 2BP + LN$; proinde hæc Curva eſt rectificabilis. Si Series continuetur, prodibunt ut prius æquationes in hoc ordine.

Æquatio Rectæ $\dot{s} : \dot{y} :: r : a$
Parabolæ $\dot{s} : \dot{y} :: r^{\frac{1}{2}} : a^{\frac{1}{2}}$
Secundæ $\dot{s} : \dot{y} :: r^{\frac{1}{3}} : a^{\frac{1}{3}}$
Tertiæ $\dot{s} : \dot{y} :: r^{\frac{1}{4}} : a^{\frac{1}{4}}$
Cujuſvis $\dot{s} : \dot{y} :: r^{\frac{1}{n}} : a^{\frac{1}{n}}$

In hac Serie primæ ſunt Recta & Parabola, unde patet dimidiam hujus ſimiliter ac prioris Seriei eſſe rectis menſurabilem: alia vero dimidia pars in rectis & arcubus Parabolicis exhiberi poteſt. In his omnibus Vis centripeta ad S eſt reciproce ut poteſtas diſtantiæ cujus Index $3-2m$, ac proinde ſemper inter duplicatam & triplicatam rationem diſtantiæ reciproce.

§ 6. Æquatio Hyperbolæ æquilateræ ad centrum eſt $\dot{s} : \dot{y} :: r^2 : a^2$, ex qua deducitur methodo directa Series hujuſmodi,

$$1.\ \dot{s} : \dot{y} :: r^2 : a^2$$

$$2.\ \dot{s} : \dot{y} :: a^2 : r^2$$

$$3.\ \dot{s} : \dot{y} :: a^{\frac{2}{3}} : r^{\frac{2}{3}}$$

$$4.\ \dot{s} : \dot{y} :: a^{\frac{2}{5}} : r^{\frac{2}{5}}$$

$$5.\ \dot{s} : \dot{y} :: a^{\frac{2}{2n-1}} : r^{\frac{2}{2n-1}}$$

Fig. 11.

Ex his Curvæ, quarum Indicum denominatores ſunt in progreſſione $-1, 3, 7, 11,$ &c. exhiberi poſſunt in rectis & arcubus Hyperbolicis; reliquæ vero in rectis & arcubus Curvæ, cujus æquatio ad axem SB (ſi x ſit abſciſſa, y vero Ordinata) eſt $\overline{xx+yy}^2 = a^2x^2 - a^2y^2$, quæque conſtruitur biſecando angulum BSL & ſumendo SN mediam proportionalem inter SB & SL.

Curvæ quæ ex Hyperbola methodo inverſa conſtrui poſſunt progrediuntur in hac Serie,

$$\text{Hyperbolæ } 1.\ \dot{s} : \dot{y} :: r^2 : a^2$$

$$2.\ \dot{s} : \dot{y} :: r^{\frac{2}{3}} : a^{\frac{2}{3}}$$

$$3.\ \dot{s} : \dot{y} :: r^{\frac{2}{5}} : a^{\frac{2}{5}}, \text{ \&c.}$$

Ubi Curvæ quarum Indicum denominatores ſunt in progreſſione 1, 5, 9, 13, &c. exprimi poſſunt in rectis & arcubus Hyperbolicis; reliquæ vero in rectis & arcubus Curvæ modo explicatæ.

Fig. 13.

Si aliæ Curvæ deſiderentur quæ alias exhiberent Series, id facillime fieri poteſt ope vel Circuli vel Rectæ: quippe ex earum aliqua omnes, in quibus $\dot{s} : \dot{y} :: a^n : r^n$, conſtrui poſſunt; ſumendo, ſi ope Circuli Problema ſit ſolvendum, BSR ad BSL ut 1 ad n, & SN in ipſa $SR = a^{\frac{n-1}{n}} \times SL^{\frac{1}{m}}$; quippe Curvæ per omnia puncta N ductæ æquatio erit $\dot{s} : \dot{y} :: a^n : r^n$. Similiter ope Rectæ conſtrui poſſunt quarum æquatio eſt $\dot{s} : \dot{y} :: r^n : a^n$.

Duas exhibuimus Series infinitas Curvarum rectis commenſurabilium; aliam arcubus circularibus, aliam Parabolicis, aliam Hyperbolicis una cum rectis menſurabiles demonſtravimus; eæ vero ad rectarum menſuram arte ſola infinita reduci poſſe videntur, ſicut æquatione ſola infinita in rectis exprimuntur.

rum *MT* & *NL* deſcribet Lineam Quarti Ordinis puncto duplice carentem.

4. Quod ſi in primo caſu hujus Conſtructionis rectæ *CMR*, *SNK*, una
Fig. 18. coincidant cum *CS*; tunc puncta *C* & *S* evadunt ſimplicia, & Curva erit
Tertii Ordinis abſque puncto duplice. *Exemplum.* Sint rectæ *BM*,
AQ, *DN*, ſibi mutuo parallelæ atque omnes perpendiculares in *CS*. Sint
quoque Anguli *RMT*, *KNL* recti, & ſi ſecundum regulam primi Caſus
Fig. 20. deſcribatur Curva, Crura *CMR*, *SNK* una coincident cum *CS*; & hac
conſtructione deſcribi poſſunt Curvæ D. *Newtoni* 10, 11, 20, 21, 40, ſecundum
varias poſitiones punctorum *C* & *S* reſpectu trium rectarum *BM*,
AQ, *DN*; Omnes vero hæ Species puncto duplice carent.

5. Lineæ vero Quarti Ordinis quæ punctum triplex habent ſic conſtrui
poſſunt. Sint tres rectæ *AQ*, *BN*, *DM* poſitione datæ; ſint etiam Anguli
QCT, *SNM* & *NML* dati & invariabiles; percurrant puncta *N* &
Fig. 21. *M* rectas *BN* & *DM*, ita ut crus *NQ* ſemper tranſeat per datum punctum
S: Revolvatur *QCT* circa *C* ita ut concurſus crurum *CK*, *SN* percurrat
tertiam rectam *AQ*; tunc concurſus crurum *CT*, *ML* deſcribet
Lineam Quarti ordinis punctum triplex habentem in *C*.

6. Oſtendi quo pacto Lineæ Quarti Ordinis deſcribi poſſunt, quæ
punctum triplex habent aut duo duplicia; Aliæ quæ unicum habent
punctum duplex ſic commode deſcribuntur. Sint tres rectæ ut prius poſitione
Fig. 22. datæ, *AQ*, *BN*, *DM*, dentur etiam Anguli *SNK*, *SML*, *RCT*;
ſint puncta *N*, *M* & *S* ſemper in eadem recta linea; Moveantur puncta
N & *M* ut prius per rectas *BN*, *DM*; Si concurſus crurum *CR*, *NK* ducatur
per rectam indefinitam *AQ*, tunc concurſus crurum *CT*, *ML* deſcribet
Lineam Quarti Ordinis habentem punctum duplex unicum in *C*.
Hæ vero duæ ultimæ Propoſitiones novas Methodus ſuppeditant lineas
Tertii Ordinis deſcribendi, tum quæ puncta duplicia habent, tum quæ iis
deſtituuntur; Eæ vero in brevi hoc Methodus noſtræ ſpecimine ſunt omittendæ.

7. Maneant Anguli atque rectæ ut in *Prop.* III. Concurſus vero
Fig. 23. nunc rectarum *MT*, *NK* ducatur per indefinitam rectam *AQ*; & Concurſus
crurum *MR* & *NL* deſcribet Lineam Quinti Ordinis punctum
qnadruplex habentem in *S*. Habeo etiam alias Methodus curvas deſcribendi
Quinti Ordinis, quæ punctum habent triplex, duplex, aut duo duplicia,
vel nulla niſi puncta ſimplicia; ſed hæc ſufficiant ad ſimplicitatem
& univerſalitatem Methodus demonſtrandam. Notandum vero in
ſpecialibus ſimplicioribus Angulorum & Rectarum circumſtantiis, Lineam
aliquando migrare in Curvam ordinis inferioris quam in *Prop.* explicatur;
Imo ſingulæ Propoſitiones Methodus ſuppeditant particulares, curvas aliqnas
ordinis cujuſcunque inferioris deſcribendi.

8. *Propoſitio Generalis.* Sumantur ad libitum Rectæ in eodem plano
ubicunque poſitæ, quarum ſit numerus (*n*) ut *BN*, *ER*, *FT*. Sumantur
Fig. 24. etiam ad libitum aliæ rectæ ut *DM*, *GL*, & *HK* *&c.* quarum ſit numerus
(*m*). Sint Anguli *CNR*, *NRT*, *RTQ*, *&c.* atque anguli *SML*,
MLK, *LKQ*, *&c.* invariati, dum puncta angularia *N*, *R*, *T*, *M*, *L*, *K*,
per-

percurrant rectas indefinitas BN, ER, FT, DM, GL, HK; Ducatur concurſus crurum TQ & KQ per rectam indefinitam AQ; Invenire ordinem curvæ quam concurſus cruris SM cum aliqua rectarum CN, NR, RT, TQ, *&c. ex. gr.* cum RT, perpetuo tanget.

In Serie rectarum CN, NR, RT, TQ, *&c.* denotet s numerum rectæ RT, cujus concurſu cum SM Curva eſt deſcribenda, a CN incluſive; qui in hoc caſu eſt ternarius: erit Curva ordinis quem exprimit numerus $sm+s+m+1$; unde in Caſu quem Figura deſignat, cum $s=m=n=3$; erit Curva ordinis 16*tæ*.

In his deſcriptionibus Rectas ſolummodo atque Angulos dari poſtulavimus: Sed facilius ſæpe ſimpliciorum Curvarum ope complexiores deſcribuntur; atque Propoſitiones his non minus Univerſales huic pertinentes inveſtigavi: Eas vero cum harum demonſtrationibus utpote prolixis impræſentiarum omitto; eaſdem poſtea publici juris facturus.

The Curve of quickeſt Deſcent, by Mr. Machin, n. 358. p. 860.

X. 1. *Invenire Curvam quam Corpus deſcendens breviſſimo tempore deſcriberet; urgente Vi Centripeta ad datum punctum tendente, quæ creſcat vel decreſcat juxta quamvis Potentiam diſtantiæ a Centro; dato nempe imo Curvæ puncto & altitudine in principio Caſus.*

Fig. 25.

Sit centrum Virium C, quo centro ad diſtantiam CB æqualem altitudini unde Corpus caſurum eſt, deſcribatur Circulus BEG, & fiat angulus BCG rectus. Ponatur A punctum Curvæ infimum, ubi axi CB occurrit ad datam diſtantiam CA. Oportet invenire punctum Q, ubi Curva celerrimi deſcenſus EQA occurrit circulo QF, ad datam aliam diſtantiam CF. Problema hoc duos habet Caſus, quorum alter pendet ab Hyperbola & Circulo, alter ab Ellipſi & Circulo.

Caſ. 1. Si fuerit Vis centripeta reciproce ut diſtantia a Centro. Sit KLM Hyperbola quævis rectangula centro C & Aſymptoto CB deſcripta, quæ occurrat normalibus BK, AM, ſuper ipſam BC ad puncta B, A erectis, in K & M; ordinatæ vero cuilibet intermediæ FL ad punctum F erectæ, in L. Fiat CD ad CG ut $\surd AFLM$ ad $\surd ABKM$, & ſit DH normalis ſuper CG: dein capiatur Sector RCB ad Aream $HDCB$ ut data Area Hyperbolica $ABKM$ ad datum Rectangulum $CA \times AM$. Tum recta RC occurret circulo FQ in puncto Q, quod quidem eſt ad Curvam celerrimi deſcenſus EQA.

Habebitur autem punctum E, a quo inciperet Corporis caſus, capiendo Sectorem BCE ad Aream Quadrantis BCG, in eadem ratione Areæ Hyperbolicæ $ABKM$ ad rectangulum ſub CA & AM contentum.

Coroll. Hinc ſi recta RC, circa centrum C revoluta, faciat Sectores RCB proportionales Areis $HDCB$, in quibus quadrata Baſium CD ſumuntur in progreſſione Arithmetica: tum rectæ CR interſecabunt Curvam EQA ad diſtantias a centro CQ, quæ decreſcant in progreſſione Geometrica.

Fig. 26.

Caſ. 2. Si vero Vis centripeta fuerit reciproce ut alia quævis Poteſtas diſtantiæ a centro; ſit $n+1$ Index iſtius Poteſtatis (ubi n poteſt eſſe Numerus quilibet integer vel fractus, affirmativus vel negativus) ſitque $H=CB$ altitudo maxima Curvæ quæſitæ EQA, $b=CA$ altitudo minima ejuſdem, & $A=CF$ altitudo alia quævis intermedia.

In recta CG capiatur CD ad CB ut $\sqrt{b^n}$ ad $\sqrt{H^n}$, atque etiam CH ad CD ut $\sqrt{A^n - b^n}$ ad $\sqrt{H^n - b^n}$. Centro C, ſemiaxibus CD, CB, deſcribatur Ellipſis BLD, cui occurrat ordinatim applicata HL in puncto L; & ducatur recta LK, quæ Ellipſin tangat in L, & Axi minori CD producto conveniat in K: dein Tangenti KL parallela ducatur NM, circulum $BEMG$ tangens in M & ipſi CD occurrens in N. Denique capiatur Sector RCB, qui ſit ad Aream $NMBLKN$, inter Circulum & Ellipſin & utriuſque Tangentes rectamque NK comprehenſam, in ratione Numeri binarii ad Numerum n. Tum recta RC interſecabit Circulum FQ in puncto Q, quod erit ad Curvam celerrimi Deſcenſus EQA.

Quod ſi fiat Sector BCE ad aream BDG, inter Ellipſeos & Circuli Quadrantes interceptam, in ratione dicta Binarii ad Numerum n, coeuntibus ſcilicet punctis L, D & M, G; (ob $A^n = H^n$) erit punctum E unde inchoaretur Caſus Corporis breviſſimo tempore deſcendentis ad A, deſcenſuque ſuo Curvam EQA deſcribentis, quam tangit recta CE in E, quamque ad angulos rectos ſecat CB in A.

Harum Conſtructionum Demonſtrationes e Celeberrimi D. *Newtoni Quadraturis*, ejuſdemque *Philoſ. Nat. Principiis* (Prop. **XXXIX.** & ſequentibus aliquibus) petitæ, alia data occaſione oſtendentur. Problema autem eſt alterius generis, Deſcribere Curvas per quas Corpora, de puncto ſummo E, ſeu principio caſus, demiſſa, celerrimo deſcenſu ad inferiora data puncta Q, urgente qualibet Vi centripeta, ferrentur; cujus quidem ſolutio in poteſtate eſt. In præſentia ſufficiat generalem hujuſmodi Curvarum tradidiſſe Ideam, earumque ad Circuli & Hyperbolæ Quadraturas relationes indicaſſe, abſque quibus eaſdem Geometrice conſtruere haud adeo proclive eſt.

The ſame, by Mr. Craig. n. 268. p. 750.

2. *Problema. Invenire Lineam Celerrimi Deſcenſus.*

Sint BC, CD duæ particulæ infinite parvæ in curva quæſita. Jam Curva illa debet eſſe talis ut tranſitus a B ad D poſt caſum ab horizontali AQ fiat in tempore minimo; quærendum itaque eſt in linea RS (ita ad AQ parallela ut differentiæ ordinatarum GC, DE ſint æquales) tale punctum C ut hoc contingat.

Jam velocitas ejus in puncto C eſt $\sqrt{LC}$ & velocitas in puncto D eſt $\sqrt{QD}$; Ergo $\frac{BC}{\sqrt{LC}}$ eſt tempus deſcenſus per BC, & etiam $\frac{CD}{\sqrt{QD}}$ eſt tempus deſcenſus per CD (per *Prop.* liv. *pag.* 158. *Newtoni*) Ergo punctum C debet eſſe tale ut $\frac{BC}{\sqrt{LC}} + \frac{CD}{\sqrt{QD}} = \text{minimo}$. Supponendo B & D eſſe fixa, ſint conſtantes $GC = DE = m$, $LC = b$, $QD = p$; indeterminatæ $BG = u$, $CE = z$; unde $\frac{\sqrt{m^2+u^2}}{\sqrt{b}} + \frac{\sqrt{m^2+z^2}}{\sqrt{p}} = \text{minimo}$; Ergo

Fig. 27.

$$\frac{u\,du}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} + \frac{z\,dz}{p^{\frac{1}{2}}\sqrt{m^2+z^2}} = 0.$$ ſed $du = -dz$ (quia $u + z = \text{conſtanti}$)

Ergo $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} = \frac{z}{p^{\frac{1}{2}}\sqrt{m^2+z^2}}$; unde patet $\frac{u}{b^{\frac{1}{2}}\sqrt{m^2+u^2}} =$ conſtanti; ſit jam Abſciſſa $AL = x$, ordinata $LC = y$; adeoque $BG = dx$, $GC = dy$, $BC = \sqrt{dx^2+dy^2}$ ſitque a linea quævis conſtans. Erit $\frac{dx}{y^{\frac{1}{2}}\sqrt{dx^2+dy^2}} = \frac{1}{\sqrt{a}}$, unde $dx\sqrt{a} = \sqrt{y}\sqrt{dx^2+dy^2}$. Sed in omni Curva dx eſt ad $\sqrt{dx^2+dy^2}$ ut Subtangens ad Tangentem: Ergo talis eſt Natura Curvæ quæſitæ, ut ejus Subtangens ſit ad Tangentem ut $\sqrt{a}$ ad $\sqrt{y}$. Quam utique Cycloidis proprietatem eſſe ſciunt omnes, quibus notum eſt Tangentem Cycloidis eſſe parallelam Chordæ arcus contermini in Circulo genitore, cujus Diameter eſt a, & cujus vertex deorſum ſpectat.

The Solid of leaſt Reſiſtance, by the ſame. ibid.

XI. *Lemma.* Invenire rationem inter reſiſtentiam, quam patitur Triangulum rectangulum AIG, & reſiſtentiam quam patitur rectangulum circumſcriptum $AIGg$, dum utrumque in fluido movetur juxta directionem Lineæ IA, ab I verſus X.

Fig. 28.

A puncto quovis B ducatur BC normalis ad AG; & Bb parallela ad AI, item BM normalis ad AI. Tum in bB capiantur $bH = \frac{CM^2}{BC}$ & $bE = BC$; & per puncta H, E ducantur rectæ HA, EA, quæ productæ ſecent Gg in K & F: Dico Reſiſtentiam Trianguli AIG eſſe ad reſiſtentiam Rectanguli $AIGg$ ut Area trianguli AKG, ad Aream Trianguli AFG. Imo & reſiſtentiam in partem quamlibet lineæ AG ad reſiſtentiam in partem correſpondentem lineæ Ag, *exem. gra.* in AB & Ab ut Area AHB ad Aream AEB. Demonſtratio pendet a Theoremate generali, quod facillime deduxi ex *Prop.* xxxv. *Newtoni*, *p.* 324.

Coroll. 1. Sint jam BG, bg partes infinite parvæ linearum AG, Ag, & producatur bB ad L; dico reſiſtentiam in BG (quam vocemus e) eſſe ad reſiſtentiam in bg (quam vocemus E) ut GL^2 ad GB^2.

Nam $e : E :: KHbg : FEbg$, id eſt $e : E :: bg \times bH : bg \times bE$ (per Lemma præcedens) Ergo $e : E :: bH : bE$, id eſt $e : E :: \frac{CM^2}{BC} : BC$ (per conſtructionem ſuperioris Lemmatis) Ergo $e : E. :: CM^2 : BC^2$. Sed $CM^2 : BC^2 :: GL^2 : GB^2$. (ob ſimilia Triangula BMC, GLB) Ergo $e : E :: GL^2 : GB^2$. Q.E.D.

Corol. 2. Reſiſtentia in partem infinite parvam GB eſt æqualis Cubo lineæ GL diviſo per Quadratum lineæ GB. Nam ſi omnes partes infinite parvæ in linea Ag ut bg ſupponantur æquales, tum Reſiſtentia in bg per ipſam bg exprimi poſſit, id eſt, $E = bg$, adeoque $E = GL$. Ergo per Corollarium primum $e : GL :: GL^2 : GB^2$; unde $e = \frac{GL^3}{GB^2}$ Q.E.D.

Corol. 3. Sit r radius & c circumferentia cujuſvis circuli, dico reſiſtentiam in conicam ſuperficiem genitam a rotatione lineolæ GB circa AI eſſe æqualem producto ex $\frac{c \times BM}{r}$ in $\frac{GL^3}{GB^2}$. Nam reſiſtentia in Conicam illam ſuperficiem eſt æqualis omnibus reſiſtentiis in lineolam GB, id eſt omnibus e; id eſt æqualis circumferentiæ circuli cujus radius eſt BM in e multiplicatæ; id eſt, reſiſtentia in Conicam illam ſuperficiem eſt æqualis $\frac{c \times BM}{r} \times e$; adeoque per *Corol.* 2. æqualis $\frac{c \times BM}{r} \times \frac{GL^3}{GB^2}$ *Q. e. D.*

Problema Invenire Lineam curvam cujus rotatione producatur Solidum rotundum, quod (dum in medio fluido ſecundum axis ſui directionem movetur) minimam patiatur Reſiſtentiam.

Fig. 29.

Sint OG, GB duæ particulæ infinite parvæ in Curva quæſita, quæ circa $A\mathcal{Q}$ rotata producat Solidum rotundum minimæ Reſiſtentiæ. Ducantur BM, GP normales ad $A\mathcal{Q}$, item BL, GN ad $A\mathcal{Q}$, & ON ad BM parallelæ. Jam $\frac{c \times BM \times GL^3}{r \times GB^2}$ eſt reſiſtentia in ſuperficiem genitam a rotatione lineolæ GB circa $A\mathcal{Q}$, & $\frac{c \times GP \times ON^3}{r \times OG^2}$ eſt reſiſtentia in ſuperficiem genitam ſimiliter ab OG per *Cor.* 3. Jam utraque hæc Reſiſtentia ſimul ſumpta debet eſſe minima ſcil. $\frac{c \times BM \times GL^3}{r \times GB^2} + \frac{c \times GP \times ON^3}{r \times OG^2} =$ minimæ. Adeoque in linea RS ita ad $A\mathcal{Q}$ parallela ut ON ſit $= GL$, quærendum eſt punctum G ut hoc contingat; quod ſupponendo puncta O & B eſſe fixa facile invenietur per notiſſimam Maximorum & Minimorum Methodum. Calculum proſequendo devenietur tandem ad $\frac{BM \times BL}{BG^4} = \frac{GP \times NG}{OG^4}$; unde patet $\frac{BM \times BL}{BG^4} =$ conſtanti; ſic ſi abſciſſa AM vocetur x, & ordinata BM, y, erit $BL = dx$, $LG = dy$ (quam conſtantem in toto hoc calculo ſuppoſui) adeoque $BG^2 = dx^2 + dy^2$, unde $\frac{y\,dx}{dx\,dx + dy\,dy^2} =$ conſtanti; Sit a linea quælibet conſtans, & proinde, ut obſervetur *Lex* homogeneorum erit $\frac{y\,dx}{dx\,dx + dy\,dy^2} = \frac{a}{dy^3}$ ut ab Illuſtriſſ. *Hoſpitalio* & celeber. *Jo. Bernouillio* inventum eſt.

The ſame, by Mr. Facio. n. 337. p. 172.

2. In adjuncta Figura ſit C Centrum Circuli Oſculantis AEF, qui cum Sectione Solidi quæſiti cujus Axis ſit ST quam intime coincidat in A. Eritque hujus Circuli Radius CA vel $u = \frac{3psx}{tt} - \frac{px}{s}$: quæ erat Solutio noſtra, in Tractatu hac de re *Londini* impreſſo.

Fig. 30.

Fiat, ut prius, AS ad Solidi Axem TS perpendicularis $= x$; cujus fiat Fluxio invariatæ Magnitudinis $AB = \dot{x}$: Sitque BE ad Axem parallela $= \dot{y}$: Rurſuſque erigatur $EG = \dot{x}$; Et erit GF ad Axem parallela $= \dot{y} + \ddot{y}$.

Erit autem p ad t ut u ſeu $\frac{3psx}{tt} - \frac{px}{s}$ ad $\frac{3sx}{t} - \frac{tx}{s}$ ſive $\frac{3\dot{y}x}{\dot{x}} -$

$-\frac{\dot{x}x}{\dot{y}}$; quod æquabitur ipſi n ſeu AD ad Axem parallelæ, poſita ſcilicet CD ad eundem Axem perpendiculari: quæ CD vocetur m.

Rurſus erit p ad s ut u ſeu $\frac{3psx}{tt} - \frac{px}{s}$ ad $\frac{3ssx}{tt} - x$, ſive $\frac{3\dot{y}\dot{y}x}{\dot{x}\dot{x}} - x$; quod æquabitur ipſi m ſeu CD. Cujus Valorem dediſſe otioſum quidem hic eſt, ſed uſum habebit in ſequentibus.

Jam vero ex Oſculantis Circuli AEF Proprietate habebitur, productis ipſis BA, BE ad alteram uſque Circumferentiæ partem, $\dot{x} \times \overline{2m + \dot{x}} = \dot{y} \times \overline{2n - \dot{y}}$.

Rurſuſque ex ejuſdem Circuli Proprietate habebitur, productis ipſis GE, GF ad alteram uſque Circumferentiæ Partem, $\dot{x} \times \overline{2m + 3\dot{x}} = \overline{\dot{y} + \ddot{y}} \times \overline{2n - 3\dot{y} - \ddot{y}}$.

Ergo, ſubducta priori hac Æquatione a poſteriori, erit $2\dot{x}\dot{x} = -2\dot{y}\dot{y} - \dot{y}\ddot{y} + 2n\ddot{y} - 3\dot{y}\ddot{y} - \ddot{y}\ddot{y}$.

Deletiſque Terminis infinite minoribus quam ſint reliqui, erit $2\dot{x}\dot{x} = -2\dot{y}\dot{y} + 2n\ddot{y}$.

Subſtitutoque ipſius n Valore, erit $\dot{x}\dot{x} = -\dot{y}\dot{y} + \frac{3x\dot{y}\ddot{y}}{\dot{x}} - \frac{x\dot{x}\ddot{y}}{\dot{y}}$: Id eſt $\underset{*}{\dot{x}^3\dot{y}} + \underset{**}{\dot{x}\dot{y}^3} - \underset{**}{3x\dot{y}\dot{y}\ddot{y}} + \underset{*}{x\dot{x}\dot{x}\ddot{y}} = 0$.

Componitur hæc Æquatio ex ſolis Indeterminatis x, $\dot{y}$, earumque Fluxionibus $\dot{x}$, $\ddot{y}$, invariabilique Quantitate $\dot{x}$, Quantitatibuſve Coefficientibus datis. Bina autem ſunt Paria Terminorum in quibus occurrunt eædem utrinque literæ, literarumque Poteſtates, niſi quatenus Quantitas Fluens per Literam unam expreſſa in Fluxionem convertitur, vel Fluxio in Fluentem. Quæ Paria Terminorum ſunt $\dot{x}^3\dot{y} + x\dot{x}\dot{x}\ddot{y}$, & $\dot{x}\dot{y}^3 - 3x\dot{y}\dot{y}\ddot{y}$; ex Terminis utique Generatoribus duobus duntaxat orta. In tota enim Æquatione nihil obſtat quominus ipſa transformetur ſcilicet Multiplicatione facta in $x^{\kappa}\dot{y}^{\lambda}$, determinatis rite ipſis Indicibus κ & λ, ut ea ratione nova Æquatio proveniens tractabilis evadat.

Ergo juxta noſtram iſtarum Transformationum Theoriam, in Generatore ex quo exoritur Terminorum Par primum unico Aſteriſco notatum, erit Numerus Dimenſionum Indeterminatæ x ad Numerum Dimenſionum Indeterminatæ $\dot{y}$, id eſt, Erit $1 + \kappa$ ad $1 + \lambda$, ut Coefficiens 1 in Termino $\dot{x}^3\dot{y}$ ad Coefficientem 1 in Termino $x\dot{x}\dot{x}\ddot{y}$. Rurſus in Generatore, ex quo exoritur Terminorum Par alterum Aſteriſco duplici notatum, Erit Numerus Dimenſionum Indeterminatæ x ad Numerum Dimenſionum Indeterminatæ $\dot{y}$, id eſt, Erit $1 + \kappa$ ad $1 + \lambda$, ut Coefficiens 1 in Termino $\dot{x}\dot{y}^3$ ad Coefficientem -3 in Termino $-3x\dot{y}\dot{y}\ddot{y}$; unde fit $\kappa = -\frac{3}{2}$, & $\lambda = -\frac{3}{2}$; ac proinde Multiplicator $x^{\kappa}\dot{y}^{\lambda} = x^{-\frac{3}{2}} \times \dot{y}^{-\frac{3}{2}}$.

Erit igitur $-x^{-\frac{1}{2}}\dot{x}^2\dot{y}^{-\frac{1}{2}} - x^{-\frac{1}{2}}\dot{y}^{\frac{3}{2}} = \pm q$ Æquatio ſuperioris Æquationis per $x^{-\frac{3}{2}}\dot{y}^{-\frac{3}{2}}$ multiplicatæ Generatrix: Eſt autem q Quantitas determinata. Quam Æquationem Generatricem, (Fluentem autem vocant alii) ſi quadraveris, ut tollantur Radices, proveniet $x^{-1}\dot{x}^4\dot{y}^{-1} + 2x^{-1}\dot{x}^2\dot{y} + x^{-1}\dot{y}^3 = qq$; id eſt $\frac{\dot{x}^4 + 2\dot{x}^2\dot{y}^2 + \dot{y}^4}{x\dot{y}} = qq$. Quæ ipſa eſt Æquatio *Newtoniana*, quam & *Joh. Bernoullius* invenit, & ipſe ego antehac erui, facillima omnium quæ ſperari poſſint ejuſdem Æquationis Inveſtigatione. Determinatur autem Quantitas qq, vel datis Poſitione Axe indefinito *TS*, Puncto *A*, & Solidi Tangente in *A*; vel datis Poſitione Puncto *A*, Centro Oſculantis circuli *C*, & *AD* ad Axem Solidi parallela.

A Problem propos'd in the Leipſick Acts, ſolv'd generally. by ---. n. 347. p. 399.

XII. Problema. *Quæritur Methodus generalis inveniendi Seriem Curvarum, quæ Curvas in ſerie alia quacumque data conſtitutas, ad angulum vel datum vel data lege variabilem ſecabunt.*

Solutio. Natura Curvarum ſecandarum dat Tangentes earundem ad interſectionum puncta quæcumque; & anguli interſectionum dant perpendicula Curvarum ſecantium; & perpendicula duo coeuntia, per concurſum ſuum ultimum, dant centrum Curvaminis Curvæ ſecantis ad punctum interſectionis cujuſcumque. Ducatur Abſciſſa in ſitu quocumque commodo, & ſit ejus Fluxio Unitas; & poſitio perpendiculi dabit Fluxionem primam Ordinatæ ad Curvam quæſitam pertinentis; & Curvamen hujus Curvæ dabit Fluxionem ſecundam ejuſdem Ordinatæ. Et ſic Problema ſemper deducetur ad æquationes. Quod erat faciendum.

Scholium. Non hujus ſed alius eſt methodi æquationes reducere, & indeterminatas ſeparare, abſolute ſi fieri poſſit, ſin minus per Series infinitas.

Another, ſolv'd, by Dr. Taylor. n. 354. p. 696.

Problema ſupra ſcriptum ediderunt Cl. V. *Leibnitius* & *Bernoulli*, quo vires Geometrarum Anglorum experirentur in Methodo Fluxionum, cujus Inventionem ſibi appropriare conatus eſt ipſe *Leibnitius*. Solutum eſt ab Anonymo noſtrate methodo generali ſupra expoſita. Illa vero minime contenti fuerunt *Leibnitius* & Fautores ejus, quin illam deriſui habuere quaſi qui illam excogitaverat non potuiſſet eam ad caſum ſpecialem applicare. *Si non viderint quomodo ex illa æquationes ſunt deducendæ*, dicit Cl. *Taylor*, *id profecto illorum imperitiæ tribuendum erit.* Paulo ante *Leibnitii* obitum prodiit Problema ſequens Problematis illius generalis Caſus particularis.

XIII. Problema. *Super recta* A G *tanquam axe, ex puncto* A *educere infinitas Curvas, qualis eſt* A B D, *ejus naturæ, ut radii Oſculi, in ſingulis punctis* B *& ubique ducti,* B O *ſecentur ab axe* A G *in* C, *in data ratione, ut nempe ſit* B O *ad* B C *ut* 1 *ad* n. [*Fig.* 31.]

Deinde conſtruendæ ſunt Trajectoriæ E B F *primas Curvas* A B D *normaliter ſecantes.*

Inventio Curvarum ſecandarum ABD.

Solutionis pars prima.

1. Ducta ordinata BH ad axem AG normali, ſint, Abſciſſa $AH = z$, Ordinata $HB = x$, Curva $AB = v$. Tum per Methodum Fluxionum directam erit $BC = \frac{\dot{v}}{\dot{z}}x$, & fluente uniformiter v, $BO = \frac{\dot{v}\dot{x}}{\ddot{z}}$. Unde per conditionem Problematis fit $BO\left(\frac{\dot{v}\dot{x}}{\ddot{z}}\right) : BC\left(\frac{\dot{v}}{\dot{z}}x\right) :: 1 : n$; adeoque $\ddot{z}x - n\dot{z}\dot{x} = 0$.

2. Collata hac æquatione cum formula Fluxionum ſecunda, in calce *Prop.* 6. Methodi Incrementorum, invenitur $\dot{z}x^{-n} = \dot{v}a^{-n}$; exiſtente a linea data, per cujus valorem poteſt Curva ABD accommodari conditioni alicui Problemati annexæ.

3. Pro $\dot{v}$ ſcripto ipſius valore $\sqrt{\dot{x}^2 + \dot{z}^2}$, migrat æquatio $\dot{z}x^{-n} = \dot{v}a^{-n}$ in hanc $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$. Unde datur z ex data x, per quadraturam Curvæ cujus abſciſſa exiſtente x eſt ordinata $\frac{x^n}{\sqrt{a^{2n} - x^{2n}}}$.

4. Sint σ & τ numeri integri, vel affirmativi vel negativi, tales ut ſit Curvarum iſto modo provenientium ſimpliciſſima, ea cujus eſt Abſciſſa y, & Ordinata $y^{\frac{1-n+2\sigma n}{2n}} \times \overline{a-y}|^{\tau - \frac{1}{2}}$; tum erit ea omnium Curvarum ſimpliciſſima, per quarum Quadraturam datur Abſciſſa z ex data Ordinata x.

5. Eſt Curva ABD Geometrica, quoties pro n ſumitur reciprocum numeri cujuſvis imparis.

6. In prædictis Curvam ABD conſideravimus ut verſus axem AG concavam, quo in caſu maxima ordinata x æqualis eſt lineæ datæ a, quam Parametrum Curvæ commode vocare licet. Et in hoc caſu Curva actu occurret Axi. Unde fluente ipſius $\frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$ debite ſumpta, hoc eſt, ita ut ſimul evaneſcant z & x, tranſibit Curva per punctum datum A, ſicut poſtulat Problema.

7. Sed ſi quæratur Curva ABD, quæ ſit verſus axem convexa, ad eundem modum pervenietur ad æquationem $\dot{z} = \frac{a^n\dot{x}}{\sqrt{x^{2n} - a^{2n}}}$; quæ etiam ex æquatione priori derivari poteſt mutando ſignum ipſius n. Et in hoc caſu eſt curva ABD Geometrica, quoties pro n ſumitur reciprocum cujuſvis numeri paris. In hoc vero caſu Ordinata omnium minima x æqualis eſt Parametro a; adeoque Curva nuſquam occurrit Axi. Quare limitatur Problema ad caſum priorem.

8. Ex præmiſſis facile colligitur Curvas omnes ABD eſſe inter ſe ſimiles, & circa punctum datum A ſimiliter poſitas, lateribus earum homologis exiſtentibus proportionalibus Parametris a.

Inventio Curvæ ſecantis.

Solutionis pars altera.

9. Ex § 2. fit $\dot{v} : \dot{z} :: a^n : x^n$. Sed eſt $BC : BH :: \dot{v} : \dot{z}$, Unde fit $BC : BH :: a^n : x^n$. Ex conditione vero Problematis eſt BC tangens

Curvæ quæſitæ EBF. Quare ſi jam ſumantur $AH\ (z)$ & $BH\ (x)$ pro coordinatis Curvæ EBF, Curva ipſa EB exiſtente r, erit, per Meth. Flux. direct. $\dot{r} : -\dot{x} :: (BC : BH ::)\ a^n : x^n$. Unde fit $\frac{x^n}{a^n} = \frac{-\dot{x}}{\dot{r}}$.

10. In Curva ABD finge æquationem $\dot{z} = \frac{\dot{x}x^n}{\sqrt{a^{2n} - x^{2n}}}$ transformari in æquationem ſignis radicalibus non affectam $\dot{z} = A\dot{x}\frac{x^n}{a^n} + B\dot{x}\frac{x^{3n}}{a^{3n}} +$ &c. Tum regrediendo ad Fluentes fiet $z = \frac{1}{n+1}A\frac{x^{n+1}}{a^n} + \frac{1}{3n+1}B\frac{x^{3n+1}}{a^{3n}} +$, &c. coefficiente nova introducta nulla, quoniam per conditionem Problematis debent ſimul naſci z & x. Hinc vice $\frac{x^n}{a^2}$ ſubſtituto ipſius valore $\frac{-\dot{x}}{\dot{r}}$ in § 9 invento, fit $z = \frac{1}{n+1}Ax\frac{-\dot{x}}{\dot{r}} + \frac{1}{3n+1}Bx\frac{-\dot{x}^3}{\dot{r}^3} +$ &c. quæ æquatio fluxionalis eſt primi gradus ad Curvam quæſitam EBF. Revocatur autem ad formulam ſimpliciorem in terminis numero finitis, modo ſequenti.

11. Fluat uniformiter r, & exiſtente a quantitate non fluente, ſit $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$. Subſtituto hoc valore ipſius $\frac{-\dot{x}}{\dot{r}}$ in æquatione noviſſime inventa, atque ducta æquatione in $\frac{s}{x}$, transformatur ea in hanc $\frac{zs}{x} = \frac{1}{n+1}A\frac{s^{n+1}}{a^n} + \frac{1}{3n+1}\times B\frac{s^{3n+1}}{a^{3n}} +$ &c. Unde capiendo Fluxiones fit $\frac{\dot{s}zx + s\dot{z}x - sz\dot{x}}{x^2} = A\dot{s}\frac{s^n}{a^n} + B\dot{s}\frac{s^{3n}}{a^{3n}} = \frac{\dot{s}s^n}{\sqrt{a^{2n} - s^{2n}}}$. Quod ultimum conſtat ex Analogia Serierum $A\dot{x}\frac{x^n}{a^n} +$ &c. & $A\dot{s}\frac{s^n}{a^n} +$ &c. Hinc pro s & $\dot{s}$ ſubſtitutis eorum valoribus ex æquatione $\frac{-\dot{x}}{\dot{r}} = \frac{s^n}{a^n}$ collectis, elicitur æquatio $n\dot{x}^2\dot{z}z - \ddot{x}x\dot{z}z - n\dot{x}x\dot{z}^2 - \ddot{x}\dot{x}x^2 = 0$. Quæ ad Fluxiones primas revocatur modo ſequenti.

12. In termino ultimo $-\ddot{x}\dot{x}x^2$ vice $\ddot{x}\dot{x}$ ſcripto ipſius valore $-\ddot{z}\dot{z}$, & æquatione deinde applicata ad $\dot{z}$, fit $n\dot{x}^2z - \ddot{x}xz - nxx\dot{z} + xx\ddot{z} = 0$. Quæ æquatio in x^{-n-1} ducta eſt Fluxio æquationis $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$; exiſtentibus a & $\dot{r}$ non fluentibus. Eſt ergo $-\dot{x}x^{-n}z + x^{1-n}\dot{z} = a^{1-n}\dot{r}$, ſeu $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{x}x^n$, æquatio fluxionalis primi gradus ad Curvam quæſitam EBF.

13. In iſta autem æquatione eſt a valor Ordinatæ BH, quando incidit punctum H in punctum A.

14. Haud proclive eſt æquationem $\overline{\dot{z}x - z\dot{x}} \times a^{n-1} = \dot{r}x^n$, manente n in terminis generalibus, revocare ad æquationem Fluentes tantum in-

volventem, vel ad quadraturam Curvarum. Sed puncta curvæ *EBF* poſſunt commode inveniri per deſcriptionem Curvæ *ABD*, & Curvæ cujuſdam Geometricæ. Per Geometricam hic intelligo Curvam, cujus æquationem non ingrediuntur Fluxiones, nec fluentes in Indicibus dignitatum. Secetur enim Curva *ABD*, cujus Parameter ſit a, in *B*, a Curva geometrica cujus æquatio eſt $a a^n x^n - z a^n x^n = x a^n \sqrt{a^{2n} - x^{2n}}$; atque erit punctum illud interſectionis *B* ad unam ex Trajectoriis quæſitis, nempe quæ tranſit per punctum *E*, exiſtente $AE = a$ & normali ipſi *AG*.

15. Hinc ſi *ABD* ſit Curva Geometrica, erit etiam *EBF* geometrica.

Scholium. Poteſt & alio modo inveniri æquatio $\overline{\dot{z} x - z \dot{x}} \times a^{n-1} = \dot{r} x^n$. Nam certa quadam Analyſi quam nunc celare ſtatuo, inveni æquationem $\frac{\dot{a}}{a} = \frac{\dot{r} r}{\dot{z} z + \dot{x} x}$. Qua comparata cum æquatione $\frac{x^n}{a^n} = \frac{-\dot{x}}{\dot{r}}$ (§ 9.) eliminando a & $\dot{a}$, tandem pervenitur ad prædictam æquationem $\overline{\dot{z} x - z \dot{x}} \times a^{n-1} = \dot{r} x^n$.

Exemplum. Ad demonſtrationem Solutionis noſtræ ſuffecerit exemplum ſimpliciſſimum. Sit itaque $n = 1$; quo in Caſu eſt *ABD* ſemicirculus diametro *AG* deſcriptus, atque eſt *EBF* item ſemicirculus deſcriptus diametro *AE*. Eſt autem in hoc Caſu $\frac{\dot{x} x^n}{\sqrt{a^{2n} - x^{2n}}} = \frac{\dot{x} x}{\sqrt{a^2 - x^2}}$. Unde in § 3. fit $\dot{z} = \frac{\dot{x} x}{\sqrt{a^2 - x^2}}$; adeoque $z = a - \sqrt{a^2 - x^2}$, quæ æquatio eſt ad Circulum diametro $AG = a$ deſcriptum, ut fieri debuit. Item pro n ſcripto 1, æquatio $\overline{\dot{z} x - z \dot{x}} \times a^{n-1} = \dot{r} x^n$ (§ 12.) migrat in hanc $\dot{z} x - z \dot{x} = \dot{r} x$. Unde exterminando $\dot{r}$ ope æquationis $\dot{r} \dot{r} = \dot{x} \dot{x} + \dot{z} \dot{z}$, fit $\frac{2 \dot{z} z x - \dot{x} z^2}{x^2} = -\dot{x}$; adeoque regrediendo ad Fluentes $\frac{zz}{x} = -x + a$, quæ æquatio eſt ad Circulum diametro $AE = a$ deſcriptum, ut etiam fieri debuit.

XIV. *Suppono Materiam omnem diviſibilem eſſe in infinitum, eamque poſſe formam quamcunque ſeu figuram induere, & ad quamcunque tenuitatem ſeu craſſitiem quamcunque exiguam reduci.*

Of the infinite Diviſibility of Matter, by Dr. John Keill, *n.* 339. *p.* 82.

Lemma. Data quavis materiæ quantitate, ex ea, vel ex quavis ejus parte, formari poteſt ſphera concava, cujus ſemidiameter ſit datæ rectæ æqualis.

Sit materiæ particula a^3 & data recta ſit b. Ratio peripheriæ circuli ad Radium ſit p ad r, dicatur ſemidiameter concavitatis x, & craſſities, pelliculæ concavitatem ſphæræ ambientis, erit $b - x$, & Cylindrus ſphæræ circumſcriptus cujus radius eſt b erit $\frac{p \times b^3}{r}$, unde ſphæra cylindro inſcripta erit $\frac{2 \times pb^3}{3r}$, Eadem ratione ſphæra cujus radius eſt x erit $\frac{2 \times px^3}{3r}$ quarum differentia $\frac{2p}{3 \times r} \times b^3 - x^3$ ponenda eſt ſphæricæ lamellæ æqualis, ſeu materiæ particulæ datæ; hoc eſt erit $\frac{2p}{3r} \times b^3 - x^3 = a^3$ ſeu $b^3 - x^3 = \frac{3ra^3}{2p}$ unde $x^3 = b^3 - \frac{3ra^3}{2p}$ & $x = \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$ adeoque craſſities lamellæ ſphæricæ ſeu $b - x$ erit $= b - \sqrt[3]{b^3 - \frac{3ra^3}{2p}}$.

Eadem ratione fieri poſſunt ex data materiæ quantitate Cubi *concavi*, Cylindri *concavi, vel corpora etiam alterius cujuſvis figuræ concavæ, quorum latera ſunt datæ rectæ æqualia.*

Theorema Primum. *Data quavis materiæ quantitate quantumvis exigua, & dato ſpatio quovis finito utcunque amplo; quod* v. gr. *ſit cubus, qui ſphæram Saturni circumſcriberet; Poſſibile eſt ut materia iſtius Arenulæ per totum illud ſpatium diffundatur, atque ipſum ita adimpleat, ut nullus ſit in eo porus cujus diameter datam ſuperet lineam.*

Sit datum ſpatium Cubus cujus latus ſit recta *AB*, diametro ſcil. orbitæ Saturni æqualis, deturque materiæ particula cujus quantitas ſit b^3, & data recta (qua pororum diametri non majores eſſe debent) ſit *d*. Dividi concipiatur recta *AB* in partes æquales rectæ *d*, quarum numerus finitus erit, cum nec recta *AB* ponitur infinite magna, nec recta *d* infinite parva: ſit numerus ille *n*, hoc eſt ſit $nd = AB$, adeoque erit $n^3 d^3$ æqualis cubo rectæ *AB*. Concipiatur item ſpatium datum dividi in cubos quorum ſingulorum latera ſunt æqualia rectæ *d*, eritque cuborum numerus n^3, & hi cubi per ſpatia *efgh* in figura repreſententur. Dividi porro ſupponatur particula b^3 in partes quarum numerus ſit n^3, & in unoquoque ſpatio cubico ponatur una harum particularum, & hac ratione materia b^3 per omne illud ſpatium diffundetur. Poteſt præterea unaquæque ipſius b^3 particula in ſua quaſi cella locata in ſphæram concavam formari, cujus diameter ſit æqualis datæ rectæ *d*; unde fiet, ut ſphæra quælibet proximam quamque tangat, & data materiæ particula utcunque exigua b^3 ſpatium datum ita adimplebit, ut nullus fiet in eo porus cujus diameter datam rectam *d* ſuperat. *Q. E. D.* Fig. 32.

Cor. Hinc dari poteſt corpus, cujus materia ſi in ſpatium abſolute plenum redigatur; ſpatium illud fieri poteſt prioris magnitudinis pars quælibet data.

Theorema Secundum. *Poſſunt eſſe duo corpora mole æqualia, quorum materiæ quantitates ſint utcunque inæquales, & datam quamvis ad ſe invicem obtineant rationem, pororum tamen ſummæ, ſeu ſpatia vacua inter corpora, ad rationem æqualitatis fere accedant.* Vel in ſtilo Carteſiano: *Spatium omne, quod a materia ſubtili intra unius corporis poros occupatur, poſſet eſſe fere æquale ſpatio quod a ſimili materia intra alterum corpus tenetur. Licet materia propria unius corporis decies millies vel centies millies ſuperat materiam propriam alterius Corporis, & Corpora ſint mole æqualia.*

Ex. gr. Sit Digitus cubicus *Auri* & Digitus cubicus *Aeris* vulgaris non condenſati. Certum eſt quantitatem materiæ in *Auro* vicies millies circiter ſuperare materiam aeris, attamen fieri poteſt, ut ſpatia in *auro* vel abſolute vacua, vel materia ſubtili repleta, ſint fere æqualia ſpatiis in aere, vel vacuis, vel materia tantum ſubtili repletis.

Sint *A* & *B* corpora duo, magnitudine æqualia: utrumque *v. gr.* ſit cubus unius digiti. Et corpus *A* decies millies ſit gravius corpore *B*, unde & corpus *A* quantitate materiæ decies millies ſuperabit corpus *B*. Ponamus jam materiæ quantitatem in *A* redigi in ſpatium abſolute plenum, quod ſit digiti cubici pars centies milleſima; (liquet enim ex corollario præcedentis Theorematis id fieri poſſe). Unde cum materia in *A* decies millies ſuperat materiam in *B*, materia illa in *B*, ſi in ſpatium abſolute plenum compingatur, occupabit tantum digiti cubici partem $\frac{1}{1000000000}$ ſeu millies decies centies milleſimam; Adeoque partes reliquæ 999999999 vel erunt abſolute vacuæ, vel materia aliqua ſubtili, qualis ſupponitur Carteſiana, tantum repletæ. Porro, cum materiæ quantitas in *A* impleat tantum digiti partem centies milleſimam, erunt in corpore *A* partes 99999 centies milleſimæ, vel vacuæ, vel materia ſubtili repletæ, hoc eſt reducendo fractionem ad denominatorem prioris fractionis, erunt in *A* partes vacuæ 999990000 millies decies centies milleſimæ. Adeoque vacuitates in *A* erunt ad vacuitates in *B*, ut numerus 999990000 ad numerum 999999999, qui numeri ſunt ad ſe invicem fere in ratione æqualitatis, nam eorum differentia parvam admodum ad ipſos numeros obtinet rationem. Adeoque ſpatia vacua, vel materia ſubtili tantum repleta, quæ ſunt in duobus corporibus *A* & *B*, eandem cum ipſis numeris ad ſe invicem rationem obtinentes, ſunt etiam fere in ratione æqualitatis. *Q. E. D.*

Corpora autem omnia eſſe rariſſima, hoc eſt pro mole ſua parvam admodum continere materiæ quantitatem, ex diaphanorum proprietatibus certiſſime conſtat; nam *Radii Lucis* inter vitrum vel aquam non ſecus ac in aere per rectas lineas diffunduntur; quæcunque luci expoſita ſit corporis Diaphani facies; Adeoque a minima quavis aſſignabili Diaphani parte, ad aliam quamvis ejuſdem partem, ſemper extenditur in his corporibus porus rectilineus, per quem tranſiverit lux, atque hoc fieri non poteſt niſi Materia Diaphani ad ejus molem parvam admodum obtineat rationem, nec fortaſſe materiæ quantitas in vitro ad ejus magnitudinem majorem habet rationem, quam magnitudo unius Arenulæ ad totam Terreni orbis molem: Hoc autem non eſſe impoſſibile, ſuperius oſtenſum eſt. Unde cum Aurum non ſit octuplo denſius Vitro; ejus quoque materia, ad propriam molem, exiguam admodum obtinebit rationem.

Hinc ratio reddi poteſt, cur effluvia magnetica eadem fere facilitate denſum Aurum & tenuem aerem pervadunt.

Ex his etiam propoſitionibus, & ex maxima lucis celeritate, ratio reddi poteſt, cur *Lucis* radii ex pluribus objectis prodeuntes & per tenue foramen tranſmiſſi, ſe mutuo non impediunt, ſed per eandem rectam in motu ſuo perſeverant. Quod per motum ſeu impulſum fluidi, plenum efficientis, vix explicari poteſt; *corpus enim omne a pluribus potentiis, ſecundum diverſas directiones, ſimul impulſum, unam tantum & determinatam directionem accipit ex omnibus compoſitam.*

To find the Center of Oſcillation, by Dr. Taylor. *n.* 337. *p.* 11.

XV. Definitio. *Eſt Centrum Oſcillationis punctum quoddam in corpore pendulo, cujus vibrationes ſingulæ eodem modo atque eodem tempore peraguntur, ac ſi illud ſolum ad eandem diſtantiam a puncto ſuſpenſionis filo ſuſpenderetur.*

Per ſe vix ſatis manifeſtum eſt in corpore aliquo dari hujuſmodi punctum: utpote cujus acceleratio debeat, (*per hanc def.*) in omnibus inclinationibus corporis penduli ad Horizontem, perinde eſſe, ac ſi a propria tantum gravitate urgeatur; reliquis particulis totius corporis ejus motum proprium haud perturbantibus. Itaque in ordine ad inventionem hujus Centri, præmittenda eſt una atque altera propoſitio, unde conſtet tale punctum dari.

Prop. 1. Prob. 1. *In corporis Oſcillantis data quavis inclinatione ad Horizontem, invenire punctum cujus acceleratio perinde ſit, ac ſi ab ipſius propria tantum gravitate urgeatur.*

Fig. 33.

Sit ABD corporis propoſiti ſectio in plano ad Horizontem perpendiculari, in quo movetur centrum gravitatis G, centro ſuſpenſionis exiſtente C. Diſtinguatur corpus in elementa priſmatica plano ABD perpendicularia, adeoque Horizonti ſemper parallela; ut patebit ex motu centri gravitatis G in plano illo ABD. Atque ob hujuſmodi ſitum, tale elementum quodvis ſpectari poteſt tanquam punctum Phyſicum p in plano eodem ABD ad punctum z locatum. Reducatur itaque corpus propoſitum in planum Phyſicum ABD conſtans ex hujuſmodi particulis p.

In hoc plano ut inveniatur punctum O, cujus acceleratio propria non mutatur ab actionibus particularum reliquarum, attendendum eſt ad vires particulæ cujuſvis ſingularis p in puncto z ſitæ. Nam ex hiſce viribus conjunctis oritur plani totius motus abſolutus; cujus ope datur motus puncti cujuſvis propoſiti; unde viciſſim invenitur punctum cujus motus eſt datus.

At urgetur particula p a vi propriæ gravitatis; quæ ſi partium cohæſio diſſolveretur, in dato tempore minimo, datam produceret accelerationem motus in perpendiculari ad Horizontem zy. Ad Cz duc normalem yx, & reſolvetur acceleratio zy in partes zx & xy. Ob corporis rigiditatem, tollitur vis zx per reſiſtentiam puncti C. At vi reliqua xy trahitur ſpatium ABD in gyrum circa punctum C; & ducta horizontali Co & perpendiculari zs, erit ea ut $\frac{Cs}{Cz}$: Nempe ob gravitatis vim datam, & ſimilia triangula xyz & sCz. Ergo vis particulæ p ad movendum ſpatium ABD eſt ut $\frac{Cs}{Cz} \times p$.

Ad has vires in unum colligendas, ſit O punctum invariabile, in linea ad libitum ducta & ad diſtantiam adhuc incognitam CO. Tum erit vis particulæ p ad movendum punctum O, ut $\frac{Cz}{CO} \times \frac{Cs}{Cz} \times p$, hoc eſt ut $\frac{Cs}{CO} \times p$. Acceleratio autem, quam tribuit p eidem puncto O, erit ut $\frac{CO}{Cz} \times \frac{Cs}{Cz}$. Itaque applicata vi illa $\frac{Cs}{CO} \times p$ ad hanc accelerationem $\frac{CO \times Cs}{Czq:}$, erit quotiens $\frac{Czq:}{COq:} \times p$ particula, quæ, ſi in ipſo puncto O ſingatur moveri cum eadem acceleratione $\frac{CO \times Cs}{Czq:}$, eundem omnino produceret motum, quem in eodem puncto O producit particula p. Hinc demum reducitur Problema ad motuum Theorema notiſſimum: Applicata enim ſumma virium $\frac{Cs}{CO} \times p$ ad ſummam particularum $\frac{Czq:}{COq:} \times p$, erit quotiens acceleratio abſoluta puncti O. Dein ducta perpendiculari O o, & poſita hac acceleratione æquali datæ accelerationi $\frac{Co}{CO}$ ipſius puncti O, dabitur diſtantia CO. Sit enim $\frac{Co}{CO} = d$, & (juxta methodum *Fluxionum*) $Cs \times p = \dot{M}$, & $Czq: \times p = \dot{C}$. Tum ob CO invariabilem erit ſumma omnium virium $\frac{Cs}{CO} \times p = \frac{M}{CO}$, & ſumma omnium particularum $\frac{Czq:}{COq:} \times p = \frac{C}{COq:}$. Unde, applicata ſumma momentorum ad ſummam corporum, erit $\frac{M}{C} \times CO = d$ adeoque $CO = \frac{dC}{M}$. Inventis igitur C & M, per *Fluxionum* methodum inverſam, dabitur CO. *Q. E. I.*

Cor. A Centro gravitatis G ad horizontalem Co duc perpendicularem Gg, & ſit corpus ipſum $ABC = A$. Tum ex notiſſima indole centri gravitatis erit $M = Cg \times A$. Unde eſt $CO = \frac{dC}{Cg \times A}$.

Prop. 2. Theor. 1. *Iiſdem poſitis, quæratur punctum* O *in recta* CG *tranſeunte per centrum gravitatis* G. *Tum erit* O *centrum Oſcillationis corporis* A.

Etenim in hoc caſu fit $\frac{Co}{CO} = \frac{Cg}{CG} = d$; Unde $CO = \left(\frac{dC}{Cg \times A}\right.$, *per Cor. Prop.* 1.$\left.\right) \frac{C}{CG \times A}$. At datur A, & dato puncto C, dantur CG & quantitas C. Unde datur CO, qualiſcunque ſit corporis Oſcillantis in- *Fig.* 34.

clinatio ad Horizontem. Ideoque *per Def. & Prob.* 1. eſt O centrum Oſcillationis corporis *A*. *Q. E. D.*

Prop. 3. Theor. 2. *Iiſdem poſitis, ſit* D *aggregatum omnium* $Gz^2 \times p$. *Tum erit* $CO = CG + \frac{D}{CG \times A}$.

Ad *CG* duc normalem *zF*, atque erit $Czq: = CGq: + Gzq: -$ $2CG \times GF$. nempe cadente *F* intra *C* & *G*. At ubi *F* cadit in *CG* producta, erit $Czq: = CGq: + Gzq: + 2CG \times Gf$: Eſt ergo $C =$ (aggregato omnium $Czq: \times p =$) aggregato omnium $CGq: \times p + Gzq: \times p$ $- 2CG \times GF \times p + 2CG \times Gf \times p$. At ob centrum gravitatis *G*, eſt aggregatum omnium $2CG \times GF \times p =$ aggregato omnium $2CG \times Gf \times p$. Quare eſt $C =$ aggregato omnium $CGq: \times p + Gzq: \times p = CGq: \times$ $A + D$. At enim *per Theor.* 1. eſt $CO = \frac{C}{CG \times A}$. Ergo $CO = CG +$ $\frac{D}{CG \times A}$. *Q. E. D.*

Cor. Hinc datur parallelogrammum $CG \times GO$. Eſt enim $GO = \frac{D}{CG \times A}$. At dantur *A* & *D*. Quare datur $CG \times GO = \frac{D}{A}$.

Prop. 4. Theor. 3. *Iiſdem poſitis, ſi in puncto* O *conſtituatur particula phyſica* $\frac{CG \times A}{CO}$, *quæ propria gravitate agitata Oſcillet circa punctum* C, *ſpatii* ABC *motus perinde omnino erit, ac ſi agitaretur ab Oſcillatione ipſius corporis* A.

Conſtat tam ex Natura centri gravitatis, quam *per Prob.* 1. Eſt enim $\frac{CG \times A}{CO}$ aggregatum omnium $\frac{Czq: \times p}{COq:} = \frac{C}{COq:}$.

Prop. 5. Prob. 2. *Datis corporis cujuſvis magnitudine* A, *centro gravitatis* G, *& puncto ſuſpenſionis* C: *Invenire ejuſdem centrum Oſcillationis* O.

Fit per *Theor.* 1. inveniendo quantitatem *C*; vel per *Theor.* 2. quærendo quantitatem *D*.

Scholium. Ad inſtituendum calculum in caſu particulari, eligenda eſt quantitas *C* vel *D*, prout ſuggerit natura figuræ propoſitæ. Dein data earum alterutra, altera item dabitur per æquationem (*Prop.* 3.) $C = CGq:$ $\times A + D$. Unde etiam dabitur pgr. $CG \times GO = \frac{D}{A}$ (*Cor. Prop.* 3.) $= \frac{C}{A} - CGq:$. Cujus ope, ex datis centro gravitatis & puncto ſuſpenſionis, datur centrum Oſcillationis per ſolam diviſionem. Quare in quolibet exemplo ſemper commodiſſimum erit hoc parallelogrammum primum eruere, vel per computum ipſius *D*, vel per quantitatem *C*, ex idonea aſſumptione centri ſuſpenſionis.

Fig. 35.

Supereſt, ut hæc exemplis aliquot illuſtremus.

Ex. 1. Sit figura propoſita Pyramis *ADC*, cujus baſis eſt pgr. *AD*, ſitque motus centri gravitatis in plano tranſeunte per verticem *C* & diametrum baſis *EF* lateri *AB* parallelam. Fig. 36.

Ad calculum commodiſſime inſtituendum, ſit ipſe vertex *C* centrum ſuſpenſionis. Tum ad modum *Prob.* 1. reducatur figura ad planum phyſicum trianguli Iſoſcelis *CEF*, in quo *ef* parallela ipſi *EF* repræſentat lineam phyſicam ex particulis *p* compoſitam. Sit $CH = a$, $HF = b$, & $Cb = x$. Tum ex natura figuræ erit $eb = \frac{bx}{a}$, & particula *p* ſita ad punctum *z* erit ut *x*; vel potius, facto $bz = v$, erit $\dot{v}\,\dot{x}$ elementi priſmatici baſis, & *p* erit ut $\dot{v}\,\dot{x}\,x$. Unde erit $\dot{C} = Czq : \times \dot{v}\,\dot{x}\,x = \dot{v}\,\dot{x}\,x^3 + \dot{x}\,\dot{v}\,v^2\,x$. Ideoque ſumma omnium $Czq : \times p$ in linea *bz* erit $v\,\dot{x}\,x^3 + \frac{\dot{x}\,x\,v^3}{3}$; & in linea *ef* (pro *v* ponendo $\frac{bx}{a}$) erit ſumma illa $\frac{6ba^2 + 2b^3}{3a^3} \times \dot{x}\,x^4$. Unde iterum capiendo fluentem, & pro *x* ſcribendo *a*, erit $C = \frac{6ba^2 + 2b^3}{15} \times a^2$. Eſt autem pyramis ipſa $A = \frac{2baa}{3}$, & diſtantia centri gravitatis *G* a vertice *C* eſt $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq : = \frac{D}{A} = CG \times GO = \frac{3a^2 + 16b^2}{80}$. Fig. 37.

Ex. 2. Sit figura propoſita Conus rectus deſcriptus rotatione trianguli iſoſcelis *ECF* circa perpendiculum *CH*.

Hic iterum ſumpto vertice *C* pro centro ſuſpenſionis, & factis $CH = a$, $HE = b$, $Cb = x$, $bz = v$, ut ſupra; erit $p = 2\,\dot{x}\,\dot{v} \times \sqrt{\frac{bb}{aa}xx - vv}$; unde $\dot{C} = 2\,\dot{v}\,\dot{x} \times \overline{xx + vv} \times \sqrt{\frac{bb}{aa}xx - vv}$. Sit *B* ſegmentum circuli diametro *ef* deſcripti, quod adjacet Abſciſſæ $bz = v$, & Ordinatæ $\sqrt{\frac{bb}{aa}xx - vv}$; tum erit ſumma omnium $Czq : \times p$ in recta $bz = 2\,\dot{x} \times \frac{4a^2 + b^2}{4a^2}x^2 B - \frac{1}{2}\dot{x}\,v \times \overline{\frac{b^2}{a^2}x^2 - v^2}\Big|^{\frac{3}{2}}$. Et quando $v = eb$, erit hæc ſumma $2\,\dot{x} \times \frac{4a^2 + b^2}{4a^2}x^2 B$; cujus duplum $\frac{4a^2 + b^2}{a^2}\dot{x}\,x^2 B$ eſt pars ipſius *C* in recta *ef*. Eſt autem area *B* ut x^2; ſit ergo $B = cx^2$; atque pars illa ipſius *C* erit $\frac{4a^2 + b^2}{a^2} \times c\,\dot{x}\,x^4$. Unde capiendo fluentem erit $C = \frac{4a^2 + b^2}{5} \times ca^3$. Eſt autem conus ipſe $A = \frac{4}{3}ca^3$, & $CG = \frac{3}{4}a$. Unde $\frac{C}{A} - CGq : = \frac{D}{A} = \frac{3a^2 + 12b^2}{80}$.

Atque ad hunc modum procedit calculus in aliis figuris, ubi rationes Cb ad be, & bz ad p ſunt magis compoſitæ.

Ex. 3. Ut pateat ratio calculi quantitatis D, ſit figura propoſita paral- Fig. 38. lelepipedon, cujus facies Horizonti perpendicularis, & parallela plano motus centri gravitatis eſt ABD. Duc diametros EF & HI, & ſit altitudo elementorum p, & ſit tr parallela HI; & $GF = a$, $GH = b$, $Gs = x$, & $sz = v$. Tum erit $\dot{D} = \dot{v}\dot{x}xx + \dot{x}\dot{v}vv$. Unde ipſius D pars in recta tr erit $2b\dot{x}x^2 + 2b^3\dot{x}$: atque iterum ſumendo fluentis duplum, erit $D = \frac{4ba^3 + 4b^3a}{3}$. Atqui eſt $A = 4ab$; unde eſt $\frac{D}{A} = \frac{aa + bb}{3}$ $= \frac{1}{12}$ DB quad.

Ex. 4 Sit ultimum exemplum in Sphæra, cujus circulus maximus Fig. 39. Btr, diameter AB, & centrum G. Tum ductis lineis ut in Schemate ſatis patent, erit $\dot{D} = Gsq : \times p + Gmq : \times p$. At ſumma omnium $Gsq : \times p$ in recta tr eſt Gsq: ductum in aream circuli diametro tr deſcripti. Item ſumma omnium $GMq : \times p$ in recta ki eſt $Gmq : \times$ aream circuli diametro ki deſcripti. Unde ſtatim conſtat eſſe $D =$ quater fluenti ipſius Gsq: in aream circuli cujus diameter eſt tr. Sit ergo c area circuli cujus radii quadratum eſt 1, & ſit $GA = a$, & $Gs = x$. Tum erit $\dot{D} = 4\dot{x}xx \times caa - cxx = 4ca^2\dot{x}x^2 - 4c\dot{x}x^4$. Unde ſumendo fluentem, & faciendo $x = a$, erit $D = \frac{8}{15}ca^5$. Eſt autem $A = \frac{4}{3}ca^3$. Unde $\frac{D}{A} = \frac{2}{5}aa$.

Ob affinitatem ſolutionis libet his ſubjungere Problema de inventione Centri Percuſſionis.

Prop. 6. Prob. 3. *Corporis cujuſvis circa datum punctum rotati, invenire Centrum Percuſſionis; punctum ſcilicet tale, ut Corpus in illud impingens, & eadem opera ſolutum a puncto ſuſpenſionis, neque huc neque illuc inclinet.*

Primum conſtat hoc punctum quæri debere in plano motus centri gra- Fig. 40. vitatis. Si enim corpus reſolvatur in elementa priſmatica plano iſti normalia, ferentur ea motu ſibi parallelo; unde momenta ex utraque parte iſtius plani erunt æqualia; adeoque per reſiſtentiam factam in hoc plano, corporis punctum nullum de eo pelletur. Sit ergo planum illud AB, ad quod reducetur corpus per contractionem elementorum priſmaticorum in particulas p ad puncta z ſitas, ut in *Prob.* 1. In hoc plano ſit C centrum rotationis; aut ſaltem ejus projectio facta per lineam perpendicularem in hoc planum demiſſam; & ſit $\mathcal{Q}$ punctum quæſitum. Per C duc ad libitum $C\xi$, in qua ſume puncta duo z, & ξ, ita ut ductis $z\mathcal{Q}$, & $\xi\mathcal{Q}$, ſit angulus $Cz\mathcal{Q}$ obtuſus, & angulus $C\xi\mathcal{Q}$ acutus: atque in punctis z & ξ ſint particulæ p & π. Tum ad $C\xi$ ductis normalibus zr & ξr, quæ ſint ad invicem ut Cz ad $C\xi$, iis repræſentabuntur velocitates abſolutæ particularum p & π. At harum velocitatum partes quæ ſunt in directionibus $z\mathcal{Q}$ & $\xi\mathcal{Q}$, tolluntur per reſiſtentiam puncti $\mathcal{Q}$. Ad $\mathcal{Q}z$ & $\mathcal{Q}\xi$ duc normales CD & Cd; & ob angulos æquales $zCD = rz\mathcal{Q}$, & $\xi Cd = r\xi\mathcal{Q}$, velocitatum partes reliquæ, in directionibus ipſis $\mathcal{Q}z$ & $\mathcal{Q}\xi$ perpendicularibus, erunt ut zD & ξd. Unde habita ratione diſtantiarum $\mathcal{Q}z$ & $\mathcal{Q}\xi$, erunt vires particularum p & π ad movendum ſpatium AB in partes contrarias, ut $Dz \times z\mathcal{Q} \times p$, & $d\xi \times \xi\mathcal{Q} \times p$. At per conditiones Problematis debent ſummæ hujuſmodi contrariarum virium eſſe inter ſe æquales.

Ob angulos ad D & d rectos, ſunt puncta D & d ad circumferentiam circuli diametro $C\mathcal{Q}$ deſcripti. Sit iſtius circuli centrum E. Tum ductis Ez & $E\xi$ circulo occurrentibus in F & I, f & i, erit $Dz \times z\mathcal{Q} = Fz \times zI = EFq: - Ezq: = E\mathcal{Q}q: - Ezq:$, & $d\xi \times \xi\mathcal{Q} = E\xi q: - E\mathcal{Q}q:$ Quare erit ſumma omnium $E\mathcal{Q}q: \times p - Ezq: \times p =$ ſummæ omnium $E\xi q: \times \pi - E\mathcal{Q}q: \times \pi$; & terminis tranſpoſitis, ſumma omnium $E\mathcal{Q}q: \times \overline{p + \pi}: =$ ſummæ omnium $Ezq: \times p + E\xi q: \times \pi$, hoc eſt, ſi p ponatur tam pro particula p intra circulum, quam pro particula π extra circulum, erit ſumma omnium $E\mathcal{Q}q: \times p =$ ſummæ omnium $Ezq: \times p$. Ad $C\mathcal{Q}$ duc normalem zs. Tum erit $Ezq: = Czq: + ECq: - \mathcal{Q}C \times Cs$. Quo valore ipſius $Ezq:$ ei ſubſtituto, & æquatione debite tractata, tandem invenies ſummam omnium $C\mathcal{Q} \times Cs \times p =$ ſummæ omnium $Czq: \times p$. Unde eſt $C\mathcal{Q} = \frac{\text{ſummæ omnium } Czq: \times p}{\text{ſumm: omnium } Cs \times p}$. At enim eſt ſumma omnium $Czq: \times p$ ipſa quantitas C in calculo centri Oſcillationis: & ſi centrum gravitatis ſit G, & ad $C\mathcal{Q}$ ducatur normalis Gg, & corpus ipſum dicatur A, erit ſumma omnium $Cs \times p = Cg \times A$. Unde eſt $C\mathcal{Q} = \frac{C}{Cg \times A}$. Sit centrum Oſcillationis O; tum per *Theor.* 1. erit $CO = \frac{C}{CG \times A}$. Unde eſt $Cg : CG :: CO : C\mathcal{Q}$. Quare per O ducta ad CO perpendicularis tranſibit per punctum $\mathcal{Q}$. *$\mathcal{Q}$.E.I.*

XVI. Lemma I. *Sint* ADFB, *&* $A\Delta\Phi B$ *Curvæ duæ, quarum relatio inter ſe hæc eſt, ut, ductis ad libitum ordinatis* $C\Delta D$, $E\Phi F$, *ſit* $C\Delta : CD :: E\Phi : EF$. *Tum ordinatis in infinitum imminutis, adeo ut coincidant Curvæ cum axe* AB; *dico quod ſit ultima ratio curvaturæ in* Δ *ad curvaturam in* D, *ut* $C\Delta$ *ad* CD.

Of the Motion of a ſtretcht String, by the ſame. n. 337. *p.* 26.

Fig. 41.

Demonſtratio. Duc ordinatam $c\delta d$ ipſi CD proximam; & ad D & Δ duc tangentes Dt & $\Delta\theta$, ordinatæ cd occurrentes in t & θ. Tum ob $c\delta : cd :: C\Delta : CD$ (per Hypotheſin) tangentes productæ ſibi invicem & axi occurrent in eodem puncto P. Unde ob triangula ſimilia CDP & ctP, $C\Delta P$ & $c\theta P$, erit $c\theta : ct :: C\Delta : CD$ ($:: c\delta : cd$, per Hyp.) $:: \delta\theta$ ($= c\theta - c\delta$) ad dt ($= ct - cd$.) Atqui ſunt curvaturæ in Δ & D, ut anguli contactus $\theta\Delta\delta$ & tDd; & ob $\delta\Delta$ & dD coincidentes cum cC, anguli iſti ſunt ut eorum ſubtenſæ $\delta\theta$ & dt, hoc eſt (per analogiam ſupra inventam) ut $C\Delta$ & CD. Quare, *&c.* *$\mathcal{Q}$.E.D.*

Lemma

Fig. 42. Lemma 2. *In aliquo articulo vibrationis ſuæ induat Nervus tenſus, inter puncta A & B, formam curvæ cujuſvis $A p \pi B$. Tum dico quod ſit incrementum velocitatis puncti alicujus P, ſeu acceleratio oriunda a vi tenſionis Nervi, ut curvatura Nervi in eodem puncto.*

Demonſtratio. Finge Nervum conſtare ex particulis rigidis æqualibus infinite parvis $p P$ & $P\pi$, *&c.* & ad punctum P erige perpendicularem PR = radio curvaturæ in P, cui occurrant tangentes $p t$ & πt in t, iis parallelæ πs & $p s$ in s, & chorda $p \pi$ in c. Tum, per Principia Mechanicæ, vis abſoluta, qua urgentur particulæ ambæ $p P$ & $P \pi$ verſus R, erit ad vim tenſionis fili, ut $s t$ ad $p t$; & hujus vis dimidium, quo urgetur particula una $p P$, erit ad Nervi tenſionem, ut $c t$ ad $t p$, hoc eſt, (ob triangula ſimilia $c t p$, $t p R$) ut $t p$ vel $P p$ ad $R t$ vel PR. Quare, ob tenſionis vim datam, erit vis acceleratrix abſoluta ut $\frac{Pp}{PR}$. Sed eſt acceleratio genita in ratione compoſita ex rationibus vis abſolutæ directe & materiæ movendæ inverſe; atque eſt materia movenda ipſa particula Pp. Quare eſt acceleratio ut $\frac{1}{PR}$, hoc eſt ut Curvatura in P. Eſt enim Curvatura reciproce ut radius circuli oſculatorii. *Q. E. D.*

Prob. 1. *Definire motum Nervi tenſi.*

In hoc Problemate & ſequentibus pono Nervum moveri per ſpatium minimum ab axe motus; ut incrementum tenſionis ex aucta longitudine, item obliquitas radiorum curvaturæ poſſint tuto negligi.

Fig. 43. Itaque extendatur Nervus inter puncta A & B; & plectro deducatur punctum z ad diſtantiam Cz ab axe AB. Tum amoto plectro, ob flexuram in puncto ſolo C, illud primum incipiet moveri (*per Lemma* 2.) At ſtatim inflexo Nervo in punctis proximis Φ & d, incipient hæc puncta etiam moveri; & deinde E & e, & ſic deinceps. Item ob magnam flexuram in C, illud punctum primo velociſſime movebitur; & exinde aucta curvatura in punctis proximis D, E, *&c.* ea continuo velocius accelerabuntur; &, eadem opera, imminuta curvatura in C, id punctum viciſſim tardius accelerabitur. Et univerſaliter, punctis juſto tardioribus magis & velocioribus minus acceleratis, tandem fiet ut viribus inter ſe rite temperatis, motus omnes conſpirent, punctis omnibus ad axem ſimul euntibus & ſimul redeuntibus, vicibus alternis ad infinitum.

Sed ut hoc fiat debet Nervus ſemper induere formam curvæ $ACDEB$, cujus curvatura in quovis puncto E eſt ut ejuſdem diſtantia ab axe $E\eta$; velocitatibus etiam punctorum C, D, E, *&c.* conſtitutis inter ſe in ratione diſtantiarum ab axe Cz, $D\vartheta$, $E\eta$, *&c.* Etenim in hoc caſu, ſpatia Cx, $D\delta$, $E\varepsilon$, *&c.* eodem tempore minimo percurſa, erunt inter ſe ut velocitates, hoc eſt ut ſpatia percurrenda Cz, $D\vartheta$, *&c.* Unde erunt ſpatia reſidua xz, $\delta\vartheta$, $\varepsilon\eta$, *&c.* inter ſe in eadem ratione. Item (*per Lemma* 2.) erunt accelerationes inter ſe in eadem ratione. Quo pacto, ſemper ſervata ratione velocitatum inter ſe eadem ac ſpatiorum percurrendorum, puncta omnia ſimul pervenient ad axem & ſimul redibunt: adeoque recte definitur curva $ACDEB$. *Q. E. D.*

Præterea, comparatis inter ſe duabus curvis $ACDEB$, & $Ax\delta\varepsilon B$, *per Lemma* 1. erunt curvaturæ in D & δ, ut diſtantiæ ab axe $D\vartheta$ & $\delta\vartheta$: adeoque *per Lemma* 2. acceleratio dati cujuſvis puncti in Nervo erit ut ejuſdem diſtantia ab axe. Unde (*per Phil. Nat. Princip. Math. Sect.* X. *Prop.* 51.) vibrationes omnes, tam maximæ quam minimæ, peragentur in eodem tempore periodico, & puncti cujuſvis motus ſimilis erit oſcillationi corporis Funipenduli in Cycloide. *Q. E. I.*

Cor. Sunt Curvaturæ reciproce ut radii circulorum oſculantium. Sit ergo a linea data, atque erit radius curvaturæ in $E = \frac{aa}{E\text{u}}$.

Prob. 2. *Datis longitudine & pondere Nervi, una cum pondere tendente; invenire tempus unius vibrationis.*

Fig. 44. Extendatur nervus inter puncta A & B per vim ponderis P, & ſit nervi ipſius pondus N, & longitudo L. Item conſtituatur nervus in poſitione $AFpCB$, & ad punctum medium C erige normalem $CS =$ radio curvaturæ in C, & occurrentem axi AB in D; & ſumpto puncto p ipſi C proximo, duc normalem pc & tangentem pt.

Ergo, ut in *Lemmate* 2, conſtat vim abſolutam qua acceleratur particula pC, eſſe ad vim ponderis P, ut ct ad pt, *i. e.* ut pC ad CS. Sed eſt pondus P ad pondus ipſius particulæ pC, in ratione compoſita ex rationibus P ad N, & N ad pondus particulæ pC, vel L ad pC, hoc eſt, ut $P \times L$ ad $N \times pC$. Quare compoſitis his rationibus, eſt vis acceleratrix ad vim gravitatis ut $P \times L$ ad $N \times CS$. Conſtituatur itaque pendulum longitudine CD: tum (per *Princip. Math. Sect. X. Prob.* 52.) erit tempus periodicum Nervi ad tempus periodicum iſtius penduli, ut $\sqrt{N} \times CS$ ad $\sqrt{P} \times L$. At (per eandem Propoſit.) data vi gravitatis longitudines pendulorum ſunt in duplicata ratione temporum periodicorum; unde erit $\frac{N \times CS \times CD}{P\ L}$, vel (pro CS ſcripto $\frac{aa}{CD}$, *per Cor. Prob.* 1.) $\frac{N \times aa}{P \times L}$ longitudo penduli cujus vibrationes ſunt iſochronæ vibrationibus Nervi.

Ad inveniendam lineam a, ſit Curvæ abſciſſa $AE = z$, & ordinata $EF = x$, & ipſa Curva $AF = v$, & $CD = b$. Tum (*per Cor. Prob.* 1.) erit radius curvaturæ in $F = \frac{aa}{x}$. At dato $\dot v$ eſt radius curvaturæ $\frac{\dot v\ \dot x}{\ddot z}$. Unde $\frac{aa}{x} = \frac{\dot v\ \dot x}{\ddot z}$; adeoque $aa\ddot z = \dot v x \dot x$: & ſumptis fluentibus $aa\dot z = \frac{\dot v xx}{2} - \frac{\dot v bb}{2} + \dot v aa$ (ubi additur data quantitas $-\frac{\dot v bb}{2} + \dot v aa$, ut fiat $\dot z = \dot v$ in puncto medio C.) Et hinc peracto calculo erit $\dot z = \frac{a^2\dot x - \frac{1}{2}b^2\dot x + \frac{1}{2}x^2\dot x}{\sqrt{a^2b^2 - a^2x^2 - \frac{1}{4}x^4 - \frac{1}{4}b^4 + \frac{1}{2}b^2x^2}}$ Evaneſcant jam b & x reſpectu a, ut coincidat curva cum axe, & fiet $\dot z = \frac{a\dot x}{\sqrt{bb - xx}}$. At centro C

&

Fig. 45.

& radio $CD = b$ deſcripto quadrante circulari DPE, & facto $CQ = x$, & erecta normali QP, atque arcu DP exiſtente y, erit $\dot{y} = \surd\,\frac{b\dot{x}}{bb-xx} = \frac{b}{a}\dot{z}$.

Unde $y = \frac{b}{a}z$, & $z = \frac{a}{b}y$. Et facto $x = b = CD$, (quo caſu etiam fit $y =$ arcui quadrantali DPE, & $z = AD = \frac{1}{2}L$) erit $\frac{1}{2}L = a \times \frac{DE}{CD}$, atque $a = L \times \frac{CD}{2DE}$. Sit ergo CD ad $2DE$ (ut diameter circuli ad circumferentiam) ut d ad c; atque erit $aa = LL \times \frac{dd}{cc}$. Subſtituto itaque hoc valore pro aa, erit $\frac{N}{P} \times L \times \frac{dd}{cc}$ longitudo penduli iſochroni ipſi Nervo. Sit ergo D longitudo cujus tempus periodicum eſt 1, atque erit $\frac{d}{c}\sqrt{\frac{N}{P} \times \frac{L}{D}}$ tempus periodicum Nervi. *Q. E. I.* Sunt enim pendulorum tempora periodica in dimidiata ratione longitudinum.

Cor. 1. Numerus vibrationum Nervi in tempore unius vibrationis penduli D eſt $\frac{c}{d} \times \sqrt{\frac{P}{N} \times \frac{D}{L}}$.

Cor. 2. Ob datum $\frac{d}{c} \times \sqrt{\frac{1}{D}}$, tempus periodicum Nervi eſt ut $\sqrt{\frac{N}{P} \times L}$. Et dato pondere P eſt tempus ut $\sqrt{N \times L}$. Item conſtitutis Nervis ex eodem filo, quo caſu fit N ut L, eſt tempus ut L.

The Proportion of Mathematical Points to each other, by Fra. Robarts, *Eſq. n.* 334. *p.* 470.

* *n.* 195. *p.* 556. Abr. *Vol.* I. *p.* 10[illegible].

XVII. It has heretofore paſs'd for a current Maxim, that all Infinites are equal. Divines and Metaphyſicians have not ſcrupled to ground many of their Arguments on that Foundation. The Poſition nevertheleſs is certainly erroneous, as * Dr. *Halley* abundantly has ſhewn, and has given divers Inſtances of infinite quantities which are in a determinate finite proportion one to another, and ſome infinitely greater one than another.

The like may be obſerv'd of infinitely ſmall quantities (*viz.*) Mathematical Points,) as the following Propoſitions will make appear.

Prop. 1. *The Points of contact between Circles and their Tangents are in Subduplicate proportion to the Diameters of the Circles.*

Fig. 46.

Let two Circles $adcb$, $afbg$, touch one another from within at the point a. Draw the Tangent paq, and parallel to it the line mn. From the point a draw the Diameter ac.

Let ac the Diameter of the greater Circle be equal to R, and ab the Diameter of the leſſer Circle be equal to S.

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Fig. 52.
Fig. 53.
Fig. 54.
Fig. 55.
Fig. 56.
Fig. 57.
Fig. 58.
Fig. A.
Fig. B.
Fig. C.

Let db the Chord of the Arch dab be equal to z, and fg the Chord of the Arch fag be equal to y, and let the Abſciſs ak be equal to x.

If the Line mn be ſuppoſed to move till it becomes coincident with the Tangent paq, the nature of a Circle will always give the following Æquations.

$$zz = 4Rx - 4xx.$$
$$yy = 4Sx - 4xx.$$

When the Line is arrived at the Tangent, z and y will become the two Points of Contact, and then $zz = 4Rx$ and $yy = 4Sx$. ($4xx$ being laid aſide as Heterogeneous to the reſt of the Æquation, by reaſon of x being become infinitely little) Therefore

$$zz : yy :: 4Rx : 4Sx :: R.S.$$

Therefore $z : y :: \surd R : \surd S$. *Q. E. D.*

Fig. 47.

Prop. II. *The Point of Contact between a Sphere and a Plane is infinitely greater than that between a Circle and a Tangent.*

Let a be the Point of Contact between the Sphere $adqf$ and the Plane bc. About the Sphere deſcribe the Cylinder $npgm$.

Draw kb to repreſent a Circle parallel to the Plane. Let the Circle be ſuppos'd to move, till it becomes coincident with the Plane. The Cylindrical Surface $khgm$ will always be equal (according to *Archimedes*) to the Spherical Surface daf.

Now when theſe Surfaces become infinitely ſmall, one terminates in the Point of Contact, and the other in the Periphery of the Baſe of the Cylinder. Therefore the Point of Contact is equal to the Periphery of the Baſe of the Cylinder (equal to a Periphery which has the ſame Diameter as the Sphere) and by conſequence is infinitely greater than any point of Contact between a Circle and a Tangent. *Q. E. D.*

Prop. III. *The Points of Contact by Spheres of different Magnitude are to one another as the Diameters of the Spheres.*

For by the ſecond Propoſition the Points of Contact are equal to the Peripheries of ſuch Diameters, whoſe proportion is the ſame as the Diameters. *Q. E. D.*

Of the Laws of Attraction and other Phyſical Principles, by Dr. John Keill. *n.* 315. *p.* 97.

XVIII. Ponenda ſunt fundamenti loco hæc tria, quibus omnis Phyſice innititur, principia. 1. Spatium inane. 2. Quantitatis in infinitum diviſibilitas. 3. Materiæ vis Attractrix. Dari ſpatium inane conſtat ex motu corporum. Quantitatis in infinitum diviſibilitatem ex continuæ quantitatis natura demonſtrant Geometræ. Materiæ ineſſe vim attractricem confirmat experientia. Ex duobus primis principiis ſequitur.

Theorema I. *Materiæ exiguæ qualibet particula poteſt ita ſpatium quantumvis largum occupare, ut pororum ſeu omnium meatuum diametri ſint data recta minores, vel ut particulæ omnes ſint a ſe invicem remotæ intervallo data recta minore.*

Theorema II. *Dari poſſunt duo corpora mole æqualia, at pondere ſeu denſitate (id eſt quantitate materiæ) utcunque inæqualia, in quibus erunt meatuum ſeu pororum ſummæ fere æquales.*

Sit *v. g.* digitus cubicus alter auri, alter aeris: quamvis materia in cubo aureo viceſies millies ſuperat materiam in cubo aereo, fieri tamen poteſt ut ſpatia vacua in digito cubico auri ſint fere æqualia ſpatiis vacuis in digito cubico aeris, ſcil. ut auri vacuitates ſint ad vacuitates aeris ut 999999 ad 1000000.

Theor. III. *Particulæ quæ aquam vel aerem vel alia ejuſmodi fluida conſtituunt (ſi modo ſe tangant) non ſunt abſolute ſolidæ, ſed ex aliis compoſitæ particulis multos meatus & poros intra ſe continentibus.*

Particulæ corporum minimæ & abſolute ſolidæ, hoc eſt vacui omnino expertes, vocentur primæ compoſitionis; Moleculæ ex pluribus hiſce particulis coaleſcentibus ortæ vocentur particulæ ſecundæ compoſitionis; Moles ex pluribus moleculis coeuntibus conflatæ, vocentur particulæ tertiæ compoſitionis; & ſic deinceps, donec tandem perventum fuerit ad particulas, e quibus corporum fit ultima compoſitio, & in quas eorundem fit prima reſolutio.

Theor. IV. *Præter vim illam Attractricem, qua Planetarum Cometarumque corpora in propriis orbitis retinentur, alia etiam ineſt materiæ potentia, qua ſingulæ, ex quibus illa conſtat, particulæ ſe invicem attrahunt, & reciproce a ſe invicem attrahuntur: quæ vis decreſcit in majore quam duplicata ratione diſtantiæ augeſcentis.*

Theorema hoc multis poteſt probari experimentis; at ratio qua minuitur vis illa, dum a ſe invicem recedunt particulæ, num ſcilicet ſit triplicata, quadruplicata, vel alia quævis diſtantiarum augeſcentium ratio, quæ major ſit duplicata, nondum æque per experimenta patet; erit fortaſſe aliquando tempus, cum accuratiore adhibita diligentia innoteſcet.

Theor. V. *Si corpus conſtet ex particulis, quarum ſingulæ vi pollent attractrice, in triplicata vel pluſquam triplicata ratione diſtantiarum decreſcente; erit vis qua ab eo corpore urgetur corpuſculum, in ipſo contactu, vel intervallo a contactu infinite exiguo, infinite major, quam ſi corpuſculum illud ad datam a dicto corpore diſtantiam locaretur.* *

Theor. VI. *Iiſdem poſitis ſi vis illa attractiva in aſſignabili diſtantia, ad Gravitatem obtineat rationem finitam; eadem, in ipſo contactu, vel in diſtantia infinite parva, vi Gravitatis erit infinite major.*

Theor. VII. *Si vero in ipſo contactu, vis corporum Attractiva ad Gravitatem obtineat rationem finitam, eadem in omni diſtantia aſſignabili eſt vi gravitatis infinite minor, adeoque evaneſcit.*

Theor. VIII. *Vis Attractiva, qua pollent ſingulæ materiæ particulæ in ipſo contactu, vim gravitatis prope in immenſum ſuperat; non tamen eſt vi Gravitatis infinite major; adeoque in data diſtantia vis illa evaneſcet.*

Vis

* *Vide Prop.* 80. & 91. *Princip. Newtoni.*

Vis igitur hæc materiæ ſuperaddita, non niſi per ſpatiola admodum perexigua diffunditur; in majoribus diſtantiis prorſus nulla eſt; unde motus corporum Cœleſtium (quæ longis intervallis a ſe invicem disjuncta ſunt) per vim hanc Attractivam nulla ratione turbari poſſunt, ſed eadem ratione continuo peraguntur, ac ſi vis illa a corporibus iis prorſus abeſſet.

Theor. IX. *Si corpuſculum aliquod corpus tangat, vis qua urgetur illud corpuſculum, hoc eſt, vis qua cum eo corpore cohæret, erit quantitati contactus proportionalis; nam partes a contactu remotiores nihil conferunt ad cohærentiam.*

Adeoque pro vario particularum contactu varii orientur cohærentiæ gradus; omnium autem maximæ ſunt vires cohærentiæ, quando ſuperficies, in quibus ſe invicem tangunt corpora, planæ exiſtunt; quo in caſu, cæteris paribus, vis qua corpuſculum cum aliis cohæret, erit ut ſuperficierum partes ſeſe tangentes.

Hinc patet ratio, cur duo marmora exactiſſime polita, & ſeſe ſecundum ſuperficies planas tangentia, a ſe invicem divelli non poſſunt, niſi a pondere, quod Gravitatem Aeris incumbentis multum ſuperat.

Hinc etiam decantatiſſimi iſtius Problematis, de cohærentia materiæ, ſolutio elici poteſt.

Theor. X. *Ea corpuſcula facillime a ſe invicem ſeparantur, quarum contactus cum aliis ſunt pauciſſimi, & minimi; quales contingere ſolent in corpuſculis Sphæricis infinite exiguis.*

Hinc fluiditatis ratio redditur.

Theor. XI. *Vis qua corpuſculum aliquod ad aliud corpus maxime propinquum attrahitur, quantitatem ſuam non mutat, ſive augeatur corporis attrahentis materia, ſive minuatur, eadem manente corporis denſitate, & corpuſculi diſtantia.*

Nam cum vires particularum Attractrices per minima tantum diffundantur ſpatia; liquet partes remotiores ad *C*, *D* & *E*, nihil conferre ad attrahendum corpuſculum *A*. Adeoque eadem vi verſus *B* trahetur corpuſculum ſive adſint hæ partes, ſive amoveantur, ſive denique aliæ ipſis conjungantur.

Fig. 48.

Theor. XII. *Si ea ſit corporis alicujus textura, ut particulæ ultimæ compoſitionis, per vim quandam externam (qualis eſt pondus eas comprimens, vel ab altero corpore proveniens ictus) a primigeniis ſuis contactibus paululum dimoveantur, nec interim in novos contactus commigrent; particulæ, per vim attractivam ſeſe mutuo petentes, ad contactus primigenios cito redibunt: iiſdem vero redeuntibus particularum corpus quodvis componentium contactibus & poſitionibus, eadem quoque redibit corporis figura; adeoque per vim attractivam corpora priſtinas quas amiſerunt figuras poſſunt denuo recuperare.*

Hinc Elaſticitatis ratio reddi poteſt. Cum autem per vim Elaſticam corpora, in ſe invicem impingentia, a ſe mutuo reſiliant (ut demonſtratum eſt in Lectionibus noſtris Phyſicis) a vi attractiva corporum oriri etiam debet eorundem a ſe invicem diſceſſus.

Theor. XIII. *Quod ſi ea ſit corporis textura, ut particulæ a prioribus contactibus per vim impreſſam dimotæ, in alios qui ejuſdem ſunt gradus immediate deveniant, corpus illud in priſtinam figuram non ſe reſtituet.*

Hinc qualis ſit textura, in qua corporum mollities conſiſtit, intelligi poteſt.

Theor. XIV. *Particulæ materiæ pro diverſa ipſarum ſtructura & compoſitione diverſis pollebunt viribus attractivis, puta non erit æque fortis attractio, cum particula datæ magnitudinis pluribus perforata ſit meatibus, ac ſi omnino ſolida & vacui expers eſſet.*

Theor. XV. *Particularum perfecte ſolidarum vires attractivæ ex figuris ipſarum multum pendent,* Nam ſi parva aliqua materiæ particula in laminam circularem indefinite exiguæ craſſitudinis formetur, & corpuſculum in recta per centrum tranſeunte & ad planum circuli Normali locetur; ſitque diſtantia corpuſculi æqualis decimæ parti ſemidiametri circuli: vis qua urgetur corpuſculum triceſies minor erit, quam ſi materia attrahens coaleſceret in Sphæram, & virtus totius particulæ ex uno quaſi puncto Phyſico diffunderetur. . Quin etiam eadem circularis lamella fortius ad ſe trahit corpuſculum, quam alia ejuſdem ponderis particula, quæ in tenuem & longum formatur Cylindrum.

Theor. XVI. *Sales ſunt corpora, quorum particulæ ultimæ compoſitionis magna vi attractiva pollent, inter quas tamen particulas plurimi interjacent meatus, particulis, quas habet aqua, ultimæ compoſitionis pervii: quæ igitur a ſalinis particulis fortiter attractæ, in eas cum impetu ruunt, & a mutuo contactu eas disjungunt, cohærentiamque ſalium diſſolvunt.*

Theor. XVII. *Si corpuſcula duo viribus attractivis decreſcentibus in triplicata aut pluſquam triplicata ratione diſtantiarum ſe mutuo petunt; erit velocitas in ſe invicem impingentium infinite major quam in dato intervallo.* Vide Prop. 39. Princip. Newtoni.

Theor. XVIII. *Corporis aqua gravioris eo uſque diminui poteſt magnitudo, ut tandem in aqua ſuſpenſum maneat, nec vi propriæ Gravitatis deſcendat.*

Hinc patet ratio, cur particulæ Salinæ, Metallicæ, & aliæ ejuſmodi, in minima redactæ, in ſuis menſtruis ſuſpenſæ hæreant.

Theor. XIX. *Corpora majora minore velocitate ad ſe invicem accedunt, quam minora.*

Fig. 49. Vis enim, qua ſe mutuo petunt corpora A & B, particulis maxime propinquis tantum ineſt; remotiorum quippe vires nullæ ſunt. Non igitur major vis adhibetur ad movenda corpora A & B quam ad particulas c & d movendas, ſed corporum eadem vi motorum velocitates ſunt corporibus reciproce proportionales: unde erit velocitas qua corpus A tendit

verſus *B*, ad velocitatem, qua particula *c*, a corpore ſoluta, verſus idem *B* tenderet, ut particula *c* ad corpus *A*. Multo igitur minor eſt velocitas corporis *A*, quam foret velocitas particulæ *c* a corpore ſolutæ.

Hinc fit, ut corporum majorum motus ſua natura adeo languidus & lentus ſit, ut ab ambiente fluido & aliis circumjacentibus corporibus plerumque impediatur. In minimis vero corpuſculis viget virtus, & ab iis perplurimi producuntur effectus: tanto plus Energiæ minoribus ineſt corporibus, quam majoribus.

Hinc patet ratio iſtius Axiomatis Chymici, Sales non agunt niſi ſoluti.

Theor. XX. *Duo corpuſcula ſeſe non contingentia, adeo ſibi vicina locari poſſunt, ut vis, qua ſe mutuo petunt, vim Gravitatis multum ſuperet.*

Theor. XXI. *Si corpuſculum in fluido locatum a particulis ambientibus undique æqualiter trahatur, nullus exinde orietur corpuſculi motus; quod ſi ab aliis particulis magis, ab aliis minus urgeatur, ad eam partem tendet corpuſculum, ubi major eſt attractio: & motus productus inæqualitati attractionis reſpondebit, ſcilicet in majori inæqualitate major erit motus, in minore minor.*

Theor. XXII. *Corpuſcula in fluido natantia & magis ſe invicem trahentia quam fluidi particulas interjectas, depulſis fluidi particulis ad ſe invicem accedent ea vi, qua ipſorum attractio mutua ſuperat attractionem particularum fluidi.*

Theor. XXIII. *Si corpus aliquod in fluido locetur, cujus partes fluidi particulas magis ad ſe trahunt, quam fluidi particulæ a ſe invicem trahuntur; ſintque in corpore meatus plurimi particulis fluidi pervii, per hos meatus fluidum illud cito ſe diffundet; & ſi partium in corpore connexio non tam firma ſit, quin ab impetu irruentium particularum ſuperari poſſit, orietur exinde corporis immerſi diſſolutio.*

Hinc ut menſtruum dato corpori diſſolvendo ſit idoneum, tria requiruntur, 1. Ut partes corporis particulas menſtrui magis ad ſe trahant, quam eæ a ſe invicem trahuntur. 2. Ut corpus habeat meatus particulis menſtrui patentes, & pervios. 3. Ut cohærentia particularum corpus conſtituentium tanta non ſit, quin ab impetu irruentium particularum menſtrui divelli poſſit. Hinc quoque conſtat particulas Spiritum Vini conſtituentes, magis a ſe invicem trahi, quam a particulis corporis ſalini in Spiritu Vini demerſi.

Theor. XXIV. *Si corpuſcula in fluido natantia, & ſe invicem petentia, Elaſtica ſint, poſt congreſſum, a ſe mutuo reſilient; & inde in alia corpuſcula rurſus impingentia, denuo reflectentur: ex quo fient innumeri alii cum aliis corpuſculis conflictus continuaque reſilitiones. Per vim autem attractivam, continuo augebitur corpuſculorum velocitas, & ſenſui patebit partium motus Inteſtinus; ſed prout fortius aut imbecillius ſe invicem trahunt corpuſcula, & pro varia, qua pollent Elaſticitate, varii erunt hi motus, & diverſis gradibus atque temporibus, fient ſenſibiles.*

Theor. **XXV.** *Si corpuſcula ſe invicem trahentia, ſe mutuo contingant, nullus orietur motus; propius enim accedere nequeunt: Si ad exiguum admodum a ſe invicem ſeponantur ſpatium, orietur motus, ſed ſi longius diſtent, non majore vi ſe invicem trahent, quam fluidi particulas interjectas; adeoque nullus producetur motus.*

Ex hiſce principiis pendent omnia Fermentationis & Effervеſcentiæ Phænomena. Hinc patet ratio cur oleum Vitrioli, cui paululum aquæ immittitur, effervеſcit atque ebullit: corpuſcula enim ſalina infuſa aqua a mutuo contactu paululum dimoventur; unde cum magis ſe invicem trahant quam aquæ particulas, & cum undique æqualiter non trahuntur, motum exinde oriri neceſſe eſt.

Hinc etiam liquet ratio, cur tanta cietur ebullitio, cum limatura Chalybis mixturæ ſupradictæ injicitur: particulæ enim chalybis magna pollent Elaſticitate, unde valida oritur reflectio. Hinc etiam videre eſt, cur menſtrua quædam fortiori vi agunt, citiuſque corpus aliquod diſſolvunt, ſi aqua dilutiora fiant.

Theor. **XXVI.** *Si corpuſcula ſe mutuo attrahentia vi Elaſtica careant, a ſe invicem non reflectuntur; ſed congeries ſeu moleculas particularum efficient, unde fiet* Coagulum: *& ſi particularum ſic coacervatarum Gravitas ſuperet Gravitatem fluidi, ſuccedet quoque* Præcipitatio. *Oriri quoque poteſt præcipitatio ex aucta vel diminuta Gravitate menſtrui, in quo natant corpuſcula.*

Theor. **XXVII.** *Si corpuſculorum ſeſe invicem attrahentium, & in fluido natantium, ea ſit figura, ut in datis quibuſdam ipſorum partibus, majore vi attractiva polleant, quam in aliis, & major ſit in iiſdem contactus; corpuſcula illa coibunt in corpora datas figuras habentia, & inde emergent Chryſtalliſationes; corpuſculorumque componentium figuræ ex data figura Chryſtalli per Geometriam determinari poſſunt.*

Theor. **XXVIII.** *Si corpuſcula magis trahantur a fluidi particulis, quam a ſe invicem; fiet ut quaſi ſe mutuo fugientes, a ſe invicem recedant, & per omne fluidum cito diffundentur.*

Theor. **XXIX.** *Si inter duas fluidi particulas aliquod intercedat corpuſculum, cujus binæ oppoſitæ facies maximis pollent viribus attractivis, hoc interjectum corpuſculum particulas fluidi ſibi agglutinabit; & plura iſtiuſmodi corpuſcula per fluidum diffuſa ejus particulas omnes in corpus firmum compingent, fluidumque in* Glaciem *reducent.*

Theor. **XXX.** *Si corpus aliquod maximam emittat effluviorum copiam, quorum vires attractrices ſunt fortiſſimæ; cum effluvia hæc corpori alicui leviuſculo appropinquent, ipſorum vires attractrices Gravitatem corporis levioris tandem ſuperabunt; & effluvia corpus illud ad ſe ſurſum trahent; cumque multo magis conferta ſunt Effluvia, in minoribus ab emittente corpore diſtantiis, quam in majoribus; corpus leve verſus denſiora Effluvia ſemper* *urgebitur, donec tandem ipſi corpori effluvia emittenti adhæreat. Hinc plurima* Electricitatis *Phænomena explicari poſſunt.*

Contra noſtram hanc de viribus attractricibus doctrinam fortaſſe objiciet aliquis; Si vis hæc attractrix omni ineſſet materiæ; corpora ponderoſiora & plus materiæ in dato ſpatio habentia, plus debere attrahere, quam corpora minus Gravia, quod experientiæ repugnat. Sed huic objectioni facile reſpondetur. Particulæ ſcilicet ultimæ compoſitionis (quibus ſolis tribuitur vis attractrix) confertim juxta ſe invicem locatæ, poſſunt corpus ponderoſum conſtituere, etiamſi ipſæ in ſe ſint rariores, quam eæ quæ corpus leve conſtituunt ultimæ compoſitionis particulæ, a ſe invicem remotiores, & plures & patentiores meatus inter ſe habentes.

Alia multa ſunt Naturæ Phænomena, quæ mihi videntur iiſdem principiis explicari poſſe, uti aſcenſus ſucci in Plantis & Arboribus, foliorum & florum determinatæ & conſtantes figuræ, eorumque virtutes ſpecificæ, *&c.* Multa quoque quæ in corpore animali quotidie occurrunt, præcipue quæ ad fluidorum curſus Secretioneſque ſpectant, ab iiſdem materiæ qualitatibus pendent, & hinc morborum *Theoria* & medicamentorum effectus optime eruuntur.

Of the Laws of Centripetal Force. by the ſame. n. 317. p. 175.

XIX. Theorema. *Si corpus urgente vi Centripeta in curva aliqua moveatur; Erit vis illa in quovis Curvæ puncto, in ratione compoſita ex directa ratione diſtantiæ corporis a centro virium, & reciproca ratione Cubi perpendicularis a Centro in rectam in eodem puncto Curvam Tangentem demiſſæ, ducti in Radium Curvaturæ quem ibi obtinet curva.*

Fig. 56.

Sit *QAO* Curva quælibet a mobili urgente vi centripeta ad punctum *S* tendente deſcripta. Sitque *AO* arcus in minimo quovis tempore percurſus, *Pm* ejus tangens, *AR* Radius circuli æquicurvi, hoc eſt cujus Peripheriæ pars minima cum Arcu *AO* coincidat. Et ſit *SP* recta a puncto *S* in tangentem perpendiculariter demiſſa; Ducantur *Om* ad *SA* & *On* ad *SP* Parallelæ. Et exponat *Om* vim qua mobile in *A* urgetur verſus *S*. Vis qua perpendiculariter a tangente recedit corpus, erit ut *On*, id eſt vis tendens verſus *R*, & faciens ut mobile, eadem qua prius velocitate latum, deſcribet circulum æquicurvum arcui *AO*, erit ad vim tendentem verſus *S*, qua corpus in curva *AO* movetur, ut *On* ad *Om*, vel ob æquiangula triangula ut *SP* ad *SA*. Sed corporum in circulis latorum vires centripetæ ſunt ut quadrata velocitatum applicata ad Radios; per *Corol. Theorem.* 4. *Princip. Newtoni.* Eſt vero velocitas reciproce ut *SP*, ſive directe ut $\frac{1}{SP}$ adeoque quadratum velocitatum erit ut $\frac{1}{SP^2}$: vis igitur ut *On*, ſive vis qua in circulo æquicurvo moveri poteſt corpus, erit ut $\frac{1}{SP^2 \times AR}$: Oſtenſum autem eſt, eſſe *SP* ad *SA* ut vis tendens verſus *R*, qua corpus in circulo æquicurvo moveri poteſt, ad vim tendentem verſus *S*: ſed eſt vis tendens verſus *R* ut $\frac{1}{SP^2 \times AR}$, adeoque cum ſit $SP : SA ::$

1

$:: \frac{1}{SP^2 \times AR} : \frac{SA}{SP^3 \times AR}$ erit vis tendens verſus S, ut $\frac{SA}{SP^3 \times AR}$. $Q. E. D.$

Cor. Si curva QAO ſit circulus, erit vis centripeta tendens verſus S, Fig. 51. ut $\frac{SA}{SP^3}$. Adeoque ſi vis centripeta tendat ad S punctum in circumferentia ſitum, erit [per 32 tertii] ang. $PAS =$ ang. AQS; adeoque ob ſimilia triangula ASP, ASQ, erit $AQ : AS :: AS : SP$: unde $SP = \frac{AS^2}{AQ}$ & $SP^3 = \frac{AS^6}{AQ^3}$ unde $\frac{SA}{SP^3} = \frac{SA \times AQ^3}{AS^6} = \frac{AQ^3}{AS^5}$, hoc eſt, ob datum AQ, erit vis reciproce ut AS^5.

Sit DAB, Ellipſis cujus Axis DB, foci F & S, AR, OR duæ perpen- Fig. 52. diculares in curvam ſibi proximæ: ducantur KL, OT in SA, & KM in (a) Prop. 3. El. 6ti. OR perpendiculares. Quia $SA : SK ::$ (*a*) $FA + SA : FS$, hoc eſt data ratione, erunt rectarum SA, SK Fluxiones AT, Kk ipſis SA, SK pro- (b) Prop. 6. partis 4tæ Sect. Con. Milnij. portionales; & eſt $AL =$ (*b*) $\frac{1}{2}$ lateris Recti $= \frac{1}{2}L$. Porro ob KA ad SP parallelam, eſt angulus $ASP = KAL = TOA$ ob ang. TAO utriuſque complementum ad rectum: quare $KA : AL :: SA : SP$, unde $SP = \frac{L}{2} \times \frac{SA}{KA}$ & $KA = \frac{L}{2} \times \frac{SA}{SP}$. Porro ob æquiangula triang. KMk, GPS & OTA, SPA.

Eſt $KM : Kk :: GP : GS :: AP : SK$
Item $Kk : AT$ $::$ $SK : SA$
Item $AT : AO$ $::$ $AP : SA$

Erit $KM : AO :: AP^2 : SA^2 :: SA^2 - SP^2 : SA^2 :: SA^2 - \frac{L^2 \times SA^2}{4AK^2} : SA^2 :: 4AK^2 - L^2 : 4AK^2$, unde $L^2 : 4AK^2 ::$ ($AO - KM : AO ::$) $AK : AR$ ac proinde $AR = \frac{4AK^3}{L^2}$. Eodem prorſus ratiocinio invenietur Radius Curvaturæ in Hyperbola æqualis $\frac{4AK^3}{L^2} = \frac{L \times SA^3}{2SP^3}$.

In Parabola vero facilior eſt calculus. Nam ob datam ſubnormalem, eſt Kk ſemper $= AT =$ Fluxioni Axis; & triangula KkM, ATO, SPA, Fig. 53. AKL, æquiangula, unde $KM : Kk :: AP : SA$, item eſt AT vel Kk: $AO :: AP : SA$, unde $KM : AO :: AP^2 : SA^2 :: SA^2 - SP^2 : SA^2 ::$ unde erit $SP^2 : SA^2 :: AO - KM : AO :: AK : AR$, ac proinde $AR = \frac{SA^2 \times AK}{SP^2}$ ſed eſt $AL = \frac{1}{2}$ lateris Recti $= \frac{1}{2}L$, & $AK : AL :: SA : SP$, quare erit $\frac{L}{2} \times \frac{SA}{AK} = SP$, & $SP^2 = \frac{L^2 \times SA^2}{4AK^2}$, quare erit $AR = \frac{4AK^3}{L^2}$, vel quoniam eſt, $AK = \frac{L \times SA}{2SP}$, erit $AR = \frac{L \times SA^3}{2SP^3}$.

Atque ex his facillima oritur conſtructio, pro determinando Radio curvaturæ in quavis Sectione Conica. Sit enim AK perpendicularis in Sectionem occurrens Axi in K, ex K ſuper AK erigatur perpendicularis

HK, cum AS producta concurrens in H. Ex H erigatur ſuper AH perpendicularis HR, erit AR radius curvaturæ. In Parabola paulo ſimplicior adhuc evadit conſtructio. Nam quoniam ex natura Parabolæ eſt $SA = SK$, & ang. AKH rectus, erit S centrum circuli per AKH tranſeuntis, unde invenitur Radius curvaturæ producendo SA in H, ut $SH = SA$, & in H erigendo perpendicularem HR; Et R erit centrum circuli oſculantis Parabolam in A. Fig. 54.

Vis Centripeta tendens ad focum Sectionis Conicæ in qua corpus movetur, eſt reciproce proportionalis quadrato diſtantiæ. Nam quoniam $AR = \frac{L \times SA^3}{2SP^3}$ erit $\frac{SA}{SP^3 \times AR} = \frac{SA \times 2SP^3}{SP^3 \times L \times SA^3} = \frac{2}{L \times SA^2}$ hoc eſt ob datam $\frac{2}{L}$ erit vis centripeta ut $\frac{1}{SA^2}$.

Sit Ellipſis BAD quam tangit in A recta GE. Sintque SP per centrum Ellipſis & KA per contactum, tranſeuntes, perpendiculares in tangentem. Erit $SP \times KA =$ quartæ parti figuræ Axis ſeu $=$ quadrato ſemiaxis minoris $= BO \times DE$. Nam ob æquiangula triang. GBO, GLA, GAK, GPS & GDE, Fig. 55.

$$\begin{array}{lcl} SP : SG & :: & BO : GO \\ SG : DG :: BG : LG & :: & GO : GA \\ DG : DE & :: & GA : AK, \end{array}$$

unde $SP : DE :: BO : AK$; & $SP \times AK = DE \times BO = \frac{1}{2} L \times SB$.

Hinc ſi Mobile moveatur in Ellipſi, vi centripeta tendente ad centrum Ellipſis, erit vis illa directe ut diſtantia; Nam eſt $\frac{SP^3 \times 4AK^3}{L^2} =$ datæ quantitati. Quia eſt $SP \times AK$ quantitas data. Vis igitur, ut $\frac{SA}{SP^3 \times AR}$, erit ut SA diſtantia.

In figura 52 demiſſa ab altero umbilico F: in Tangentem Perpendiculari FI. Ob æquiangula Triangula SAP, FAI, erit $SA : SP :: FA : FI = \frac{SP \times FA}{SA}$ unde erit $SP \times FI = \frac{SP^2 \times FA}{SA} =$ quadrato ſemiaxis minoris: unde ſi Axis major vocetur b, minor autem $2d$, erit $SP^2 = \frac{d^2 SA}{b - SA}$ & $SP = \frac{dSA^{\frac{1}{2}}}{\sqrt{b - SA}}$.

In Hyperbola autem eſt $SP = \frac{dSA^{\frac{1}{2}}}{\sqrt{b + SA}}$.

In Parabola eſt $SP = \sqrt{dSA}$, poſito ejus latere recto $= 4d$.

Quoniam eſt $TA^2 : TO^2 :: AP^2 : SP^2 :: SA^2 - SP^2 : SP^2 :: SA^2 - \frac{d^2 SA}{b - SA} : \frac{d^2 SA}{b - SA} :: SA - \frac{d^2}{b - SA}\ \frac{d^2}{b - SA} :: bSA - SA^2 - d^2 : d^2$, erit $\sqrt{bSA - SA^2 - d^2} : d :: TA : TO$, cumque ſit $TA = SA$, erit $TO = \frac{dSA}{\sqrt{bSA - SA^2 - d^2}}$.

Sit jam QAO quælibet curva, cujus arcus minimus ſit AO, tangentes in punctis A & O, AP, Op. Radius Curvaturæ AR, Perpendiculares in tangentes ſint SP, Sp, erit $\frac{SA \times TA}{fP} = AR$. Nam ob æquiangula

Fig. 56. triangula eſt $fP : AO :: PA : RA$ & $AO : TA :: SA : PA$; unde ex æquo erit $fP : TA$ vel $S\dot{A} :: SA : RA$, eſt vero $fP = S\dot{p}$, quare erit $RA = \frac{SA \times S\dot{A}}{S\dot{p}}$.

Hinc ſi diſtantia SA, in ſuam Fluxionem ducatur, & dividatur per Fluxionem perpendicularis, habebitur radius Curvaturæ; Quo Theoremate facile determinatur Curvatura in Radialibus curvis. Exempli Gratia, Sit AQ, Spiralis Nautica; quoniam angulus SAP datur, ratio quoque SA ad SP dabitur; ſit illa ratio a ad b, erit $SP = \frac{bSA}{a}$ & $S\dot{p} = \frac{bS\dot{A}}{a}$ & $AR = \frac{SAS\dot{A}}{S\dot{p}} = \frac{aSA}{b}$, unde facile conſtabit, Spiralis Nauticæ Evolutam eſſe eandem Spiralem, in alia poſitione.

Quoniam $AR = \frac{SAS\dot{A}}{S\dot{p}}$, erit $\frac{SA}{SP^3 \times AR} = \frac{S\dot{p}}{SP^3 \times S\dot{A}}$. Atque hinc rurſus, ex data relatione SA ad SP, facile invenietur lex vis centripetæ.

Exemplum. Sit VAB Ellipſis cujus focus S, Axis major $VB = b$, Axis minor $= 2d$, latus Rectum $= 2R$. Sitque VaQ alia curva, ita ad hanc relata, ut ſit perpetuo angulus VSA angulo VSa proportionalis, & ſit

Fig. 57. $Sa = SA$. Quæritur lex vis centripetæ tendentis ad S, qua corpus in curva VaQ moveri poteſt.

Quoniam ang. VSA eſt ad VSa, in data ratione; horum angulorum incrementa erunt in eadem ratione, ſitque ea ratio m ad n; unde erit $ot = \frac{n \times OT}{m}$. Eſt autem $OT = \frac{dS\dot{A}}{\sqrt{bSA - SA^2 - d^2}}$ unde erit $ot = \frac{ndS\dot{A}}{m\sqrt{bSA - SA^2 - d^2}}$.

Quoniam autem eſt $SA^2 + SP^2 : SP^2 :: ta^2 + ot^2 : ot^2 :: S\dot{A}^2 + \frac{n^2 d^2 S\dot{A}^2}{m^2\, bSA - SA^2 - d^2} : \frac{n^2 d^2 S^2}{m^2\, bSA - SA^2 - d^2} :: 1 + \frac{n^2 d^2}{m^2 \times bSA - SA^2 - d^2} : \frac{n^2 d^2}{m^2 \times bSA - SA^2 - d^2} :: m^2 bSA - m^2 SA^2 - m^2 d^2 + n^2 d^2 : n^2 d^2$, unde erit $\sqrt{m^2 bSA - m^2 SA^2 - m^2 d^2 + n^2 d^2} : nd :: SA : SP$, & $SP = \frac{ndSA}{\sqrt{m^2 bSA - m^2 SA^2 - m^2 d^2 + n^2 d^2}}$. Cujus ut habeatur fluxio pro $m^2 bSA - m^2 SA^2 - m^2 d^2 + n^2 d^2$, ſcribatur x; & erit $SP = \frac{ndSA}{\sqrt{x}}$, & $SP^3 = \frac{n^3 d^3 SA^3}{x^{\frac{3}{2}}}$; & eſt $\dot{x} = m^2 bS\dot{A} - 2m^2 SAS\dot{A}$; & $S\dot{p} = ndS\dot{A} \times x^{-\frac{1}{2}}$

$-\frac{1}{2}\frac{nASA\dot{x}}{x^{\frac{3}{2}}}$, & reducendo partes ad eundem denominatorem; erit $S\dot{P} = \frac{ndS\dot{A}\,x - \frac{1}{2}ndSA\dot{x}}{x^{\frac{3}{2}}}$. Et in numeratore, loco x & $\dot{x}$, ponendo ipſorum valores, & ordinando fit $SP = \frac{ndS\dot{A}\times\overline{\frac{1}{2}m^2bSA - m^2d^2 + n^2d^2}}{x^{\frac{3}{2}}}$, unde erit $\frac{S\dot{P}}{SP^3\times S\dot{A}} = \frac{\frac{1}{2}m^2bSA - m^2d^2 + n^2d^2}{n^2d^2SA^3}$. Sed eſt $\frac{S\dot{P}}{SP^3\times S\dot{A}}$, ut vis centripeta, quare erit vis, ut $\frac{\frac{1}{2}m^2bSA - m^2d^2 + n^2d^2}{n^2d^2SA^3}$ vel ob datam n^2d^2 in denominatore erit vis, ut $\frac{\frac{1}{2}m^2bSA - m^2d^2 + n^2d^2}{SA^3}$, vel loco d^2 ponendo $\frac{bR}{2}$ erit vis ut $\frac{\frac{1}{2}m^2bSA - \frac{1}{2}m^2bR + \frac{1}{2}n^2bR}{SA^3}$, ſeu ob datam $\frac{b}{2}$, ut $\frac{m^2SA - Rm^2 + Rn^2}{SA^3} = \frac{m^2}{SA^2} + \frac{Rn^2 - Rm^2}{SA^3}$. Quæ omnia exacte coincidunt cum iis quæ a Domino *Newtono* de vi centripeta corporis in eadem curva moti, traduntur, in *Prop.* 44. *Princip.*

Quoniam vis Centripeta tendens ad punctum *S*, qua urgente corpus in curva moveri poteſt, eſt ſemper ut $\frac{S\dot{P}}{SP^3\times S\dot{A}}$; hinc ex data lege vis Centripetæ, Inveniri poteſt relatio SA ad SP, ac proinde per methodum Tangentium Inverſam, exhiberi poteſt Curva quæ data vi Centripeta deſcribi poſſit.

Sit verbi gratia Vis reciproce ut diſtantiæ Dignitas quælibet m, hoc eſt, ſit $\frac{S\dot{P}}{SP^3\times S\dot{A}} = \frac{b}{a^2SA^m}$, erit $\frac{S\dot{P}}{SP^3} = \frac{bS\dot{A}}{a^2SA^m}$, & capiendo harum fluxionum fluentes; erit $\frac{1}{2}\ SP^{-2} = \frac{bSA^{1-m}\mp e}{\overline{m-1}\times a^2}$, unde erit $\frac{\frac{m-1}{2}\times a^2}{bSA^{1-m}\mp e} = SP^2$, & multiplicando tam numeratorem, quam denominatorem fractionis, per SA^{m-1}; & loco $\frac{m-1}{2}\ a^2$ ponendo d^2, fit $\frac{d^2SA^{m-1}}{b\mp eSA^{m-1}} = SP^2$; quare erit $SP = \frac{d\sqrt{SA^{m-1}}}{\sqrt{b\mp eSA^{m-1}}}$.

Quod ſi quantitas conſtans e ſit nihilo æqualis erit $SP \frac{\sqrt{SA^{m-1}}}{\sqrt{b}}$.

Adeoque ſi vis reciproce ut diſtantiæ quadratum, poni poteſt $SP = \frac{\sqrt{d^2SA}}{\sqrt{b}}$, & curva erit Parabola cujus latus rectum eſt $\frac{4d^2}{b}$, vel poteſt eſſe $SP = d\times\frac{\sqrt{SA}}{\sqrt{b-SA}}$, & curva erit Ellipſis, vel denique poteſt eſſe $SP = d\times\frac{\sqrt{SA}}{\sqrt{b\times SA}}$, & curva evadit Hyperbola.

Si

Si vis ſit reciproce ut diſtantiæ cubus, ſupponi poteſt, ut SP ſit $= \frac{dSA}{b}$ & curva ſit ſpiralis Nautica, vel fieri poteſt ut ſit $SP = \frac{dSA}{\sqrt{b - eSA^2}}$, & Curva erit eadem cum ea cujus conſtructionem a ſectore hyperbolæ petit Dominus *Newtonus*; vel poteſt eſſe $SP = \frac{dSA}{\sqrt{b + eSA^2}}$, & ejus Curvæ conſtructionem per Sectores Ellipticos tradit idem *Newtonus*, *Cor.* 3. *Prop.* 1. *lib.* 1. *Princip.*

Si vis centripeta ſit reciproce ut diſtantia; relatio inter SA & SP, æquatione Algebraica definiri nequit, Curva tamen per Logarithmicam vel per quadraturam Hyperbolæ conſtruitur, ſit enim $SP = \frac{d}{\sqrt{b - L.SA}}$, ubi $L.SA$ deſignat Logarithmum ipſius SA.

Moveatur jam corpus in Curva QAO, urgente vi centripeta tendente
Fig. 50. ad S; & Celeritas corporis in A dicatur C; celeritas autem qua corpus urgente eadem vi centripeta, in eadem diſtantia, in circulo moveri poteſt, dicatur c. Conſtat ex Theoremate primo, quod ſi SA exponat vim Centripetam tendentem ad S; vis Centripeta tendens ad R, qua urgente, corpus cum celeritate C, circulum cujus radius eſt AR deſcribet; per SP exponetur. Corporum autem circulos deſcribentium vires Centripetæ ſunt ut velocitatum quadrata ad circulorum radios applicata, quare erit $SP : SA :: \frac{C^2}{AR} : \frac{c^2}{SA}$, unde erit $SP \times AR : SA^2 :: C^2 : c^2$ & $C : c :: \sqrt{SP \times AR} : SA$.

Si SP cum SA coincidat, ut fit in figurarum verticibus, erit $C : c :: \sqrt{AR} : \sqrt{SA}$. Quod ſi curva ſit Sectio Conica AR, radius curvaturæ in ejus vertice eſt æqualis dimidio lateris recti $= \frac{1}{2}L$, ac proinde erit velocitas corporis in vertice Sectionis, ad velocitatem corporis in eadem diſtantia circulum deſcribentis, in dimidiata ratione lateris recti, ad diſtantiam illam duplicatam.

Quoniam eſt $AR = \frac{SA \times S\dot{A}}{S\dot{P}}$, erit $C^2 : c^2 :: \frac{SP \times SA \times S\dot{A}}{S\dot{P}} : SA^2 :: \frac{SP \times S\dot{A}}{S\dot{P}} : SA :: SP \times S\dot{A} : SA \times S\dot{P}$, adeoque ex data relatione SP ad SA, dabitur ratio C ad c. Exempli Gratia, Si vis ſit reciproce ut diſtantiæ dignitas m, hoc eſt ſit $\frac{S\dot{P}}{SP^3 \times S\dot{A}} = \frac{b}{a^2 SA^m}$ & erit $S\dot{P} : \frac{bSP^3 \times S\dot{A}}{a^2 SA^m}$, adeoque erit $C^2 : c^2 :: SP \times S\dot{A} : \frac{bSP^3 \times SA \times S\dot{A}}{a^2 SA^m} :: a^2 SA^{m-1} : bSP^2$. Unde ſi ponatur $SP^2 = \frac{d^2 SA^{m-1}}{b} = \frac{m-1}{2} \frac{a^2 SA^{m-1}}{b}$, erit $C^2 : c^2 :: a^2 SA^{m-1} : \frac{m-1}{2} a^2 SA^{m-1} :: m-1 : 2$ ac proinde erit $C : c :: \sqrt{2} : \sqrt{m-1}$.

Quod ſi ponatur $SP^2 = \frac{d^2 SA^{m-1}}{b - eSA^{m-1}} = \frac{\frac{m-1}{2} a^2 SA^{m-1}}{b - eSA^{m-1}}$ fiet C^2 ad c^2, ut $a^2 SA^{m-1}$ ad $\frac{\frac{m-1}{2} a^2 bSA^{m-1}}{b - eSA^{m-1}}$, hoc eſt ut $b - eSA^{m-1}$ ad $\frac{m-1}{2} b$, ſed eſt ratio $b - eSA^{m-1}$, ad $\frac{m-1}{2} \times b$, minor ratione b ad $\frac{m-1}{2} b$, ſeu ratione 2 ad $m - 1$, unde erit C ad c in minore ratione quam eſt $\sqrt{2}$ ad $\sqrt{m-1}$.

Similiter, ſi capiatur $SP = \frac{d^2 SA^{m-1}}{b + eSA^{m-1}}$, invenietur eſſe C ad c in majore ratione quam eſt $\sqrt{2}$ ad $\sqrt{m-1}$.

Cor. Si corpus in Parabola moveatur, & vis Centripeta tendat ad focum S, erit velocitas corporis ad velocitatem corporis in eadem diſtantia circulum deſcribentis ubique ut $\sqrt{2}$, ad 1, nam in eo caſu eſt $m = 2$ & $m - 1 = 1$. Velocitas corporis in Ellipſi eſt ad velocitatem corporis in circulo ad eandem diſtantiam moti, in minore ratione quam $\sqrt{2}$ ad 1. Velocitas in Hyperbola eſt ad velocitatem in circulo in majore ratione, quam $\sqrt{2}$ ad 1.

Si Corpus in Spirali Nautica deferatur, eſt ejus velocitas ubique æqualis velocitati corporis in eadem diſtantia circulum deſcribentis, nam in eo caſu eſt $m = 3$ & $m - 1 = 2$.

Problema. *Poſito quod vis Centripeta (cujus quantitas abſoluta nota eſt,) ſit reciproce ut diſtantiæ quadratum, & projiciatur corpus ſecundum datam rectam cum data velocitate. Invenire curvam in qua movetur corpus.*

Fig. 56.

Projiciatur Corpus ſecundum datam rectam AB, cum data velocitate C. Et quoniam quantitas abſoluta vis centripetæ nota eſt, dabitur inde velocitas qua corpus poſſit circulum ad diſtantiam SA deſcribere urgente eadem vi; eſt enim æqualis ei quæ acquiritur, dum corpus vi illa uniformiter applicata urgente, cadit per $\frac{1}{2} SA$. Sit illa velocitas c. Ex A in AB, erigatur perpendicularis AK, & in ea capiatur AR, quarta proportionalis ipſis c^2 C^2 & $\frac{SA^2}{SP}$, & erit AR radius curvaturæ in A. Ex R in AS demittatur perpendicularis RH & ex H in AR perpendicularis HK, & ducta recta SK, dabit axis poſitionem; Fiat angulus FAK = angulo SAK. Et ſi FA ſit ad SK Parallela, figura in qua movetur corpus erit Parabola. Si autem Axi SK occurrat in F; & puncta S & F cadant ad eandem partem puncti K, figura erit Hyperbola; ſin ad contrarias partes cadant puncta S & K, erit figura Ellipſis, unde focis S & F & Axe $= SA \pm FA$ deſcribetur ſectio, in qua corpus movebitur.

XX. *Cum* D. J. Bernoulli *objectiones quaſdam edidiſſet contra Solutionem* Newtonianam *hujuſce Problematis, adjecta Solutione a ſeipſo inventa;* Cl. Keillius, *Objectionibus ejus dilutis, Solutionem aliam edit, repetens pauca quæ ſupra tradita ſunt.*

A Solution of the inverſe Problem of Centripetal Forces, by Dr. Keill. n. 340. p. 91.

Sit VIL Curva quævis, quam corpus urgente vi centripeta ad centrum C tendente deſcribit: hanc Curvam in duobus punctis infinite vicinis I & K tangant rectæ IP, Kp, ad quas e centro demittantur perpendiculares CP, Cf; centro item C deſcribantur KE, ID, & ducatur CI.

Erit vis centripeta ut Quantitas $\frac{Pp}{PC^3 \times IN}$ Quod Theorema licet in prædicto loco demonſtravimus, ecce aliam ejus demonſtrationem. Ex K ducantur Km ad CP & Kn ad CI parallelæ. Et ob æquiangula triangula ICP, IKN, nKM. Itemque ob IKm & IpP æquiangula, Erit,

Ip vel $IP : IK :: pP : Km$

$PC : IP :: Km : mn$

$IN : IK :: mn : nK$ unde ex æquo fiet

$PC \times IN : IK^2 :: pP : nK$, & erit $nK = \frac{pP \times IK^2}{PC \times IN}$. Præterea tempus quo deſcribitur arcus IK eſt ut Area ſeu triangulum ICK, vel ejus duplum $PC \times IK$; adeoque ſi tempus detur erit $PC \times IK$ quantitas conſtans. Dato autem tempore, vis centripeta eſt ut lineola Kn quæ ſub urgente vi illa deſcribitur, adeoque vis centripeta eſt ut lineola illa Kn ducta in quantitatem conſtantem $\frac{1}{PC^2 \times IK^2}$, hoc eſt, erit vis centripeta ut $\frac{1}{PC^2 \times IK^2} \times \frac{Pp \times IK^2}{PC \times IN}$ ſeu ut quantitas $\frac{Pp}{PC^3 \times IN}$. Quod erat demonſtrandum.

Velocitas corporis in quovis loco eſt ut via in minimo quovis tempore percurſa directe & ut tempus illud inverſe; adeoque & ut $IK \times \frac{1}{PC \times IK}$, hoc eſt, velocitas erit reciproce ut Perpendicularis e centro in Tangentem.

Si diſtantia corporis a centro dicatur x, & Perpendicularis in tangentem p, erit $IN = \dot{x}$ & $Pp = \dot{p}$ & vis centripeta exponi poteſt per quantitatem $\frac{f^4 \dot{p}}{p^3 \dot{x}}$, aſſumendo quantitatem quamlibet pro f^4.

Adeoque ſi cum * Domino *Bernoullio* vim centripetam nominemus φ, erit $\frac{f^4 \dot{p}}{p^3 \dot{x}} = \varphi$ & $\frac{f^4 \dot{p}}{p^3} = \dot{x}\varphi$; & capiendo harum quantitatum fluentes erit $\frac{f^4}{2p^2} =$ Fluenti quantitatis $\dot{x}\varphi$.

At cum velocitas corporis ſit reciproce ut perpendicularis p, ejus quadratum exponi poteſt per $\frac{f^4}{2p^2}$. Si itaque velocitas dicatur v, erit $v^2 = \frac{f^4}{2p^2} =$ Fluenti quantitatis $\dot{x}\varphi$. Quod ſi A ſit locus de quo cadere debet corpus ut acquirat in D vel I velocitatem v, deque loco corporis D erigatur perpendicularis $DF = \varphi$ erit rectangulum $DE \times DF = \dot{x}\varphi$. Sit jam BFG linea curva cujus ordinatæ exponant vires centripetas, ſeu quantitates φ. Fluens quantitatis $\dot{x}\varphi$ erit Area curvilinea $ABFD = v^2 = \frac{f^4}{2p^2}$, adeoque erit v ut Areæ $ABFD$ latus quadratum. Quod ſi velocitas ea ſit

qua

* Comm. Phyſ. Mathem. Par. An. 1710.

Fig. II.

O
O
A
B
V
I
D
F
N
E
G
n
m
K
P
p
f
L
C

Fig. VI

Fig. V.

Fig. III

H
T
V
P
n
m
o
G
R
Y
K
X
I
c
N
q
C
Q
S
L

Fig. VII.

Fig. VIII.

Fig. IV

G
F
M
S
V
L
E
Y
X
I
K
R
T

J. Senex sculp^{t}.

quæ ab infinita diſtantia cadendo acquiritur, erit v^2 ſeu fluens ipſius $\dot{x}\,\phi$ æquale areæ *ODFO* indefinite protenſæ.

Hinc ſemper dabitur quantitas p in terminis finitis, quando Area illa curvilinea terminis finitis exponi poteſt. Sit, verbi gratia, vis centripeta reciproce ut diſtantiæ dignitas m, hoc eſt, ſit $\dot{x}\,\phi = \frac{g\dot{x}}{x^m}$. Si velocitas corporis ſit ea quæ acquiritur cadendo ab infinita diſtantia, erit $v^2 = \frac{g}{m-1 \times x^{m-1}} = \frac{f^4}{2p^2}$ & in hiſce omnibus caſibus Area indefinite protenſa eſt quantitas finita. Poteſt autem corpus in trajectoria revolvi velocitate cujus quadratum vel majus fieri poteſt, vel minus quantitate $\frac{g}{m-1 \times x^{m-1}}$, vel huic æquale. Adeoque erit $v^2 = \frac{f^4}{2p^2} = \frac{g}{m-1\, x^{m-1}} \pm e^2$.

Hinc urgentibus his viribus tria Curvarum genera deſcribi poſſunt; prout e^2 eſt quantitas poſitiva vel negativa vel nulla.

V.G. Si Velocitas major ſit ea quæ acquiritur ab infinita diſtantia cadendo, fit $\frac{f^4}{2p^2} = \frac{g}{m-1\, x^{m-1}} + e^2$: ſi velocitas ſit minor, erit $\frac{f^4}{2p^2} = \frac{g}{m-1\, x^{m-1}} - e^2$: ſi æqualis, erit $\frac{f^4}{2p^2} = \frac{g}{m-1\, x^{m-1}}$.

Sit $\frac{1}{2} f^4 = a^2 e^2$ & $\frac{1}{m-1} \times g = b^2 e^2$. Et ſi velocitas corporis ſit ea quæ ab infinito cadendo acquiritur, erit $p^2 = \frac{a^2 x^{m-1}}{b^2}$ ſeu $p = \frac{a x^{\frac{m-1}{2}}}{b}$.

At ſi velocitas major ſit aut minor hac velocitate, fiet uti oſtenſum eſt $\frac{f^4}{2p^2} = \frac{g}{m-1\, x^{m-1}} \pm e^2 = \frac{\frac{1}{m-1} g \pm e^2 x^{m-1}}{x^{m-1}}$. Unde pro $\frac{1}{2} f^4$ & $\frac{g}{m-1}$ ponendo earum valores $a^2 e^2$ & $b^2 e^2$, erit $\frac{a^2 e^2}{p^2} = \frac{b^2 e^2 \pm e^2 x^{m-1}}{x^{m-1}}$ ſeu $\frac{a^2}{p^2} = \frac{b^2 \pm x^{m-1}}{x^{m-1}}$; & fiet $p^2 = \frac{a^2 x^{m-1}}{b^2 \pm x^{m-1}}$.

Adeoque ſi Vis centripeta ſit reciproce ut cubus diſtantiæ, hoc eſt, ſi ſit $m = 3$ & $m - 1 = 2$. Erit $p^2 = \frac{a^2 x^2}{b^2}$, vel $p^2 = \frac{a^2 x^2}{b^2 + x^2}$, vel denique $p^2 = \frac{a^2 x^2}{b^2 - x^2}$.

In primo caſu conſtat Curvam eſſe Spiralem Logarithmicam: nam fit $p = \frac{ax}{b}$, & $b : a :: x : p$. adeoque ob conſtantem rationem b ad a, erit angulus *CIP* ubique conſtans.

Ponamus jam eſſe $p^2 = \frac{a^2 x^2}{b^2 + x^2}$ & ex hac ſuppoſitione tres oriuntur diverſæ Curvarum ſpecies, prout a major eſt quam b, aut ei æqualis, aut minor. Fig. 3.

Et primo ſit a major quam b. Centro C & ad diſtantiam quamvis datam deſcribatur circulus HYX, cui rectæ CK, CI productæ occurrant in Y & X. Et eſt $IN^2 : KN^2 :: IP^2 : PC^2$ & ita $CI^2 - PC^2 : PC^2 :: x^2 - p^2 : p^2$ $:: x^2 - \frac{a^2 x^2}{b^2 + x^2} : \frac{a^2 x^2}{b^2 + x^2} :: 1 - \frac{a^2}{b^2 + x^2} : \frac{a^2}{b^2 + x^2} :: b^2 + x^2 - a^2 : a^2$. Quare erit $\sqrt{x^2 + b^2 - a^2} : a :: IN : KN :: x : \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}} = KN$. Et quoniam eſt a major quam b, erit $b^2 - a^2$ quantitas negativa. Sit illa $-c^2$, unde fit $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$. Dicatur radius circuli $HY\,b$, & eſt CK $: KN :: CY : YX$ hoc eſt $x : \frac{a\dot{x}}{\sqrt{x^2 - c^2}} :: b : \frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = YX = \dot{y}$, ſi arcus HY vocetur y.

Sit $x = \frac{c^2}{z}$ unde $\dot{x} = -\frac{c^2 \dot{z}}{z^2}$, & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Item erit $x^2 - c^2 = \frac{c^4}{z^2} - c^2 = \frac{c^4 - c^2 z^2}{z^2} = \frac{c^2}{z^2} \times c^2 - z^2$: unde $\sqrt{x^2 - c^2} = \frac{c}{z} \times \sqrt{c^2 - z^2}$: quibus valoribus ſubſtitutis, erit $\frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = \frac{-ba\dot{z}}{c\sqrt{c^2 - z^2}}$. Sit $a : c :: n : 1$. hoc eſt, ſit $a = nc$, & fiet XY ſeu $\dot{y} = -\frac{nb\dot{z}}{\sqrt{c^2 - z^2}}$. Eſt vero $\frac{nb\dot{z}}{\sqrt{c^2 - z^2}}$ ad $-\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nb ad c; hoc eſt in ratione data: adeoque eorum fluentes, ſi ſimul incipiunt, erunt in eadem ratione, hoc eſt erit HY ſeu y ad fluentem quantitatis $\frac{c\dot{z}}{\sqrt{c^2 - z^2}}$ ut nb ad c.

Quod ſi centro C radio $CV = c$ deſcribatur circulus VL, & CG ſit $= z$, & $no = \dot{z}$, fiet arcus $mn = -\frac{c\dot{z}}{\sqrt{c^2 - z^2}} =$ fluxioni arcus Qm, quando fluxio eſt quantitas poſitiva: ſed quando eſt negativa, ejus fluens eſt arcus Vm prioris complementum. Arcus enim ejuſque complementum eandem habent quantitatem fluxionem denotantem, diverſis tantum ſignis affectam; quia creſcente uno decreſcit alter.

Hinc eſt HY ad Vm ut nb ad c: ſed eſt CV ad CH ut $Ve : HY$, hoc eſt $c : b :: Ve : \frac{b \times Ve}{c} = HY$, quare erit $\frac{b \times Ve}{c} : Vm :: nb : c$, unde $Ve : Vm$ $:: n : 1$.

Præterea ex natura circuli erit $CG : CV :: CV : CY$, quando mY circulum tangit: hoc eſt erit $z : c :: c : \frac{c^2}{z} = CY = x$. Hinc ſi capiatur angulus VCe ad angulum VCm ut n ad 1, & producatur Ce ad K ut ſit CK = ſecanti CT, erit K punctum in Curva quæſita.

Hic obiter notandum eſt, ſi n ſit numerus, hoc eſt ſi ſit a ad c vel a ad $\sqrt{a^2-b^2}$ ut numerus ad numerum, Curva VI fiet Algebraica: nam in hoc caſu relatio mG ad ſinum anguli VCe æquatione definitur, & inde habebitur relatio ſinus anguli VCe ad CT vel CK per æquationem determinatam, & inde demum dabitur æquatio quæ exprimet relationem inter ordinatam & interceptam a puncto C incipientem. Harum Curvarum ordines & gradus in Scala æquationum Algebraica diverſi erunt pro magnitudine numeri n. In his omnibus Curvis ſic deſcriptis Aſymptoti poſitio hac ratione determinatur: Fiat angulus VCL ad rectum angulum ut n ad 1. In eo angulo diſtantia corporis a centro evadit infinita. Jam quad. perpendicularis in Tangentem $PC = \frac{a^2x^2}{b^2+x^2}$, ubi x eſt infinita, fit $PC^2 = \frac{a^2x^2}{x^2}$, ſeu $PC = a$. Ducatur itaque CR ad CL perpendicularis & æqualis rectæ a, & ſi per R ducatur RS rectæ CL parallela, hæc Curvam tanget ad infinitam diſtantiam, ſeu erit Curvæ Aſymptotos.

Si corpus in quavis harum Curvarum deſcendendo, ad Apſidem imam pervenerit; Hinc rurſus aſcendet in infinitum, & aliam Curvam priori ſimilem, ſeu potius ejuſdem Curvæ ſimilem portionem aſcendendo deſcribet.

Curvæ hæ poſſunt pluribus revolutionibus circa centrum torqueri, priuſquam ad Aſymptoton convergere incipiant, & motus angularis rectæ CK erit æqualis totidem rectis quot numerus n conſtat Unitatibus. *v. g.* ſi n ſit 100, perficientur viginti quinque integræ revolutiones priuſquam diſtantia a centro evadat infinita.

Aucto numero n, eadem manente a, minuitur c: eſt enim $\frac{a}{n} = c$ & $\frac{a^2}{n^2} = c^2 = a^2 - b^2$, unde fiet $n^2 - 1 \times a^2 = n^2 b^2$. Et proinde fiet $a^2 : b^2 :: n^2 : n^2 - 1$; adeoque ſi b^2 ad æqualitatem accedat ipſius a^2, perveniet quoque $n^2 - 1$ ad rationem æqualitatis cum n^2, & proinde augebitur n & in eadem ratione minuetur c. Ponatur itaque eſſe b^2 fere æquale ipſi a^2; adeo ut cum differentia ſit infinite parva, fiat n numerus infinite magnus, & radius circuli c fiet infinite parvus, ſeu circulus in ſuum centrum contrahetur. At ſic evaneſcente c, non pariter evaneſcit CT, ſi angulus VCM ſit propemodum rectus: nam in omni circulo, etiam minimo, ſecans anguli recti eſt quantitas infinita. Curva itaque hæc, ob n numerum infinitum, infinitis numero revolutionibus centrum ambibit, priuſquam ad Aſymptoton convergere incipiet.

Evaneſcente autem c fit $b = a$ & $p = \frac{ax}{\sqrt{x^2+a^2}}$. Et quoniam in omni caſu eſt $\dot{y} = \frac{ba\dot{x}}{x\sqrt{x^2+c^2}}$, evaneſcente c fiet $\dot{y} = \frac{ba\dot{x}}{x^2}$, unde capiendo Fig. 4.
Fluentes fiet $y = \frac{ba}{x}$ ſeu $xy = ba =$ datæ quantitati.

Hæc Curva eſt Spiralis Hyperbolica, quæ plures habet notabiles proprietates. Si ducatur radius quilibet *CIT* Curvæ occurrens in *I*, & peripheriæ circuli in *T*, & ex *C* ad *CI* excitetur perpendicularis *CT*, atque *IT* tangat Curvam in *I*, & rectæ *CT* occurrat in T: erit *CT* conſtans recta, æqualis ſcil. arcui *VE*; qua proprietate Logarithmicam æmulatur, cum *CT* Curvæ Subtangens dici poſſit. Sit enim Radius circuli $CE = b$, arcus $VE = a$, dicatur *CI* x, & *VT* ſit y. Quia eſt $ba = x \times y$ erit $\frac{ba}{x} = y$ & $\frac{ba\dot{x}}{x^2} = \dot{y}$. Porro eſt $CT : CI :: TX : NK$ hoc eſt $b : x :: \frac{ba\dot{x}}{x^2} : NK$: quæ proinde eſt $\frac{a\dot{x}}{x}$. Et quoniam eſt $IN : NK :: CI : CT$, hoc eſt $\dot{x} : \frac{a\dot{x}}{x} :: x : CT$, erit $CT = a$.

Si centro *C*, intervallo quovis *CG*, deſcribatur circuli arcus *GF*, hic arcus inter rectam *CV* & curvam interceptus erit ſemper æqualis conſtanti rectæ *CT* vel a. Nam quoniam eſt $VL \times CF = CV \times VE$, erit $VL : VE :: CV : CF :: VL : GF$ unde æquantur *VE* & *GF*. Si ad *CG* ex *C* excitetur normalis $CR = VE$ vel *FG* vel a, & per *R* agatur *RS* rectæ *CV* parallela, erit *RS* Curvæ Aſymptotos. Nam eſt recta *MS* æqualis arcui *GF*, & proinde *FS* diſtantia Curvæ ab *RS* eſt ſemper æqualis exceſſui quo arcus ſuperat ſuum ſinum: at cum diſtantia creſcat in infinitum, exceſſus ille minuetur in infinitum, & fiet tandem data quavis recta minor, & proinde *RS* erit Curvæ Aſymptotos.

Sit jam b major quam a; & ſimiliter, ut in priore caſu, invenietur $KN = \frac{a\dot{x}}{\sqrt{x^2 + b^2 - a^2}}$: at quoniam b ſuperat a, erit $c^2 = b^2 - a^2$ quantitas poſitiva, & *KN* fiet $= \frac{a\dot{x}}{\sqrt{x^2 + c^2}}$ & ponendo radium circuli $HT = b$, invenietur $XT = \frac{ha\dot{x}}{x\sqrt{x^2 + c^2}}$. Ponatur $x = \frac{c^2}{z}$, & erit $\dot{x} = -\frac{c^2\dot{z}}{z^2}$ & $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$. Erit quoque $x^2 = \frac{c^4}{z^2}$ & $x^2 + c^2 = \frac{c^4}{z^2} + c^2 = \frac{c^4 + c^2z^2}{z^2} = \frac{c^2}{z^2} \times \overline{c^2 + z^2}$: unde $\sqrt{x^2 + c^2} = \frac{c}{z} \times \sqrt{c^2 + z^2}$. His itaque valoribus ſubſtitutis fit $\frac{ha\dot{x}}{x\sqrt{x^2 + c^2}} = -\frac{baz}{c\sqrt{c^2 + z^2}} = -\dot{y}$. Nam tale ſumi poteſt initium arcus *HT*, ut ſimul cum Fluente quantitatis $\frac{-ba\dot{z}}{c\sqrt{c^2 + z^2}}$ creſcat & decreſcat. Fiat $nc = a$ & erit $\frac{nb\dot{z}}{\sqrt{c^2 + z^2}} = \dot{y}$, & $\frac{\frac{1}{2}nb^2\dot{z}}{\sqrt{c^2 + z^2}} = \frac{1}{2}b\dot{y} =$ ſectori *CXT*.

Eſt autem $\frac{\frac{1}{2}nb^2\dot z}{\sqrt{c^2+z^2}} : \frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}} :: nb^2 : c^2$, hoc eſt in data ratione. Adeoque erit ſector CXT ad $\frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}}$ ſemper in data ratione. Harum itaque quantitatum fluentes erunt in eadem ratione, cum ſimul incipere ponantur. Fluens autem ſectoris CXT eſt ſector CVT, & fluens quantitatis $\frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}}$ eſt ſector Hyperbolæ, quod ſic oſtenditur.

Centro C ſemiaxe tranſverſo $CV = c$ deſcribatur Hyperbola æquilatera, & ex duobus punctis vicinis D & F ordinentur ad axem conjugatum rectæ DB, EF; ducantur item CD, CF. Et incrementum ſeu fluxio trianguli BCD æquale erit $BE \times BD$ — ſectore DCF: unde ſector DCF (qui eſt Fluxio ſectoris CVD) æqualis erit $BE \times BD$ — incremento trianguli BCD. Et ſi BC dicatur z, ob Hyperbolam, eſt $BD^2 = BC^2 + CV^2 = z^2 + c^2$: unde $BD = \sqrt{c^2+z^2}$, & $BE \times BD = \dot z \times \sqrt{c^2+z^2}$. Triangulum autem BCD eſt $\frac{1}{2}z \times \sqrt{c^2+z^2}$, cujus fluxio eſt $\frac{1}{2}\dot z \times \sqrt{c^2+z^2} + \frac{\frac{1}{2}\dot z \times z^2}{\sqrt{c^2+z^2}}$. Subtrahatur hæc quantitas ab $\dot z \times \sqrt{c^2+z^2}$, & reſtabit ſector Hyperbolæ minimus $CDF = \frac{1}{2}\dot z \times \sqrt{c^2+z^2} - \frac{\frac{1}{2}\dot z \times z^2}{\sqrt{c^2+z^2}} = \frac{\frac{1}{2}\dot z \times \overline{c^2+z^2} - \frac{1}{2}\dot z \times z^2}{\sqrt{c^2+z^2}}$ $= \frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}}$. Adeoque fluens ſectoris CDF eſt æqualis fluenti quantitatis $\frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}}$. Proinde erit ſector CVD fluens quantitatis $\frac{\frac{1}{2}c^2\dot z}{\sqrt{c^2+z^2}}$. Præterea DT recta tangat Hyperbolam & occurrat Axi conjugato in T. Eſt ex natura Hyperbolæ $BC : CV :: CV : CT$, hoc eſt $z : c :: c : \frac{c^2}{z} = CT = x$. Atque hinc oritur conſtructio quæ ſequitur.

Fig. 5.

Centro C ſemiaxe tranſverſo CV, deſcribatur Hyperbola æquilatera Vm, item circulus Ve. Capiatur ſector circularis CVe ad ſectorem Hyperbolicum CVm ut n ad 1; tangat Hyperbolam in m recta Tm, occurrens Axi conjugato in T; producatur Ce ad k ut ſit $Ck = CT$, & punctum k erit in Curva quæſita. Nempe talis eſt ea Curva, ut ſi Ck dicatur x, Perpendicularis a C in tangentem ejus demiſſa erit ſemper æqualis $\frac{ax}{\sqrt{b^2+x^2}}$. Quando x eſt infinita evaneſcit b^2, & perpendicularis fit $= a$, & tunc coincidit CR cum CV. Si itaque capiatur in axe conjugato $CR = a$, & ducatur RS ipſi CV parallela, erit hæc Curvæ Aſymptotos.

Fig. 6.

Si eo uſque augeatur a ut fiat quantitas $b - a$ infinite parva, tunc evaneſcet c^2, & quantitas $\frac{ba\dot x}{x\sqrt{x^2+c^2}}$ fit $\frac{ba\dot x}{x^2} = \dot y$. Unde ſi capiantur harum quantitatum fluentes, habebimus $\frac{ba}{x} = y$, & $ba = xy$, hoc eſt rectangulum ſub arcu circulari & diſtantia Curvæ a centro erit ſemper data quantitas; atque hac ratione migrabit curva in ſpiralem Hyperbolicam. Eſt itaque ſpiralis Hyperbolica Curva media ſeu quaſi limes, inter eas Curvas quæ conſtruuntur per ſectores circulares & eas quæ conſtruuntur per ſectores Hyperbolicos. Itaque ſpiralis illa Hyperbolica concipi poteſt formari vel per ſectorem Circuli aut Ellipſis, vel per ſectorem Hyperbolæ, cujus Axis tranſverſus minuitur in infinitum, & in eadem ratione augetur numerus n.

Ad eum jam devenimus caſum ubi velocitas corporis minor eſt ea quæ acquiritur cadendo ab infinita diſtantia, & ubi $p^2 = \frac{a^2 x^2}{b^2 - x^2}$. Et hic, ſimili ratiocinio ac in priori caſu, invenietur $KN = -\frac{a\dot{x}}{\sqrt{b^2 - a^2 - x^2}}$, ubi neceſſe eſt ut ſit b^2 majus quam a^2. Hinc ſi $b^2 - a^2$ dicatur c^2, fit $KN = -\frac{a\dot{x}}{\sqrt{c^2 - x^2}}$; & proinde XY ſeu $\dot{y} = \frac{ba\dot{x}}{x\sqrt{c^2 - x^2}}$.

Sit jam $x = \frac{c^2}{z}$ & fiet $\frac{\dot{x}}{x} = -\frac{\dot{z}}{z}$ ſeu $\frac{ba\dot{x}}{x} = -\frac{ba\dot{z}}{z}$ & $c^2 - x^2$ erit $= \frac{c^2}{z^2} \times z^2 - c^2$, quibus valoribus ſubſtitutis fit $\frac{-ba\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{ba\dot{x}}{x\sqrt{x^2 - c^2}} = -\dot{y}$. Nam tale ponendum eſt initium arcus VX, ut ſimul cum fluente quantitatis $\frac{ba\dot{z}}{c\sqrt{z^2 - c^2}}$ incipiat: unde erit $\frac{\frac{1}{2} b^2 a\dot{z}}{c\sqrt{z^2 - c^2}} = \frac{1}{2} b\dot{y} =$ ſectori $CXY = \frac{\frac{1}{2} nb^2\dot{z}}{\sqrt{z^2 - c^2}}$, ponendo $nc = a$. Eſt vero $\frac{\frac{1}{2} nb^2\dot{z}}{\sqrt{z^2 - c^2}}$ ad $\frac{\frac{1}{2} c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nb^2 ad c^2, hoc eſt in ratione conſtanti: Quare harum quantitatum Fluentes ſunt in eadem ratione, hoc eſt Fluens quantitatis $\frac{1}{2} b\dot{y}$ ſeu $\frac{\frac{1}{2} nb^2\dot{z}}{\sqrt{z^2 - c^2}}$ erit ad fluentem quantitatis $\frac{\frac{1}{2} c^2\dot{z}}{\sqrt{z^2 - c^2}}$ ut nb^2 ad c^2. Eſt autem fluens quantitatis $\frac{1}{2} b\dot{y} =$ ſectori CVX, & Fluens quantitatis $\frac{\frac{1}{2} c^2\dot{z}}{\sqrt{z^2 - c^2}}$ eſt ſector Hyperbolæ, quod ſic oſtenditur.

Fig. 7.

Centro C ſemiaxe tranſverſo $CV = c$ deſcribatur Hyperbola æquilatera, & ex duobus punctis infinite vicinis B & D ad axem ordinentur duæ rectæ BE, DF; ducantur item CB, CD. Et erit Fluxio ſeu incrementum trianguli $CBE =$ triangulo $CBD + BE \times EF$; unde triangulum CBD, ſeu ſector minimus CBD, erit $=$ incremento trianguli $CBE - BE \times EF$. Dicatur CE, z, & erit $BE = \sqrt{z^2 - c^2}$, & $BE \times EF = \dot{z}\sqrt{z^2 - c^2}$. Eſt quoque triangulum $CBE = \frac{1}{2} z\sqrt{z^2 - c^2}$, cujus Fluxio eſt $\frac{1}{2}\dot{z} \times \sqrt{z^2 - c^2} + \frac{\frac{1}{2}\dot{z} \times z^2}{\sqrt{z^2 - c^2}}$; a quo ſi ſubtrahatur quantitas $\dot{z} \times \sqrt{z^2 - c^2}$, fit ſector minimus $CBD = \frac{\frac{1}{2}\dot{z} \times z^2}{\sqrt{z^2 - c^2}} - \frac{1}{2}\dot{z} \times \sqrt{z^2 - c^2} = \frac{\frac{1}{2}\dot{z} \times z^2 - \frac{1}{2}\dot{z} \times \overline{z^2 - c^2}}{\sqrt{z^2 - c^2}} =$

$\frac{1}{2} c^2$

$\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$: unde conſtat ſectorem CBE eſſe fluentem quantitatis $\frac{\frac{1}{2}c^2\dot{z}}{\sqrt{z^2-c^2}}$. Præterea ſi BT tangens Hyperbolam Axi tranſverſo occurrat in T, ex natura Hyperbolæ fit $CE : CV :: CV : CT$, hoc eſt $z : c :: c : \frac{c^2}{z} = CT = x$. Fig. 8.

Hinc deducimus ſequentem conſtructionem. Centro C, ſemiaxe tranſverſo $CV = c$, deſcribatur Hyperbola æquilatera VB, & circulus CeG ex centro C. Ad hyperbolam ducatur recta CB, & hyperbolæ Tangens BT axi tranſverſo occurrat in T. Capiatur circuli ſector CVe, qui ſit ad ſectorem Hyperbolicum CVB ut n ad 1. In Ce capiatur $CK = CT$, & erit K punctum in Curva quæſita, cujus perpendiculum e centro C ad Tangentem in K demiſſum, ſi CK dicatur x, eſt æquale $\frac{ax}{\sqrt{b^2-x^2}}$.

Et in hac Curva, urgente vi centripeta quæ ſit reciproce ut cubus diſtantiæ, movebitur corpus, ſi ſecundum directionem Tangentis cum juſta velocitate exeat. Qualis autem debet eſſe velocitas quæ faciat ut corpus harum Curvarum quamvis deſcribat, ſic invenietur.

Cum velocitas qua corpus in trajectoria quacunque movetur ſit reciproce ut quantitas p, aſſumendo conſtantem quamvis a, ea ſemper exponi poteſt per $\frac{a}{p}$. Et ſi ad Axem CV ordinentur rectæ quæ ſint reciproce ut cubi diſtantiarum a centro, ſeu ut vires centripetæ, & hac ratione formetur Figura curvilinea, ejus Area indefinite extenſa ſemper exponi poteſt per $\frac{b^2}{x^2}$, ut ex Quadraturis conſtat. At Area illa eſt ut quadratum velocitatis quæ acquiritur ab infinita diſtantia cadendo, adeoque velocitas hoc caſu acquiſita erit ut $\frac{b}{x}$. Hinc ſi velocitas illa dicatur y, & velocitas qua corpus in Trajectoria movetur dicatur v, taleſque aſſumantur quantitates a & b, ut in una aliqua a centro diſtantia ſit $y : v :: \frac{b}{x} : \frac{a}{p}$, erit ubique in omnibus diſtantiis $y : v :: \frac{b}{x} : \frac{a}{p} :: p : \frac{ax}{b}$. Unde ſi $y = v$, erit $p = \frac{ax}{b}$, & Curva hac velocitate deſcripta erit Spiralis Nautica; vel Circulus exiſtente $p = x$ & $a = b$.

Si y ſit major quam v, tunc p major erit quam $\frac{ax}{b}$; eritque illa, ut ex præcedentibus conſtat, $= \frac{ax}{\sqrt{b^2-x^2}}$. Curva autem conſtruetur per ſectorem Hyperbolicum, ut in ultimo caſu oſtenſum fuit, ubi diſtantia corporis a centro per concurſum Tangentis Hyperbolæ cum Axe tranſverſo determinatur. Si y ſit minor quam v, at in tantilla ratione ut maneat b major quam a, Curva formabitur per eundem ſectorem hyperbolicum. At diſtantia corporis a centro deſumitur ex concurſu Tangentis cum Axe conjugato.

Si ſit $y : v :: p : x$, erit in eo caſu $a = b$, & Curva evadit Spiralis Hyperbolica, ubi eſt $p = \frac{ax}{\sqrt{a^2 + x^2}}$. Hinc ſi de loco quovis projiciatur corpus ſecundum datam rectam, cum ea velocitate quæ ſit ad velocitatem ab infinito cadendo acquiſitam, ut diſtantia corporis a centro ad perpendicularem e centro ad lineam directionis demiſſam, movebitur illud corpus in Spirali Hyperbolica. Si denique ſit v tanto major quam y, ut ſit etiam a major quam b, Curva conſtruetur per Sectores Circulares. Atque hac ratione data velocitate ſemper determinari poſſit relatio quantitatum a & b, ac proinde Curva deſcribetur in qua corpus cum illa velocitate movebitur: & viciſſim data Curva, ſeu datis quantitatibus a & b, invenietur velocitas qua Curva illa deſcribitur,

Omnium Curvarum Areæ (ſi circulum excipias) quæ urgente hac vi centripeta deſcribi poſſunt, ſunt perfecte quadrabiles. Nam primo, in ſpirali Logarithmica, quia eſt $p = \frac{ax}{b}$, erit $KN = \frac{a\dot{x}}{\sqrt{b^2 - a^2}} = \frac{a\dot{x}}{c}$, ponendo $b^2 - a^2 = c^2$: *vid. Fig.* II. adeoque erit triangulum $CKI = \frac{\frac{1}{2}ax\dot{x}}{c}$, cujus Fluens eſt $\frac{ax^2}{4c}$ = Areæ Curvæ.

Fig. 2.

Si p ſit $\frac{ax}{\sqrt{b^2 + x^2}}$, & a major quam b, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{x^2 - c^2}}$, unde $KN \times \frac{1}{2} CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2 - c^2}}$, cujus Fluens eſt $\frac{1}{2} a \times \sqrt{x^2 - c^2}$ = Areæ Curvæ. At ſi a minor ſit quam b, fit $KN = \frac{a\dot{x}}{\sqrt{x^2 + c^2}}$, & $KN \times \frac{1}{2} CI = \frac{\frac{1}{2}ax\dot{x}}{\sqrt{x^2 + c^2}}$, cujus Fluens eſt $\frac{1}{2} a \sqrt{x^2 + c^2} - \mathfrak{Q}$ = Areæ Curvæ. Ponatur $x = 0$, & fiet $\frac{1}{2} ac - \mathfrak{Q} = 0$, unde $\mathfrak{Q} = \frac{1}{2} ac$, & Area Curvæ fit $= \frac{1}{2} a \sqrt{x^2 + c^2} - \frac{1}{2} ac$.

In Spirali Hyperbolica evaneſcit quantitas c, & Area Curvæ fit $\frac{1}{2} ax$.

Si p ſit $= \frac{ax}{\sqrt{b^2 - x^2}}$, oſtenſum eſt eſſe $KN = \frac{a\dot{x}}{\sqrt{c^2 - x^2}}$, unde $\frac{1}{2} CI \times KN = \frac{\frac{1}{2} a \times x\dot{x}}{\sqrt{c^2 - x^2}}$, cujus fluens eſt $\mathfrak{Q} - \frac{1}{2} a \sqrt{c^2 - x^2}$ = Areæ. Fiat $x = 0$, & erit $\mathfrak{Q} - \frac{1}{2} ac = 0$, ſeu $\mathfrak{Q} = \frac{1}{2} ac$; unde erit Area Curvæ ſemper æqualis $\frac{1}{2} ac - \frac{1}{2} a \sqrt{c^2 - x^2}$. Fiat $c^2 - x^2 = 0$ ſeu $c = x$, & Area curvæ fit $\frac{1}{2} ac$. Unde ſi initium Areæ non capiatur ab initio ipſius x, ſeu ubi x eſt $= 0$, ſed ubi $x = c$ eſt maxima, hoc eſt ſi Area ab V incipiat, (*vid. Fig.* 7.) erit Area ſemper æqualis $\frac{1}{2} a \sqrt{c^2 - x^2}$.

Fig. 7.

De Areis quas deſcribunt corpora radiis ad centrum ductis, urgente vi centripeta quæ ſit reciproce ut diſtantiarum cubi, ſequentia adnotavit Collega meus peritiſſimus Geometriæ Profeſſor *Halleius*. Nempe ſi corpora diverſos circulos vel diverſas Spirales Hyperbolicas hac lege deſcribunt; erunt areæ ſectorum, tam in Circulis quam in Spiralibus illis omnibus, æqualibus temporibus deſcriptæ, ſemper æquales: Nam velocitates corporum in circulis motorum ſecundum hanc legem, debent eſſe radiis ſeu diſtantiis reciproce proportionales, adeoque arcus ſimul percurſi erunt quoque in eadem radiorum reciproca ratione, unde ſtatim patebit ſectores ſimul deſcriptos eſſe æquales.

v. Fig. 3.

In reliquis omnibus Curvis cum ſit velocitas ad velocitatem corporis in eadem diſtantia in circulo moti ut $\frac{a}{b} \times x$ ad p, ſeu ut $\frac{a}{x} \times IK$ ad KN; interea dum corpus in Trajectoria percurrit Lineolam IK, corpus aliud in Circulo in eadem diſtantia motum percurret arcum $\frac{b}{a} \times KN$; & Area Sectoris Circuli & Trajectoriæ ſimul deſcriptæ erunt $\frac{b}{a} \times KN \times \frac{1}{2} CN$ & $KN \times \frac{1}{2} CN$, quæ duæ Areæ ſunt in ratione data, ſcil. ut b ad a. Adeoque ubi eſt $a = b$, uti ſit in ſpirali Hyperbolica, Area ſic deſcripta erit ſemper æqualis Areæ Sectoris circularis in æquali tempore deſcriptæ.

Analytical Solution of certain infiniteſimal Equations, by Mr. De Moivre. *n.* 309. *p.* 2368.

XX. Sit n numerus quicunque, y quantitas incognita, ſive Æquationis Radix quæſita, ſitque a quantitas quævis omnino cognita, ſive ut vocant Homogeneum Comparationis: Atque horum inter ſe relatio exprimatur per Æquationem $ny + \frac{nn-1}{2 \times 3} ny^3 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} ny^5 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} \times \frac{nn-25}{6 \times 7} ny^7$, *&c.* $= a$.

Ex hujus ſeriei natura manifeſtum eſt, quod ſi n ſumatur numerus aliquis impar (integer ſcilicet, nec refert utrum ſit affirmativus vel negativus) tunc ſeries ſponte ſua terminabitur, & Æquatio ſit una ex ſupra præfinitis, cujus Radix eſt.

$$(1)\ y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}+a}}.$$

$$\text{vel}\,(2)\ y = \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}+a} - \tfrac{1}{2}\sqrt{\sqrt{1+aa}-a}$$

$$\text{vel}\,(3)\ y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \tfrac{1}{2}\sqrt[n]{\sqrt{1+aa}-a}:$$

$$\text{vel}\,(4)\ y = \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}} - \frac{\frac{1}{2}}{\sqrt[n]{\sqrt{1+aa}-a}}:$$

Exempli gratia, ſit hujus Æquationis poteſtatis quintæ $5y + 20y^3 + 16y^5 = 4$ Radix invenienda, quo in caſu erit $n = 5$, & $a = 4$. Radix juxta formam primam erit $y=\frac{1}{2}\sqrt[5]{\sqrt{17}+4}-\frac{\frac{1}{2}}{\sqrt[5]{\sqrt{17}+4}}$, quæ in numeris vulgaribus expeditiſſime explicari poteſt ad hunc modum. Eſt $\sqrt{17}+4=8.1231$, cujus Logarithmus 0.9097164, & hujus pars quinta 0.1819433, huic reſpondens numerus eſt $1.5203=\sqrt[5]{\sqrt{17}+4}$. Ipſius vero 0.1819433 Complementum Arithmeticum eſt 9.8180567, cui reſpondet numerus $0.6577=\frac{1}{\sqrt[5]{\sqrt{17}+4}}$. Igitur horum numerorum ſemidifferentia $0.4313=y$.

Hic venit Obſervandum quod loco Radicis generalis, non incommode ſumeretur $y=\frac{1}{2}\sqrt[n]{2a}-\frac{\frac{1}{2}}{\sqrt[n]{2a}}$, ſi quando numerus a reſpectu unitatis, ſit ſatis magnus, ut ſi Æquatio fuerit $5y+20y^3+16y^5=682$, erit Log. $2a=3.1348143$, cujus pars quinta 0.6269628, & huic reſpondens numerus 4.236. Complementi autem Arithmetici 9.3730372 numerus eſt 0.236 & horum numerorum ſemidifferentia $2=y$.

Atqui præterea, ſi in Æquatione præcedenti ſigna alternatim ſint affirmantia & negantia, vel quod eodem redit, ſi ſeries obvenerit hujus modi $ny+\frac{1-nn}{2\times3}ny^3+\frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}ny^5+\frac{1-nn}{2\times3}\times\frac{9-nn}{4\times5}\times\frac{25-nn}{6\times7}ny^7$, &c. $=a$. Erit hujus Radix

$$(1)\ y=\tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}}+\frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}.$$

$$\text{vel}\,(2)\ y=\tfrac{1}{2}\sqrt[n]{a+\sqrt{aa-1}}+\tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}.$$

$$\text{vel}\,(3)\ y=\frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}}+\tfrac{1}{2}\sqrt[n]{a-\sqrt{aa-1}}$$

$$\text{vel}\,(4)\ y=\frac{\frac{1}{2}}{\sqrt[n]{a-\sqrt{aa-1}}}+\frac{\frac{1}{2}}{\sqrt[n]{a+\sqrt{aa-1}}}.$$

Hic autem Notandum, quod ſi $\frac{n-1}{2}$ numerus extiterit impar, Radicis inventæ ſignum in ei contrarium permutandum eſt.

Proponatur Æquatio $5y-20y^3+16y^5=6$, unde $n=5$ & $a=6$. Erit Radix $=\frac{1}{2}\sqrt[5]{6+\sqrt{35}}\ \frac{+\frac{1}{2}}{\sqrt[5]{6+\sqrt{35}}}$. Vel quoniam $6+\sqrt{35}=11.916$, erit hujus logarithmus 1.0761304 & ejus pars quinta 0,2152561, Complementum vero Arithmeticum 9.7847439. Horum Logarithmorum numeri ſunt 1.6415 & 0.6091 reſpective, quorum ſemiſumma $1.1253=y$.

Verum ſi acciderit ut a ſit minor unitate, tunc Radicis forma ſecunda, ut quæ propoſito eſt magis conveniens, præ reliquis ſeligenda eſt. Sic ſi Æquatio fuerit $5y - 20y^3 + 16y^5 = \frac{61}{64}$, erit $y = \frac{1}{2}\sqrt[5]{\frac{61}{64} + \sqrt{\frac{-375}{4096}}} + \frac{1}{2}\sqrt[5]{\frac{61}{64} - \sqrt{\frac{-375}{4096}}}$. Et quidem ſi Binomialium Radix quintana ullo pacto extrahi queat, prodibit Radix proba & poſſibilis, etſi expreſſio ipſa impoſſibilitatem mentiatur. Binomialis vero $\frac{61}{64} + \sqrt{\frac{-375}{4096}}$ Radix quintana eſt $\frac{1}{4} + \frac{1}{4}\sqrt{-15}$, & Binomialis $\frac{61}{64} - \sqrt{\frac{-375}{4096}}$ Radix itidem quintana eſt $\frac{1}{4} - \frac{1}{4}\sqrt{-15}$; quorum Binomialium ſemiſumma $= \frac{1}{4} = y$.

Si autem extractio iſta vel non peragi poſſet, vel etiam difficilior videretur, res ubique confici poteſt per Tabulam ſinuum naturalium ad modum ſequentem.

Ad Radium 1 ſit $a = \frac{61}{64} = 0.95112$ ſinus arcus cujuſdam, qui proinde erit 72°: 23′ cujus pars quinta (eo quod $n = 5$) eſt 14°: 28′; hujus ſinus $0.24981 = \frac{1}{4}$ proxime. Nec ſecus procedendum in Æquationibus graduum ſuperiorum.

XXI. In *Phil. Tranſ.* * N°. 210. Dr. *Halley* has publiſh'd a compendious and uſeful Method of extracting the Roots of adfected Æquations of the common Form, in Numbers. This Method proceeds by aſſuming the Root deſir'd, nearly true, to one or two Places in Decimals (which is done by Geometrical Conſtruction, or ſome other convenient way) and correcting the Aſſumption, by comparing the Difference between the true Root and the aſſum'd, by means of a new Equation, whoſe Root is the Difference, and which he ſhews how to form from the Equation propos'd, by ſubſtituting the Value of the Root ſought, partly in known, and partly in unknown Terms.

Abridg. Vol. I. p. 81.

An Attempt to improve the Method of Approximating in extracting the Roots of Equations in Numbers, by Dr. Taylor. *n.* 352. *p.* 610.

In doing this, he makes uſe of a Table of Products, (which he calls *Speculum Analyticum*) by which he computes the Co-Efficients in the new Equation for finding the Difference mention'd. This Table, I obſerv'd, was form'd in the ſame manner from the Equation propos'd, as the Fluxions are, taking the Root ſought for the only flowing Quantity, its Fluxion for Unity, and after every Operation dividing the Product ſucceſſively by the Numbers, 1, 2, 3, 4, &c. Hence I ſoon found that this Method might eaſily and naturally be drawn from *Cor.* 2. *Prop.* 7. of my *Methodus Incrementorum*, and that it was capable of a further degree of Generality; it being applicable, not only to Equations of the common Form, (*viz.* ſuch as conſiſt of Terms wherein the Powers of the Root ſought are poſitive and integral, without any Radical Sign) but alſo to all Expreſſions in general, wherein any thing is propoſed as given which by any known Method might be computed; if *vice verſa*, the Root were conſider'd as given: ſuch as are all Radical Expreſſions of Binomials, Trinomials, or of any other Nomial, which may be computed by the Root given, at leaſt by Logarithms, whatever be the Index of the Power of that Nomial; as likewiſe Expreſ-

ſions of Logarithms, of Arches by the Sines or Tangents, of Areas of Curves by the *Abſciſſa*'s or any other Fluents, or Roots of Fluxional Equations, *&c.*

For the ſake of this great Generality, it may not be improper to ſhew how this Method is derived from the foreſaid *Corollary.* Therefore z and x being two flowing quantities (whoſe Relation to one another may be expreſt by any Equation whatſoever) by this *Corollary*, while z by flowing uniformly becomes $z + v$, x will become $x + \frac{\dot{x}}{1.\dot{z}} v + \frac{\ddot{x}}{1.2\dot{z}^2} v + \frac{\dddot{x}}{1.2.3\ddot{z}^3} v^3 + \&c.$ or $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1 \times 2} + \frac{\dddot{x}v^3}{1.2.3} + \&c.$ for $\dot{z}$ putting 1.

Hence if y be the Root of any Expreſſion formed of y and known Quantities, and ſuppoſed equal to nothing, and z be a part of y, and x be formed of z and the known Quantities, in the ſame manner as the Expreſſion made equal to nothing is formed of y; and let y be equal to $z + v$: the difference v will be found by Extracting the Root of this expreſſion $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3} + \&c. = 0$. For in this Caſe z being become $z + v = y$, x which is now become $x + \dot{x}v + \frac{\ddot{x}v}{2} + \&c.$ muſt become equal to nothing.

The Root v in the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} + \frac{\dddot{x}v^3}{1.2.3} + \&c. = 0$, is to be found upon the Suppoſition of its being very ſmall with reſpect to z, (as it muſt be, if z be taken tolerably exact) by which means the Terms $\frac{\dddot{x}v^3}{1.2.3} + \frac{\ddddot{x}v^4}{1.2.3.4} + \&c.$ may be neglected, upon account of their ſmallneſs with reſpect to the other Terms, ſo as to leave the Equation $x + \frac{\dot{x}v}{1} + \frac{\ddot{x}v^2}{1.2} = 0$, for finding the firſt approximation of v.

By extracting the Root of this Equation, we have $v = \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$. That is,

Firſt, $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Sec. $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}} - \frac{\dot{x}}{\ddot{x}}$, if $-x + \dot{x}v + \frac{\ddot{x}v^2}{2} = 0$.

Third. $\sqrt{\frac{\dot{x}}{\ddot{x}}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} - \frac{2x}{\ddot{x}}}$, if $x - \dot{x}v + \frac{\ddot{x}v^2}{2}, \&c. = 0$.

Fourth. $\sqrt{\frac{\dot{x}}{\ddot{x}}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}}}$, if $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$, &c. $= 0$.

This approximation gives v exact to twice as many places as there are true Figures in z, and therefore trebles the number of true Figures in the Expreſſion of y by $z + v$, which may be taken for a new Value of z, for computing a ſecond v, ſeeking other Values of x, $\dot{x}$, $\ddot{x}$, &c. Tho' when z is tolerably exact (which it may be eſteem'd when it contains two or three or more true Figures in the Value of y, according to the Number of Figures the Root is propoſed to be computed to,) the Calculation may be reſtor'd without ſo much trouble, only by taking $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{1.2.3.4\ddot{x}}v^4}$ &c. inſtead of $\sqrt{\frac{\dot{x}^2}{\ddot{x}^2} \pm \frac{2x}{\ddot{x}}}$ taking every time for v its Value laſt computed.

From the ſame Equation $x + \ddot{x}v + \frac{\ddot{x}v^2}{2} + \frac{\dddot{x}v^3}{1.2.3} +$ &c. $= 0$, may be gather'd alſo a rational Form, *viz.* $v = \frac{-x}{\dot{x} - \frac{x\ddot{x}}{2\dot{x}}}$. For neglecting the Terms $\frac{\dddot{x}v^3}{1.2.3}$, &c. we have $v = \frac{-x}{\dot{x} + \frac{\ddot{x}}{2}v}$ which is nearly $= \frac{-x}{\dot{x}}$. Therefore in the Diviſor inſtead of v writing $\frac{-x}{\dot{x}}$ we have more exactly $v = \frac{-x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, that is

1. $\frac{-x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, when $x + \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

2. $\frac{x}{\dot{x} + \frac{\ddot{x}x}{2\dot{x}}}$, when $-x + \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

3. $\frac{x}{\dot{x} - \frac{\ddot{x}x}{2\dot{x}}}$, when $x - \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

4. $\frac{-x}{\dot{x} + \frac{\ddot{x}x}{2\dot{x}}}$, when $-x - \dot{x}v + \frac{\ddot{x}v^2}{2}$ &c. $= 0$.

This *Formula* will alſo triplicate the number of true Figures in z. And the Calculation may be repeated, after every Operation, taking for a Diviſor $\dot{x} \pm \frac{\ddot{x}}{2} v + \frac{\dddot{x} v^2}{1.2.3} + \frac{\ddddot{x} v^3}{1.2.3.4}$ + &c. inſtead of $\dot{x} + \frac{\ddot{x} x}{2 \dot{x}}$.

Dr. *Halley* has fully explain'd the manner of uſing both theſe *Formula*'s in Æquations of the common Form; wherefore I ſhall be the ſhorter in explaining two or three Examples of another ſort.

Ex. 1. Let it be propoſed to find the Root of this Equation $\overline{y^2+1}|^{\sqrt{2}} + y - 16 = 0$. In this Caſe for y writing z, and for 0 writing x, we have $\overline{z^2+1}|^{\sqrt{2}} + z - 16 = x$. Whence by taking the Fluxions, we have $\dot{x} = 2\sqrt{2} \times z \times \overline{z^2+1}|^{\sqrt{2}-1} + 1$, and $\ddot{x} = 2\sqrt{2} \times 8 - 4\sqrt{2}\, z^2 \times \overline{z^2+1}|^{\sqrt{2}-2}$. For finding the firſt Figures of the Root y, for $\sqrt{2}$ take $\frac{3}{2}$, and we have the Equation $\overline{y^2+1}|^{\frac{3}{2}} + y - 16 = 0$, which being expanded gives $y^6 + 3y^4 + 2y^2 + 32y - 255 = 0$.

By this Equation I find that for the firſt ſuppoſition we may take $z = 2$. Therefore in order to find v, let us now make $\sqrt{2} = \frac{7}{5}$, (which is nearer than before) and we have $x = \overline{z^2+1}|^{\frac{7}{5}} + z - 16 = \overline{2^2+1}|^{\frac{7}{5}} - 14 = 5^{\frac{7}{5}} - 14 = -4{,}48$; $\dot{x} = 10{,}66$; $\ddot{x} = 4{,}72$. Whence by the ſecond rational Form $v = \cfrac{4{,}48}{10{,}66 + \cfrac{4{,}72 \times 4{,}48}{2 \times 10{,}66}} = 0{,}38$; which muſt be too big, becauſe $\frac{7}{5} \sqsubset \sqrt{2}$, and therefore will require a larger Value of y to exhauſt the Equation, than where $\sqrt{2}$ is exact. For the ſecond ſuppoſition therefore, let us take $z = 2{,}3$, and make $\sqrt{2} = 1{,}4142136$, and by help of the Logarithms we ſhall have $\overline{z^2+1}|^{\sqrt{2}} = 13{,}47294$, whence $x = -0{,}22706$; $\dot{x} = 14{,}93429$, and $\ddot{x} = 5{,}18419$. Hence by the 2*d* irrational *Formula* $v = \sqrt{\frac{14{,}93429^2}{5{,}18419^2} + \frac{0{,}45412}{5{,}18419}} - \frac{14{,}93429}{5{,}18419} = 0{,}01516$, which gives $y = z + v = 2{,}31516$, which is true to ſix Places. If you deſire it more exact than to the extent of the Tables of Logarithms, taking $z = 2{,}31516$ for the next Suppoſition, the Calculation muſt be repeated by computing of $\overline{zz+1}|^{\sqrt{2}}$ to a ſufficient number of Places; which muſt be done by the Binomial Series, or by making a Logarithm on purpoſe, true to as many places as are neceſſary.

Ex. 2. For another Example, let it be required to find the Number whoſe Logarithm is 0,29, ſuppoſing we had no other Table of Logarithms but Mr. *Sharp*'s of 200 Logarithms to a great many places. This amounts to the reſolving this Equation $ly = 0{,}29$, or $ly - 0{,}29 = 0$. Hence therefore we have $x = lx - 0{,}29$, $\dot{x} = \frac{a}{z}$ (*a* being the *Modulus* belonging to the Table we uſe, *viz.* 0,4342944819, &c.) $\ddot{x} = \frac{-a}{z^2}$ $\dddot{x} = \frac{2a}{z^3}$, $\ddddot{x} =$

$-6a$

$\frac{-6a}{z^4}$ &c. In this Caſe becauſe $\ddot{x}$ has a negative Sign, changing the Signs of all the Coefficients, the Canon for v will be found in the fourth Caſe, which in the irrational Form gives $v = \frac{\dot{x}}{\ddot{x}} - \sqrt{\frac{\dot{x}^2}{\ddot{x}^2} + \frac{2x}{\ddot{x}} - \frac{2\dddot{x}}{2.3\ddot{x}}v^3 - \frac{2\ddddot{x}}{2.3.4\ddot{x}}v^4 \text{ \&c.}} = z - \sqrt{z^2 + \frac{2lz - 0{,}58}{a} \times z^2 + \frac{2v^3}{3z} - \frac{2v^4}{4z^2} + \frac{2v^5}{5z^3} \text{ \&c.}}$ In this Caſe to avoid often dividing by z, it will be moſt convenient to compute $\frac{v}{z}$, which is got from this Equation $\frac{v}{z} = 1 - \sqrt{1 + \frac{2lz - 0{,}58}{a} + \frac{2v^3}{3z^3} - \frac{2v^4}{4z^4} + \frac{2v^5}{5z^5}}$, &c. The neareſt Logarithm, in the Tables propoſed, to the propoſed Logarithm 0,29 is 0,2900346114, its Number being 1,95. Therefore for the firſt ſuppoſition taking $z = 1{,}95$, we have $x\ (= lz - 0{,}29 = 0{,}2900346114 - 0{,}29) = 0{,}0000346114$, and $\frac{2lz - 0{,}58}{a} = \frac{0{,}0000692228}{0{,}4342944819} = 0{,}00015939139$, and $1 + \frac{2lz - 0{,}58}{a} = 1{,}00015939139$. Whence for the firſt approximation we have $\frac{v}{z} = 1 - \sqrt{1{,}00015939139} = -0{,}00007969247$, and $v = -0{,}00015540032$, and $y = z + v = 1{,}94984459968$. Which is true to eleven places, and may eaſily be corrected by the Terms $\frac{2v^3}{3z}$ &c. which I leave to the Readers curioſity.

Being upon the Subject of Approximations, it may not be amiſs to ſet down here two Approximations I have formerly hit upon. The one is a Series of Terms for expreſſing the Root of any Quadratick Equation: and the other is a particular Method of Approximating in the invention of Logarithms which has no occaſion for any of the Tranſcendental Methods, and is expeditious enough for making the Tables without much trouble.

A general Series for expreſſing the Root of any Quadratick Equation.

Any Quadratick Equation being reduc'd to this Form $xx - mqx + my = 0$, the Root x will be expreſt by this Series of Terms.

$$x = \frac{y}{q} + A \times \frac{1}{\frac{mq^2}{y} - 2} + B \times \frac{1}{a^2 - 2} + C \times \frac{1}{b^2 - 2} + D \times \frac{1}{c^2 - 2}$$

&c. Which muſt be thus interpreted.

1. The Capital Letters A, B, C, &c. ſtand for the whole Terms with their Signs, preceding thoſe wherein they are found, as $B = A \times \frac{1}{\frac{mq^2}{y} - 2}$.

2. The little Letters a, b, c, &c. in the Diviſors, are equal to the whole Diviſors of the Fraction in the Terms immediately preceding; thus $b = a^2 - 2$.

For an Example of this, let it be required to find $\surd 2$. Putting $\surd 2 = x + 1$, we have $x^2 + 2x - 1 = 0$. which being compared with the general *Formula*, gives $mq = -2$, and $my = -1$: therefore for m taking -1, we have $q = 2$, and $y = 1$, which Values ſubſtituted in the Series give $x = \frac{1}{2} - \frac{1}{2 \times 6} - \frac{1}{2 \times 6 \times 34} - \frac{1}{2 \times 6 \times 34 \times 1154} - \frac{1}{2 \times 6 \times 34 \times 1154 \times 1331714}$ &c. The Fractions here wrote down giving the Root true to twenty three Places.

A new Method of computing Logarithms.

This Method is founded upon theſe Conſiderations.

1. That the Sum of the Logarithms of any two Numbers is the Logarithm of the Product of thoſe two Numbers Multiplied together.

2. That the Logarithm of Unite is nothing; and conſequently that the nearer any Number is to Unite, the nearer will its Logarithm be to 0. 3*dly*. That the Product by Multiplication of two Numbers, whereof one is bigger, and the other leſs than Unite, is nearer to Unite than that of the two Numbers which is on the ſame ſide of Unite with its ſelf; for Example the two Numbers being $\frac{2}{3}$ and $\frac{4}{3}$, the Product $\frac{8}{9}$ is leſs than Unite, but nearer to it than $\frac{2}{3}$, which is alſo leſs than Unite. Upon theſe Conſiderations, I found the preſent Approximation; which will be beſt explain'd by an Example. Let it therefore be propoſed to find the Relation of the Logarithms of 2 and of 10. In order to this, I take two Fractions $\frac{128}{100}$ and $\frac{8}{10}$, *viz.* $\frac{\overline{2|}^{7}}{10^{2}}$ and $\frac{\overline{2}\,^{3}}{10|^{1}}$ whoſe Numerators are Powers of 2, and their Denominators Powers of 10; one of them being bigger, and the other leſs than 1. Having ſet theſe down in Decimal Fractions in the firſt Column of the Table annext, againſt them in the ſecond Column I ſet A and B for their Logarithms, expreſſing by an Equation the manner how they are Compounded of the Logarithms of 2 and 10, for which I write $l\,2$ and $l\,10$. Then Multiplying the two Numbers in the firſt Column together, I have a third Number 1,024, againſt which I write C for its Logarithm, expreſſing likewiſe by an Equation in what manner C is formed of the foregoing Logarithms A and B. And in the ſame manner the Calculation is continued; only obſerving this *Compendium*, that before I Multiply the two laſt Numbers already got in the Table, I conſider what Power of one of them muſt be uſed to bring the Product the neareſt to Unite that can be. This is found, after we have gone a little way in the Table, only by dividing the Differences of the Numbers from Unite one by the other, and taking the Quotient with the neareſt, for the Index of the Power wanted. Thus the two laſt Num-

bers in the Table being 0,8 and 1,024, their Differences from Unit are 0,200 and 0,024; therefore $\frac{0,200}{0,024}$ gives 9 for the Index; wherefore Multiplying the ninth Power of 1,024 by 0,8, I have the next Number 0,990352031429, whoſe Logarithm is $D = 9C + B$. In ſeeking the Index in this manner by Diviſion of the Differences, the Quotient ought generally to be taken with the leaſt: but in the preſent Caſe it happens to be the moſt, becauſe inſtead of the Difference between 0,8 and 1, we ought ſtrictly to have taken the Difference between the reciprocal 1,25 and 1, which would have given the Index 10; and that would be too big, becauſe the Product by that means would have been bigger than 1, as 1,024 is. Whereas this Approximation requires that the Numbers in the firſt Column be alternately greater and leſs than 1, as may be ſeen in the Table.

1, 280000000000	$A = 7\,l\,2 - 2\,l\,10$	$l\,2$	$> 0,28$
0, 800000000000	$B = 3\,l\,2 - l\,10$		$< 0,33$
1, 024000000000	$C = B + A = 10\,l\,2 - 3\,l\,10$		$> 0,300$
0, 990352031429	$D = 9\,C + B = 93\,l\,2 - 28\,l\,10$		$< 0,30107$
1, 004336277664	$E = 2\,D + C = 196\,l\,2 - 59\,l\,10$		$> 0,301020$
0, 998959536107	$F = 2\,E + D = 485\,l\,2 - 146\,l\,10$		$< 0,3010309$
1, 000162894165	$G = 4\,F + E = 2136\,l\,2 - 643\,l\,10$		$> 0,30102996$
0, 999936281874	$H = 6\,G + F = 13301\,l\,2 - 4004\,l\,10$		$< 0,301029997$
1, 000035441215	$I = 2\,H + G = 28738\,l\,2 - 8651\,l\,10$		$> 0,3010299951$
0, 999971720830	$K = I + H = 42039\,l\,2 - 12655\,l\,10$		$< 0,3010299959$
1, 000007161046	$L = K + I = 70777\,l\,2 - 21306\,l\,10$		$> 0,30102999562$
0, 999993203514	$M = 3\,L + K = 254370\,l\,2 - 76573\,l\,20$		$< 0,30102999567$
1, 000000364511	$N = M + L = 325147\,l\,2 - 97879\,l\,10$		$> 0,301029995 6635$
0, 999999764687	$O = 18\,N + M = 6107016\,l\,2 - 1838395\,l\,10$		$< 0,3010299956640$
Com. Ar. 235313			
$0 = 364511\,O + 235313\,N = 2302585825187\,l\,2 - 693147400972\,l\,10$			$> 0,3010299956 63987$

When I have in this manner continued the Calculation, till I have got the Numbers ſmall enough, I ſuppoſe the laſt Logarithm to be equal to nothing. Which gives me an Equation, from which having got away the Letters by means of the foregoing Equations, I have the relation of the Logarithms propoſed. In this manner if I ſuppoſe $G = 0$, I have $2136\,l\,2 - 643\,l\,10 = 0$. Which gives the Logarithm of 2 true in ſeven Figures, and too big in the Eighth; which happens becauſe the Number correſponding with G is bigger than Unite.

There is another Expedient which renders this Calculation ſtill ſhorter. It is founded upon this Conſideration, that when x is very ſmall $\overline{1+x}|^n$ is very nearly $1 + nx$. Hence if $1 + x$, and $1 - z$ are the two laſt Numbers already got in the firſt Column of the Table, and their Powers $\overline{1+x}|^m$ and $\overline{1-z}|^n$ are ſuch as will make the Product $\overline{1+x}|^m \times \overline{1+z}|^n$ very near to Unite, m and n may be found thus: $\overline{1+x}|^m = 1 + mx$, and $\overline{1-z}|^n = 1 - nz$, and conſequently $\overline{1+x}|^m \times \overline{1-z}|^n = 1 + mx - nz - mnzx$, or (neglecting $mnzx$) $1 + mx - nz$. Make this equal to 1, and we have $m : n :: z : x :: l\,\overline{1-z} : l\,\overline{1+x}$. Whence $x\,l\,\overline{1-z} + z\,l\,\overline{1+x} = 0$. To give an Example of the Application of this, let 1,024 and 0,990352 be the laſt Numbers in the Table, their Logarithms being C and D. Then we have $1,024 = 1 + x$, and $0,990352 = 1 - z$, and conſequently $x = 0,024$, and $z = 0,009648$. Whence the Ratio $\frac{z}{x}$ in the leaſt Numbers is $\frac{201}{500}$. So that for finding the Logarithms propoſed we may have $500\,D + 201\,C = 48510\,l\,2 - 14603\,l\,10 = 0$, which gives $l\,2 = 0,3010307$, which is too big in the laſt Figure; but it is nearer the truth, than what is got from the Logarithm F ſuppoſed equal to nothing. So that by this means we have ſaved four Multiplications, which were neceſſary to find the Number 9989595 &c. correſpondent to F, and which muſt have been had if we would make the Logarithm true to the ſame Number of places without this *Compendium*.

Of infinite Series's, by Mr. Monmort. *n.* 353. *p.* 633.

XXI. *Prop.* I. *Prob.* Invenire ſummam terminorum quot libuerit Seriei hujus $a \times \overline{a+n} \times \overline{a+2n} \times \&c. \times \overline{a+\overline{p-1}\,n} + \overline{a+n} \times \overline{a+2n} \times \overline{a+3n} \times \&c. \times \overline{a+pn} + \overline{a+2n} \times \overline{a+3n} \times \overline{a+4n} \times \&c. \times \overline{a+\overline{p+1}\,n} + \overline{a+3n} \times \&c.$ Ubi eſt n differentia data, tam inter Factores continuos, a, $a + 2n$, &c. ejuſdem cujuſvis termini, quam inter Factores homologos terminorum diverſorum in Serie continuata; atque deſignat p numerum factorum hujuſmodi in quovis termino.

Solutio. Per x deſignetur primus Factorum in ultimo terminorum quorum ſumma requiritur, atque ſumma illa erit

$$\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \&c. \times \overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n} \quad \textit{Q.E.I.}$$

Ex. 1. Proponatur Series numerorum naturalium $1 + 2 + 3 + 4 +$ &c. & invenienda ſit ſumma tot terminorum quot ſunt unitates in nu-

mero z, qui in hoc caſu eſt etiam ultimus terminorum quorum ſumma requiritur. In hoc itaque caſu ſunt $a=1$, $n=1$, $p=1$, & $x=z$. Unde fit $x \times \overline{x+n} \times \&c. \times \overline{x+pn} = z \times \overline{z+1}$, $\overline{a-n} \times a \times \&c. \times \overline{a+\overline{p-1}\,n} = 0 \times 1$, atque $\overline{p+1}\,n = 2 \times 1$; adeoque ſumma quæſita eſt $\frac{z \times \overline{z+1}}{2}$.

Ex. 2. Invenienda ſit ſumma tot terminorum, quot ſunt unitates in numero z, Seriei $1+3+6+10+\&c.$ Numerorum Triangularium. Numeri 1, 3, 6, 10, *&c.* in hac Serie ſic ſcribi poſſunt $\frac{1\times2}{2}$, $\frac{2\times3}{2}$, $\frac{3\times4}{2}$, $\frac{4\times5}{2}$, *&c.* Hoc pacto, ſepoſito diviſore dato 2, Series revocatur ad formam Propoſitionis, exiſtentibus $a=1$, $n=1$, & $p=2$, $x=z$. Unde ſumma Seriei duplicata eſt $\frac{x\times\overline{x+1}\times\overline{x+2} - 0\times1\times2}{3} = \frac{x\times\overline{x+1}\times\overline{x+2}}{3}$; adeoque habita ratione diviſoris 2, Summa Seriei ipſius eſt $\frac{x\times\overline{x+1}\times\overline{x+2}}{2\times3}$, vel $\frac{z\times\overline{z+1}\times\overline{z+2}}{2\times3}$, in hoc caſu exiſtente x eodem ac z. Ad eundem modum inveniuntur ſummæ cæterorum numerorum figuratorum, quorum formulæ jam vulgo innoteſcunt.

Ex. 3. Sint $a=1$, $n=2$, $p=3$, ut ſit Series propoſita $1\times3\times5+3\times5\times7+5\times7\times9+\&c.$ In hoc itaque caſu formula ſummæ fit $\frac{x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6} - \overline{1-2}\times1\times3\times5}{4\times2} = \frac{x\times\overline{x+2}\times\overline{x+4}\times\overline{x+6}+15}{8}$. Verbi gratia ſi quæratur ſumma decem terminorum, fit $x=19$ (nempe terminus decimus in Serie Arithmetice proportionalium, 1, 3, 5, 7, *&c.*) adeoque ſumma eſt $\frac{19\times21\times23\times25+15}{8} = 28680$. Propoſitio vero ſic demonſtratur.

Demonſtratio. Sit Series quantitatum $A, B, C, D, E,$ *&c.* quarum differentiæ conſtituant Seriem $a, b, c, d,$ *&c.* (nempe ut ſint $a=B-A$, $b=C-B$, $c=D-C$, *&c*) Hinc ſtatim colligitur eſſe $a+b=C-A$, $a+b+c=D-A$, $a+b+c+d=E-A$: & in genere aggregatum quotlibet terminorum Seriei $a, b, c, d,$ *&c.* æquale eſt termino proxime inſequenti Seriei $A, B, C, D, E,$ *&c.* mulctato termino primo A. Pro $A, B, C,$ *&c.* ſume terminos $\frac{\overline{a-n}\times a\times\&c.\times\overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n}$, $\frac{a\times\overline{a+n}\times\&c.\times\overline{a+pn}}{\overline{p+1}\,n}$, $\frac{\overline{a+n}\times\overline{a+2}\,n\times\&c.\times\overline{a+\overline{p+1}\,n}}{\overline{p+1}\,n}$, *&c.* hoc eſt, valores ſucceſſivos ipſius $\frac{x\times\overline{x+n}\times\&c.\times\overline{x+pn}}{\overline{p+1}\,n}$; & eorum differentiæ, pro $a, b, c, d,$ *&c.* ſumendæ, erunt $a\times\overline{a+n}\times\&c.\times\overline{a+\overline{p-1}\,n}$, $\overline{a+n}\times\overline{a+2n}\times\&c.\times\overline{a+pn}$, *&c.* qui ſunt ipſiſſimi termini Seriei propoſitæ. Sed comparando has

Series, ſi terminus aliquis Seriei poſterioris ſit $x \times \overline{x+n} \times \&c. \times \overline{x+\overline{p-1}\,n}$, conſtat terminum uno ulteriorem in Serie priori fore $\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn}}{\overline{p+1}\,n}$. Summa itaque Seriei poſterioris uſque terminum $x \times \overline{x+n} \times \&c. \times \overline{x+\overline{p-1}\,n}$ incluſive eſt $\frac{x \times \overline{x+n} \times \&c. \times \overline{x+pn} - \overline{a-n} \times a \times \&c. \times \overline{a+\overline{p-1}\,n}}{\overline{p+1}\,n}$ *Q. E. D.*

Scholium. 1. In hac propoſitione continetur particula quædam Methodi incrementorum, de qua ante biennium librum edidit D. *Brook Taylor Soc. Reg. Lond. Secr.* mihi amicitia conjunctiſſimus. Librum ipſum adeat qui de ea methodo plura ſcire velit: ad inſtitutum noſtrum ſufficit obſervare quanta interſit affinitas inter Methodum hanc & Methodum Fluxionum ſeu differentialem. Nam ut in Methodo differentiali, ad inveniendum differentiale ipſius x dignitatis x^m, unum latus x convertendum eſt in differentiam dx; & ortum ducendum eſt in dignitatis Indicem m, ut ſit $m\,dx\,x^{m-1}$ differentiale quæſitum; ſic in Methodo Incrementorum *Ad inveniendum Incrementum facti hujuſmodi* $x \times \overline{x+n} \times \overline{x+2n}$, (*ubi factores* x, $x+n$, $x+2n$, *ſunt in progreſſione Arithmetica, cujus differentia communis eſt ipſius* x *Incrementum datum* n,) *Factorum minimus* x *convertendus eſt in Incrementum, & ortum ducendum eſt in numerum Factorum, ut ſit* $3n \times \overline{x+n} \times \overline{x+2n}$ *Incrementum quæſitum, numero Factorum in caſu expoſito exiſtente* 3. Sic etiam ipſius $x \times \overline{x+n}$ Incrementum fit $2n \times \overline{x+n}$.

2. Incrementa etiam Reciprocorum hujuſmodi Factorum inveniuntur per eandem regulam; hoc nempe obſervato, quod cum ſit Diviſio contrarium Multiplicationis, vice ablationis minimi Factorum, ſit jam addendus alius factor adhuc uno Incremento major; item quod Factorum numerus ſit ſcribendus cum ſigno negativo. Hoc pacto ipſius $\frac{1}{x}$ Incrementum fit $\frac{-1 \times n}{x \times \overline{x+n}}$; ipſius $\frac{1}{x \times \overline{x+n}}$ Incrementum fit $\frac{-2 \times n}{x \times \overline{x+n} \times \overline{x+2n}}$; & ſic de aliis hujuſmodi. Hoc facile probatur ſumendo differentias inter Integralium valores duos continuos.

3. Inſiſtendo veſtigiis Methodi directæ, hinc colliguntur præcepta Methodi inverſæ, quibus inveniuntur Integralia Incrementorum oblatorum. *Applicetur* enim *Incrementum oblatum ad lateris Incrementum datum; addatur Factor adhuc uno Incremento minor, & applicetur ortum ad numerum Factorum ſic auctorum.* Sic *e. g.* oblato Incremento $n \times x \times \overline{x+n} \times \overline{x+2n}$, fit primo $x \times \overline{x+n} \times \overline{x+2n}$; deinde $\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}$, addito Factore $\overline{x-n}$; denique $\frac{\overline{x-n} \times x \times \overline{x+n} \times \overline{x+2n}}{4}$, quod eſt Integrale quæſitum. Hoc quidem ubi Factores ſunt Multiplicantes; Ubi vero Factores occupant locum diviſoris, mutatis mutandis, regula hæc eſt, *Applicetur Incrementum oblatum ad lateris incrementum datum; rejiciatur Factorum maximus, & applicetur ortum ad numerum Factorum relictorum cum ſigno negativo.* Exempli gratia oblato Incremento

$\frac{n}{x \times \overline{x+n} \times \overline{x+2n}}$, ſit primo $\frac{1}{x \times \overline{x+n} \times \overline{x+2n}}$, deinde $\frac{1}{x \times \overline{x+n}}$, denique $\frac{1}{-2 \times x \times \overline{x+n}}$, ſeu $\frac{-1}{2x \times \overline{x+n}}$, quod eſt Integrale quæſitum.

4. In caſu hoc noviſſimo Integrale inventum, cum ſigno contrario, æquale eſt ſummæ omnium Incrementorum in Serie in infinitum continuata; *v. g.* eſt $\frac{1}{2x \times \overline{x+n}} = \frac{n}{x \times \overline{x+n} \times \overline{x+2n}} + \frac{n}{\overline{x+n} \times \overline{x+2n} \times \overline{x+3n}} + \frac{n}{\overline{x+2n} \times \overline{x+3n} \times \overline{x+4n}}$ + *&c.* Nam in hoc caſu, facto x tandem infinito, evaneſcit $\frac{1}{2x \times \overline{x+n}}$, hoc eſt, ultimus terminorum $A, B, C,$ *&c.* ſit nihil; & ob contrarietatem ſignorum Integralis & Incrementi, vice $-A$ exprimitur aggregatum per $+A$.

Lemma 1. Per X deſignetur terminus quilibet in Serie quavis numerorum $M, N, O, P,$ *&c*; per x deſignetur locus termini iſtius X in Serie illa (*v. g.* ut ſit $x = 1$, quando deſignat X terminum primum M; ſit $x = 2$, quando deſignat X terminum ſecundum N, & ſic de cæteris) & ſint terminorum M, N, O, P prima differentiarum primarum b, c prima differentiarum ſecundarum, d prima tertiarum, e prima quartarum, & ſic porro. Tum erit $X = M + b \times \frac{x-1}{1} + c \times \frac{x-1}{1} \times \frac{x-2}{2} + d \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} + e \times \frac{x-1}{1} \times \frac{x-2}{2} \times \frac{x-3}{3} \times \frac{x-4}{4}$ + *&c.* Sequitur hoc ex tabula æquationum *pag.* 66. tractatus noſtri *Eſſay d' Analyſe, &c.*

Lemma 2. Iiſdem poſitis, per z deſignetur terminus quilibet in Serie Arithmetice proportionalium $a, \overline{a+n}, \overline{a+2n},$ *&c.* & ſit jam $X = A + Bz + Cz \times \overline{z+n} + Dz \times \overline{z+n} \times \overline{z+2n} + Ez \times \overline{z+n} \times \overline{z+2n} \times \overline{z+3n}$ + *&c.* Tum ipſorum $A, B, C, D, E,$ *&c.* valores erunt

$$A = M + b \times \frac{-a}{n} + c \times \frac{-a}{n} \times \frac{-a-n}{2n} + d \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} + e \times \frac{-a}{n} \times \frac{-a-n}{2n} \times \frac{-a-2n}{3n} \times \frac{-a-3n}{4n} + \&c.$$

$$B = \frac{1}{n} \times b + c \times \frac{-a-n}{n} + d \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} + e \times \frac{-a-n}{n} \times \frac{-a-2n}{2n} \times \frac{-a-3n}{3n} \&c.$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times c + d \times \frac{-a-2n}{n} + e \times \frac{-a-2n}{n} \times \frac{-a-3n}{2n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} d + e \times \frac{-a-3n}{n} + \&c.$$

$$E = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \frac{1}{4n} e + \&c.$$

Ordo formandi coefficientes ipſorum b, c, d, e, &c. in his valoribus, per ſe eſt ſatis manifeſtus.

Demonſtratio. Quoniam per x & z deſignantur termini correſpondentes progreſſionum Arithmeticarum $1, 2, 3, 4,$ &c. & $a, a+n, a+2n, a+3n,$ &c. indicabit $x-1$ numerum differentiarum n qui in z continetur, ut ſit $z = a + \overline{x-1}\, n$. Hinc fit $x-1 = \frac{z-a}{n}$, $x-2 = \frac{z-n-a}{n}$, $x-3 = \frac{z-2n-a}{n}$, &c. Subſtituendo itaque hos valores $x-1$, $x-2$, $x-3$, &c in Serie Lemmatis præcedentis, & terminis in ordinem redactis, prodeunt ipſorum A, B, C, &c. valores exhibiti.

Cor. Ubi $a = n$, prodeunt A, B, C, D, &c. per formulas ſimpliciores, nempe

$$A = M - b + c - d + e \ \&c.$$

$$B = \frac{1}{n} \times \overline{b - 2c + 3d - 4e \ \&c.}$$

$$C = \frac{1}{n} \times \frac{1}{2n} \times \overline{c - 3d + 6e \ \&c.}$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times \overline{d + 4e \ \&c.}$$

Lemma 3. Symbolis X & x eodem modo interpretatis ac in Lemmate primo, ſint q, r, s, t, u, &c. generatores Trianguli Arithmetici, cujus lineam tranſverſam occupat Series M, N, O, P, Q, &c. in ordine nempe inverſo, ut ſit q $(= M)$ generator ultimus, r penultimus, s antepenultimus, & ſic porro. Tum erit

$$X = q + r \times \frac{x-1}{1} + s \times \frac{x-1}{1} \times \frac{x}{2} + t \times \frac{x-1}{1} \times \frac{x}{2} \times \frac{x+1}{3} + \&c.$$

Conſtat ex contemplatione ipſius Trianguli Arithmetici, quam exhibuimus *pag.* 63. tractatus *Eſſay d' Analyſe*, &c. ubi idem fuſius explicatur.

Lemma 4. Iiſdem poſitis, & Symbolo z eodem modo interpretato ac in *Lem.* 2. ſi ſit $X = A + Bz + Cz \times \overline{z+n} + \&c.$ ut in *Lem.* 2. erunt coefficientium A, B, C, D, &c. valores:

$$A = q + r \times \frac{-a}{n} + s \times \frac{-a}{n} \times \frac{-a+n}{2n} + t \times \frac{-a}{n} \times \frac{-a+n}{2n} \times \frac{-a+2n}{3n} + \&c.$$

$$B = \frac{1}{n} \times r + s \times \frac{-a}{n} + t \times \frac{-a}{n} \times \frac{-a+n}{2n} + \&c.$$

C=

$$C = \frac{1}{n} \times \frac{1}{2n} \times s + t \times \frac{\overline{-a}}{n} + \&c.$$

$$D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n} \times t + \&c.$$

Ordo coefficientium in his valoribus eſt manifeſtus, & demonſtratur Lemma ad modum *Lemmatis* 2.

Cor. 1. Ubi $a = n$, coefficientes $A, B, C, D,$ *&c.* prodeunt per formulas ſimpliciores, nempe

$$A = q - r, \qquad C = \frac{1}{n} \times \frac{1}{2n} \times \overline{s - t}$$

$$B = \frac{1}{n} \times \overline{r - s}, \qquad D = \frac{1}{n} \times \frac{1}{2n} \times \frac{1}{3n}\, \overline{t - u}, \ \&c.$$

Cor. 2. Unde ſi generatorum $q, r, s, t, u,$ *&c.* aliquot ſint inter ſe æquales, exhibebitur X per formulam ſimpliciorem, evaneſcentibus aliquot coefficientium $A, B, C, D,$ *&c.*

Sic exempli gratia, propoſita Serie numerorum 4, 69, 530, 2676, 10350, *&c.* qui conſtituunt lineam decimam tranſverſam in Triangulo Arithmetico cujus generatores tres priores ſunt 54, −18, 5, & ſeptem poſteriores ſunt æquales 4; exiſtente $a = 1 = n$, Terminus X exhibetur per formulam quatuor tantum terminorum. $-\frac{z}{1} \cdot \frac{z+1}{2} \cdot \frac{z+2}{3}\, \&c. \times \frac{z+6}{7} + 23\, \frac{z}{1} \cdot \frac{z+1}{2}\, \&c. \times \frac{z+6}{7} - 72\, \frac{z}{1} \cdot \frac{z+1}{2}\, \&c. \times \frac{z+7}{8} + 54\, \frac{z}{1} \cdot \frac{z+1}{2}\, \&c. \times \frac{z+8}{9}$, evaneſcentibus coefficientibus ſex primis A, B, C, D, E, F.

Prop. II. *Prob.* Invenire ſummam quotlibet terminorum Seriei

$$\frac{M}{a \times \overline{a+n} \times \&c. \times a + \overline{p-1}\, n} + \frac{N}{\overline{a+n} \times \&c. \times \overline{a+pn}} + \frac{O}{\overline{a+2n} \times \&c. \times a + \overline{p+1}\, n} + \&c.$$

ubi numeratores $M, N, O,$ *&c.* conſtituunt Seriem quamlibet terminorum, quorum differentiæ, vel primæ, vel ſecundæ, vel aliæ quædam dantur; vel quod perinde eſt, qui conſtituunt lineam quamvis tranſverſam in dato quovis triangulo Arithmetico; Denominatores autem conſtituunt Seriem in *Prop.* I. exhibitam.

Solutio. Per X deſignetur primus factorum $a, a+n, a+2n,$ *&c.* in denominatore ejuſdem termini, ut ſint X & z iidem ac in *Lemm.* præmiſſis, adeoque deſignetur terminus quilibet Seriei per $\frac{X}{z \times \overline{z+n} \times \&c. \times z + \overline{p-n}}$

Per *Lem.* 2, vel per *Lem.* 4. (prout magis commodum videatur vel differentias, vel generatores trianguli Arithmetici adhibere,) reſolvatur X in Multinomium $A + B \times z + Cz \times \overline{z+n} + Dz \times \overline{z+n} \times \overline{z+2n} + \&c.$

Hoc pacto (terminis multinomii ad denominatorem $z \times \overline{z+n} \times \&c. \times \overline{z+\overline{p-n}}$, applicatis) terminus quilibet Seriei revocabitur ad formulam $\frac{A}{z \times \overline{z+n} \times \&c. \times \overline{z+\overline{p-1}\,n}} + \frac{B}{\overline{z+n} \times \&c. \times \overline{z+\overline{p-1}\,n}} + \frac{C}{\overline{z+2n} \times \&c. \times \overline{z+\overline{p-1}\,n}} + \&c.$

Unde (per Scholium 4 *Prop.* I.) aggregatum totius Seriei, a termino $\frac{X}{z \times \overline{z+n} \times \&c. \times \overline{z+\overline{p-1}\,n}}$ incluſive in infinitum continuatæ, eſt $\frac{A}{\overline{p-1} \times n \times z \times \overline{z+n} \times \&c. \times \overline{z+\overline{p-2}\,n}} + \frac{B}{\overline{p-2} \times n \times \overline{z+n} \times \&c. \times \overline{z+\overline{p-2}\,n}} + \frac{C}{\overline{p-3} \times n \times \overline{z+2n} \times \&c. \times \overline{z+\overline{p-2}\,n}} + \&c.$ quare ſi dematur hoc aggregatum ab ejuſdem aggregati valore quando $z=a$, reſiduum erit ſumma omnium terminorum ante terminum $\frac{X}{z + \&c.}$, hoc eſt, tot terminorum quot ſunt unitates in $\frac{z-a}{n}$. *Q. E. I.*

Ex. 1. Sit primum exemplum in Serie $\frac{5}{3.5.7.9.11.13} + \frac{41}{5.7.9.11.13.15} + \frac{131}{7.9.11.13.15.17} + \frac{275}{9.11.13.15.17.19} + \frac{473}{11.13.15.17.19.21} + \&c.$ Sunt hic $a=3$, $n=2$, $p=6$, $M=5$, & capiendo differentias numeratorum inveniuntur $b=36$, $c=54$, $d=0=e=\&c.$ Hinc in Lemmate ſecundo ſunt $A=5+36 \times \frac{-3}{2} + 54 \times \frac{-3}{2} \times \frac{-5}{4} = \frac{209}{4}$, $B=\frac{1}{2} \times 36 + 54 \times \frac{-5}{2} = \frac{-99}{2}$, $C=\frac{1}{2} \times \frac{1}{4} \times 54 = \frac{27}{4}$, $D=0=E=\&c.$ Summa itaque totius Seriei eſt $\frac{209}{4 \times 5 \times 2 \times 3.5.7.9.11} + \frac{-99}{2 \times 4 \times 2 \times 5.7.9.11} + \frac{27}{4 \times 3 \times 2 \times 7.9.11} = \frac{283}{80 \times 3.5.7.9.11}$ atque ſumma terminorum numero $\frac{z-3}{2}\left(=\frac{z-a}{n}\right)$ eſt $\frac{283}{80 \times 3.5.7.9.11} - \frac{209}{40 \times z.\overline{z+2}.\overline{z+4}.\overline{z+6}.\overline{z+8}} + \frac{99}{16 \times \overline{z+2} \times \overline{z+4}.\overline{z+6}.\overline{z+8}} - \frac{27}{24 \times \overline{z+4}.\overline{z+6}.\overline{z+8}}$ Quærantur *v. g.* octo termini; tum exiſtente $\frac{z-3}{2}=8$ fit $z=19$, quo valore in formula adhibito, prodit ſumma $\frac{155891}{2.3.3.3.3.5.5.5.7.11.19.23}$.

Iidem Numeratores occupant lineam tertiam tranſverſam in Triangulo Arithmetico

$$\begin{array}{l} 54 \cdot 54 \cdot 54 \cdot 54 \cdot 54 \cdot 54 \cdot \&c. \\ -18 \cdot 36 \cdot 90 \cdot 144 \cdot 198 \cdot \&c. \\ \quad 5 \cdot 41 \cdot 131 \cdot 275 \cdot \&c. \end{array}$$

Unde in formula *Lem.* 4. ſunt generatores $q = 5$, $r = -18$; $s = 54$, $t = 0 = \&c.$ & prodeunt coefficientes $A = 5 - 18 \times \frac{-3}{2} + 54 \times \overline{\frac{-3}{2} \times \frac{-3+2}{4}} = \frac{209}{4}$, $B = \frac{1}{2} \times -18 + 54 \times \frac{-3}{2} = \frac{-99}{2}$, $C = \frac{1}{2} \times \frac{1}{4} \times 54 = \frac{27}{4}$, $D = 0 = E = \&c.$ iidem ac ſupra.

Ex. 2. Sit Series $\frac{4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 \cdot 11} + \frac{69}{2 \cdot 3 \cdot \&c. \ 12} + \frac{530}{3 \cdot 4 \cdot \&c. \ 13} + \frac{2676}{4 \cdot 5 \cdot \&c. \ 14} + \frac{10350}{5 \cdot 6 \cdot \&c. \ 15} + \&c.$ Ubi ſunt $a = 1$, $n = 1$, $p = 11$, atque Numeratores conſtituant Seriem in *Corol.* 20. *Lem.* 4. exhibitam. Applicando itaque valorem X in *Corol.* illo ad denominatorem $z \times \overline{z+1} \times \&c. \times \overline{z+10}$, fit Seriei propoſitæ Terminus

$$\frac{-1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \times \overline{z+6} \cdot \overline{z+7} \cdot \overline{z+8} \cdot \overline{z+9} \cdot \overline{z+10}}$$

$$+ \frac{23}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \times \overline{z+7} \cdot \overline{z+8} \cdot \overline{z+9} \cdot \overline{z+10}}$$

$$- \frac{72}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \times \overline{z+8} \cdot \overline{z+9} \cdot \overline{z+10}}$$

$$+ \frac{54}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \times \overline{z+9} \times \overline{z+10}}.$$

Adeoque per hanc *Prop.* ſumma Seriei a termino illo in infinitum continuatæ eſt

$$\frac{-1}{4 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \times \overline{z+6} \cdot \overline{z+7} \cdot \overline{z+8} \cdot \overline{z+9}}$$

$$+ \frac{23}{3 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \times \overline{z+7} \cdot \overline{z+8} \cdot \overline{z+9}}$$

$$- \frac{72}{2 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \times \overline{z+8} \cdot \overline{z+9}}$$

$$+ \frac{54}{1 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \times \overline{z+9}}.$$

Itaque pro z ſumpto 1, fit ſumma totius Seriei $\frac{305}{12 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}$. Et in genere ſumma terminorum numero $\frac{z-1}{1}$, eſt

$$\frac{305}{12 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10}$$

$$+ \frac{1}{4 \times 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \times \overline{z+6} \cdot \overline{z+7} \cdot \overline{z+8} \cdot \overline{z+9}}$$

$$= \frac{23}{3 \times 1 . 2 . 3 . 4 . 5 . 6 . 7 . \times \overline{z+7} . \overline{z+8} . \overline{z+9}}$$

$$+ \frac{72}{2 \times 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 \times \overline{z+8} \times \overline{z+9}}$$

$$- \frac{54}{1 \times 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9 \times \overline{z+9}}.$$

Scholium 1. In computandis ſummis hujuſmodi Serierum, calculus plerumque levior eſt adhibitis generatoribus trianguli Arithmetici, quam ſi adhibeantur differentiæ. Libet itaque hac occaſione oſtendere quomodo ex datis differentiis inveniri poſſunt generatores Trianguli Arithmetici.

Sunto itaque ω primus Seriei terminus, a differentia ultima data, b prima differentiarum penultimarum, c prima antepenultimarum, & ſic porro d, e, *&c.* atque ſint t, u, x, y, *&c.* generatores quæſiti Trianguli Arithmetici, cujus lineam tranſverſam ordine p occupet Series propoſita. Tum (quod ex contemplatione Trianguli Arithmetici facile conſtat) ſunt.

$$a = t$$

$$b = \frac{p-1}{1} t + u$$

$$c = \frac{p-1}{1} \times \frac{p-2}{2} t + \frac{p-2}{1} u + x$$

$$d = \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3} t + \frac{p-2}{1} \times \frac{p-3}{2} u$$

$$+ \frac{p-3}{1} x + y \text{ &c.}$$

Unde colliguntur generatorum valores

$$t = a$$

$$u = b - \frac{p-1}{1} t$$

$$x = c - \frac{p-1}{1} \times \frac{p-2}{2} t - \frac{p-2}{1} u$$

$$y = d - \frac{p-1}{1} \times \frac{p-2}{2} \times \frac{p-3}{3} t - \frac{p-2}{1} \times \frac{p-3}{2} u$$

$$- \frac{p-3}{1} x \text{ &c.}$$

Ultimus autem generator æqualis eſt Seriei termino primo ω.

2. D^{nus} de *Monſoury Abbas Orbacenſis* mihi amiciſſimus, & ruri vicinus, poſtquam cum eo hæc communicaveram, aliam invenit hujus Problematis Solutionem, cujus formulam ob ejus miram ſimplicitatem hic referre juvat. Itaque in Serie numeratorum ſint ω terminus primus, b prima differentiarum primarum, c prima ſecundarum, d prima tertiarum, & ſic porro; atque ſit termini primi Denominator $z \times \overline{z+n} \times \text{&c.} \times \overline{z+\overline{p-1}\,n}$; Tum ſumma totius Seriei in infinitum continuatæ exhibebitur per formulam $\dfrac{\omega}{n \times \overline{p-1} \times z \times \overline{z+n} \times \text{&c.} \times \overline{z+\overline{p-2}\,n}}$

+

$$+\frac{b}{n^2\times\overline{p-1}\times\overline{p-2}\times\overline{z+n}\times\&c.\times\overline{z+p-2n}}+$$

$$\frac{c}{n^3\times\overline{p-1}\times\overline{p-2}\times\overline{p-3}\times\overline{z+2n}\times\&c.\times\overline{z+p-2n}}+\&c.$$

Sit exemplum in Serie $\frac{5}{3\,.\,5\,.\,\&c.\,13}+\frac{41}{5\,.\,7\,.\,\&c.\,15}+\frac{131}{7\,.\,9\,.\,\&c.\,17}+\frac{275}{9\,.\,11\,.\,\&c.\,19}+\&c.$ cujus ſummam jam exhibuimus. In hoc caſu ſunt $a=5$, $b=36$, $c=54$, $d=0=e=\&c.$ Unde per formulam ſumma Seriei integræ fit $\frac{5}{2\,.\,5\times 3\,.\,5\ldots 11}+\frac{36}{4\,.\,5\,.\,4\times 5\ldots 11}+\frac{54}{8\,.\,5\,.\,4\,.\,3\times 7\ldots 11}=\frac{283}{80\times 3\,.\,5\ldots 11}$, ut per formulam noſtram exhibetur. Si quæratur ſumma ejuſdem Seriei incipientis a termino decimo $\frac{2273}{21\ldots 31}$, in eo caſu $a=2273$, $b=522$, $c=54$, & ſumma eſſet $\frac{2273}{2\,.\,5\times 21\ldots 29}+\frac{522}{4\,.\,5\,.\,4\times 23\ldots 29}+\frac{54}{8\,.\,5\,.\,4\,.\,3\times 25\ldots 29}$.

Hæc formula eſt commodiſſima, & ſummam exhibet nullo fere negotio, quoties quæritur ſumma Seriei integræ, & differentiæ non ſunt nimis multæ. Sed ubi plures ſunt differentiæ, & quæritur non Series integra, ſed termini tantum initiales aliquammulti, formulæ noſtræ ſunt commodiores.

3. Quando Serierum termini formantur tantum per Multiplicationem, nec afficiuntur diviſoribus variabilibus, ſummæ ſemper exhiberi poſſunt per Methodum in *Prop.* I. traditam, ſint licet formulæ quantumlibet compoſitæ. Nam poſſunt ſemper revocari ad terminos in forma quam poſtulat Propoſitio illa. Sic ſi differentiæ ipſorum z & x ſint m & n, & deſignetur terminus Seriei per $z\,x$; hic terminus revocabitur ad formam $\overline{a-n}\,z+\frac{n}{m}\,z\times\overline{z+m}$; cujus Integrale datur *per Prop.* I; nempe quoniam $d\,x=n$, & $d\,z=m$, eſt $d\,x=d\,z\times\frac{n}{m}$; unde regrediendo ad integralia fit $x=\frac{n}{m}\,z+a$ (adjecto invariabili a, ut habeatur ratio relationis inter z & x in Seriei termino primo,) quod ſic ſcribi poteſt $\overline{a-n}+\frac{n}{m}\times\overline{z+m}$, ut deinde in z ductum induat formam requiſitam. Et ad eundem modum procedere licet in aliis caſibus ejuſmodi. Sed ubi formulæ oblatæ diviſoribus afficiuntur, eædem ac in Calculo integrali, ut vocant, difficultates occurrunt, eadem induſtria ſuperandæ. Nec tamen ſemper ſuperari poſſunt. Nam præterquam quod vix certo ſciri poſſit quæ debeat relatio intercedere inter Numeratorem fractionis & Denominatorem, ut formula oblata ad Integrale revocari poſſit; ſæpe etiam difficillimum eſt

explorare an adſit jam talis relatio in formula iſta, aut ſi deſit, an introduci poſſit. Quicquid ego in hac materia potiſſimum inveni, continetur in tribus ſequentibus propoſitionibus.

Prop. III. *Prob.* Creſcentibus, z, u, y, x, *&c.* per differentias datas n, m, l, o, *&c.* invenire valorem numeratoris integri N, ut exiſtente Denominatore $z \cdot \overline{z+n} \cdot \&c.\ \overline{z+pn} \times u \cdot \overline{u+m} \cdot \&c.\ \overline{u+qm} \times y \cdot \overline{y+l} \cdot \&c.\ \overline{y+rl} \times x \cdot \overline{x+o}\ \&c.\ \overline{x+so} \cdot \&c.$ Fractio ad Integrale revocari poſſit.

Solutio. Fiat $N = \overline{z+pn} \times \overline{u+qm} \times \overline{y+rl} \times \overline{x+so} \times \&c. - zuyx\ \&c.$ atque Integrale erit fractio, cujus Denominator $z \cdot \overline{z+n} \cdot \&c.\ \overline{z+\overline{p-1}\,n} \times u \cdot \overline{u+m} \cdot \&c.\ \overline{u+\overline{q-1}\,m} \times y \cdot \overline{y+l} \cdot \&c.\ \overline{y-\overline{x-1}\,l} \times \overline{x+o} \cdot \&c.\ \overline{x+\overline{s-1}\,o} \times \&c.$ exiſtente 1 Numeratore

Differentia enim hujus fractionis eſt fractio cujus numerator eſt ipſius N valor exhibitus, & denominator idem eſt ac denominator propoſitus, ut fieri debuit.

Ex. 1. Sit denominator propoſitus $z \times \overline{z+2} \times u \times \overline{u+3}$. In hoc caſu ſunt $n=2$, $m=3$, $p=1$, $q=1$; adeoque eſt $N = \overline{z+2} \times \overline{u+3} - zu = 3z+2u+6$, & per $\dfrac{3z+2u+6}{z \cdot \overline{z+2} \times u \cdot \overline{u+3}}$ repreſentatur terminus Seriei ſummabilis, cujus nempe in infinitum continuatæ ſumma exhibetur per $\dfrac{1}{zu}$. Sint verbi gratia, ipſorum z & u primus valor communis 1, atque Series ſummabilis erit $\dfrac{11}{1 \cdot 3 \times 1 \cdot 4} + \dfrac{23}{3 \cdot 5 \times 4 \cdot 7} + \dfrac{35}{5 \cdot 7 \times 7 \cdot 10} + \&c.$ quippe cujus totius ſumma eſt 1. Per p deſignetur ordo termini cujuſvis in hac Serie, erit $p = \dfrac{z-1+2}{2} = \dfrac{u-1+3}{3}$, adeoque $z = 2p-1$, & $u = 3p-2$; quibus valoribus pro z & u ſcriptis, deſignabitur terminus per formulam $\dfrac{12p-1}{\overline{2p-1} \times \overline{2p+1} \times \overline{3p-2} \times \overline{3p+1}}$. Summa autem terminorum omnium ante terminum illum, hoc eſt terminorum initialium numero $\dfrac{z-1}{2} = p-1$, eſt $1 - \dfrac{1}{zu}, = \dfrac{zu-1}{zu}$, hoc eſt $\dfrac{6pp-7p+1}{\overline{2p-1} \times \overline{3p-2}}$. Quare pro p ſcripto $p+1$, erit $\dfrac{p \times \overline{6p+5}}{\overline{2p+1} \times \overline{3p+1}}$ aggregatum tot terminorum initialium quot ſunt unitates in p.

Ex. 2. Iiſdem manentibus z, u, n, m, ſit denominator $z \cdot \overline{z+2} \cdot \overline{z+4} \times u \cdot \overline{u+3}$. Tum per formulam numerator erit $\overline{z+4} \times \overline{u+3} - zu = 3z + 4u + 12$, & ſumma Seriei exhibebitur per formulam $\dfrac{1}{z \cdot \overline{z+2} \times u}$. Sit ipſorum z & u primus valor communis 1, & hinc elicietur Series $\dfrac{19}{1 \cdot 3 \cdot 5 \times 1 \cdot 4} + \dfrac{37}{3 \cdot 5 \cdot 7 \times 4 \cdot 7} + \dfrac{55}{5 \cdot 7 \cdot 9 \times 7 \cdot 10} + \&c. = \tfrac{1}{3}$.

Scholium. In Seriebus jam expoſitis eadem ubique eſt differentia inter factores continuos ejuſdem cujuſvis termini, ac inter factores homologos terminorum continuorum. In ſequentibus exempla quædam ſunt Serierum, quarum ſummæ in terminis numero finitis exhiberi poſſunt, quamvis ea regula non obſervetur.

Prop. IV. *Prob.* Creſcente z per differentias datas qn, invenire numeratorem integrum N, ut ad Integrale revocari poſſit fractio, cujus Denominator fit ex certo numero p terminorum z, $z+n$, $z+2n$, &c. Arithmetice proportionalium in invicem ductorum. Debet autem eſſe q numerus integer minor quam factorum numerus p.

Solutio. Erit $N = \overline{z+p-1\,n} \times \overline{z+p-2\,n} \times \&c. \times \overline{z+p-q\,n} - z \times \overline{z+n} \times \&c. \times \overline{z+q-1\,n}$, Integrale exiſtente $\dfrac{1}{z \times \overline{z+n} \times \&c. \times \overline{z+p-q-1\,n}}$. Demonſtratur ad modum propoſitionis præcedentis.

Sumptis ad libitum n, p, q, & primo valore z, hinc oriuntur infinitæ Series ſummabiles, cujuſmodi ſunt Series tres ſequentes.

$$A = \frac{5}{1.2.3.4} + \frac{9}{3.4.5.6} + \frac{13}{5.6.7.8} + \frac{17}{7.8.9.10}\ \&c.$$

$$B = \frac{1}{1.2.3.4.5} + \frac{4}{4.5.6.7.8} + \frac{9}{7.8.9.10.11} + \frac{16}{10.11.12.13.14} + \&c.$$

$$C = \frac{1}{1.2.3.4.5} + \frac{14}{5.6.7.8.9} + \frac{55}{9.10.11.12.13} + \frac{140}{13.14.15.16.17} + \&c.$$

Has Series jampridem communicavi cum primariis quibuſdam Geometris, à quibus minime contemni videntur. Sic ad me ſcribit peritiſſimus Geometra D. *Nicolaus Bernoulli* in epiſtola data 25 *Julii* 1716. "Vous me ferez un extreme plaiſir, *Monſieur*, de me communiquer la "Solution de voſtre probleme, *Etant donnée une ſuitte des Fractions dont* "*les Numerateurs ſoient des nombres figurés quelconque, & dont les Deno*"*minateurs ſoient formés du produit d'un nombre egal de Facteurs qui* "*ſoient en Progreſſion Arithmetique, trouver la ſomme*; & principale"ment comment vous avez trouvé ces deux formules $\dfrac{p}{24 \times \overline{4p+1}}$, "$\dfrac{p \cdot \overline{p+1}}{12 \times \overline{3p+1} \times \overline{3p+2}}$. Hæ formulæ ſpectant ad Series C & B, deſignante p numerum terminorum, quorum ſumma requiritur. Sic etiam ad me ſcribit D. *Taylor* in epiſtola data 22 *Aug.* 1716. "Ut & qua ratione "incidiſti in ſummationem Serierum a te exhibitarum, præſertim loquor "de Serie $\dfrac{1}{1.2.3.4.5} + \dfrac{4}{4.5.6.7.8} + \dfrac{9}{7.8.9.10.11} + \&c.$ quæ vide"tur eſſe altioris indaginis.

Sed ut ad exempla jam redeamus. In Serie *A* ſunt $p=4$, $q=2$, $n=1$, primo valore z exiſtente 1. Eſt itaque $\overline{z+3} \times \overline{z+2} - z \times \overline{z+1} = 2 \times \overline{2z+3}$ formula, unde (rejecto dato numero 2) derivantur numeratores 5, 9, 13, 17, &c. Formula etiam ſummæ eſt $\frac{1}{z \times \overline{z+1}}$. Quare habita ratione numeri 2, quem ex numeratoribus rejecimus, ſumma totius Seriei, a termino in quo eſt z in infinitum continuatæ, exhibetur per formulam $\frac{1}{2z \times \overline{z+1}}$; adeoque ſumma Seriei integræ eſt $\frac{1}{2 \times 1 \times 2} = \frac{1}{4}$.

In Serie *B* ſunt $n=1$, $p=5$, $q=3$, primo valore z exiſtente 1. Eſt itaque $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} - z \times \overline{z+1} \times \overline{z+2} = 6 \times \overline{z+2}|^2$. Ipſius autem $\overline{z+2}$ valores continui ſunt 3, 6, 9, &c. qui quoniam omnes ſunt diviſibiles per 3, ponendo $z+2=3x$, fit $N = 6 \times \overline{3x}|^2 = 6 \times 9x^2 = 54x^2$, ipſius x valoribus continuis exiſtentibus 1, 2, 3, &c. Rejecto itaque numero dato 54, hinc prodeunt numeratores 1, 2^2, 3^2, &c. hoc eſt 1, 4, 9, &c. Formula etiam Integralis eſt $\frac{1}{z \times \overline{z+1}}$; quare habita ratione numeri 54 quem ex numeratoribus rejecimus, ſumma Seriei a termino in quo eſt z in infinitum continuatæ eſt $\frac{1}{54z \times \overline{z+1}}$. Unde ſumma Seriei integræ eſt $\frac{1}{108}$.

In Serie denique *C* ſunt $n=1$, $p=5$, $q=4$, & primus valor $z=1$. Unde fit $N = \overline{z+4} \times \overline{z+3} \times \overline{z+2} \times \overline{z+1} - z \times \overline{z+1} \times \overline{z+2} \times \overline{z+3} = 4 \times \overline{z+1} \times \overline{z+2} \times \overline{z+3}$. Valores autem N per hanc formulam prodeuntes ſemper poſſunt dividi per $4 \times 2 \times 3 \times 4 = 96$. Ergo hoc diviſore rejecto prodeunt numeratores 1, 14, 55, 140, &c. Et formula Summæ, habita ratione numeri 96, eſt $\frac{1}{96z}$. Adeoque Summa Seriei integræ eſt $\frac{1}{96}$.

Scholium 1. Per Propoſitiones has duas noviſſimas nullo negotio inveniri poſſunt Series quot libuerit ſummabiles. Et viciſſim oblata Serie hujus ſpeciei, ſi ſummari poteſt, ejus ſumma plerumque revocatur ad alterutram ex his Propoſitionibus. In examine tamen ſolertia eſt opus. Optime autem procedit ſi termini Seriei oblatæ revocentur ad formulam *Prop.* II. Sic *e. gr.* propoſita Serie $\frac{7}{3.5.7.9.11} + \frac{11}{7.9.11.13.15} + \frac{15}{11.13.15.17.19} +$ &c. Denominatores ſic ſcribi poſſunt $3.7.11 \times 5.9.7.11.15 \times 9.13.11.15.19 \times 13.17.$ &c.

Unde juxta *Prop.* III. fit $n=4$, $m=4$, $p=2$, $q=1$, primus valor $z=3$, primus valor $u=5$. Hinc formula Numeratoris invenitur $4 \times z + 2u + 8$, Eſt autem $z + 2u + 8$ ſemper diviſibile per 3; quare rejectis diviſoribus datis 4 & 3, per hanc formulam prodeunt Numeratores 7, 11, 15, 15, *&c.* iidem ac Numeratores in Serie propoſita, quæ proinde ſummabitur per illam propoſitionem.

2. Cum Series illas *A*, *B*, *C*, communicaveram cum D. *Taylor*, reſcripſit ſe earum ſummas inveniſſe primam quidem *A* & tertiam *C*, eas revocando ad caſus ſimplices Methodi Incrementorum, tertiam *C*, *e.g.* revocavit ad hanc formam $\frac{1}{24}\times\frac{1}{1.5}+\frac{1}{5.9}+\frac{1}{9.13}+\frac{1}{13.17}$ *&c.* ut habeatur ſumma per præcepta tradita in *Scholio Prop.* 1. In Serie autem ſecunda *B*, cum hoc non æque ſucceſſit, ſequenti uſus eſt Analyſi, quam, ipſius venia jam impetrata, ob ejus eximiam elegantiam huc transferre non piget. “ Seriei iſtius terminus [in Stylo ejus] exhibetur per formulam $\frac{\overline{z+2}\times\underset{\cdot}{z}}{27z\times\overline{z+1}\times\underset{,}{z}\times\overline{\underset{,}{z}+1}}$; pro $z+3$ in denominatore ſcripto $\underset{,}{z}$, quoniam

“ eſt $\underset{\cdot}{z}=3$. Pone $\frac{B}{27C}$ æquale eſſe Integrali quæſito, hoc eſt $\frac{B}{C}$ eſſe Inte-

“ grale ipſius $\frac{\overline{z+2}\times\underset{\cdot}{z}}{z.\overline{z+1}\times\underset{,}{z}.\overline{\underset{,}{z}+1}}$, ſepoſito diviſore dato 27. Ipſius au-

“ tem $\frac{B}{C}$ incrementum eſt $\frac{\underset{\cdot}{B}C-B\underset{\cdot}{C}}{C\underset{,}{C}}$. Debet ergo $\frac{\underset{\cdot}{B}C-B\underset{\cdot}{C}}{C\underset{,}{C}}$ i-

“ dem eſſe ac $\frac{\overline{z+2}\times\underset{\cdot}{z}}{z.\overline{z+1}\times\underset{,}{z}.\overline{\underset{,}{z}+1}}$. Comparando denominatores inve-

“ nitur $C=z\times\overline{z+1}$. Hinc itaque ſumendo incrementa fit $C=2z\underset{\cdot}{z}+\underset{\cdot}{z}^2$

“ $+\underset{\cdot}{z}$ ($=2\underset{\cdot}{z}z+4\underset{\cdot}{z}$, quoniam eſt $\underset{\cdot}{z}=3$,) His valoribus in locum C & $\underset{\cdot}{C}$

“ ſubſtitutis prodit $\underset{\cdot}{B}C-B\underset{\cdot}{C}=\overline{zz+z}\,\underset{\cdot}{B}-2\underset{\cdot}{z}\times\overline{z+2}\,B$, quod debet

“ eſſe idem ac $\overline{z+2}\times\underset{\cdot}{z}$. Sit $B=a+v$, exiſtente a ipſius B parte in-

“ variabili, & v parte variabili. Tum ſumendo incrementa fit $\underset{\cdot}{B}=\underset{\cdot}{v}$.

“ Unde ad invenienda a & v habetur æquatio $\overline{zz+z}\,\underset{\cdot}{v}-2\underset{\cdot}{z}\times\overline{z+2}\times$

“ $\overline{a+v}=\overline{z+2}\times\underset{\cdot}{z}$, quæ ſic ſcribi poteſt $\overline{zz+z}\,\underset{\cdot}{v}-2\underset{\cdot}{z}\times\overline{z+2}\,v=\underset{\cdot}{z}\times$

“ $\overline{z+2}\times\overline{1+2a}$, vel etiam $C\underset{\cdot}{v}-\underset{\cdot}{C}v=\underset{\cdot}{z}\times\overline{z+2}\times\overline{1+2a}$. Pone

“ $1+2a=0$ (unde fit $a=\frac{-1}{2}$,) & fit $C\underset{\cdot}{v}-\underset{\cdot}{C}v=0$; ubi fieri poteſt

“ $v=0$, (quoniam æquationis termini ſinguli afficiuntur vel ab v, vel ab

“ $\underset{\cdot}{v}$) Hinc ergo fit $B=a=\frac{-1}{2}$, adeoque $\frac{B}{C}=\frac{-1}{2z\times\overline{z+1}}$. Unde ha-

“ bita ratione diviſoris 27, Integrale quæſitum fit $\frac{-1}{54\times z\times\overline{z+1}}$. Sed

“ & comparando æquationem $C\underset{\cdot}{v}-\underset{\cdot}{C}v=0$ cum formula generali

“ $\frac{\underset{\cdot}{B}C-B\underset{\cdot}{C}}{C\underset{,}{C}}=0$, inde etiam concludere licet eſſe $\frac{v}{C}=$ quantitati datæ,

“ (quoniam ipſius incrementum eſt o.) Unde pro *n* ſumpto quovis numero
“ dato, fit $v = nC$, atque $B = -\frac{1}{2} + nC$. Quo pacto Integrale
“ quæſitum fit $\frac{B}{C} = \frac{-\frac{1}{2}+nC}{C} = \frac{-1}{2C} + n$, quod ab Integrali prius in-
“ vento differt quantitate data *n*. Hoc inde fit, quod, ut in quadratura
“ Curvarum Area inventa augeri poteſt vel minui area data, ſic in Me-
“ thodo incrementorum Integrale inventum augeri poteſt vel minui
“ quantitate data. Per Integrale autem primum, ubi deeſt *n*, exhibe-
“ tur ſumma Seriei in infinitum continuatæ.

Prop. V. Creſcente z per unitates, & exiſtentibus $a, b, c,$ &c. numeris datis integris, quorum nullæ inter ſe æquantur; invenire Integrale ipſius $\frac{1}{z \times \overline{z+a} \times \overline{z+b} \times \overline{z+c} \times \&c.}$

Solutio. Ducendo tam numeratorem quam denominatorem fractionis in terminos $z+1$, $z+2$, &c. $z+a+1$, $z+a+2$, &c. $z+b+1$, $z+b+2$, &c. $z+c+1$, $z+c+2$, &c. in denominatore deficientes, revocetur Denominator ad formulam $z \times \overline{z+1} \times \overline{z+2} \times \&c.$ denominatoris in *Prop.* I. *Schol. n.* 3. Deinde revocetur Numerator ad formam $A + Bz + Cz \times \overline{z+1} + Dz \times \overline{z+1} \times \overline{z+2} + \&c.$ Tum applicando terminos ad Denominatorem novum $z \times \overline{z+1} \times \overline{z+2} \times \&c.$ revocetur fractio ad hanc formam $\frac{A}{z \times \overline{z+1} \times \&c.} + \frac{B}{\overline{z+1} \times \overline{z+2} \times \&c.} + \frac{C}{\overline{z+2} \times \overline{z+3} \times \&c.} + \frac{D}{\overline{z+3} \times \overline{z+4} \times \&c.}$ &c. Unde denique quæratur Integrale per *Schol. Prop.* I. *n.* 3.

Ratio Solutionis per ſe ſatis eſt manifeſta.

Scholium 1. Hujus Solutionis tota difficultas latet in revocatione numeratoris ad formam requiſitam, quod tamen quomodo ſit faciendum uno exemplo patebit. Proponatur itaque factum $\overline{z+2} \times \overline{z+3} \times \overline{z+7}$, quod ad formam propoſitam ſit revocandum. Terminos itaque evolvo gradatim ut ſequitur. Factorem primum $z+2$ ſic ſcribo $2+z$, cujus terminum primum 2 duco in $3+z$, unde fit $6+2z$: Terminum ſecundum z duco in $2+\overline{z+1}$ $(=\overline{z+3})$ unde fit $2z + z \times \overline{z+1}$. Dein facta in unam ſummam colligendo, fit $\overline{z+2} \times \overline{z+3} = \begin{matrix} 6+2z \\ +2z \end{matrix} + z \times \overline{z+1} = 6 + 4z + z \times \overline{z+1}$. Supereſt ut hoc ducatur in $z+7$. Itaque terminum primum 6 duco in $7+z$ $(=z+7)$ unde fit $42+6z$; terminum ſecundum $4z$ duco in $6+\overline{z+1}$ $(=z+7)$ unde fit $24z + 4z \times \overline{z+1}$; terminum tertium $z \times \overline{z+1}$ duco in $5+\overline{z+2}$ $(=z+7,)$ unde fit $5z \times \overline{z+1} + z \times \overline{z+1} \times \overline{z+2}$. Factis itaque in unum collectis ut prius, fit $\overline{z+2} \times \overline{z+3} \times \overline{z+4} = 42 + 30z + 9z \times \overline{z+1} + z \times \overline{z+1} \times \overline{z+2}$. Et ad eundem modum procedere licet in aliis caſibus.

2. Sit autem exemplum Propoſitionis in fractione $\frac{1}{z \times \overline{z+2} \times \overline{z+5}}$.

Reſtituendo factores $z+1$, $z+3$, $\overline{z+4}$ in Denominatore deficientes, fractio fit $\frac{\overline{z+1}\times\overline{z+3}\times\overline{z+4}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$. Revocandus itaque eſt Numerator $\overline{z+1}\times\overline{z+3}\times\overline{z+4}$ ad formam requiſitam. Itaque per methodum jam traditam fit primo $\overline{z+1}\times\overline{z+3}=1\times\overline{3+z}+z\times\overline{2+z+1}=3+z+2z+z\times\overline{z+1}=3+3z+z\times\overline{z+1}$. Deinde $\overline{z+1}\times\overline{z+3}\times\overline{z+4}=3\times\overline{4+z}+3z\times\overline{3+z+1}+z\times\overline{z+1}\times\overline{2+z+2}=12+3z+9z+3z\times\overline{z+1}+2z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}=12+12z+5z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Applicando hoc factum ad Denominatorem $z\times\overline{z+1}\times\&c.\times\overline{z+5}$ fractio tandem revocatur ad hanc formam $\frac{12}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{12}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{5}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}\times\overline{z+5}}+\frac{1}{\overline{z+3}\times\overline{z+4}\times\overline{z+5}}$ Cujus denique Integrale eſt

$$\frac{-12}{5z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-12}{4\cdot\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-5}{3\cdot\overline{z+2}\times\overline{z+3}\times\overline{z+4}}+\frac{-1}{2\cdot\overline{z+3}\times\overline{z+4}}.$$

3. Quando duo tantum ſunt factores z & $z+a$, exhibebitur etiam Integrale per formulam $\frac{1}{2}-\frac{1-a}{2z\times\overline{z+1}}-\frac{\overline{1-a}\times\overline{2-a}}{3z\times\overline{z+1}\times\overline{z+2}}-\frac{\overline{1-a}\times\overline{2-a}\times\overline{3-a}}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}}$ &c. Seriem nempe continuando donec abrumpatur per evaneſcentiam terminorum. Si Factores duo ſint z & $z-a$ exhibebitur Integrale per formulam $\frac{-1}{z-1}-\frac{-1+a}{2\cdot\overline{z-1}\cdot\overline{z-2}}-\frac{\overline{-1+a}\times\overline{-2+a}}{3\cdot\overline{z-1}\cdot\overline{z-2}\cdot\overline{z-3}}-\&c.$ Poteſt idem Integrale exprimi utroque modo; prout fractionis oblatæ factor vel minor vel major ſumatur pro z.

4. Si primus valor z ſit $a+1$, migrabit formula poſterior in hanc $\overline{\frac{-1}{a}\times\frac{1}{1}\times\frac{1}{2}\times\frac{1}{3}+\&c.}$ uſque $\frac{1}{a}$ incluſive, qua, cum ſigno contrario, exhibetur ſumma Seriei $\frac{1}{1\times\overline{1+a}}+\frac{1}{2\times\overline{2+a}}+\frac{1}{3\times\overline{3+a}}+\&c.$ in infinitum continuatæ. Sit *e. gr.* $a=1$, atque Series erit $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\&c.=\frac{1}{1}\times\frac{1}{1}=1$. Si $a=2$, erit Series $\frac{1}{1\times 3}+\frac{1}{2\times 4}+$

$+ \frac{1}{3\times5} +$ &c. $= \frac{1}{2} \times \frac{1}{1} + \frac{1}{2} = \frac{3}{4}$; Si $a = 3$, Series erit $\frac{1}{1\times4}$ $+ \frac{1}{2\times5} + \frac{1}{3\times6} + \frac{1}{4\times7}$ &c. $= \frac{1}{3} \times \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{18}$.

5. Ex eadem Serie $\frac{1}{1\times\overline{1+a}} + \frac{1}{2\times\overline{2+a}} + \frac{1}{3\times\overline{3+a}} +$ &c. pro diverſo valore a oriuntur Series plures forma ſatis elegantes, quarum nonnullas Lectori ob oculos ſiſtere, credo, ingratum non erit.

Si pro a ſumantur ſucceſſive numeri pares, 2, 4, 6, 8, &c. Series erunt

$$\text{Si } a = 2)\ \frac{1}{1\times\overline{1+2}} + \frac{1}{2\times\overline{2+2}} + \frac{1}{3\times\overline{3+2}} + \frac{1}{4\times\overline{4+2}} + \&c.$$

$$4)\ \frac{1}{1\times\overline{1+4}} + \frac{1}{2\times\overline{2+4}} + \frac{1}{3\times\overline{3+4}} + \frac{1}{4\times\overline{4+4}} + \&c.$$

$$6)\ \frac{1}{1\times\overline{1+6}} + \frac{1}{2\times\overline{2+6}} + \frac{1}{3\times\overline{3+6}} + \frac{1}{4\times\overline{4+6}} + \&c.$$

$$8)\ \frac{1}{1\times\overline{1+8}} + \frac{1}{2\times\overline{2+8}} + \frac{1}{3\times\overline{3+8}} + \frac{1}{4\times\overline{4+8}} + \&c.$$

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{9-4} + \frac{1}{16-4} + \frac{1}{25-4} + \frac{1}{36-4} + \&c.$$

$$\frac{1}{16-9} + \frac{1}{25-9} + \frac{1}{36-9} + \frac{1}{49-9} + \&c.$$

$$\frac{1}{25-16} + \frac{1}{36-16} + \frac{1}{49-16} + \frac{1}{64-16} + \&c.$$

$$\text{Vel } \frac{1}{4-1} + \frac{1}{9-1} + \frac{1}{16-1} + \frac{1}{25-1} + \&c.$$

$$\frac{1}{4+1} + \frac{1}{9+3} + \frac{1}{16+5} + \frac{1}{25+7} + \&c.$$

$$\frac{1}{4+3} + \frac{1}{9+7} + \frac{1}{16+11} + \frac{1}{25+15} + \&c.$$

$$\frac{1}{4+5} + \frac{1}{9+11} + \frac{1}{16+17} + \frac{1}{25+23} + \&c.$$

Si pro a ſumantur ſucceſſive numeri impares 1, 3, 5, 7, &c. Series erunt

$$a = 1)\ \frac{1}{1\times\overline{1+1}} + \frac{1}{2\times\overline{2+1}} + \frac{1}{3\times\overline{3+1}} + \frac{1}{4\times\overline{4+1}} + \&c.$$

$$3)\ \frac{1}{1\times\overline{1+3}} + \frac{1}{2\times\overline{2+3}} + \frac{1}{3\times\overline{3+3}} + \frac{1}{4\times\overline{4+3}} + \&c.$$

$$5)\ \frac{1}{1\times\overline{1+5}} + \frac{1}{2\times\overline{2+5}} + \frac{1}{3\times\overline{3+5}} + \frac{1}{4\times\overline{4+5}} + \&c.$$

$$7)\ \frac{1}{1\times\overline{1+7}} + \frac{1}{2\times\overline{2+7}} + \frac{1}{3\times\overline{3+7}} + \frac{1}{4\times\overline{4+7}} + \&c.$$

$$\text{Vel } \frac{1}{2} \times \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{3-1} + \frac{1}{6-1} + \frac{1}{10-1} + \frac{1}{15-1} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{6-3} + \frac{1}{10-3} + \frac{1}{15-3} + \frac{1}{21-3} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{10-6} + \frac{1}{15-6} + \frac{1}{21-6} + \frac{1}{28-6} + \text{\&c.}$$

$$\text{Vel } \frac{1}{2} \times \frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{6+0} + \frac{1}{10+0} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{1+1} + \frac{1}{3+2} + \frac{1}{6+3} + \frac{1}{10+4} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{1+2} + \frac{1}{3+4} + \frac{1}{6+6} + \frac{1}{10+8} + \text{\&c.}$$

$$\frac{1}{2} \times \frac{1}{1+3} + \frac{1}{3+6} + \frac{1}{6+9} + \frac{1}{10+12} + \text{\&c.}$$

6. Ante aliquot annos **D. *Jac. Bernoulli*** Geometra inſignis invenit ſummam Seriei cujuſlibet, cujus Numeratores conſtituunt Seriem æqualium, Denominatores vero conſtituunt, vel Seriem quadratorum dato aliquo quadrato Q minutorum, vel Seriem Triangulorum, dato aliquo Triangulo T minutorum. Hæc invenit ille obſervando quod hujuſmodi Series oriantur ex ablatione Seriei Harmonice proportionalium truncatæ ab eadem Serie integra; nempe ita ut numerus terminorum deficientium in Serie truncata, ſit, vel duplus lateris dati quadrati Q, vel duplus unitate auctus lateris dati Trianguli T. Idem etiam obſervavit fruſtra quæri ſummam Seriei reciprocæ Quadratorum. Hoc idem etiam verum eſt de reciprocis Cuborum, vel aliarum quarumlibet dignitatum numerorum in progreſſione Arithmetica. Ratio eſt, quod nulla intercedit differentia inter factores denominatorum, quod ad hujuſmodi ſummationes ſemper requiri conſtat ex Methodo ſumendi differentias in *Scholio Prop.* I. jam explicata. Nam ſi per formulam aliquam exhiberi poſſet ſumma quæſita, differentia iſtius formulæ exhiberet terminos Seriei propoſitæ: ſed in tali differentia denominator ſemper afficitur per factores ab invicem diverſos, quod quoniam in Seriebus prædictis non obtinet, ſummæ Serierum hujuſmodi in terminis finitis haberi nequeunt. Ad eundem fere modum, argumento petito a *Prop.* III. & IV. demonſtrari poteſt ſummas Serierum exhiberi non poſſe in terminis numero finitis, quarum Numeratores conſtituunt Seriem æqualium. Denominatores vero conſtant ex certo numero terminorum in progreſſione Arithmetica, maximo factore cujuſvis termini minore exiſtente quam factor minimus in termino proxime inſequenti, cujuſmodi eſt Series $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \text{\&c.}$

7. Jam liceret regulas nonnullas tradere, quas pro caſibus quibuſdam ſingularibus concinnavi; ſed hæc nos longius abducerent: Sufficiat itaque quæ generaliora ſunt explicaſſe, & ſimul monuiſſe, ad novæ hujuſce Serierum infinitarum doctrinæ provectionem nihil magis facere, quam ſi

excogitentur formulæ generaliores ſummarum, ex quarum differentiis, per regulas ſupra traditas computatis, deinde conficiantur Canones quantitatum ſummabilium; ita fere ut jam factum eſt in Calculo Integrali, *h. e.* in Stylo *Newtoniano*, in Methodo Fluxionum.

8. Reſtituendo factores in Denominatore deficientes potuiſſet præſens Problema revocari ad *Propoſitionem* II. Sed & in terminis generalioribus proponi poteſt, nempe pro Numeratore ſumpta quavis Formula, cujus differentia aliqua datur. Sub ea tamen conditione ut dimenſiones Denominatoris ad minimum binario ſuperent Dimenſiones Numeratoris; alias enim ſumma Seriei in terminis numero finitis haberi nequit. Sit hujus rei exemplum in Serie $\frac{1}{1.3.5.7} + \frac{4}{2.4.6.8} + \frac{9}{3.5.7.9} + \frac{16}{4.6.8.10} +$ *&c.* ubi Numeratores ſunt numerorum naturalium quadrata. Applicando tum Numeratores tum Denominatores ad numeros naturales, Series revocatur ad formam ſimpliciorem $\frac{1}{3.5.7} + \frac{2}{4.6.8} + \frac{3}{5.7.9} + \frac{4}{6.8.10} +$ *&c.* Per p deſignatis numeris naturalibus 1, 2, 3, 4, *&c.* terminus Seriei deſignabitur per formulam $\frac{p}{\overline{p+2}\times\overline{p+4}\times\overline{p+6}}$; vel per formulam $\frac{z-2}{z\times\overline{z+2}\times\overline{z+4}}$, nempe pro $p+2$ ſcripto z. Quoniam progrediendo de termino in terminum augetur z per unitates, reſtituendi ſunt factores in denominatore deficientes $z+1$, $z+3$, & hoc pacto revocatur terminus Seriei ad formulam $\frac{\overline{z-2}\times\overline{z+1}\times\overline{z+3}}{z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}}$. Per methodum in hac Propoſitione jam explicatam revocatur numerator ad formam $-6-6z-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$. Unde habita ratione denominatoris Terminus revocatur ad formam $\frac{-6}{z\times\overline{z+1}\times\&c.\times\overline{z+4}} + \frac{-6}{\overline{z+1}\times\overline{z+2}\times\overline{z+3}\times\overline{z+4}} + \frac{-1}{\overline{z+2}\times\overline{z+3}\times\overline{z+4}} + \frac{1}{\overline{z+3}\times\overline{z+4}}$. Adeoque ſumendo Integrale fit $\frac{6}{4z\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}} + \frac{6}{3\times\overline{z+1}\times\overline{z+2}\times\overline{z+3}} + \frac{1}{2\times\overline{z+2}\times\overline{z+3}} + \frac{-1}{z+3}$; quo, ſub ſigno contrario, exhibetur ſumma Seriei in infinitum continuatæ, incipientis a termino $\frac{z-2}{z\times\overline{z+2}\times\overline{z+4}}$. Summa itaque Seriei integræ incipientis a termino $\frac{1}{3.5.7}$ eſt $\frac{31}{240}$.

Si per *Prop.* II. procedere eſſet animus, ex formula $\overline{z-2}\times\overline{z+1}\times\overline{z+3}$ collectis numeratoribus primis 24, 70, 144, 252, ſumendo eorum differentias haberentur $46=b$, $28=c$, $6=d$, $e=0=$ *&c.* exiſtente $M=24$;

unde per *Lem.* 2. prodiret formula $-6-6z-z\times\overline{z+1}+z\times\overline{z+1}\times\overline{z+2}$, qua deſignatur Terminus, eadem ac ſupra; atque pergendo per *Prop.* II. haberetur ſumma.

Prop. VI. *Prob.* Invenire ſummam quotlibet terminorum Seriei Fractionum, quarum Numeratores & Denominatores conſtituunt lineas duas quaſvis tranſverſas in Triangulo Arithmetico *Paſchalii*; nempe cujus generatores ſunt unitates.

Solutio. Per n deſignetur Ordo Seriei Numeratorum in Triangulo Arithmetico, & ſit p differentia inter ordinem Numeratorum & Denominatorum, & per q deſignetur numerus terminorum quorum ſumma requiritur. Tum ſi Denominatores ſint plurium dimenſionum quam ſunt Numeratores, Summa exhibebitur per formulam primam ſequentem; ſi dimenſiones Numeratorum plures ſint quam dimenſiones Denominatorum, Summa exhibebitur per formulam ſecundam.

Formula I. $\dfrac{n+p-1}{p-1}-\dfrac{n\,.\,\overline{n+1}\,.\,\overline{n+2}\,.\,\&c.\,\overline{n+p-1}}{\overline{p-1}\times\overline{n+q}\,.\,\overline{n+q+1}\,.\,\&c.\,\overline{n+q+p-2}}$

Formula II. $-\dfrac{n-p-1}{p+1}+\dfrac{\overline{q+n-1}\,.\,\overline{q+n-2}\,.\,\&c.\,\overline{q+n-p-1}}{\overline{p+1}\times\overline{n-1}\,.\,\overline{n-2}\,.\,\&c.\,\overline{n-p}}$

Ex. 1. Inveniendum ſit aggregatum ſex primorum terminorum Seriei $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+\frac{20}{84}+\frac{35}{210}+\frac{56}{462}+$ &c. ubi Numeratores conſtituunt lineam quartam, Denominatores conſtituunt lineam ſeptimam in Triangulo Arithmetico. Sunt itaque $n=4$; $p=3$; $q=6$; & quoniam dimenſiones Denominatorum ſuperant dimenſiones Numeratorum, dabitur ſumma per Formulam primam; nempe $\frac{4+3-1}{3-1}-\frac{4\,.\,5\,.\,6}{\overline{3-1}\times\overline{4+6}\times\overline{4+7}}$ ſive $3-\frac{6}{11}=2\frac{5}{11}$.

Ex. 2. Quæratur ſumma ſex primorum terminorum Seriei $\frac{1}{1}+\frac{7}{4}+\frac{28}{10}+\frac{84}{20}+\frac{210}{35}+\frac{462}{56}+$ &c. cujus termini ſunt terminorum Seriei prioris reciproci. Sunt itaque $n=7$, $p=3$, $q=6$, adeoque per formulam ſecundam ſumma fit $-\frac{3}{4}+\frac{12\,.\,11\,.\,10\,.\,9}{4\times 6\,.\,5\,.\,4}=24$.

Scholium 1. Formulas in hac propoſitione exhibitas ante biennium communicavi cum Viris celeberrimis *Moivreo* & *Bernoulliis*. Facile autem derivari poſſunt ex præceptis in *Prop.* I. traditis. Sit exemplum in Serie priori $\frac{1}{1}+\frac{4}{7}+\frac{10}{28}+$ &c. Per p deſignato loco Termini in Serie hac, exhibetur Terminus per formulam $\frac{4\,.\,5\,.\,6}{\overline{p+3}\,.\,\overline{p+4}\,.\,\overline{p+5}}$. Unde regrediendo ad Integrale, ſumma Seriei incipientis a termino illo exhibetur

tur per formulam $\frac{4.5.6}{2\times\overline{p+3}\times\overline{p+4}}$; adeoque pro p ſumpto 1, Series integra fit $\frac{4.5.6}{2.4.5}=3$, atque ſumma primorum ſex terminorum fit $3-\frac{4.5.6}{2.10.11}$, omnino ut per formulam jam exhibetur.

2. In formula prima ſumma Seriei in infinitum continuatæ eſt $\frac{n+p-1}{p-1}$, evaneſcente jam parte altera formulæ. Sed in caſu formulæ ſecundæ ſumma hæc eſt infinitum quid, cujus ſpecies, reſpectu numeri infiniti q, exhibetur per formulæ partem alteram, quæ in hoc caſu fit $\frac{q^{p+1}}{\overline{p+1}\times\overline{n-1}.\overline{n-2}.\&c.\overline{n-p}}$.

3. De hujuſmodi Seriebus in epiſtola data menſe *Maio* 1716, ſic ad me ſcripſit Vir. Ill. *D. Leibnitius*, quem magno Scientiarum damno nobis nuper ereptum lugemus. “ Il me ſemble qu'autrefois j'ay auſſi ſommé “ quelques Series ou ſuittes comme $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{35}+\frac{6}{56}+$ “ &c. Le terme de cette ſuitte exprimé Analytiquement eſt “ $\frac{x}{x.\overline{x+1}.\overline{x+2}\times\frac{1}{1}.\frac{1}{2}.\frac{1}{3}}=\frac{1.2.3}{\overline{x+1}.\overline{x+2}}=\frac{6}{xx+3x+2}$. On demande “ donc la ſomme d'une ſuitte donnée, dont un terme ſoit “ $\frac{ll}{xx+3lx+2ll}$ ou x ſignifie les nombres naturales 1, 2, 3, 4, &c. & l “ ſignifie l'Unité, ou la difference des x. Suppoſons que le terme de la “ ſuitte ſommatrice demandée ſoit $\frac{fx}{mx+nl}=\frac{\odot}{\text{☽}}$. Or Diff. $\frac{\odot}{\text{☽}}=-\frac{\odot}{\text{☽}}+$ “ $\frac{\odot+d\odot}{\text{☽}+d\text{☽}}=\frac{\text{☽}\,d\odot-\odot\,d\text{☽}}{\text{☽}.\text{☽}+\text{☽}\,d\text{☽}}$: ſed $d\odot=fdx$, & $d\text{☽}=mdx=ml$; “ donc la Difference de $\frac{\odot}{\text{☽}}$ eſt $=\frac{nfll}{\begin{array}{l}mmxx+2mnlx+nnll\\ \qquad +mmlx+mnll\end{array}}$. Maintenant il “ faut faire $\frac{nfll}{\begin{array}{l}mmxx+2mnlx+nnll\\ \qquad +mmlx+mnll\end{array}}=\frac{mfll}{mmxx+3mmlx+2mmll}$ c'eſt “ a dire, il faut identifier ces deux formules, où la donnée eſt multipliée “ per $\frac{nf}{mm}$: donc égalant les termes reſpectifs, puiſque les xx convien- “ nent, on aura par les x, $2n+m=3m$, c'eſt a dire il y aura $m=n$, & “ par les abſolus on aura $nn+mn=2mm$, ce qui donne encore $m=n$; “ donc l'identification reüſſit, & nous pouvons faire $n=m=l=1$, & “ $f=1$ (car f demeure arbitraire) & le terme de la ſuitte ſommatrice ſe- “ ra $\frac{x}{x+1}$, car diff. $\frac{x}{x+1}$ donne $-\frac{x}{x+1}+\frac{x+1}{x+2}=\frac{1}{xx+3x+2}$, &

“ par conſequente $\frac{6x}{x+1}$ donne la ſomme des $\frac{x}{x\cdot\overline{x+1}\cdot\overline{x+2}\times\frac{1}{1}\cdot\frac{1}{2}\cdot\frac{1}{3}}$
“ 3, 4, $\frac{9}{2}$, $\frac{24}{5}$, 5, $\frac{36}{7}$, *&c. Series ſummatrix, cujus terminus* $\frac{6x}{x+1}$.
“ $\frac{1}{1}+\frac{2}{4}+\frac{3}{10}+\frac{4}{20}+\frac{5}{35}+$ *&c. Series ſummanda, cujus terminus*
“ $\frac{x}{x\cdot\overline{x+1}\cdot\overline{x+2}\times\frac{1}{1}\times\frac{1}{2}\cdot\frac{1}{3}}$. Et pour s'en ſervir aux ſommations, les
“ 5 termes, *par Ex.* de la ſuitte donnée ſeront $\frac{36}{7}-3=\frac{15}{7}$: Et generalement
“ la ſomme des termes juſqu'a quelque terme
“ $\frac{x}{x\cdot\overline{x+1}\cdot\overline{x+2}\times\frac{1}{1}\cdot\frac{1}{2}\cdot\frac{1}{3}}$, excluſivement, ſera $\frac{6x}{x+1}-3$: Et pour la
“ ſomme de la ſuitte entiere a l'infinie, x devient infini, & $\frac{6x}{x+1}$
“ $=6$: donc la ſomme de toutte la ſuitte eſt $6-3=3$, comme vous
“ l'avez trouvé.

“ Cette methode eſt le calcul des differences appliqué aux Nombres; &
“ il faut vous avouer qu'avant que de l'appliquer aux Figures, & même
“ avant que d'avoir été Geometre, Je le prattiquai en quelque façon dans
“ les nombres; ayant trouvé encore jeune garçon que les ſuites dont les
“ Numerateurs fuſſent des Unites, & dont les Denominateurs fuſſent les
“ Nombres figurés, comme Triangulaires Pyramidaux *&c.* etoient les
“ differences 1$^{\text{eres}}$, 2$^{\text{es}}$, 3$^{\text{emes}}$, *&c.* multipliées par les conſtantes de la ſuitte
“ $\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+$ *&c.* & par conſequent ſommables. Mais quand
“ je devins un peu Geometre & Analyſte, Je vis qu'il y avoit moyen
“ de venir a bout de telles ſommations par une Methode generale, autant
“ qu'il etoit poſſible: & que le calcul des differences eſtoit encore
“ plus commode dans la Geometrie que dans les Nombres, puis qu'il y a
“ plus d'evanouiſſements, & que les differences repondent aux Tangentes,
“ les ſommes aux Quadratures. Cette methode generalle de
“ chercher la ſuitte ſommatrice de la ſuitte donnée, quand elle eſt
“ poſſible, reüſſit toujours, quand le terme de la ſuitte donnée exprimé
“ Analytiquement n'a point la quantité variable enveloppé dans une
“ racine, ny entrant dans l'expoſant; & alors, on peut toujours determiner
“ la ſuitte ſommatrice, ou prouver qu'il eſt impoſſible d'en trouver.
“ Et la choſe reüſſit même bien ſouvent, lors même que la variable
“ entre dans l'Expoſant. Mais comme il y a quelquefois des Quadratures
“ particulieres de quelques portions d'une Figure, dont on ne ſçauroit
“ donner la Quadrature generalle ou la Figure quadratrice; de même
“ on peut trouver quelquefois la ſomme de toute la ſuitte, ou d'un
“ certaine partie, quoy qu'on ne puiſſe pas trouver la ſomme de chaque
“ partie; & alors il faut avoir recours a des Methodes particulieres, dont

" on n'eſt pas toujours le maiſtre, noſtre Analyſe n'eſtant pas encore por-
" tée a ſa perfection.

Prop. VII. *Prob.* Invenire ſummam Seriei cujus Numeratores conſtituunt lineam quamlibet erectam in Triangulo Arithmetico *Paſchalii*, Denominatores vero conſtituunt lineam quamlibet tranſverſam.

Solutio. Deſignetur ordo lineæ erectæ per p, ordo lineæ tranſverſæ per q, & ſit m aggregatum tot terminorum primorum in linea erecta ordinis $p+q-1$ quot ſunt unitates in $q-1$, atque ſumma quæſita erit $\overline{2^{p+q-2}-m} \times \frac{1 . 2 . 3 . \&c. \overline{q-1}}{p . \overline{p+1} . \&c. \overline{p+q-2}}$.

Ex. 1. Proponatur Series $\frac{1}{1}+\frac{5}{4}+\frac{10}{10}+\frac{10}{20}+\frac{5}{35}+\frac{1}{56}$. Ubi Numeratores conſtituunt lineam ſextam erectam, Denominatores occupant lineam quartam tranſverſam. In hoc itaque caſu ſunt $p=6$, $q=4$, $p+q-1=9$, $q-1=3$, adeoque $m=1+8+28=37$ *i.e.* tribus terminis primis lineæ nonæ erectæ. Unde fit ſumma quæſita $\overline{2^{8}-37} \times \frac{1 . 2 . 3}{6 . 7 . 8}=\frac{219}{56}$.

Ex. 2. Conſtituant Numeratores lineam centeſimam erectam, & ſint Denominatores Numeri Trigonales, qui occupant lineam tertiam tranſverſam. Tum erunt $p=100$, $q=3$, $m=102$ atque adeo ſumma quæſita fit $\overline{2^{101}-102} \times \frac{1 . 2}{100 . 101}$.

Cor. Si $q=2$, formula fit $\frac{2^{p}-1}{p}$, qua exhibetur aggregatum primi termini, una cum ſemiſſe ſecundi, triente tertii, quadrante quarti, & ſic porro, lineæ cujuſvis erectæ ordinis p Trianguli Arithmetici *Paſchalii*. Sic *v. gr.* eſt $\frac{1}{1}+\frac{5}{2}+\frac{10}{3}+\frac{10}{4}+\frac{5}{5}+\frac{1}{6}=\frac{2^{6}-1}{6}=10\frac{1}{2}$.

Prop. VIII. *Prob.* Invenire ſummam ejuſdem Seriei, quando terminorum ſigna ſunt alternatim + & −.

Solutio. Summa quæſita exhibetur per formulam ſimpliciſſimam $\frac{q-1}{p+q-2}$.

Ex. Invenienda ſit ſumma Seriei $\frac{1}{1}-\frac{6}{9}+\frac{15}{45}-\frac{20}{165}+\frac{15}{495}-\frac{6}{1287}+\frac{1}{3003}$, ubi Numeratores conſtituunt lineam ſeptimam erectam, Denominatores conſtituunt nonam tranſverſam. In formula itaque pro p & q ſcriptis 7 & 9, fit ſumma $\frac{8}{14}$.

Manente eadem Serie Numeratorum (nempe linea ſeptima erecta), ſi pro Serie Denominatorum ſumantur ſucceſſive lineæ tranſverſæ 2^da^, 3^tia^, 4^ta^, *&c.* Summæ erunt $\frac{1}{7}$, $\frac{2}{8}$, $\frac{3}{9}$, $\frac{4}{10}$, $\frac{5}{11}$, *&c.* quæ ſic poſſunt ſcribi, bi, $\frac{1}{7}$, $\frac{7}{28}$, $\frac{28}{84}$, $\frac{84}{210}$, $\frac{210}{462}$, &c. ubi tam Numeratores, quam Denominatores excerpuntur ex linea tranſverſa ordinis ſeptimi. Idem eveniret ſi loco ſeptimæ, Numeratores conſtituiſſent aliam quamlibet lineam erectam ordinis p; Summæ quippe orirentur ex applicatione terminorum lineæ tranſverſæ ejuſdem ordinis p ad terminos proxime ſequentes in eadem linea.

Propoſitiones hæ duæ noviſſimæ potius elegantes ſunt quam utiles; quare Formularum noſtrarum demonſtrationem Lectoris ſolertia inveſtigandam relinquimus, ad Propoſitionem ultimam jam properantes, quæ tertiam continet Serierum ſpeciem, ob uſum multiplicem ſatis inſignem.

Lemma 5. Sit Series quævis $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. cujus terminorum Denominatores conſtituunt progreſſionem quamlibet Geometricam b, b^2, b^3, b^4, &c. Sint etiam Numeratorum primus A $(=M)$ prima differentiarum primarum B, prima ſecundarum C, prima tertiarum D, quartarum E, & ſic porro; & ſint $\frac{\alpha}{b}$, $\frac{\beta}{b^2}$, $\frac{\gamma}{b^3}$, $\frac{\delta}{b^4}$, &c. reſpective, aggregata, Unius, Duorum, Trium, Quatuor, vel plurium terminorum Seriei $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, &c. atque ſint Numeratorum primus a $(=\alpha)$ prima differentiarum primarum b, prima ſecundarum c, prima tertiarum d, & ſic porro: & ſit $b-1=q$. Tum ipſorum $a, b, c, d,$ &c. valores erunt,

$$\begin{aligned}
a &= A = \alpha = M\\
b &= bA + B\\
c &= q\,bA + bB + C\\
d &= q^2\,bA + q\,bB + bC + D
\end{aligned}$$

& ſic porro.

Demonſtratio. Satis conſtat eſſe $a=\alpha=A=M$.

Termini $\frac{M}{b}$, $\frac{N}{b^2}$, $\frac{O}{b^3}$, $\frac{P}{b^4}$, &c. Numeratoribus $M, N, O, P,$ &c. expreſſis per $A, B, C, D,$ &c. transformantur in terminos $\frac{A}{b}$, $\frac{A+B}{b^2}$, $\frac{A+2B+C}{b^3}$, $\frac{A+3B+3C+D}{b^4}$ &c. Unde colligendo ſummas terminorum, inveniuntur Numeratores $\alpha, \beta, \gamma, \delta,$ &c. nempe

$$\begin{aligned}
\alpha &= A\\
\beta &= \overline{b+1}\,A + B\\
\gamma &= \overline{b^2+b+1}\,A + \overline{b+2}\,B + C\\
\delta &= \overline{b^3+b^2+b+1}\,A + \overline{b^2+2b+3}\,B + \overline{b+3}\,C + D, \text{ \&c.}
\end{aligned}$$

Unde ſumendo differentias fiunt

$$\begin{aligned}
b &= bA + B.\\
c &= q\,bA + bB + C
\end{aligned}$$

$d=qqbA+qbB+bC+D$

& ſic porro, ut in Propoſitione exhibentur.

Cor. 1. Si Numeratorum $M, N, O, P,$ &c. differentia vel prima vel ſecunda, vel alia quædam detur, terminis omnibus poſt primos aliquot in Serie $A, B, C, D,$ &c. evaneſcentibus, Differentiæ, $b, c, d,$ &c. tandem incurrent in Progreſſionem Geometricam in ratione 1 ad q. Exempli gratia, ſi detur Numeratorum $M, N, O, P,$ &c. differentia prima B, erunt $c, d,$ &c. in ratione continua Geometrica 1 ad q; ut conſtat per ipſorum valores $qbA+bB$, $qqbA+qbB$, &c. exiſtentibus $C=0=D=$ &c.

Cor. 2. Ordo autem primæ differentiarum $B, C, D,$ &c. quæ hoc modo evaneſcunt, idem eſt ac ordo differentiæ vel b, vel c, &c. unde incipit Progreſſio illa Geometrica. Sic ſi $B=0=C=$ &c. erunt $b, c, d,$ &c. in Progreſſione Geometrica; ſi $C=0=D=$ &c. erunt $c, d,$ &c. in Progreſſione Geometrica. Et ſic porro.

Lemma 6. Iiſdem poſitis ſit r terminus unde incipit Progreſſio Geometrica in Serie differentiarum $b, c, d,$ &c. & per $p+1$ deſignetur ordo Termini in Serie $\frac{\alpha}{b}, \frac{\beta}{b^2}, \frac{\gamma}{b^3}, \frac{\delta}{b^4},$ &c. Tum Terminus ille deſignabitur per fractionem cujus Denominatore exiſtente b^{p+1} Numerator eſt

$$a+bp+cp\times\frac{p-1}{2}+dp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.+\frac{r}{q^n}$$

$$\times\overline{b^p-1-qp-q^2p\times\frac{p-1}{2}-q^3p\times\frac{p-1}{2}\times\frac{p-2}{3}-\&c.}$$

nempe per n deſignato ordine differentiæ evaneſcentis in Serie $B, C, D,$ &c. ut & Numero terminorum $a+bp,$ &c. item terminorum $-1-qp,$ &c.

Demonſtratio. Per **Lemma** 1. Termini iſtius Numerator exhibetur per formulam $a+bp+cp.\frac{p-1}{2}+dp\times\frac{p-1}{2}.\frac{p-2}{3}+$ &c. ($p+1$ ſubeunte vices x in Lemmate iſto).

Ergo ſi ſit, *ex. gr.* $n=2$, per **Lemm.** 5. *Cor.* 2. erunt $c, d,$ &c. in ratione continua 1 ad q. Numerator itaque in hoc caſu eſt

$$a+bp+cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+cq^2p\times\frac{p-1}{2}\times\frac{p-2}{3}$$

$\times\frac{p-3}{4}+$ &c. Sed ſi termini $cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+$ &c. ducantur in $\frac{q^2}{c}$, & productui addantur termini $1+qp$, prodibit Series qua exprimitur binomii $1+q$ dignitas $\overline{1+q}|^p=b^p$. Ergo productum illud æquale eſt b^p-1-qp; adeoque termini $cp\times\frac{p-1}{2}+cqp\times\frac{p-1}{2}\times\frac{p-2}{3}+$ &c. $=\frac{c}{q^2}\times\overline{b^p-1-qp}$. Quo pacto Numerator fit $a+bp+\frac{c}{q^2}\times\overline{b^p-1-qp}$, exiſtentibus duobus terminis $a+bp$, ut &

duobus $-1-qp$, juxta ſenſum Propoſitionis, quoniam $n=2$. Atque eadem eſt demonſtratio in aliis caſibus. De Denominatore vero per ſe ſatis conſtat.

Prop. IX. *Prob.* Invenire ſummam quotlibet terminorum Seriei cujuſvis $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, &c. cujus terminorum Denominatores conſtituunt progreſſionem quamlibet Geometricam h, h^2, h^3, h^4, &c. Numeratores autem ſunt quantitates differentia aliqua conſtanti gaudentes.

Solutio. Sunto Numeratorum M, N, O, P, &c. primus A, prima differentiarum primarum B, prima ſecundarum C, prima tertiarum D, & ſic porro; & ſit ipſorum A, B, C, D, &c. numerus n, atque $h-1=q$, Tum fiat $a=A\ (=M)$. $b=hA+B$, $c=qhA+hB+C$, $d=q^2hA+qhB+hC+D$, &c. ut ſint tot termini a, b, c, d, &c. quot ſunt unitates in $n+1$. Terminorum iſtorum ultimus dicatur r, atque per $p+1$ deſignetur numerus terminorum $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$, &c. quorum ſumma requiritur; Dico ſummam illam exhiberi per fractionem, cujus Denominatore exiſtente h^{p+1}, Numerator eſt

$$a+bp+cp\times\frac{p-1}{2}+dp\times\frac{p-1}{2}\times\frac{p-2}{3}+\&c.+\frac{r}{q^n}\times\overline{h^p-1-qp-q^2p\times\frac{p-1}{2}-q^3p\times\frac{p-1}{2}\times\frac{p-2}{3}-\&c.-q^{n-1}p\times\frac{p-1}{2}\times\&c.}$$

Demonſtratio. Nam (per *Lem.* 6.) per hanc formulam repræſentatur terminus ordine $p+1$ Seriei $\frac{\alpha}{h}, \frac{\beta}{h^2}, \frac{\gamma}{h^3}, \frac{\delta}{h^4}$, &c. qui terminus (per conſtructionem *Lemmatis* 5.) æqualis eſt aggregato terminorum numero $p+1$ Seriei propoſitæ $\frac{M}{h}, \frac{N}{h^2}, \frac{O}{h^3}, \frac{P}{h^4}$. *Q. E. D.*

Ex. 1. Invenienda ſit ſumma novem terminorum Seriei $\frac{1}{2}, \frac{2}{4}, \frac{3}{8}, \frac{4}{16}$, &c. Sunt in hoc caſu $h=2$, $q\ (=h-1)=1$, $p+1=9$, $p=8$, $A=1$, $B=1$, $C=0$, $=D=$ &c. adeoque $n=2$, (quoniam ſunt duo A, B,) Hinc fit $a\ (=A)=1$, $b\ (=hA+B=2\times1+1)=3$, $c\ (=qhA+hB+C=2\times1+2\times1+0)=4=r$, Adeoque per formulam fit ſumma quæſita

$$\frac{1+3\times8+\frac{4}{1^2}\times\overline{2^8-1-1\times8}}{2^9}=\frac{1013}{512}.$$

Ex. 2. Quæratur ſumma ſex terminorum Seriei $1\times3+3\times3^2+6\times3^3+10\times3^4+15\times3^5+21\times3^6+$ &c. In hoc caſu ſunt $h=\frac{1}{3}$, $q=$

$\frac{-2}{3}$, $p+1=6$, $p=5$, $A=1$, $B=2$, $C=1$, $D=0=E=$ &c. adeoque $n=3$, atque $a=1$, $b=\frac{1}{3}+2=\frac{7}{3}$, $c=\frac{-2}{9}+\frac{2}{3}+1=\frac{13}{9}$, $d=\frac{4}{27}-\frac{4}{9}+\frac{1}{3}=\frac{1}{27}=r$. Unde ſumma quæſita fit $=19956$, ſive

$$\frac{1+\frac{7}{3}\times 5+\frac{13}{9}\times 5\times\frac{4}{2}+\frac{-1}{8}\times\frac{1}{3^5}-1+\frac{2}{3}\times 5-\frac{4}{9}\times 5\times\frac{4}{2}}{\frac{1}{3}\big|^{6}}.$$

Cor. 1. Ejuſdem Seriei, a termino primo $\frac{M}{b}$ in infinitum continuatæ, ſumma exhibetur per formulam ſimpliciſſimam $\frac{A}{b-1}+\frac{B}{\overline{b-1}|^2}+\frac{C}{\overline{b-1}|^3}+\frac{D}{\overline{b-1}|^4}$ &c.

Cor. 2. Si $b=2$, Seriei totius in infinitum continuatæ ſumma habetur ſola additione terminorum A, B, C, D, &c. Et hæc ſumma eadem eſt ac ſumma lineæ erectæ reſpondentis termino primo A, in Triangulo Arithmetico, cujus lineam tranſverſam occupant Numeratores M, N, O, P, &c. Quod facile conſtat ex contemplatione Trianguli. Si itaque fuerint M, N, O, &c. Numeri figurati cujuſvis ordinis n, ſumma Seriei $\frac{M}{2}+\frac{N}{4}+\frac{O}{8}+\frac{P}{16}+$ &c. æqualis erit Numeri binarii dignitati $2|^{n-1}$. Sic Series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+$ &c. $=2^{1-1}=1$, ut vulgo notum; Series $\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+$ &c. $=2^{n-1}=2$; Series $\frac{1}{2}+\frac{3}{4}+\frac{6}{8}+\frac{10}{16}+$ &c. $=2^{3-1}=2^2=4$, & ſic porro.

Scholium. Celeb. D. *Jac. Bernoulli*, in Tractatu ſuo de Seriebus infinitis, ſolvit illud Problema. "Invenire ſummam Seriei infinitæ Fractio-
" num quarum Denominatores creſcunt in Progreſſione quacunque Geo-
" metrica, Numeratores vero progrediuntur vel juxta Numeros natura-
" les, 1, 2, 3, 4, &c. vel Trigonales 1, 3, 6, 10, &c. vel Pyramidales 1,
" 4, 10, 20, &c. aut juxta Quadratos 1, 4, 9, 16, &c. aut Cubos 1, 8, 27,
" 64, &c. eorumve multiplices." Ipſius ſolutionem conſulat Lector. Aliam vero, & quidem multo generaliorem invenit D. *Nic. Bernoulli* illius Nepos, eamque (poſtquam ei hæc miſeram, ſed ſine demonſtratione) mecum communicare dignatus eſt, in epiſtola data 18° *Septembris* 1715, miris quidem inventis refertiſſima, qualibus me crebro dignatur vir Doctiſſimus. De hoc vero Problemate ſic ſcribit. "Pour la ſomme d'un
" nombre determiné n de termes de la ſuitte de voſtre Theoreme 7. [Co-

“ *rollarium* primum eſt hujus Propoſitionis] j'ay trouvé cette formule
“ $\frac{1}{m^n} \times \frac{m-1}{m-1}a + \frac{A-n}{m-1}b + \frac{B-n\cdot\frac{n-1}{2}}{m-1}c + \frac{C-n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}}{m-1}d$
“ + *&c.* ou les Lettres *A*, *B*, *C*, *&c.* marquent les Cœfficients des ter-
“ mes immediatement precedents. Et en mettant dans cette formule
“ $p+1$ pour *n*, b^m pour *m*, & en multipliant tout encore par e^{m-1}, on
“ a la ſolution de voſtre *Prob.* IX$^{\text{me}}$”. Et me monuit Vir peritiſſimus hanc ſuam formulam generalem in noſtram particularem (*Cor.* 1. hujus propoſitionis) migrare quando $n = \infty$; quippe tum evaneſcunt $1, n, n\cdot\frac{n-1}{2}, n\cdot\frac{n-1}{2}\cdot\frac{n-2}{3}$, *&c.* reſpectu ipſorum m^n, *A*, *B*, *C*, *&c.* adeo ut Series in eo caſu ſit $\frac{1}{m-1}a + \frac{A}{m-1}b + \frac{B}{m-1}c + \&c.$ quæ omnino coincidit cum noſtra $\frac{a}{m-1} + \frac{b}{\overline{m-1}|^{2}} + \frac{c}{\overline{m-1}|^{3}} + \&c.$

Of the ſame, by Dr. Taylor, *ib. p.* 675.

2. *Definitiones.* 1. Quantitatis cujuſvis variabilis valorem præſentem deſigno litera ſimpliciter ſcripta, ut x; valores præcedentes lineolis eidem literæ ex parte ſuperiori poſitis, ſequentes lineolis ex parte inferiori ſcriptis. Ut vi hujus Definitionis ſint $\overset{\prime\prime}{x}, \overset{\prime}{x}, x, \underset{\prime}{x}, \underset{\prime\prime}{x}$, ejuſdem variabilis valores quinque continui, exiſtente x valore præſenti, $\overset{\prime}{x}$ proxime præterito, $\overset{\prime\prime}{x}$ ſecundo præterito; $\underset{\prime}{x}$ proxime, atque $\underset{\prime\prime}{x}$ ſecundo futuro. Et ſic de aliis. Ad eundem modum ſunt interpretandæ lineolæ quæ incrementis apponuntur. Sic ſunt $\overset{\prime\prime}{\underset{..}{x}}, \overset{\prime}{\underset{..}{x}}, \underset{..}{x}, \underset{\prime\,..}{x}, \underset{\prime\prime\,..}{x}$, ipſius $\underset{..}{x}$ valores quinque continui; ut ſit $\overset{\prime\prime}{\underset{..}{x}}$ incrementum ſecundum ipſius $\overset{\prime\prime}{x}$, ſit $\overset{\prime}{\underset{..}{x}}$ incrementum ſecundum ipſius $\overset{\prime}{x}$. Et ſic de aliis.

Cor. Vi hujus Definitionis, $\overset{\prime}{x} + \overset{\prime}{\underset{.}{x}} = x$, $x + \underset{.}{x} = \underset{\prime}{x}$, $\underset{.}{x} + \underset{..}{x} = \underset{\prime\,.}{x}$. Et ſic de aliis hujuſmodi.

Quando uſu venit ut variabilis quantitas, puta x, ſpectanda ſit tanquam Incrementum, ejus Integrale deſigno litera inter uncos [ ] incluſa. Iſtius etiam Integralis $[x]$ Integrale (vel ipſius x integrale ſecundum,) deſigno numero binario uncorum priori ſuperimpoſito, ut $\overset{2}{[}x]$. Iſtius etiam Integralis Integrale (vel ipſius x Integrale tertium,) ad eundem modum deſigno numero ternario, ut $\overset{3}{[}x]$. Et ſic deinceps. Unde vi hujus Definitionis conſtituunt $\overset{3}{[x]}$, $\overset{2}{[x]}$, $[x]$, x Seriem terminorum, quorum quilibet eſt ipſum immediate præcedentis incrementum primum, ut ſit $\overset{2}{[x]} = \overset{3}{[\underset{.}{x}]}$, $[x] = \overset{2}{[\underset{.}{x}]}$, $x = [\underset{.}{x}]$.

Lemma. Facti $x\,v$ ex Multiplicatione duorum variabilium x & v, incrementum eſt $\underset{.}{x}\,v + \underset{,}{x}\,\underset{.}{v}$.

Nam auctis variabilibus per propria incrementa, fit novum productum $\overline{x+\underset{.}{x}} \times \overline{v+\underset{.}{v}}$, ſive $x\,v + \underset{.}{x}\,v + \overline{x+\underset{.}{x}} \times \underset{.}{v}$, hoc eſt $x\,v + \underset{.}{x}\,v + \underset{,}{x}\,\underset{.}{v}$ (pro $x+\underset{.}{x}$ ſcripto $\underset{,}{x}$ *per Def.* 1.) Unde dempto priori producto $x\,v$, reſtat Incrementum $x\,\underset{.}{v} + \underset{,}{x}\,\underset{.}{v}$.

Prop. I. *Theor.* Ejuſdem Facti $x\,v$ Incrementum vel primum, vel ſecundum, vel tertium, vel aliud quodvis, cujus ordo deſignatur per ſymbolum n, exhibetur per formulam hanc generalem

$$\underset{n}{x}\,v + n\,\underset{,\;n-1}{x}\,\underset{.}{v} + n \times \frac{n-1}{2}\,\underset{,,\;n-2}{x}\,\underset{:}{v} + n \times \frac{n-1}{2} \times \frac{n-2}{3}\,\underset{,,,\;n-3}{x}\,\underset{\vdots}{v} + \&c.$$

In hac formula hæc ſunt obſervanda, 1^mo^ Terminorum numeri coefficientes 1, n, $n \times \frac{n-1}{2}$, $n \times \frac{n-1}{2} \times \frac{n-2}{3}$ &c. iidem ſunt ac in binomii dignitate n. 2^do^ Numeri n, $n-1$, $n-2$, $n-3$, &c. ipſis x infraſcripti deſignant numeros punctorum quibus definiuntur Incrementa. 3^tio^ Lineolæ $,$ $,,$ $,,,$ &c. ipſis x infraſcriptæ, interpretandæ ſunt per Def. 1. 4^to^ In quovis Termino numerus punctorum ipſis x & v ſimul infraſcriptorum, eſt n. Sit *v. g.* $n = 4$: tum per formulam, ipſius $x\,v$ incrementum quartum prodit $\underset{::}{x}\,v + 4\,\underset{,\,\vdots}{x}\,\underset{.}{v} + 6\,\underset{,,\,:}{x}\,\underset{:}{v} + 4\,\underset{,,,\,.}{x}\,\underset{\vdots}{v} + \underset{,,,,}{x}\,\underset{::}{v}$.

Theorema hoc generale demonſtrari poteſt per Inductionem, incrementis continuo ſumptis juxta formam in Lemmate præcedenti traditam. Sed & collecta forma Seriei ex hujuſmodi calculo, Theorema etiam demonſtrari poteſt per Methodum Incrementorum, ad eum modum cujus ſpecimen mox dabimus in demonſtratione Propoſitionis tertiæ.

Prop. II. *Theor.* Ipſius $x\,v$ Integrale primum $[x\,v]$ exhibetur per Seriem $[x]\,v - \overset{2}{[\underset{,}{x}]}\,\underset{.}{v} + \overset{3}{[\underset{,,}{x}]}\,\underset{:}{v} - \overset{4}{[\underset{,,,}{x}]}\,\underset{\vdots}{v}$ &c.

Series autem ita terminatur, ut ſit $[x\,v] = [x]\,v - \Big[[\underset{,}{x}]\,\underset{.}{v}\Big] = [x]\,v - \overset{2}{[\underset{,}{x}]}\,\underset{.}{v} + \Big[\overset{2}{[\underset{,,}{x}]}\,\underset{:}{v}\Big] = \&c.$

Nam ſumendo incrementa reſtituitur propoſitum $x v$.

Cor. 1. Datis duobus ex iſtis $[x]$, $[xv]$, $\left[[\underset{\prime}{x}]\underset{\cdot}{v}\right]$, datur tertium. Item datis tribus ex iſtis $[x]$, $\overset{2}{[\underset{\prime}{x}]}$, $[xv]$, $\left[\overset{2}{[\underset{\prime\prime}{x}]}\underset{\cdot}{v}\right]$, datur quartum. Et ſic porro.

Cor. 2. Si $\underset{\cdot}{v}=0$, datur $[xv]$ ex dato $[x]$. Si $\underset{\cdot\cdot}{v}=0$ datur $[xv]$ ex datis duobus $[x]$, & $\overset{2}{[x]}$, Si $\underset{\therefore}{v}=0$, datur $[xv]$, ex datis tribus $[x]$, $\overset{2}{[x]}$, $\overset{3}{[x]}$. Et ſic porro.

Ex. 1. Sit exemplum hujus formulæ in inventione Integralis ipſius $\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$, dato nempe $\underset{\cdot}{z}$, atque exiſtente $\underset{\therefore}{v}=0$, qui caſus eſt ſpecialis Propoſitionis ſecundæ Tractatus præcedentis D^ni^ *Monmort*. Facto itaque $x=\frac{1}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}$, ſunt $[x]=\frac{-1}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}}$, $\overset{2}{[\underset{\prime}{x}]}=\frac{1}{2\underset{\cdot}{z}\times3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}}$, atque $\overset{3}{[\underset{\prime\prime}{x}]}=\frac{-1}{1\underset{\cdot}{z}\times2\underset{\cdot}{z}\times3\underset{\cdot}{z}\underset{\prime\prime}{z}}$. Unde per formulam fit $[xv]$, hoc eſt $\frac{v}{z\underset{\prime}{z}\underset{\prime\prime}{z}\underset{\prime\prime\prime}{z}}=-\frac{v}{3\underset{\cdot}{z}z\underset{\prime}{z}\underset{\prime\prime}{z}}-\frac{\underset{\cdot}{v}}{2\underset{\cdot}{z}\times3\underset{\cdot}{z}\underset{\prime}{z}\underset{\prime\prime}{z}}-\frac{\underset{\cdot\cdot}{v}}{1\underset{\cdot}{z}\times2\underset{\cdot}{z}\times3\underset{\cdot}{z}\underset{\prime\prime}{z}}$.

Ex. 2. Sit aliud exemplum in inventione Integralis ipſius na^{z}, ubi eſt $z=1$, atque datur a. Tum pro x ſumpto a^{z}, & pro v ſumpto n, fit $x=a^{\underset{\prime}{z}}$ hoc eſt $x=ax$, ſeu $x+\underset{\cdot}{x}=ax$, adeoque $\underset{\cdot}{x}=\overline{a-1}\,x$, atque $x=\frac{\underset{\cdot}{x}}{a-1}$ Regrediendo itaque ad Integralia fit $[x]=\frac{x}{a-1}$; item $\overset{2}{[x]}=\frac{[x]}{a-1}=\frac{x}{\overline{a-1}|^{2}}$, item $\overset{3}{[x]}=\frac{x}{\overline{a-1}|^{3}}$; & ſic porro. Adeoque (quoniam $\underset{\prime}{x}=ax$,) ſunt $[x]=\frac{x}{a-1}$, $[\underset{\prime}{x}]=\frac{ax}{\overline{a-1}|^{2}}$, $[\underset{\prime\prime}{x}]=\frac{a^{2}x}{\overline{a-1}|^{3}}$, &c. Unde per formulam prodit $[na^{z}]=\frac{a^{z}n}{a-1}-\frac{a^{z+1}\underset{\cdot}{n}}{\overline{a-1}|^{2}}+\frac{a^{z+2}\underset{\cdot\cdot}{n}}{\overline{a-1}|^{3}}$, &c.

In hoc exemplo continetur Solutio Problematis, de quo agit D^nus^ de *Monmort* in Propoſitione nona. Coincidit autem formula cum ea quam exhibet ille in Corollario primo ejuſdem Propoſitionis.

Scholium. Poſſunt etiam ex hac formula alii derivari valores Integralis quæſiti, pro vario modo quo interpretantur Incrementi propoſiti factores. Sic in exemplo ſecundo integrale ipſius na^{z} exhiberi poteſt per formulam

Iam $a^x [n] - \overline{a-1}\, a^x \overset{2}{[\underset{,}{n}]} + \overline{a-1}|^2\, a^x \overset{3}{[\underset{,,}{n}]} -$ &c. pro x nempe ſumpto n, & pro v ſumpto a^x. Sed de his fortaſſe alia occaſione fuſius dicemus.

Prop. III. *Theor.* Ejuſdem xv Integrale, vel primum, vel ſecundum, vel tertium, vel aliud quodvis cujus ordo deſignatur ſymbolo n, exhibetur per Seriem in hac forma generali prodeuntem $\overset{n}{[xv]} = \overset{n}{[x]}v - n \overset{n+1}{[\underset{,}{x}]}\underset{\cdot}{v} + n \times \frac{n+1}{2} \overset{n+2}{[\underset{,,}{x}]}\underset{\cdot\cdot}{v} - n \times \frac{n+1}{2} \times \frac{n+2}{3} \overset{n+3}{[\underset{,,,}{x}]}\underset{\because}{v} +$ &c.

Collecta forma Seriei ex Propoſitione præcedenti, Coefficientes $1, -n, n \times \frac{n+1}{2}, -n \times \frac{n+1}{2} \times \frac{n+2}{3}$, &c. ſic inveniuntur per Methodum Incrementorum. Pone $\overset{n}{[xv]} = A \overset{n}{[x]}v + B \overset{n+1}{[\underset{,}{x}]}\underset{\cdot}{v} + C \overset{n+2}{[\underset{,,}{x}]}\underset{\cdot\cdot}{v} + D \overset{n+3}{[\underset{,,,}{x}]}\underset{\because}{v} +$ &c. Tum aucto n incremento ſuo $\underset{,}{n} = 1$, atque ipſis A, B, C, D, &c. incrementis ſuis contemporaneis $\underset{\cdot}{A}, \underset{\cdot}{B}, \underset{\cdot}{C}, \underset{\cdot}{D}$, &c. ut jam evadant $\underset{,}{n}$, $\underset{,}{A}, \underset{,}{B}, \underset{,}{C}, \underset{,}{D}$, &c. fiet novum Integrale (quod Integrale eſt ipſius $\overset{n}{[xv]}$,) $\overset{n+1}{[xv]} = \underset{,}{A} \overset{n+1}{[x]}v + \underset{,}{B} \overset{n+2}{[\underset{,}{x}]}\underset{\cdot}{v} + \underset{,}{C} \overset{n+3}{[\underset{,,}{x}]}\underset{\cdot\cdot}{v} + \underset{,}{D} \overset{n+4}{[\underset{,,,}{x}]}\underset{\because}{v} +$ &c. Hujus itaque Incrementum primum coincidere debet cum Integrali prius poſito. Sumptis ergo incrementis, fit $\overset{n}{[xv]} = \underset{,}{A} \overset{n}{[x]}v \begin{matrix} +\underset{\prime}{A} \\ +\underset{,}{B} \end{matrix} \overset{n+1}{[\underset{,}{x}]}\underset{\cdot}{v} \begin{matrix} +\underset{\prime}{B} \\ +\underset{,}{C} \end{matrix} \overset{n+2}{[\underset{,,}{x}]}\underset{\cdot\cdot}{v} \begin{matrix} +\underset{\prime}{C} \\ +\underset{,}{D} \end{matrix} \overset{n+3}{[\underset{,,,}{x}]}\underset{\because}{v} +$ idem ac Integrale prius poſitum. Itaque terminos homologos inter ſe comparando fit 1^{mo} $\underset{,}{A} = A$. Unde eſt A datum quid. Sed ubi $n = 0$, eſt $A = 1$, ergo $A = 1$. 2^{do}. $B = \underset{,}{B} + \underset{,}{A}$, hoc eſt $B = B + \underset{\cdot}{B} + 1$, ſeu $\underset{\cdot}{B} = -1 = -\underset{\cdot}{n}$. Ergo regrediendo ad Integralia, fit $B = -n + a$. Sed ubi $n = 0$, eſt $B = 0$. Ergo $a = 0$, atque $B = -n$. 3^{tio}. $C = \underset{,}{C} + \underset{,}{B}$, hoc eſt $\underset{\cdot}{C} = \underset{,}{n}$. Regrediendo itaque ad integralia fit $C = \frac{n\underset{,}{n}}{2} + b$. Sed ubi $n = 0$, eſt $C = 0$. Ergo $b = 0$, atque $C = \frac{n\underset{,}{n}}{2}$, hoc eſt

$n \times \frac{n+1}{2}$. 4^{to}. Ad eundem modum invenitur $D = -n \times \frac{n+1}{2} \times \frac{n+2}{3}$. Et ſic pergendo inveniuntur cæteri Coefficientes.

Scholium 1. In hac Propoſitione comparata cum Propoſitione prima, cernitur ſingularis quædam relatio Incrementa inter & Integralia. Ut enim in Arithmetica vulgari, Multiplicatio & Diviſio ſunt invicem ita contrariæ, ut ſi Multiplicatio deſignetur per Indicem affirmativum, Diviſio deſignabitur per Indicem cum ſigno negativo; ſic etiam in Methodo Incrementorum, ſi Incrementum deſignetur per Indicem affirmativum, Index negativus Integrale ſiſtet. Sic in Propoſitione prima, ſi pro n ſumatur Numerus binarius 2, per formulam exhibebitur ipſius xv incrementum ſecundum, nempe $\underset{..}{x}\,v + 2\,\underset{\prime .}{x}\,\underset{.}{v} + \underset{\prime\prime}{x}\,\underset{..}{v}$; Sed ſi pro n ſumatur numerus negativus -2, ut jam quæratur ipſius xv incrementum (ita loqui liceat) negative ſecundum, (quod idem eſt ac Integrale ſecundum) prodeunt coefficientes iidem ac ſi ſumatur n affirmative in Propoſitione præſenti: atque interpretatis inſuper ipſis $\underset{-2}{x}$, $\underset{\prime -3}{x}$, $\underset{\prime\prime -4}{x}$, &c. per $\overset{2}{[x]}$, $\overset{3}{[\underset{\prime}{x}]}$, $\overset{4}{[\underset{\prime\prime}{x}]}$, &c. Series fit omnino eadem ac per Propoſitionem præſentem prodit, ubi quæritur Integrale ſecundum.

2. Ex his autem formulis quaſi ſua ſponte procedunt formulæ Propoſitionum undecimæ atque duodecimæ Libri de Methodo Incrementorum. Nam pro incrementis ſcribe Fluxiones, atque evaneſcentibus incrementis fiant jam omnes x, $\underset{\prime}{x}$, $\underset{\prime\prime}{x}$, $\underset{\prime\prime\prime}{x}$, &c. inter ſe æquales, atque migrabit ſtatim hæc Propoſitio ſecunda in illam undecimam, atque præſens tertia in illam duodecimam. Quod quidem exemplum ſatis inſigne eſt Methodi *Newtonianæ*, qua colligit ille rationes Fluxionum ex rationibus ultimis Incrementorum evaneſcentium, vel ex primis naſcentium.

Præcedentium impreſſioni intentus dum in animo illa ſæpius revolvo, ſubiit Artificium illud quo jam olim uſus eſt D. *Jac. Bernoulli* in inventione quarundam Serierum, ope Progreſſionis Harmonicæ cujus meminit D. de *Monmort* in *Scholio 6. Prop.* V. præcedente, commode etiam applicari poſſe ad inventionem ipſius *Monmortii* Propoſitionum $2^{dæ}$, $3^{tiæ}$, $4^{tæ}$, $5^{tæ}$, atque id genus aliarum aliquanto fortaſſe generaliorum. Hoc in ſequentibus paucis oſtendiſſe credebam Lectori non fore ingratum.

Theorema. Sit Progreſſio Arithmetica p, $p+n$, $p+2n$, &c. cujus termini ſinguli ſucceſſive deſignentur per x, & ſunto b, c, d, &c. quivis multiplices differentiæ datæ n terminorum Progreſſionis iſtius Arithmeticæ. Sint A, B, C, D, &c. Numeri quilibet dati, & conſtituantur fractiones quotvis $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, &c. Pro x ſucceſſive ſcriptis valoribus ſuis p, $p+n$, $p+2n$, &c. ex harum fractionum qualibet oritur Series Harmonice proportionalium. Sic *v. g.* ex fractione prima $\frac{A}{p}$, oritur Series $\frac{A}{p}$, $\frac{A}{p+n}$, $\frac{A}{p+2n}$, &c. Dico quod aggregatum quotlibet hujuſmodi Serierum in infinitum continuatarum in terminis numero finitis exhiberi poteſt, ſi modo fuerit numeratorum A, B, C, D, &c. aggregatum æquale nihilo. Duobus exemplis hoc fiet manifeſtum.

Ex. Sint duæ tantum fractiones $\frac{A}{x}$, atque $\frac{-A}{x+3n}$, exiſtente $b=3n$. Scribantur Series harmonicæ ex his formulis ortæ, eo ordine, ut termini, in quibus ſunt denominatores æquales, ſibi invicem reſpondeant, & collectis ſummis terminorum homologorum, prodibit aggregatum Serierum in terminis numero finitis, ut in calculo appoſito videre eſt.

$$\begin{array}{l}
\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\frac{A}{p+4n}+\&c.=\text{Seriei ortæ ex }\frac{A}{x}\\
\qquad\qquad\qquad\qquad\quad+\frac{-A}{p+3n}+\frac{-A}{p+4n}+\&c.=\text{Seriei ex }\frac{-A}{x+3n}\\
\hline
\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\;0\;+\;0\;+\&c.=\text{Aggreg. Serierum.}
\end{array}$$

Ex. 2. Sint tres fractiones $\frac{A}{x}$, $\frac{B}{x+2n}$, $\frac{C}{x+3n}$, exiſtentibus $b=2n$, $c=3n$, atque $A+B+C=0$. In hoc caſu Calculus ſic ſe habet.

$$\begin{array}{l}
\frac{A}{p}+\frac{A}{p+n}+\frac{A}{p+2n}+\frac{A}{p+3n}+\ldots+\&c.=\text{Seriei ortæ ex }\frac{A}{x}\\
\qquad\qquad\qquad+\frac{B}{p+2n}+\frac{B}{p+3n}+\ldots+\&c.=\text{Seriei ex }\frac{B}{x+2n}\\
\qquad\qquad\qquad\qquad\quad+\frac{C}{p+3n}+\ldots+\&c.=\text{Seriei ex }\frac{C}{x+3n}\\
\hline
\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}+\frac{A+B+C=0}{p+3n}+\&c.=\text{Aggregato Serierum.}
\end{array}$$

Ubi etiam prodit aggregatum Serierum in terminis numero finitis, nempe $\frac{A}{p}+\frac{A}{p+n}+\frac{A+B}{p+2n}$, ob Numeratorum A, B, C, aggregatum æquale nihilo. Et ad eundem modum demonſtratur Theorema in aliis caſibus quibuſvis.

Cor. 1. Ex his principiis derivari poſſunt innumeræ Series in infinitum continuatæ, in terminis tamen numero finitis ſummabiles.

Caſ. 1. Sint $\frac{A}{x}$ & $\frac{-A}{x+b}$ formulæ duarum Serierum harmonicarum, quarum aggregatum prodit in terminis numero finitis per ſuperius demonſtrata. Tum, formulis iſtis in unam ſummam collectis, fit $\frac{Ab}{x\times\overline{x+b}}$ formula Seriei ſummabilis. Sint *v. gr.* $A=\frac{1}{6}$, $p=1$, $n=2$, atque $b=3n=6$. Tum formulæ Serierum harmonicarum erunt $\frac{1}{6x}$, & $\frac{-1}{6\times x+6}$, formula Seriei compoſitæ ſummabilis erit $\frac{1}{x\times\overline{x+6}}$, Serie illa exiſtente $\frac{1}{1\times 7}+\frac{1}{3\times 9}+\frac{1}{5\times 11}+\frac{1}{7\times 13}+$ &c. atque ſumma Seriei, per calculum in præmiſſis demonſtratum, erit $\frac{1}{6\times 1}+\frac{1}{6\times 3}+\frac{1}{6\times 5}$. Sint tres formulæ Serierum harmonicarum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$ (exiſtente $A+B+C=0$, ut ſit Serierum aggregatum finitum per præmiſſa.) Tum formulis in unam ſummam collectis fit

$$\frac{A\times\overline{x+b}\times\overline{x+c}+B\times x\times\overline{x+c}+C\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}},$$

ſeu (terminis revocatis ad formam factorum x, $x\times\overline{x+b}$, $x\times\overline{x+b}\times\overline{x+c}$,)

$$\frac{Acb+\overline{Ac+\overline{c-b}B}\times x+\overline{A+B+C}\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}},$$

hoc eſt (ob $A+B+C=0$) $\frac{Acb+\overline{Ac+B\times\overline{c-b}}\times x}{x\times\overline{x+b}\times\overline{x+c}}$, formula Seriei ſummabilis. Si quatuor ſint Fractiones $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, (exiſtente $A+B+C+D=0$) ad eundem modum invenietur formula Seriei ſummabilis

$$\frac{Abcd+\overline{Acd+B\times\overline{c-b}\times\overline{d-b}}\times x+\overline{Ad+B\times\overline{d-b}+C\times\overline{d-c}}\times x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$$

Et ſic pergere licet ad formulas adhuc magis compoſitas.

Caſ. 2. Et ſi plures ſint formulæ Serierum hujuſmodi ſummabilium, quarum denominatorum factores excerpantur ex diverſis progreſſionibus Arithmeticis, ex iſtarum formularum quotvis in unam ſummam additione, conficietur formula nova Seriei ſummabilis. Sint *e. gr.* formulæ duæ Serierum ſummabilium $\frac{1}{x\times\overline{x+3}}$ & $\frac{1}{z\times\overline{z+2}}$, excerptis x ex Progreſſione Arithmetica 1, 2, 3, 4, &c. z ex Progreſſione Arithmetica 1, 3, 5, &c. Tum ex his formulis in unam ſummam collectis fiet formula nova

$$\frac{z\times\overline{z+2}+x\times\overline{x+3}}{x\times\overline{x+3}\times z\times\overline{z+2}},$$

vel (expoſito z per x & numeros datos)

$$\frac{\overline{2x-1}\times\overline{2x+1}+x\times\overline{x+3}}{x\times\overline{x+3}\times\overline{2x-1}\times\overline{2x+1}}.$$

Cor. 2. Hinc omnis Series in infinitum continuata ſummabilis eſt, cujus termini deſignantur per Fractionem, cujus denominatoris factores excerpuntur ex data qualibet Progreſſione Arithmetica, numerator autem eſt multinomium, cujus dimenſiones ſunt ad minimum binario pauciores, quam ſunt dimenſiones Denominatoris. Nam omnis hujuſmodi fractio reſolvi poteſt in tot fractiones ſimplices, quot ſunt dimenſiones (hoc eſt, quot ſunt factores) Denominatoris, quarum numeratorum aggregatum eſt nihil. Sit exempli gratia, formula oblata $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$. Pone hanc formulam æquari aggregato fractionum $\frac{A}{x} + \frac{B}{x+b} + \frac{C}{x+c} + \frac{D}{x+d}$. Tum fractionibus iſtis in unam ſummam collectis fiet $Abcd + \overline{Acd + B\,\overline{c-b}\times\overline{d-b}}\times x + \overline{Ad + B\times\overline{d-b} + C\times\overline{d-c}}\times x\times\overline{x+b} + \overline{A+B+C+D}\times x\times\overline{x+b}\times\overline{x+c}$ applicatum ad $x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d} = \frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$.

Unde per comparationem terminorum homologorum fit $Abcd = \alpha$, $Acd + B\times\overline{c-b}\times\overline{d-b} = \beta$. $Ad + B\times\overline{d-b} + C\times\overline{d-c} = \gamma$, $A+B+C+D = 0$, adeoque $A = \frac{\alpha}{bcd}$, $B = \frac{\beta - Acd}{\overline{c-b}\times\overline{d-b}}$ $C = \frac{\gamma - Ad - B\times\overline{d-b}}{d-c}$, $D = -A-B-C$. Quo pacto formula oblata reſolvitur in fractiones ſimplices $\frac{\alpha}{bcdx} + \frac{\beta - Acd}{\overline{c-b}\times\overline{d-b}\times\overline{x+b}} + \frac{\gamma - Ad - B\times\overline{d-b}}{\overline{d-c}\times\overline{x+c}} + \frac{-A-B-C}{x+d}$, ex quibus ortarum Serierum aggregatum, hoc eſt, ſumma Seriei ortæ ex formula oblata $\frac{\alpha+\beta x+\gamma x\times\overline{x+b}}{x\times\overline{x+b}\times\overline{x+c}\times\overline{x+d}}$, per jam dicta prodit in terminis numero finitis. Quod vero dimenſiones numeratoris in formula oblata debeant eſſe binario ad minimum pauciores, quam ſunt dimenſiones Denominatoris, hinc conſtat, quod in reductione fractionum $\frac{A}{x}$, $\frac{B}{x+b}$, $\frac{C}{x+c}$, $\frac{D}{x+d}$, quilibet numerator A, B, C, D, ducitur in omnes denominatores excepto uno, nempe ſuo; unde prodeunt Numeratoris Dimenſiones unitate pauciores quam ſunt dimenſiones Denominatoris. Sed per æquationem $A + B + C + D = 0$ perit altiſſima dimenſio in numeratore; Unde ſuperſunt Numeratoris Dimenſiones ad minimum binario pauciores quam ſunt dimenſiones Denominatoris. Ad hoc vero Corollarium revocari poſſunt D. de *Monmort* Propoſitiones 2^{da} & 5^{ta}.

Cor. 3. Item oblata formula juxta *Caſ.* 2. *Cor.* 1. adhuc magis compoſita, ex iiſdem principiis perſpici poteſt an ſit Series ſummabilis. Sint progreſſiones duæ Arithmeticæ 1, 3, 5, *&c.* 2, 4, 6, *&c.* quarum termini homologi deſignentur per x & z, & ſit formula Seriei oblata $\frac{\alpha+\beta x+\gamma x^2}{x\times\overline{x+2}\times z\times\overline{z+2}}$, vel (pro z ſcripto $x+1$, & factoribus Denominatoris in ordinem coactis) $\frac{\alpha+\beta x+\gamma x^2}{x\times\overline{x+1}\times\overline{x+2}\times\overline{x+3}}$. Pone formulam hanc æquari aggregato formularum $\frac{P}{x\times\overline{x+2}}$, $\frac{Q}{\overline{x+1}\times\overline{x+3}}$, Serierum per ſuperius dicta ſummabilium, ut (formulis his noviſſimis in unam ſummam collectis) ſit $\frac{P\times\overline{x+1}\times\overline{x+3}+Q\times x\times\overline{x+2}}{x\times\overline{x+1}\times\overline{x+2}\times\overline{x+3}}$ ſeu $\frac{3P+\overline{4P+2Q}\,x+\overline{P+Q}\,x^2}{x\times\overline{x+1}\times\overline{x+2}\times\overline{x+3}}=\frac{\alpha+\beta x+\gamma x^2}{x\times\overline{x+1}\times\overline{x+2}\times\overline{x+3}}$: Hinc comparando terminos homologos oriuntur æquationes $3P=\alpha$, $4P+2Q=\beta$, $P+Q=\gamma$. Unde eliminatis P & Q per debitas operationes Analyticas, prodit æquatio $2\alpha-3\beta+\gamma=0$, qua definitur relatio quæ inter coefficientes α, β, γ intercedere debet, ut Series orta ex formula oblata $\frac{\alpha+\beta x+\gamma x^2}{x\times\overline{x+1}\times\overline{x+2}\times\overline{x+3}}$ ſit ſummabilis. Ad eundem modum ſi formulæ oblatæ Denominatoris factores excerpantur ex tribus Progreſſionibus Arithmeticis, invenientur duæ æquationes quibus definiantur relationes coefficientium Numeratoris, ut ſit Series ſummabilis. Si quatuor ſint Progreſſiones Arithmeticæ, Coefficientium relatio definietur per tres æquationes. Et ſic porro. Et in hujuſmodi formulis ut ſint Series ſummabiles, hæc inſuper obſervanda ſunt, Primo ut Numeratorum dimenſiones ſint ad minimum binario pauciores quam ſunt dimenſiones Denominatorum. Deinde ut ex ſingulis Progreſſionibus Arithmeticis excerpantur ad minimum duo factores Denominatoris. Denique, quod ſi ſint duo vel plures factores Denominatoris inter ſe æquales, ponendum ſit tot etiam Progreſſiones Arithmeticas, ex quibus excerpuntur, eſſe inter ſe æquales. Præmiſſis attentius perpenſis, hæc obvia erunt. Ad hoc vero Corollarium facile revocantur D. de *Monmort* Propoſitiones 3$^{\text{tia}}$ & 4$^{\text{ta}}$.

Sir Iſaac Newton's *Differential Method illuſtrated, by Mr.* James Sterling. *n.* 362 *p.* 1050.

XXII. Arithmeticæ pars præcipua conſiſtit in invenienda in numeris quantitate quacunque determinata; cum vero quantitatum & numerorum natura non patiatur ut omnes quantitates exhibeantur in numeris accurate, neceſſe habemus ad Approximationes confugere. Hoc eſt ubi quantitatum valores mathematice accurati nequeunt obtineri, quærenda ſunt ii qui ab accuratis diſtant minus data quavis differentia.

Quicquid hac de re a Veteribus ad nos pervenit, vel eſt particulare, ut Methodus eorum reducendi Æquationes Quadraticas; vel ſaltem uſibus generalibus male deſtinatum, ut Methodus Exhauſtionum. *Vieta* quidem primus erat qui aliquid generale in hac arte aſſequutus eſt: quippe invenit methodum reducendi Æquationes Rationales, quæ ſolæ tunc in uſu erant. In hac acquievere omnes Geometræ ex ejus temporibus uſque ad ea *Newtoni*. Hic ex Interpolationibus primo pervenit ad Series: quas poſtea ad reductionem Æquationum omnium omnino generum univerſaliter applicuit. Hæc autem methodus procedit per quantitatum naſcentium & evaneſcentium rationes primas & ultimas, ſeu ſi ita loqui liceat, per quantitatum coincidentium differentias infinite parvas. Sed & ulterius promovit *Newtonus* hanc methodum; docuitque qua ratione approximandum ſit ad quantitates quæ determinantur per regularem ſeriem terminorum, non per Æquationem ut vulgo fit. Atque ſic poſuit fundamenta calculi hujus Differentialis, qui procedit per quantitatum differentias cujuſcunque magnitudinis; ideoque eſt methodo Serierum univerſalior. Per haſce artes *Newtonianas* univerſa doctrina Approximationum reducitur ad ſolutionem Problematis, *Invenire Lineam Geometricam quæ per data quotcunque puncta tranſibit.* Ex hujus inquam ſolutione inveniuntur radices Æquationum quarumcunque, & etiam quantitates quarum relationes ad alias datas per nullas Æquationes hactenus notas poſſunt exprimi. Exiſtimo igitur *Newtonum* perduxiſſe methodum Approximandi ad ſummum perfectionis faſtigium; dum ex unico ſimpliciſſimo principio totam hanc doctrinam longe lateque patentem deducit.

Author noſter, in Epiſtola ad *Oldenburgum, Octob.* 24. 1676. data, mentionem fecit de methodo expedita ducendi Lineam Parabolicam per data quotcunque puncta; qua dixit ſe uſum fuiſſe ubi Series ſimplices non ſunt ſatis tractabiles. Et hanc methodum primo publicavit in Lemmate quinto Libri tertii *Principiorum.* Atque in Lectionibus publicis, circa idem tempus quo dicta Epiſtola ſcripta eſt, *Cantabrigiæ* habitis, expoſuit modum generalem determinandi Curvas cujuſcunque generis quæ tranſibunt per totidem data puncta quot earum natura patitur. Hæ Lectiones ſub titulo *Arithmeticæ Univerſalis* anno 1707. publicatæ ſunt, ubi habetur methodus exemplis illuſtrata in ſectionibus Conicis. Anno vero 1711. tandem prodiit, inter alios ejuſdem Authoris tractatus, ipſa Methodus Differentialis plenius quam ante expoſita, cum fundamento ejus demonſtrato.

Archimedes in methodo Exhauſtionum, *Cavallerius* in methodo Indiviſibilium, & *Walliſius* noſter in Arithmetica infinitorum, poſuerunt fundamenta doctrinæ de determinanda quantitate quæſita per locum quem obtinet inter terminos in data Serie: at qua ratione approximandum eſſet ad valores quantitatum ſic determinatarum, horum nemo docuit; Hoc primus & ſolus perfecit *Newtonus:* atque exinde haud parum ampliata eſt univerſa Analyſis. Nam ſicut ante hoc inventum, ea Problemata Arithmetica ſola pro ſolutis habebantur, ubi relatio quantitatis quæſitæ ad alias datas definiebatur Æquatione, jam pro ſolutis habenda ſunt non minus ea, in quibus quantitas quæſita locum datum ſortitur inter terminos datæ Seriei; ſiquidem numeri deſiderati non minus accurate obtinentur per Methodum Differentialem, quam per extractionem Radicum: hiſce vero habitis, parum intereſt quomodo ad eos deventum eſt. Et experientia multiplex docuit, quod plurima Problemata ad Æquationes ægre deducuntur, dum ad methodum Differentialem facillime. Qualis eſt ex multis aliis toties decantata Circuli Quadratura; quam tam perfectam, mea opinione, *Walliſius* in Arithmetica Infinitorum exhibuit quam *Archimedes* illam Parabolæ.

☞ Propoſitio. *Invenire Lineam Parabolicam quæ tranſibit per extrema Ordinatorum quotcunque æquidiſtantium.*

q
Q
p
A
A2
A3
A4
O
A5
A6
A7
P
A8
A9

B *B2* *B3* *B4* *B5* *B6* *B7* *B8*
C *C2* *C3* *C4* *C5* *C6* *C7*
D *D2* *D3* *D4* *D5* *D6*
E *E2* *E3* *E4* *E5*
F *F2* *F3* *F4*
G *G2* *G3*
H *H2*
I

Caſus Primus. Deſignent *A*, *A2*, *A3*, *A4*, *A5*, *A6*, *A7*, *A8*, *A9*, *&c.* Ordinatas æquidiſtantes inſiſtentes Abſciſſæ in dato angulo. Collige earum differentias *B*, *B2*, *B3*, *B4*, *B5*, *B6*, *B7*, *B8*, *&c.* harumque differentias *C*, *C2*, *C3*, *C4*, *C5*, *C6*, *C7*, *&c.* harumque differentias *D*, *D2*, *D3*, *D4*, *D5*, *D6*, *&c.* harumque differentias *E*, *E2*, *E3*, *E4*, *E5*, *&c.* harumque *F*, *F2*, *F3*, *F4*, *&c.* Et ſic porro. Differentiæ autem colligi debent auferendo priores ſemper de poſterioribus. Hoc eſt ponendo $B = A2 - A$, $B2 = A3 - A2$, $B3 = A4 - A3$, $B4 = A5 - A4$, $B5 = A6 - A5$, &c. Tum $C = B2 - B$, $C2 = B3 - B2$, $C3 = B4 - B3$, $C4 = B5 - B4$, *&c.* deinde $D = C2 - C$, $D2 = C3 - C2$, $D3 = C4 - C3$, &c. Et ſimiliter ſunt omnes differentiæ ſequentes colligendæ. Vel ſint α, β, γ, δ, ϵ, ζ, η, &c. æquales *A*, *A2*, *A3*, *A4*, *A5*, *A6*, *A7*, &c. Eritque $A = \alpha$, $B = \beta - \alpha$, $C = \gamma - 2\beta + \alpha$, $D = \delta - 3\gamma + 3\beta - \alpha$, $E = \epsilon - 4\delta + 6\gamma - 4\beta + \alpha$, $F = \zeta - 5\epsilon + 10\delta - 10\gamma + 5\beta - \alpha$, $G = \eta - 6\zeta + 15\epsilon - 20\delta + 15\gamma - 6\beta + \alpha$, &c. In hiſce valoribus numerales Coefficientes ipſorum α, β, γ, δ, ϵ, *&c.* generantur ut in dignitatibus integris Binomii $\overline{1-z}|^{0}$, $\overline{1-z}|^{1}$, $\overline{1-z}|^{2}$, $\overline{1-z}|^{3}$, $\overline{1-z}|^{4}$, &c. Scribendo numeros 1, 2, 3, 4, 5, *&c.* in Serie $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. ſucceſſive pro *n*. Sit jam *PQ* quælibet Ordinata reliquis intermedia, & *AP* ejus diſtantia ab Ordinata prima *A* appelletur *z*, tum erit

$$\begin{aligned} PQ = A &+ \\ B \times \tfrac{z}{1} &+ \\ C \times \tfrac{z}{1} \times \tfrac{z-1}{2} &+ \\ D \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} &+ \\ E \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} &+ \\ F \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} \times \tfrac{z-4}{5} &+ \\ G \times \tfrac{z}{1} \times \tfrac{z-1}{2} \times \tfrac{z-2}{3} \times \tfrac{z-3}{4} \times \tfrac{z-4}{5} \times \tfrac{z-5}{6} &+ \text{\&c.} \end{aligned}$$

Adeoque ſignum ipſius z mutandum eſt, quando PQ cadit ad alteras partes Ordinatæ primæ, ut pq.

Caſus Secundus. Sit jam $A5$ Ordinata in medio omnium; pone $A = B4 + B5$, $B = D3 + D4$, $C = F2 + F3$, $D = H + H2$, *&c.* & $a = C4$, $b = E3$, $c = G2$, $d = I$, *&c.* id eſt, ſi ſint $A6 = \alpha$, $A7 = \beta$, $A8 = \gamma$, $A9 = \delta$, *&c.* $A4 = \kappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, *&c.* Pone $A = \alpha - \kappa$, $B = \beta - 2\alpha + 2\kappa - \lambda$, $C = \gamma - 4\beta + 5\alpha - 5\kappa + 4\lambda - \mu$, $D = \delta - 6\gamma + 14\beta - 14\alpha + 14\kappa - 14\lambda + 6\mu - \nu$, *&c.* $a = \alpha - 2\,A5 + \kappa$, $b = \beta - 4\alpha + 6\,A5 - 4\kappa + \lambda$, $c = \gamma - 6\beta + 15\alpha - 20\,A5 + 15\kappa - 6\lambda + \mu$, $d = \delta - 8\gamma + 28\beta - 56\alpha + 70\,A5 - 56\kappa + 28\lambda - 8\mu + \nu$, *&c.* Et dicatur $A5\,P$, z, tum erit

$$\begin{aligned} PQ = A5 + \frac{Az + azz}{1.2} &+ \\ \frac{2Bz + bzz}{1.2} \times \frac{zz-1}{3.4} &+ \\ \frac{3Cz + czz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} &+ \\ \frac{4Dz + dzz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} &+ \\ \frac{5Ez + ezz}{1.2} \times \frac{zz-1}{3.4} \times \frac{zz-4}{5.6} \times \frac{zz-9}{7.8} \times \frac{zz-16}{9.10} &+ \text{\&c.} \end{aligned}$$

Caſus Tertius. Sint jam $A4$, $A5$, Ordinatæ duæ in medio omnium. Pone $A = \frac{A4 + A5}{2}$, $B = \frac{C3 + C4}{2}$, $C = \frac{E2 + E3}{2}$, $D = \frac{G + G2}{2}$, &c. $a = B4$, $b = D3$, $c = F2$, $d = H$, &c. Vel ſint $A5 = \alpha$, $A6 = \beta$, $A7 = \gamma$, $A8 = \delta$, &c. $A4 = \kappa$, $A3 = \lambda$, $A2 = \mu$, $A = \nu$, &c.

Deinde erunt $2A = \alpha + \kappa$, $2B = \beta - \alpha - \kappa + \lambda$, $2C = \gamma - 3\beta + 2\alpha + 2\kappa - 3\lambda + \mu$, $2D = \delta - 5\gamma + 9\beta - 5\alpha - 5\kappa + 9\lambda - 5\mu + \nu$, &c. Et $a = \alpha - \kappa$, $b = \beta - 3\alpha + 3\kappa - \lambda$, $c = \gamma - 5\beta + 10\alpha - 10\kappa + 5\lambda - \mu$, $d = \delta - 7\gamma + 21\beta - 35\alpha + 35\kappa - 21\lambda + 7\mu - \nu$, &c. Et ſit O punctum medium inter $A4$, $A5$, atque appelletur OP, z; eritque Ordinata

$$PQ = \frac{A + az}{4^0} +$$
$$\frac{3B + bz}{4^1} \times \frac{4zz - 1}{2 \cdot 3} +$$
$$\frac{5C + cz}{4^2} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} +$$
$$\frac{7D + dz}{4^3} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} +$$
$$\frac{9E + ez}{4^4} \times \frac{4zz - 1}{2 \cdot 3} \times \frac{4zz - 9}{4 \cdot 5} \times \frac{4zz - 25}{6 \cdot 7} \times \frac{4zz - 49}{8 \cdot 9} + \&c.$$

In hiſce duobus etiam caſibus z eſt negativa, quando Ordinata PQ cadit ad alteras partes initii Abſciſſæ. Et in omnibus tribus caſibus diſtantia communis Ordinatarum ponitur unitas.

Omnes tres caſus demonſtrantur facillime per calculum. In caſu primo pro PQ ſcribo ſucceſſive α, β, γ, δ, ε, &c. & pro z interea 0, 1, 2, 3, 4, *&c.* quæ ſunt longitudines Abſciſſæ ordine ſequentes; & provenient æquationes,

$\alpha = A$, $\beta = A + B$, $\gamma = A + 2B + C$, $\delta = A + 3B + 3C + D$, $\varepsilon = A + 4B + 6C + 4D + E$, &c.

$\beta - \alpha = B$, $\gamma - \beta = B + C$, $\delta - \gamma = B + 2C + D$, $\varepsilon - \delta = B + 3C + 3D + E$, &c.

$\gamma - 2\beta + \alpha = C$, $\delta - 2\gamma + \beta = C + D$, $\varepsilon - 2\delta + \gamma = C + 2D + E$, &c.

$\delta - 3\gamma + 3\beta - \alpha = D$, $\varepsilon - 3\delta + 3\gamma - \beta = D + E$, &c.

$\varepsilon - 4\delta + 6\gamma - 4\beta + \alpha = E$, &c.

Hæ Æquationes, capiendo earum differentias, nullo labore reſolvuntur, uti videre eſt. Et dant eoſdem ipſorum A, B, C, D, *&c.* valores, qui antea poſiti ſunt in ſolutione. Et ad eundem modum demonſtrantur caſus duo reliqui.

Harum trium ſerierum unaquæque converget ad valorem Ordinatæ PQ, ubi Ordinatarum datarum differentiæ ſunt juſtæ magnitudinis. At ubi non convergunt, aliæ artes adhibendæ ſunt. Sed impræſentiarum de hujus Propoſitionis uſu pauca adjiciamus.

Deſignent α, β. γ, δ, ε, ζ, η θ, κ λ &c. terminos quoſcunque æquidiſtantes, quorum differentiæ ſunt perexiguæ, & relationes, quas inter ſe obtinent, definiuntur quamproxime per Æquationes ſequentes, quæ oriuntur

tur capiendo differentias & differentias differentiarum continuo, & ponendo eas æquales nihilo.

$$\begin{array}{l}
\alpha-\beta=0\\
\alpha-2\beta+\gamma=0\\
\alpha-3\beta+3\gamma-\delta=0\\
\alpha-4\beta+6\gamma-4\delta+\varepsilon=0\\
\alpha-5\beta+10\gamma-10\delta+5\varepsilon-\zeta=0\\
\alpha-6\beta+15\gamma-20\delta+15\varepsilon-6\zeta+\eta=0\\
\alpha-7\beta+21\gamma-35\delta+35\varepsilon-21\zeta+7\eta-\theta=0\\
\alpha-8\beta+28\gamma-56\delta+70\varepsilon-56\zeta+28\eta-8\theta+\kappa=0\\
\alpha-9\beta+36\gamma-84\delta+126\varepsilon-126\zeta+84\eta-36\theta+9\kappa-\lambda=0:\\
\qquad\&c.
\end{array}$$

Hæc Tabula in uſum reſervanda eſt, ut conſulatur quoties opus ſit. Quod autem hæ Æquationes vel obtinent accurate, vel ad verum approximant, ubi differentiæ terminorum ſunt parvæ, patet ex demonſtratione caſus primi Propoſitionis.

Aſſumatur quælibet Series $\frac{1}{101}$, $\frac{1}{102}$, $\frac{1}{103}$, $\frac{1}{104}$, $\frac{1}{105}$, $\frac{1}{106}$, &c. Et quæratur terminus qui ſtat proximus ante $\frac{1}{101}$: patet quod ille eſt $\frac{1}{100}$; videamus ergo qualem hæc methodus exhibebit eundem. Repræſentet α terminum quæſitum, eritque

$$\begin{array}{l}
\frac{1}{101}=\beta=0099,0099,0099,0,\\
\frac{1}{102}=\gamma=0098,0392,1568,7,\\
\frac{1}{103}=\delta=0097,0873,7864,1,\\
\frac{1}{104}=\varepsilon=0096,1538,4615,4,\\
\frac{1}{105}=\zeta=0095,2380,9523,8,\\
\frac{1}{106}=\eta=0094,3396,2264,2,
\end{array}
\quad \text{Æquatio}\left\{\begin{array}{l}
\text{1ma}\\ \text{2da}\\ \text{3tia}\\ \text{4ta}\\ \text{5ta}\\ \text{6ta}
\end{array}\right\}\text{dat }\alpha\left\{\begin{array}{l}
0099,0099,0099,0,\\
0099,9805,8629,3,\\
0099,9994,3455,0,\\
0099,9999,7824,8,\\
0099,9999,9895,8,\\
0099,9999,9993,1,
\end{array}\right.$$

Patet ergo quod hæc methodus continue approximat. Si terminorum differentiæ fuiſſent minores, valores acceſſiſſent citius ad verum, & contra tardius quando differentiæ ſunt majores. Hinc ſi in Tabulis numericis deſit terminus, poteſt is per hanc methodum inſeri.

Hoc modo etiam prodeunt ipſiſſimæ Series Specioſæ, quæ per alias methodos prodire ſolent. Proponatur $\overline{1+zz}|^{-1}$ Ordinata Curvæ quadrandæ: Ea eſt prima in ſerie regulari $\overline{1+zz}|^{-1}$, $\overline{1+zz}|^{0}$, $\overline{1+zz}|^{1}$, $\overline{1+zz}|^{2}$, $\overline{1+zz}|^{3}$, &c. Ordinatarum, quæ omnes præter primam dant ſuas areas z, $z+\frac{1}{3}z^3$, $z+\frac{2}{3}z^3+\frac{1}{5}z^5$, $z+\frac{3}{3}z^3+\frac{3}{5}z^5+\frac{1}{7}z^7$, *&c.* conſtituentes novam ſeriem cujus primus terminus erit Area quæſita: quæ ideo invenietur ponendo pro ea α, & pro reliquis in ſuo Ordine β, γ, δ, ε, *&c.* Prima Æquatio dat $\alpha=z$, ſecunda $\alpha=z-\frac{1}{3}z^3$, tertia $\alpha=z-\frac{1}{3}z^3+\frac{1}{5}z^5$, quarta $\alpha=z-\frac{1}{3}z^3+\frac{1}{5}z^5-\frac{1}{7}z^7$, &c. Eſt ergo univerſim area quæſita $z-\frac{1}{3}z^3+\frac{1}{5}z^5-\frac{1}{7}z^7+\frac{1}{9}z^9-\frac{1}{11}z^{11}$, &c. Eſtque hæc Series arcus ad Tangentem z, in circulo radium habente unitati æqualem. Eam invenit *Jacobus Gregorius* noſter, & cum *Collinio* communicavit initio anni 1671. a quo, mediante *Oldenburgo* ad *Leibnitium* delata eſt.

Sit jam &c. $e, d, c, b, a, P, \alpha, \beta, \gamma, \delta, \epsilon$, &c. Series utrinque excurrens in infinitum, ubi dantur omnes termini præter P in medio omnium. Sit $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \epsilon + e$, &c. atque erit

$$\begin{aligned} P = {} & \frac{A}{2} + \\ & \frac{A-B}{6} + \\ & \frac{5A-8B+3C}{60} + \\ & \frac{7A-14B+9C-2D}{140} + \\ & \frac{42A-96B+81C-32D+5E}{1260} + \\ & \frac{66A-165B+165C-88D+25E-3F}{2772} + \\ & \frac{429A-1144B+1287C-832D+325E-72F+7G}{24024} + \&c. \end{aligned}$$

Inveſtigatur hæc Series ex Æquationibus, excerpendo alternas in quibus numerus terminorum eſt impar. Nam earum differentiæ relinquent terminos in hac Serie; quæ itaque ad libitum produci poteſt.

Sit $\overline{1+z}|^{-1}$ Ordinata Hyperbolæ, & quæratur Area ejus quæ jacet ſupra Abſciſſam z, quando ea evadit unitas. Hæc Ordinata eſt media in Serie Ordinatarum, &c. $\overline{1+z}|^{-5}$, $\overline{1+z}|^{-4}$, $\overline{1+z}|^{-3}$, $\overline{1+z}|^{-2}$, $\overline{1+z}|^{-1}$, $\overline{1+z}|^{0}$, $\overline{1+z}|^{1}$, $\overline{1+z}|^{2}$, $\overline{1+z}|^{3}$ &c. æquidiſtantium, hinc inde excurrente in infinitum. Adeoque Areæ ab hiſce Ordinatis genitæ conſtituent ſeriem conſimilem, cujus medius terminus erit Area quæſita; quæ proinde obtinebitur per Seriem modo expoſitam. Quando z eſt unitas, ut in caſu præſente, areæ curvarum evadunt, &c. $\frac{15}{64}$, $\frac{7}{24}$, $\frac{3}{8}$, $\frac{1}{2}$, & 1, $\frac{3}{2}$, $\frac{7}{3}$, $\frac{15}{4}$, &c. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{3}{2} + \frac{3}{8} = \frac{15}{8}$, $C = \frac{7}{3} + \frac{7}{24} = \frac{21}{8}$, $D = \frac{15}{4} + \frac{15}{64} = \frac{255}{64}$, &c. Hiſce in Serie ſubſtitutis, prodit P, id eſt, area Hyperbolæ, $\frac{3}{4} - \frac{3}{48} + \frac{3}{480} - \frac{3}{4480} + \&c.$ id eſt, $\frac{3}{4} - \frac{A}{4 \cdot 3} - \frac{2B}{4 \cdot 5} - \frac{3C}{4 \cdot 7} - \frac{4D}{4 \cdot 9} - \frac{5E}{4 \cdot 11} - \&c.$ Ubi jam A, B, C, D, &c. more *Newtoniano*, deſignant terminos in ſuo ordine ab initio. Calculum appono.

TERMINI.

Affirmativi	*Negativi.*
7500,0000,0000,0000,0	0625,0000,0000,0000,0
62,5000,0000,0000,0	6,6964,2857,1428,5
7440,4761,9047,6	845,5086,5800,8
97,5586,9130,8	11,3818,4731,9
1,3390,4086,1	1585,7062,8
188,7745,5	22,5708,7
2,7085,0	3260,2
393,4	47,5
5,7	7
+7563,2539,3930,7494,1	−0631,7821,3370,8041,1

Summam negativam ſubducens ab affirmativa, habeo pro Area, id eſt, pro Logarithmo Hyperbolico Binarii, numerum 6931,4718,0559,9453.

Pro conſtructione Tabularum quarumvis numericarum percommoda eſt Series quæ ſequitur. Deſignent &c. $e, d, c, b, a, \alpha, \beta, \gamma, \delta, \varepsilon$, &c. terminos alternos in Serie utrinque ſerpente in infinitum; Pone $A = \alpha + a$, $B = \beta + b$, $C = \gamma + c$, $D = \delta + d$, $E = \varepsilon + e$, &c. Et terminus inter α & a erit

$$\frac{A}{2} +$$
$$\frac{1}{1} \times \frac{A - B}{2^4} +$$
$$\frac{1 \cdot 3}{1 \cdot 2} \times \frac{2A - 3B + C}{2^7} +$$
$$\frac{1 \cdot 3 \cdot 5}{1 \cdot 2 \cdot 3} \times \frac{5A - 9B + 5C - D}{2^{10}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7}{1 \cdot 2 \cdot 3 \cdot 4} \times \frac{14A - 28B + 20C - 7D + E}{2^{13}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} \times \frac{42A - 90B + 75C - 35D + 9E - F}{2^{16}} +$$
$$\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} \times \frac{132A - 297B + 275C - 154D + 54E - 11F + G}{2^{19}} + \&c.$$

Hæc Series ſequitur ex caſu tertio Propoſitionis, ponendo $z = 0$. Coefficientes numerales literarum ſic producuntur; exempli gratia, in quarto termino coefficiens literæ penultimæ C eſt 5; pone $5 + 1 = n$, & numeri qui proveniunt ex multiplicatione terminorum $1 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} \times$ &c. erunt 1, 6, 15, 20, &c. Horum differentiæ 5, 9, 5, ſunt numeri quæſiti. Atque adeo Series ad libitum produci poteſt.

Datis Logarithmis numerorum 46, 48, 50, 52, 54, 56, 58 & 60; invenire Logarithmum numeri 53 qui conſiſtit in medio omnium. Pone $l\,52 + l,54 = A = 3,4483,9710,34$, $l,50 + l,56 = B = 3,4471,5803,13$, $l,48 + l,58 = C = 3,4446,6923,08$, $l,46 + l,60 = D = 3,4409,0908,19$. Hiſce valoribus in Serie ſcriptis, primi quatuor termini dabunt 1,7242,2586,96 pro Logarithmo numeri 53. Et eadem ratione invenire licet quemvis alium intermedium.

In Conſtructione ergo Tabularum ſufficit primo quærere aliquos terminos in debitis diſtantiis, nam reliqui poſſunt hoc modo interſeri. Etenim continuo ſunt intercalandi termini primo inventi, uſque dum perventum fuerit ad ultimos qui deſiderantur. Hoc modo habebitur tota Tabula ex datis paucis terminis ſub initio pro fundamento operationis. Sed non convenit ut termini quos primo quærimus, ſint omnes per totam Tabulam æquidiſtantes; nam ſi omittimus alternos ubi eorum differentia eſt maxima, poſſumus alibi per ſaltum omittere duos, tres, viginti aut forte plures terminos. Numerus autem terminorum inter duos datos conſiſtentium, qui omittuntur, debet ſemper eſſe aliquis ſequentium 1, 3, 7, 15, 31, 63, *&c.* dummodo volumus inſerere eos per hanc Seriem; hoc vero neutiquam incommodabit opus.

Poſſunt autem pro Praxi termini in unam ſummam colligi, ut factum vides in hac Tabella. Prima expreſſio eſt primus terminus; ſecunda eſt ſumma primi & ſecundi; tertia eſt ſumma primi, ſecundi & tertii: & ſic porro.

$$2 \,\Big|\, \frac{A}{2}$$

$$4 \,\Big|\, \frac{9A - B}{16}$$

$$6 \,\Big|\, \frac{150A - 25B + 3C}{256}$$

$$8 \,\Big|\, \frac{1225A - 245B + 49C - 5D}{2048}$$

$$10 \,\Big|\, \frac{39690A - 8820B + 2268C - 305D + 35E}{65536}$$

Sic datis aliquibus terminis alternis, intermedii confeſtim dabuntur per haſce expreſſiones, nulla ratione habita naturæ Tabulæ particularis. Nam hæ regulæ ſunt eædem in omnibus. Areæ curvarum ſunt proxime æquales areis Parabolicæ figuræ quæ tranſit per extrema Ordinatarum ſuarum. Sed quoniam laborioſum nimis eſſet ſemper recurrere ad Parabolam, computavi Tabulam ſequentem, qua Areæ directe exhibentur ex datis Ordinatis.

$$1 \Big| \frac{A}{1}R$$

$$3 \Big| \frac{A+4B}{6}R$$

$$5 \Big| \frac{7A+32B+12C}{90}R$$

$$7 \Big| \frac{41A+216B+27C+272D}{840}R$$

$$9 \Big| \frac{989A+5888B-928C+10496D-4540E}{28350}R$$

$$11 \Big| \frac{16067A+106300B-48525C+272400D-260550E+427368F}{598752}R.$$

Hic numerus Ordinatarum eſt impar, A eſt ſumma primæ & ultimæ, B ſecundæ & penultimæ, C tertiæ & antepenultimæ; & ſic porro, uſque dum deventum ſit ad eam in medio omnium, quæ per ultimam literam in quaque expreſſione repræſentatur. R eſt baſis ſeu pars Abſciſſæ inter primam & ultimam Ordinatam interceptæ. Expreſſiones ſunt Areæ contentæ inter Curvam, Baſin & Ordinatas hinc inde extremas. Tabulam pro pare numero Ordinatarum non appoſui, quoniam Area cæteris paribus ex impare earum numero accuratius definitur.

Quæratur area quæ generatur ab Ordinata $\overline{1+zz}|^{-1}$ & jacet ſupra Abſciſſam z quando ea evadit unitas. In $\overline{1+zz}|^{-1}$, pro z ſcribe $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$; & prodibunt undecim Ordinatæ $1, \frac{100}{101}, \frac{25}{26}, \frac{100}{109}, \frac{25}{29}, \frac{4}{5}, \frac{25}{34}, \frac{100}{149}, \frac{25}{41}, \frac{100}{181}, \frac{1}{2}$. Hinc eſt $A = 1 + \frac{1}{2} = \frac{3}{2}$, $B = \frac{100}{101} + \frac{100}{181} = \frac{28200}{18281}$, $C = \frac{25}{26} + \frac{25}{41} = \frac{1675}{1066}$, $D = \frac{100}{109} + \frac{100}{149} = \frac{25800}{16241}$, $E = \frac{25}{29} + \frac{25}{34} = \frac{1575}{986}$, $F = \frac{4}{5}$. Hiſce valoribus ſubſtitutis in ultima expreſſione, & unitate pro R, invenies aream eſſe 785398187. Juſtus eſt hic numerus in ſeptima figura, in octava verum ſuperans Binario.

Si undecim Ordinatæ non dent aream ſatis exactam, erige plures; & concipe aream diviſam eſſe in plures partes, quarum quamque ſeorſum quærens habebis pro lubitu juſtam.

Valor ipſius $\overline{1+\mathfrak{Q}}|^{n}$ exprimi poteſt per quamcunque trium ſerierum ſequentium.

$$\overline{1+\mathfrak{Q}}|^{n} = 1 +$$

$$\mathfrak{Q} \times \frac{n}{1} +$$

$$\mathfrak{Q}^2 \times \frac{n}{1} \times \frac{n-1}{2} +$$

$$\mathfrak{Q}^3 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} +$$

$$\mathfrak{Q}^4 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} +$$

$$\mathfrak{Q}^5 \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} \times \frac{n-3}{4} \times \frac{n-4}{5} + \&c.$$

Vel $\overline{1+Q}|^{n} = 1 +$

$$R \times \frac{n}{1} +$$
$$R^2 \times \frac{n}{1} \times \frac{n+1}{2} +$$
$$R^3 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} +$$
$$R^4 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} +$$

$R^5 \times \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3} \times \frac{n+3}{4} \times \frac{n+4}{5} +$ *&c.* poſito ſcilicet $R = \frac{1+Q}{Q}$. Vel

$\overline{1+Q}|^{n} = 1 +$

$$\frac{2+\overline{n+1}\times Q}{\overline{1+Q}|^{1}} \times Q \times \frac{n}{1.2} +$$
$$\frac{4+\overline{n+2}\times Q}{\overline{1+Q}|^{2}} \times Q^3 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} +$$
$$\frac{6+\overline{n+3}\times Q}{\overline{1+Q}|^{3}} \times Q^5 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} +$$
$$\frac{8+\overline{n+4}\times Q}{\overline{1+Q}|^{4}} \times Q^7 \times \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} +$$
$$\frac{10+\overline{n+5}\times Q}{\overline{1+Q}|^{5}} \times Q^9 \ \frac{n}{1.2} \times \frac{nn-1}{3.4} \times \frac{nn-4}{5.6} \times \frac{nn-9}{7.8} \times \frac{nn-16}{9.10} + \text{\&c.}$$

Primæ duæ Series demonſtrantur per Caſum primum Propoſitionis. Nam ſi $\overline{1+Q}|^{0}$, $\overline{1+Q}|^{1}$, $\overline{1+Q}|^{2}$, $\overline{1+Q}|^{3}$, $\overline{1+Q}|^{4}$, *&c.* deſignent Ordinatas totidem æquidiſtantes in Parabolica figura, erit $\overline{1+Q}|^{n}$ ejuſdem Ordinata, cujus diſtantia a $\overline{1+Q}|^{0}$ eſt n. Et ſic prodit Series prima. At ſi in alia Parabola $\overline{1+Q}|^{0}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-4}$, *&c.* ſint æquidiſtantes Ordinatæ, erit $\overline{1+Q}|^{n}$ Ordinata in eadem, cujus diſtantia a $\overline{1+Q}|^{0}$ eſt $-n$; ſic proveniet Series ſecunda. Sit jam in tertia Parabola *&c.* $\overline{1+Q}|^{-4}$, $\overline{1+Q}|^{-3}$, $\overline{1+Q}|^{-2}$, $\overline{1+Q}|^{-1}$, $\overline{1+Q}|^{0}$, $\overline{1+Q}|^{1}$, $\overline{1+Q}|^{2}$, $\overline{1+Q}|^{3}$, $\overline{1+Q}|^{4}$, *&c.* Series Ordinatarum æquidiſtantium hinc inde progrediens in infinitum, eritque in eadem $\overline{1+Q}|^{n}$ Ordinata, diſtantia n a termino medio $\overline{1+Q}|^{0}$ remota. Et ſic provenit Series tertia per Caſum Secundum Propoſitionis. Prima abrumpit quando eſt n integer & affirmativus, ſecunda quando eſt n integer & negativus, & tertia in caſu utroque abrumpit. Per harum quamque radices numerales commode evolvuntur in Series. Tertia reliquis multo citius convergit: ejus terminus ſecundus adhiberi poteſt pro correctione, ubi fit extractio per repetitionem calculi.

Halleius in ſua methodo conſtruendi Logarithmos, ex prima harum ſerierum demonſtrat Seriem *Mercatoris* pro Quadratura Hyperbolæ. Sit ejus Ordinata $\overline{1+z}|^{-1}$, vel $\overline{1+z}|^{n-1}$, exiſtente n numero infinite parvo; unde per methodos Quadrandi, area quæ jacet ſupra Abſciſſam z, id eſt, Logarithmus numeri $1+z$, erit $\frac{\overline{1+z}|^{n}-1}{n}$. Eſt vero per primam Seriem $\overline{1+z}|^{n} = 1 + \frac{n}{1}z + \frac{n}{1} \times \frac{n-1}{2} z^2 + \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3} z^3 +$ &c. adeoque in caſu præſente, ubi eſt n infinite parvus, eſt $\overline{1+z}|^{n} = 1 + \frac{n}{1} z - \frac{n}{2} z^2 + \frac{n}{3} z^3 - \frac{n}{4} z^4$ &c. quo ſubſtituto in valore areæ, ea prodit $z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \frac{1}{5}z^5 -$ &c. quæ eſt Series *Mercatoris.*

Similiter per Seriem ſecundam prodit hæc regula; Sit datus numerus $1+z$, pone $R = \frac{z}{1+z}$, eritque ejus Logarithmus $R + \frac{1}{2}R^2 + \frac{1}{3}R^3 + \frac{1}{4}R^4 + \frac{1}{5}R^5 +$ &c.

Per Seriem tertiam provenit ſequens regula. Sit quilibet numerus R, pone $z = \frac{\overline{R-1}|^2}{2R}$, eritque ejus Logarithmus $\frac{RR-1}{2R} - \frac{1}{3}Az - \frac{2}{5}Bz - \frac{3}{7}Cz - \frac{4}{9}Dz - \frac{5}{11}Ez -$ &c. Ubi A, B, C, D, E, &c. more *Newtoniano* deſignant terminos Seriei ſicut ab initio. Hæc Series, ut ea ex qua deducitur, reliquis duabus multis vicibus celerius approximat: eſtque eadem generalius expreſſa quam, ex fundamento haud abſimili, pro inventione Logarithmi Binarii prius dedimus.

Methodus inveniendi valores Serierum Arithmeticarum utcunque tarde convergentium.

In aliquibus Seriebus ſumma terminorum haberi nequit niſi ad pauciſſima figurarum loca, dummodo præter ſimplicem eorum additionem aliæ artes non adhibeantur. Proponatur jam Series quælibet cujus termini omnes iiſdem ſignis afficiuntur, & quorum proximi continue tendunt eſſe inter ſe æquales; quales ſunt ſequentes $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} + \frac{1}{7 \cdot 8} +$ &c. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} +$ &c. Collige ſummam aliquot terminorum ſub initio, ii proxime addendi ſint α, β, γ, δ, ε, ζ, &c. In numeris proximis ſit $r = \frac{\alpha\gamma - \beta\beta}{\alpha\beta - 2\alpha\gamma + \beta\gamma}$, & quantitatum $\alpha \times \frac{\alpha + r\beta}{\alpha - \beta}$, $\alpha + \beta \times \frac{\beta + r\gamma}{\beta - \gamma}$, $\alpha + \beta + \gamma \times \frac{\gamma + r\delta}{\gamma - \delta}$, $\alpha + \beta + \gamma + \delta \times \frac{\delta + r\varepsilon}{\delta - \varepsilon}$, $\alpha + \beta + \gamma + \delta + \varepsilon \times \frac{\varepsilon + r\zeta}{\varepsilon - \zeta}$, &c. differentiæ ſint a, b, c, d, e, &c. Deinde in numeris proximis ſit $s =$

ac

$\frac{ac-bb}{ab-2ac+bc}$, & ipſorum $a\times\frac{a+sb}{a-b}$, $a+b\times\frac{b+sc}{b-c}$, $a+b+c\times\frac{c+sd}{c-d}$, $a+b+c+d\times\frac{d+se}{d-e}$, &c. differentiæ ſint A, B, C, D, &c. & ſit $t=\frac{AC-BB}{AB-2AC+BC}$: atque ſic procede quoad libuerit. Tum erit $\alpha+\beta+\gamma+\delta+\varepsilon+$ &c. $=\alpha\times\frac{\alpha+r\beta}{\alpha-\beta}+a\times\frac{a+sb}{a-b}+A\times\frac{A+tB}{A-B}+$ &c. atque ultra duos primos terminos hujus novæ Seriei raro opus erit progredi.

Ut ſi deſideretur valor Seriei $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+\frac{1}{7.8}+$ &c. collige primos 21 terminos, quorum ſummam reperio fore 6813,8410,1885. Termini proxime addendi ſunt $\alpha =$,0005,2854,1226, $\beta =$,0004,8309,1787, $\gamma =$,0004,4326,2411, $\delta =$,0004,0816,3265, &c. Hinc ſit $r=1$ proxime, & $\alpha\times\frac{\alpha+r\beta}{\alpha-\beta} =$,0117,6449,6282, $a = -$,0000,0017,5096, $b = -$,0000,0014,7410, $c = -$,0000,0012,4986, &c. Unde $s=\frac{1}{3}$ prope, & $a\times\frac{a+sb}{a-b} = -$,0000,0141,8111, quem propter ſignum negativum ſubduco ab $\alpha\times\frac{\alpha+r\beta}{\alpha-\beta}$, & remanet 0117,6307,8171: hic additus ſummæ primo inventæ 6813,8410,1885, dat pro ſumma totius Seriei numerum 6931,4718,0056, qui juſtus eſt in nona decimali; at ante duas haſce correctiones ſumma erat juſta in prima figura ſola. Si animus ſit propius ſcopum attingere, pergendum erit ad approximationes ſequentes. Si termini Seriei diverſa habeant ſigna, conjungendi ſunt, ut omnes eadem tandem habeant, ut in Serie $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-$ &c. conjunctis terminis ea evadit $\frac{2}{1.3}+\frac{2}{5.7}+\frac{2}{9.11}+\frac{2}{13.15}+$ &c. Sed hic notandum eſt quod differentiæ, a, b, c, d, e, &c. ut & A, B, C, D, &c. colligi debent ſubducendo quantitates antecedentes de ſubſequentibus. Et in omnibus hujuſmodi Seriebus ſi p, q, r, repræſentent tres terminos ordine ſequentes, p primum, q ſecundum, r tertium, & rectangulum $\frac{p+r}{2}\times q$ non ſit majus pr, valor Seriei erit infinite magnus: at magnitudinis ſemper finitæ ubi accidit contrarium. Poteſt hæc regula nonnunquam fallere, ubi termini p, q, r parum diſtant ab initio Seriei, at ſi conſiſtant inter eos ab initio aliquantum remotos, evadet regula certiſſima.

Ad alia Serierum genera debent aliæ regulæ adhiberi. Sit Series regulatium Polygonorum Circulo Inſcriptorum, exiſtente Radio unitate.

$$\begin{array}{l|l}
H = 2{,}0000{,}0000{,}0000{,}000 & 4 \\
G = 2{,}8284{,}2712{,}4746{,}190 & 8 \\
F = 3{,}0614{,}6745{,}8920{,}718 & 16 \\
E = 3{,}1214{,}4515{,}2258{,}051 & 32 \\
D = 3{,}1365{,}4849{,}0545{,}938 & 64 \\
C = 3{,}1403{,}3115{,}6954{,}752 & 128 \\
B = 3{,}1412{,}7725{,}0932{,}772 & 256 \\
A = 3{,}1415{,}1380{,}1144{,}299 & 512
\end{array}$$

Dicatur jam ultimum Polygonum A, penultimum B, antepenultimum C, & reliqua in ſuo ordine retrorſum D, E, F, *&c.* atque area Circuli quæſita erit $A + \frac{A-B}{3} + \frac{4A-5B+C}{3.15} + \frac{64A-84B+21C-D}{3.15.63} + \frac{4096A-5440B+1428C-85D+E}{3.15.63.255} +$ *&c.* Ubi ſi pro $A, B, C, D, E,$ *&c.* ſcribantur proprii valores, primi quatuor termini dabunt 3,1415,9265, 3589,790 pro area Circuli. Hæc autem Series eſt generalis, ex natura Circuli neutiquam dependens: applicabilis eſt quotieſcunque numerorum approximantium differentiæ priores ſunt poſteriorum quaſi quadruplæ. Factores in Denominatoribus ſunt dignitates integræ numeri 4 unitatibus minutæ: quibus datis, coefficientes literarum in diverſis terminis formantur ex multiplicatione continua numerorum $1, \frac{n}{3}, \frac{n-3}{15}, \frac{n-15}{63}, \frac{n-63}{255},$ *&c.* Ubi pro n ſubſtituendus eſt ultimus Factorum in Denominatore.

Ultima quantitatum $x-1$, $2\sqrt[2]{x}-2$, $4\sqrt[4]{x}-4$, $8\sqrt[8]{x}-8$, $\sqrt[16]{x}-16$, *&c.* æqualis eſt Logarithmo numeri x. Pro x ſcribe 2, & per repetitam extractionem radicis quadratæ exibunt numeri.

$M = 1{,}0000{,}0000{,}0000{,}0000.$
$L = 8284{,}2712{,}4746{,}1901.$
$I = 7568{,}2864{,}0010{,}8843.$
$H = 7240{,}6186{,}1322{,}0613.$
$G = 7083{,}8051{,}8838{,}6214.$
$F = 7007{,}0875{,}6931{,}7337.$
$E = 6969{,}1430{,}7308{,}8294.$
$D = 6950{,}2734{,}2438{,}7611.$
$C = 6940{,}8641{,}2851{,}8363.$
$B = 6936{,}1658{,}4759{,}4014.$
$A = 6933{,}8182{,}9699{,}9493.$

Dicatur ultimus numerorum A, penultimus B, & ſic retro, atque Logarithmus quæſitus erit $A + \frac{A-B}{1} + \frac{2A-3B+C}{1.3} + \frac{8A-14B+7C-D}{1.3.7} +$

$+\frac{64A-120B+70C-15D+E}{1.3.7.15}$ + &c. Primi quinque termini dant 6931,4718,0559,9457 pro Logarithmo Hyperbolico Binarii. Et quomodo hæc Series procedit in infinitum facile colligitur ex eo quod de priore diximus: eſtque etiam univerſalis, proprietates Hyperbolæ minime reſpiciens.

Extenditur quoque Methodus hæcce Differentialis ad Reſolutionem Æquationum & alia quamplurima quorum hic non fit mentio.

XXVI. § 1. Æquationis Cubicæ Univerſalis

$$x^3 = \begin{array}{l} 3px^2 + 3qx + 2r, \\ \quad\; - 3p^2 \quad + p^3 \\ \qquad\qquad\quad - 3pq \end{array}$$

Radices Tres ſunt,

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

A univerſal Solution of Cubick and B quadratick Æquations, viz. *Analytical, Geometrical, and Mechanical. by* Mr. J. Colſon, *n.* 309. *p.* 2353.

Vel ut Calculus Arithmeticus facilior ac paratior evadat, ſi poſueris Binomii irrationalis $r + \sqrt{r^2 - q^3}$ Radicem Cubicam eſſe $m + \sqrt{n}$, erunt ejuſdem Æquationis Radices tres $x = p + 2m$, & $x = p - m \pm \sqrt{-3n}$.

Igitur data Æquatione quavis Cubica, inter ejus hujuſque Æquationis Univerſalis terminos ſingulos inſtituenda eſt comparatio, quo pacto facillime invenientur ipſæ p, q, r; & hiſce cognitis, innoteſcent Æquationis datæ Radices omnes. Hujus vero Solutionis Exempla ſint ſequentia in Numeris.

1. Æquationis Cubicæ $x^3 = 2x^2 + 3x + 4$ ſit Radix x indaganda. Erit primo juxta præſcriptum $3p = 2$, ſive $p = \frac{2}{3}$. Secundo $3q - (3p^2)\frac{4}{3} = 3$, ſive $q = \frac{13}{9}$. Tertio $2r\ (+\overline{p^2 - 3q} \times p) - \frac{70}{27} = 4$, ſive $r = \frac{89}{27}$, & $r^2 - q^3 = \frac{212}{27}$. Et propterea $x = \frac{2}{3} + \sqrt[3]{\frac{89}{27} + \sqrt{\frac{212}{27}}} + \sqrt[3]{\frac{89}{27} - \sqrt{\frac{212}{27}}}$. Reliquæ duæ Radices ſunt impoſſibiles.

2. In Æquatione $x^3 = 12x^2 - 41x + 42$, erit primo $3p = 12$, ſive $p = 4$. Secundo $3q - (3p^2)\,48 = -41$, ſive $q = \frac{7}{3}$. Tertio $2r + (\overline{p^2 - 3q} \times p)\,36 = 42$, ſive $r = 3$; Et inde $r^2 - q^3 = -\frac{100}{27}$. At Binomii ſurdi $3 + \sqrt{-\frac{100}{27}}\ (= r + \sqrt{r^2 - q^3})$ Radix Cubica, per Metho-

dos ex Arithmetica petendas extracta, eſt $-1+\sqrt{-\frac{4}{3}}$, $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-2=)\,2$, vel etiam $x=(p-m\pm\sqrt{-3n}=4+1\pm(\sqrt{4})\,2=)\,7$ vel 3. Vel rurſus, ejuſdem Binomii $3+\sqrt{-\frac{100}{27}}$. Radix alia Cubica (tres enim agnoſcit) eſt $\frac{3}{2}+\sqrt{-\frac{1}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4+3=)\,7$, & etiam $x=(p-m\pm\sqrt{-3n}=4-\frac{3}{2}\pm(\sqrt{\frac{1}{4}})\,\frac{1}{2}=3$ vel 2. Vel denuo, ejuſdem Binomii $3+\sqrt{-\frac{100}{27}}$ Radix Cubica tertia eſt $-\frac{1}{2}-\sqrt{-\frac{25}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-1=)\,3$, atque etiam $x=(p-m\pm\sqrt{-3n}=4+\frac{1}{2}\pm(\sqrt{\frac{25}{4}})\,\frac{5}{2}=)\,7$ vel 2.

3. In Æquatione $x^3=-15x^2-84x+100$, erit $p=-5$, $q=-3$, $r=135$; & Binomii $135+\sqrt{18252}$ Radix Cubica eſt $3+\sqrt{12}$. Igitur Radix $x=-5+6=1$, & $x=-5-3\pm\sqrt{-36}=-8+\sqrt{-36}$, impoſſibiles.

4. In Æquatione $x^3=34x^2-310x+1012$, erit $p=\frac{34}{3}$, $q=\frac{226}{9}$, $r=\frac{5536}{27}$; & Binomii $\frac{5536}{27}+\sqrt{\frac{707560}{27}}$ Radix Cubica eſt $\frac{16}{3}+\sqrt{\frac{10}{3}}$. Igitur Radix $x=\frac{34}{3}+\frac{32}{3}=22$, & $x=\frac{34}{3}-\frac{16}{3}\pm\sqrt{-10}=6\pm\sqrt{-10}$, impoſſibiles.

5. In Æquatione $x^3=28x^2+61x-4048$, erit $p=\frac{28}{3}$, $q=\frac{967}{9}$, $r=-\frac{25010}{27}$; & Binomii $-\frac{25010}{27}+\sqrt{-382347}$ Radix Cubica eſt $\frac{41}{6}+\sqrt{-\frac{243}{4}}$. Igitur $x=\frac{28}{3}+\frac{41}{3}=23$, & $x=\frac{28}{3}-\frac{41}{6}\pm(\sqrt{\frac{729}{4}})\,\frac{27}{2}=16$ vel -11.

6. In Æquatione $x^3=-x^2+166x-660$, erit $p=-\frac{1}{3}$, $q=\frac{499}{9}$, $r=-\frac{9658}{27}$; & Binomii $-\frac{9658}{27}+\sqrt{-\frac{1147205}{27}}$ Radix Cubica eſt $-\frac{22}{3}+\sqrt{-\frac{5}{3}}$. Igitur $x=-\frac{1}{3}-\frac{44}{3}=-15$, & $x=-\frac{1}{3}+\frac{22}{3}\pm\sqrt{5}=7\pm\sqrt{5}$, irrationales.

7. In Æquatione $x^3 = 63x^2 + 99673x + 9951705$, erit $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; & Binomii $6031680 + \sqrt{-\frac{47887175043136}{27}}$ Radix Cubica eſt $183 + \sqrt{-\frac{529}{3}}$. Igitur $x = 21 + 366 = 387$, & $x = 21 - 183 \pm (\sqrt{529})\ 23 = -139$ vel 185.

Nec ſecus in cæteris procedendum: Inveſtigatur autem Theorema ad modum ſequentem. Pono Æquationis cujuſdam Cubicæ Radicem $z = a + b$, & cubice multiplicando proveniet $z^3 = (a^3 + 3a^2b + 3ab^2 + b^3 =)\ a^3 + 3ab \times \overline{a+b} + b^3$. Jam loco ipſius $a + b$ valorem ejus z ſubſtituendo, fiet $z^3 = 3abz + a^3 + b^3$, quæ eſt Æquatio Cubica ex Radice $z = a + b$ conſtructa, cui terminus ſecundus deeſt. Ut hæc vero ad formam magis commodam magiſque concinnam revocentur, ſumo Æquationem $z^3 = 3qz + 2r$, quæ poſthac ipſius $z^3 = 3abz + a^3 + b^3$, vices gerat. Igitur tranſmutatione hujus in illam, fiet primo $3q = 3ab$, ſive $q^3 = a^3b^3$; & ſecundo $2r = a^3 + b^3$, ſive $2ra^3 = (a^6 + a^3b^3 =)\ a^6 + q^3$. Et ſoluta hac æquatione quadratica, erit $a^3 = r + \sqrt{r^2 - q^3}$, & hinc $b^3 = (2r - a^3 =)\ r - \sqrt{r^2 - q^3}$: Atque igitur tandem $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Et propterea in Æquatione Cubica $z^3 = 3qz + 2r$ erit Radix $z = (a + b =)\ \sqrt[3]{r^2 + \sqrt{r - q^3}} + \sqrt[3]{r^2 - \sqrt{r - q^3}}$.

At vero hæc Radix revera triplex eſt, pro triplici valore quem induere poteſt, & $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$. Cujuſvis enim quantitatis Radix Cubica triplex erit, & ipſius Unitatis Radix Cubica vel eſt 1, vel $-\frac{1}{2} + \frac{1}{2}\sqrt{-3}$, vel $-\frac{1}{2} - \frac{1}{2}\sqrt{-3}$. Atque id adeo, propterea quod harum alicujus Cubus ſit Unitas. Igitur ſi $1 \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ aut $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ $(= \sqrt[3]{1 \times \overline{r + \sqrt{r^2 - q^3}}} = \sqrt[3]{1} \times \sqrt[3]{r + \sqrt{r^2 - q^3}})$ Radicem aliquam [quam ſupra nominavimus $m + \sqrt{n}$, aut $1 \times \overline{m + \sqrt{n}}$,] Cubi $r + \sqrt{r^2 - q^3}$ deſignet; ipſæ $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}}$ [i. e. $\frac{-1 + \sqrt{-3}}{2} \times \overline{m + \sqrt{n}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \overline{m + \sqrt{n}}$] alias duas ejuſdem Cubi Radices deſignabunt. Similiter & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$, $\frac{-1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$ & $\frac{-1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$, [i. e. $\overline{m - \sqrt{n}}$, $\frac{-1 + \sqrt{-3}}{2} \times \overline{m - \sqrt{n}}$, $\frac{-1 - \sqrt{-3}}{2} \times \overline{m - \sqrt{n}}$,] tres Cubicæ Radices erunt Apotomes $r - \sqrt{r^2 - q^3}$. Atque has Radices debite connectendo, fiet $z = \sqrt[3]{r + \sqrt{r^2 - q^3}} +$

$\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=\overline{m+\sqrt{n}}+\overline{m-\sqrt{n}}=2m$,] $z=\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=\frac{-1+\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}+\frac{-1-\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}=-m+\sqrt{-3n}$,] & $z=\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$ [i. e. $z=\frac{-1-\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}+\frac{-1+\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}=-m-\sqrt{-3n}$,] quæ tres erunt Radices Æquationis Cubicæ $z^3=3qz+2r$. Debite autem connectuntur Radices iſtæ ad modum præcedentem, quippe quæ ſic connexæ, & more vulgari in ſe invicem continue ductæ, Æquationem $z^3=3qz+2r$ reſtituunt. Denique fac $z=x-p$, & Æquatio fiet $x^3-3px^2+3p^2x-p^3=3qx-3pq+2r$, quæ univerſalis eſt, & cujus Radices evadunt ut ſupra fuerunt exhibitæ.

Hic obiter notatu dignum eſt, quod Æquationis Cubicæ cujuſcunque Radices omnes ſint poſſibiles & reales, quoties Binomii membrum irrationale $\sqrt{r^2-q^3}$ impoſſibilitatem in ſe complectitur; hoc eſt, quoties q eſt quantitas affirmativa, & ſimul cubus ejus major eſt quadrato ex latere r. At ſi membrum iſtud $\sqrt{r^2-q^3}$ ſit poſſibile, hoc eſt ſi q ſit quantitas uegativa, aut etiam ſi affirmativæ cubus ſit minor quadrato ex latere r, tunc unicam tantum agnoſcit Æquatio Radicem poſſibilem & realem, reliquæque duæ erunt impoſſibiles.

In hoc Theoremate ſi fiat $p=0$, hoc eſt, ſi deſit Æquationis terminus ſecundus, tunc deventum erit ad caſum Regularum quæ dicuntur *Cardani*, cujus ſolutio continetur in præcedentibus.

§ 2. Æquationis Biquadraticæ Univerſalis

$$\begin{array}{l} x^4=4px^3+2qx^2+8rx+4s, \\ \qquad\qquad\;\; -4p^2\quad -4pq-q^2 \end{array}$$

Radices quatuor ſunt $x=p-a\pm\sqrt{p^2+q-a^2-\frac{2r}{a}}$, & $x=p+a\pm\sqrt{p^2+q-a^2+\frac{2r}{a}}$, Ubi a^2 eſt Radix Æquationis Cubicæ

$$\begin{array}{l} a^6=p^2a^4-2pra^2+r^2 \\ \qquad\;\; +q\qquad -s \end{array}$$

Jam data Æquatione quavis Biquadratica, inter ejus hujuſque Æquationis Univerſalis terminos ſingulos inſtituenda eſt comparatio, quo pacto citiſſime invenientur ipſæ p, q, r, s; & hiſce cognitis, non latebit valor ipſius a, ex Theoremate ſuperiori inveniendus, & tum demum innoteſcent Æquationis datæ Radices omnes.

Huic Solutioni illuſtrandæ Exemplum unum aut alterum ſufficiat.

1. Æquationis Biquadraticæ $x^4=8x^3+83x^2-162x-936$ ſint Radices extrahendæ. Erit primo juxta præſcriptum $4p=8$, ſive $p=2$. Secundo $2q-(4p^2)\,16=83$, ſive $q=\frac{99}{2}$. Tertio $8r-(4pq)\,396=$

=

-162, ſive $r=\frac{117}{4}$. Quarto $4s-(q^2)\frac{9801}{4}=-936$, ſive $s=\frac{6057}{16}$. Hinc $p^2+q=\frac{107}{2}$, $2pr+s=\frac{7929}{16}$, $r^2=\frac{13689}{19}$, & proinde $a^6=\frac{107}{2}a^4-\frac{7929}{16}a^2+\frac{13689}{16}$. Jam ut Æquatio hæc aliquatenus Cubica in Radices ejus reſolvatur, ad Theorema præcedens recurrendum eſt, in quo erit $p=\frac{107}{2}$, $q=\frac{22009}{144}$, $r=\frac{2903923}{1728}$, & $r^2-q^3=-\frac{11940075}{16}$. Atqui Binomii $\frac{2903923}{1728}+\sqrt{-\frac{11940075}{16}}$ Radix Cubica eſt $-\frac{53}{12}+\sqrt{-\frac{400}{3}}$ & propterea $a^2=\frac{107}{6}-\frac{53}{6}=9$, & etiam $a^2=\frac{107}{6}+\frac{53}{12}\pm(\sqrt{400})\,20=\frac{169}{4}$ vel $\frac{9}{4}$: Vel quod perinde eſt, Æquationis præmiſſæ reverа Cubo-Cubicæ ſex Radices ſunt $a=\pm 3$, $a=\pm\frac{13}{2}$, & $a=\pm\frac{3}{2}$, quarum quævis indiſcriminatim propoſito noſtro faciet ſatis. Puta ſi in præſenti caſu fiat $a=3$, erit juxta Theorema $x=(p-a\pm\sqrt{p^2+q-a^2-\frac{2r}{a}}=2-3\pm\sqrt{4+\frac{99}{2}-9-\frac{39}{2}}=-1\pm(\sqrt{25})\,5=)\,4$ vel -6, & $x=(p+a\pm\sqrt{p^2+q-a^2+\frac{2r}{a}}=2+3\pm\sqrt{4+\frac{99}{2}-9+\frac{39}{2}}=5\pm(\sqrt{64})\,8=)\,13$ vel -3, quæ ſunt Æquationis datæ Radices quatuor.

2. In Æquatione $x^4=20x^3+252x^2-6592x+21312$, erit $p=5$, $q=176$, $r=-384$, & $s=13072$. Hinc $p^2+q=201$, $2pr+s=9232$, & $r^2=147456$; & inde $a^6=201a^4-9232a^2+147456$. Jam in Theoremate pro Cubicis erit $p=67$, $q=\frac{4235}{3}$, & $r=65219$; eritque Binomii $65219+\sqrt{\frac{38889307072}{27}}$ Radix Cubica $\frac{77}{2}+\sqrt{\frac{847}{12}}$. Igitur $a^2=67+77=144$, ſive $a=12$; & proinde $x=5-12\pm\sqrt{25+176-144+64}=-7+(\sqrt{121})\,11=4$ vel -18, & $x=5+12\pm\sqrt{25+176-144-64}=17\pm\sqrt{-7}$, impoſſibiles.

Hujus autem Theorematis Inventio eſt hujuſmodi. Ex duarum Æquationum Quadraticarum $z^2+2az-b=0$, & $z^2-2az-c=0$, in ſe invicem multiplicatione, Æquationem conficio Biquadraticam $z^4=\overline{4a^2+b+c}\times z^2+\overline{2ac-2ab}\times z-bc$, cui terminus ſecundus deeſt, quamque huic Æquationi $z^4=ez^2+fz+g$ ſtatuo æquipollere. Unde primo $4a^2+b+c=e$, ſive $b=e-4a^2-c$. Secundo $2ac-2ab=4$, hoc eſt, $2ac - 2ae + 8a^3 + 2ac = f$, ſive $c = \frac{f}{4a} + \frac{e}{2} - 2a^2$, & inde $b = (e - 4a^2 - c =) - \frac{f}{4a} + \frac{e}{2} - 2a^2$. Tertio $-bc = g$, ſive $-\frac{f^2}{16a^2} + \frac{e^2}{4} - 2ea^2 + 4a^4 = -g$, hoc eſt, $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$, quæ Æquatio quaſi Cubica eſt, ex Radice a^2 & notis vel aſſumptis $e\,f$, g conflata. Ea vero Radix per Theorema ſuperius exhiberi poteſt, & eodem Calculo innoteſcent ipſæ b & c. At Æquationum $z^2 + 2az - b = 0$ & $z^2 - 2az - c = 0$ Radices ſunt $z = -a \pm \sqrt{a^2 + b}$ & $z = a \pm \sqrt{a^2 + c}$, ſive $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - 4a}$, & $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, quæ proinde erunt Radices Equationis $z^4 = ez^2 + fz + g$; cognita videlicet a vel a^2 ex Equatione $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea + \frac{f^2}{64}$. Jam ut Equatio iſta evadat univerſalis, & omnibus ſuis terminis inſtructa, fac $z = x - p$, eritque $x^4 - 4px^3 + 6p^2x^2 - 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$, item & $x = p - a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$ & $x = p + a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$. Tandem concinnitatis & compendii gratia, fac $e = 2q + 2p^2$ & $f = 8r$; tum $x^4 - 4px^3 + 4p^2x^2 = 2qx^2 - 4pqx + 2p^2q + p^4 + 8rx - 8pr + g$, $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, & $a^6 = \overline{p^2 + q} \times a^4 - \overline{\tfrac{1}{4}g + \tfrac{1}{4}p^4 + \tfrac{1}{2}p^2q + \tfrac{1}{4}q^2} \times a^2 + r^2$. Denique fac $g = 4s - q^2 + 8pr - p^4 - 2p^2q$, & fiunt Equationes præcedentes

$$\begin{array}{llllllll} x^4 = 4px^3 & +2qx^2 & +8rx & +4s & \&\; a^6 = & p^2a^4 & -2pra^2 & +r^2. \\ & -4p^2 & -4pq & -q^2 & & +q & -s & \end{array}$$

Scilicet omnia evadunt ut ſupra ſunt poſita.

§ 3. Hactenus de Æquationum Cubicarum & Biquadraticarum Reſolutione Analytica. Quoniam autem earundem *Effectio Geometrica* per Parabolam vulgo tradi ſolet, & nonnullis in pretio eſt, ipſam συνθετικῶς, & quidem univerſalius, non pigebit hic exhibere.

Data Æquatione qua[illegible]vis vel Cubica vel Biquadratica, inſtituenda eſt comparatio inter terminos ejus, terminoſque reſpondentes hujus Æquationis

$$\begin{array}{lllll} x^4 = & \frac{2p}{q}x^3 & +\frac{4pr}{q}x^2 & +\frac{2p2}{q}x & +p^2, \\ & -4r & -4r^2 & -\frac{2ps}{q} & -q^2 \\ & & +2s & +4rs & -s^2 \\ & & -1 & -2q & +t^2 \end{array}$$

quo pacto facile ſatis

eruentur ipſæ p, q, r, s, t; earum interim una aliqua utcunque pro lubitu aſſumpta. Tum in Parabola quavis data AVB, cujus Vertex principalis *Plate* 3. *Fig.* 1. V, Axis VS, & Axi perpendicularis VT; capiatur $VS = p$ verſus interiora Parabolæ, & in angulo SVT inſcribatur $ST = q$, quæ producta Parabolam ſecet in punctis binis N & O. Biſecetur ON in M, & per M agatur MA Axi parallela & Parabolæ occurrens in A. Ipſi ON parallela ducatur AL, ut ſit AL Latus rectum Parabolæ ad Diametrum AM, ſitque hæc eadem Unitas. In AL (utrinque ſi opus eſt producta) capiatur $AG = r$, & a puncto G ducatur GR Axi parallela, quæ Parabolam ſecet in B, a quo capiatur $BR = s$. A noviſſime invento puncto R ducatur RE ipſi VT parallela & æqualis, quæ ſiniſtram verſus jaceat reſpectu ipſius R ſi q ſit quantitas affirmativa, at verſus dextram ſi q ſit negativa. Atque idem de ipſis AG & BR intelligatur, quæ ad contrarias itidem partes duci debent, ſi modo valores ipſarum r & s prodeant negativi. Denique Centro E & Radio $EC = t$ deſcribatur Circulus $CK\kappa c$, qui Parabolam in totidem ſecabit punctis, quot ſunt Æquationis datæ Radices reales. Etenim a punctis iſtis C, K, *&c.* ducantur CP, $\kappa\pi$, *&c.* ipſi ST parallelæ, & ad rectam GR (ſi opus eſt productam) terminatæ, eritque harum quævis x, ſeu Æquationis datæ Radix quæſita; eæ ſcilicet ad dextram jacentes erunt Radices affirmativæ, quæ vero ad ſiniſtram ſunt poſitæ erunt Radices negativæ. Punctum contactus, ſi quod fuerit, hic ſumitur pro interſectionis punctis duobus ad invicem viciniſſimis.

Inter Æquationes Cubicas & Biquadraticas ita conſtructas hoc tantum intercedit diſcriminis, quod in prioribus, ob terminum ultimum in præcedente Æquatione deficientem, ſemper ſit $p^2 - q^2 - s^2 + t^2 = 0$, ſive $t = \sqrt{s^2 + q^2 - p^2}$. Igitur Centro E & Radio EB $(= \sqrt{BRq + (ERq)\ STq - VSq})$ deſcripto Circulo $CK\kappa c$, Radicum una CP in priori conſtructione in nihilum abit.

Hæc autem demonſtrantur ad modum ſequentem. Manentibus jam conſtructis, & producta CP, ſi opus eſt, donec ſecat AM in H, erit CH Ordinata Parabolæ ad Diametrum AH, & proinde $CHq = AL \times AH = AH$, ob $AL = 1$. At $CH = CP + AG$, & $AH = GB + BP$, & propterea $CPq + 2AG \times CP + AGq = GB + BP$; ſed ob naturam Parabolæ erit $AGq = GB$, unde $CPq + 2AG \times CP = BP$. Jam a puncto C ad ipſam BP demittatur normalis CD, quæ occurrat etiam ipſi EI, ad BP actæ parallelæ, in puncto I. Propter ſimilia Triangula CDP & TVS, erit $DP = \frac{VS \times CP}{ST}$ & $CD = \frac{VT \times CP}{ST}$, & proinde $CPq + 2AG \times CP = BP = DP + BD = \frac{VS \times CP}{ST} + BR - IE$, ſive $CPq + 2AG \times CP - \frac{VS}{ST}CP - BR = -IE$. Aſt $IEq = CEq - CIq = CEq - CDq - VTq - 2CD \times VT = CEq - \frac{VTq \times CPq}{STq} - VTq - \frac{2VTq \times CP}{ST}$

=

$= (\text{ob } VTq = STq - SVq)\ CEq - CPq + \frac{SVq}{STq} CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST} CP$, quæ igitur æqualis erit Quadrato ex Latere $CPq + 2AG \times CP - \frac{VS}{ST} CP - BR$. Atque hæc Æquatio, ad terminos p, q, r, s, t, revocata, ipſiſſima fit Æquatio propoſita.

Hinc liquet, quod eadem quævis Æquatio Biquadratica innumeras per Parabolam conſtructiones ſortiri poſſit, pro indefinito valore quantitatis iſtius, quam ad arbitrium aſſumi poſſe jam diximus. Sed caſus eſt ſimpliciſſimus faciendo $VS = p = 0$, & migrat conſtructio, ſi rem ipſam ſpectes, in vulgarem iſtam, in qua Radicum repræſentatrices rectæ *CP*, *&c.* ſunt ad Axem perpendiculares. Æquatio autem fit

$$x^4 = -4rx^3 \begin{array}{l} -4r^2x^2 \\ +2s \\ -1 \end{array} \begin{array}{l} +4rsx \\ -2q \\ \end{array} \begin{array}{l} -q^2, \\ -s^2 \\ +t^2 \end{array}$$

quæ facile conſtruitur ut ſupra.

§ 4. Sed ne Parabolæ deſcriptio Organica difficilis nimium videatur, in promptu eſt Artificium quoddam Mechanicum, ope Fili penduli pondere inſtructi peractum, cujus auxilio quam exactiſſime & facillime Æquatio noviſſima conſtrui poteſt, & proinde Æquationum quarumcunque Cubicarum & Biquadraticarum Radices inveniri; idque ſine ullo linearum ductu niſi Rectarum & Circuli. Conſtructio autem, quam appellare libet *Mechanicam*, eſt ad hunc modum.

Plate 3. Fig. 2. Contra Parietem erectum, vel planum aliud quodvis Horizonti perpendiculare, ad punctum aliquod *F* ſuſpendatur filum tenuiſſimum & flexile *FP*; pondere quovis *P* ad extremitatem *P* appenſo. In hoc filo notetur punctum aliquod *N*, a puncto ſuſpenſionis *F* ſatis remotum; vel filo parvulus, ſi id mavis, innectatur Nodus *N*. Et ſumpta utcunque *NP* pro Unitate, ad punctum medium *A* ducatur (in plano prædicto) recta *AQ* Horizonti parallela, & utrinque quantum ſatis producta. Hiſce generaliter paratis, pro particulari jam applicatione fac $AQ = r$; ipſis q, r, s, t, ut ſæpius inculcatum, vel Arithmetice vel Geometrice, pro datæ cujuſvis Æquationis exigentia, in Æquatione noviſſima prius determinatis. Tunc Acu vel Stylo tenuiſſimo, aut etiam cuſpide Circini admodum gracili, flectatur filum a loco ſuo ad punctum quoddam *B*, ita ut punctum *N* cadat in noviſſime invento puncto *Q*. In *BQ* ab iſto *B* capiatur $BR = s$, & in *R* ad ipſam *BR* perpendicularis erigatur $ER = q$. Verum enimvero iſtæ *AQ*, *BR*, *RE* ad contrarias partes ab earum initiis cadere debent, ſi forte valores ipſarum *r*, *s*, *q* prodeant negativi. Denique in puncto invento *E* figatur Circini crus unum, &, ad diſtantiam $EZ = t$ extentum, agatur crus alterum in orbem, ſecumque circumducat filum *FZP*. Hac fili circulatione pondus *P* nunc aſcendet nunc deſcendet motu reciproco, ut & Nodus *N* nunc ſupra rectam *AQ* extabit, nunc vero infra eandem deprimetur. Quoties autem reperietur Nodus ille *N* in ipſa *AQ*, puta in punctis *D*, *d*, Δ, δ, abſcindet is rectas *DQ*, *dQ*, Δ*Q*, δ*Q*, quæ erunt

Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ vero ad ſiniſtram Radices negativæ. Demonſtratio eſt manifeſta ex præcedentibus, habita tantum ratione Parabolæ per puncta B, C, c, κ, K tranſeuntis. Nam poſito F foco Parabolæ, (cujus diſtantia a Vertice eſt $\frac{1}{4} ON$,) notum eſt quod lineæ omnes ut $FB + BQ$, $FC + CD$, *&c.* eandem ubique conficiant ſummam.

Atque ex principiis hic poſitis proclive erit Inſtrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujuſmodi Æquationum quarumcunque Radices nullo fere negotio inveniri poſſint, & præ oculis exhiberi.

Rules for correcting the common computus of Intereſt. by Mr. Tho. Watkins. *n.* 340. *p.* 111.

1. *Of Compound Intereſt.*

XXVII. The Suppoſition whereon the Method of computing by Compound Intereſt is founded; *viz.* That all Intereſt Money, Rents, *&c.* are or may be conſtantly receiv'd, and put out again at Intereſt, the Moment they become due, without any Charge, or Trouble, being impracticable; therefore all Computations by this Method (except of Fee-Simples or other Perpetuities) muſt needs be erroneous. Thus for Inſtance, the Amount of a Sum of Money, or Annuity, for want of Deductions out of the Profits, for the unavoidable Trouble, Charge, and Delay in the Management, will be too great: and for the ſame reaſon, the preſent value of a Sum of Money payable in any time to come, will be too little; alſo the preſent value of an Annuity (being only the Amount of the difference between the Annuity, and Intereſt of the ſaid preſent value) will be too much. But in long terms of Years, as that difference becomes leſs, ſo does the Error, as the term is greater; and in Fee-Simples it vaniſhes; the contrary to which happens in Amounts of Sums of Money, and Annuities.

All which is propos'd to be rectify'd, only by a juſt reduction of the Rate, and Annuity; (which is done by deducting ſo much *per Cent.* thereout, as the whole Trouble and Charge of Management is ſuppos'd to amount to, and reducing the Remainder, by a Diſcount equivalent to the ſuppos'd loſs of time) and then by working with the Rate ſo reduc'd for Sums of Money, and with the Rate and Annuity reduc'd in the like proportion for Annuities, according to the common Method of Compound Intereſt; as follows. Put r for the rate of Intereſt of 1 *l.* c for the Charge and Trouble of the Management of 1 *l.* Then is $r - cr =$ the Rate after deducting the ſaid Charge, $=$ (putting d for $1 - c$) dr. And for the Diſcount put t for the time loſt, that is for ſuch part of the Period of time in which the Payments are made (whether Yearly, ½ Yearly, Quarterly, or otherwiſe) as is ſuppos'd to be ſpent in receiving and putting them out again at Intereſt. Then, dtr being $=$ the Intereſt of 1 *l.* for that time; ſay, as $1 + dtr : 1 :: dr : \frac{dr}{1 \times dtr} =$ (putting e for $1 + dtr$) $\frac{dr}{e}$, which is equal to the reduc'd Rate, near enough for practice, for which put r. But if the utmoſt accuracy be requir'd, the Diſcount itſelf muſt be made with regard to the like loſs of time, which is done by a Series of Diſcount rais'd thus; $e\,(=1+tdr):tdr::dr:\frac{td^2r^2}{e}::\frac{td^2r^2}{e}:\frac{t^2d^3r^3}{e^2}$ &c. Whence $dr-\frac{td^2r^2}{e}+\frac{t^2d^3r^3}{e^2}-\frac{t^3d^4r^4}{e^3}$ &c. $=$(putting q for $\frac{tdr}{e}$) $\overline{1-q+q^2-q^3\ \&c.}\times dr.=\frac{dr}{e}\,(=\overline{1-q}\times dr)+\overline{q^2-q^3\ \&c.}\times dr$, is $=\mathfrak{r}$, $=$ the true Rate reduced. Put $s=1+\mathfrak{r}$, $n=$ the time, $p=$ the preſent Sum or Value, $m=$ the Amount. Then will $\overline{1+\frac{dr}{e}+\overline{q^2-q^3\ \&c.}\times dr}|^{n}\times p=\overline{1+\mathfrak{r}}|^{n}\times p=ps^{n}$, be exactly $=m$: But $\overline{1+\frac{dr}{e}}|^{n}\times p=m$ is ſufficient for practice. And for the Amounts and preſent values of Annuities.

Put $A=$ Annuity *per annum.*
$a=$ Annuity $\frac{1}{2}$ yearly, quarterly, &c.
$R=$ Yearly rate of Intereſt of 1 *l.*
$r=$ Rate $\frac{1}{2}$ yearly, quarterly, &c.
$\mathfrak{r}=$ Reduc'd rate yearly, $\frac{1}{2}$ yearly, quarterly, &c.
$n=$ Number of Years, $\frac{1}{2}$ years, quarters, &c.

Then will $A\frac{\mathfrak{r}}{R}=a\frac{\mathfrak{r}}{r}$ be $=$ reduc'd Annuity taken yearly, $\frac{1}{2}$ yearly, quarterly, or otherwiſe; and by Compound Intereſt 'twill be $\frac{\overline{1+\mathfrak{r}}|^{n}-1}{\mathfrak{r}}\times a\frac{\mathfrak{r}}{r}=\frac{\overline{1+\mathfrak{r}}|^{n}-1}{r}a=\frac{\overline{1+\mathfrak{r}}|^{n}-1}{R}A=\frac{s^{n}-1}{R}A=m$, and $\frac{s^{n}-1}{Rs^{n}}A\,(=\frac{m}{s^{n}})=p$. Whence the Theorems for ſolving all the other Caſes are eaſily deduc'd. And if the Rate be requir'd, when 'tis for a Sum of Money, the Solution is obvious: when for the Amount or Value of an Annuity, ſince $\overline{1+r}|^{n}=\frac{mr+a}{a}=\frac{a}{a-pr}$ are the Equations whence Theorems for the Rate are uſually deriv'd, which by this Correction become $\overline{1+\mathfrak{r}}^{n}=\frac{mr+a}{a}=\frac{a}{a-pr}$. That the ſame $\mathfrak{r}$ may be had on both ſides the Equation, put $u\mathfrak{r}$ for r, and 'twill be $\overline{1+\mathfrak{r}}|^{n}=\frac{mu\mathfrak{r}+a}{a}=\frac{a}{a-pu\mathfrak{r}}$; then, by the Rate aſſum'd as near the truth as may be, find the value of $u\,(=\frac{1}{d}+tr=\frac{1}{d-td\mathfrak{r}})$ and in any Theorem for the Rate, putting mu for m, and pu for p, the reſult will be the Rate reduced nearly: and by repeated Operations correcting $\mathfrak{r}$ and thereby u, the true $\mathfrak{r}$, and thence R the whole Rate may be found.

The only difficulty that remains, is the right aſſuming the Quantities c and t, the impoſſibility of doing which with perfect Exactneſs, I ſuppoſe to be the reaſon why neither this, nor any Method of Correction to the like purpoſe, has yet been taken notice of by the Writers on this Subject; and what may therefore be very likely to be objected to this. But the ſame Objection I take to be of equal force againſt the Eſtimates of any other Uncertainties whatſoever, as Eſtates for Lives, Inſurances, *&c.*

Firſt then, for the Quantity c, which is put for the Trouble and Charge of Management, *viz.* of collecting and placing out the Money on good Security, together with all Contingencies attending the ſame, as travelling Charges, Expences, Attorney's Bills, *&c.* of all which, the principal Article is the Charge of Collection, or Receiver's Fees, which is commonly a fix'd Rate, cuſtomarily allow'd in the Place, or upon the Eſtate itſelf out of which the Purchaſe is made, if it be a Rent; and for Intereſt Money, or any other Annuity, the like Eſtimate is to be made, whether the Proprietor acts for himſelf, or by another. Then for the Charge of placing out the Money at Intereſt when receiv'd, though this be for the moſt part defray'd by the Borrower, yet becauſe it highly concerns the Lender to ſee it be ſecurely done, there are uſual Allowances made to Agents and Scriveners, to encourage their Care and Fidelity therein; beſides the Time, Expence and Trouble of the Proprietor himſelf, in finiſhing Contracts, inſpecting Securities, *&c.* and whatever is ſav'd in this Article, we muſt ſuppoſe to be fully made up by an Equivalent Degree of Riſque in the Security.

In the next place, for the loſs of time; if the uſual times of Payment of the particular Rent or Annuity to be purchaſed, with a moderate degree of diligence in the Manager, and the uſual indulgence practis'd by Men of Buſineſs in this Caſe towards one another be obſerv'd, a reaſonable Eſtimate may be alſo made of the loſs of time; in which 'tis to be noted, that Intereſt Money being uſually paid in ſmall Sums, when any Sum of Money to be made up of ſeveral ſuch Payments, is intended to be put out at Intereſt, the whole muſt lie dead till the laſt Payment be made; alſo that the Principal lies dead ſometimes as well as the Intereſt; and that on the other hand, to ſave time, Borrowers may be found out, and treated with during the time of Collection; but this Advantage is in a great Meaſure loſt by the difficulty of fixing the time or *Quantum* of a Loan, till the whole be paid in. Note alſo, that if the Charge of Collection, or loſs of time, on the Rent, or Annuity, of any particular Eſtate or Place, be found to differ from that of the Intereſt of the Purchaſe Money, and ſo much exactneſs be required, as that the Computation be made with regard to ſuch difference: It muſt be done, either by aſſuming a proper Medium for both, or more accurately thus: For the reduc'd Rate of the $\left\{\begin{matrix}\text{Annuity}\\ \text{Intereſt-Money}\end{matrix}\right\}$ put $\left\{\begin{matrix}\mathfrak{z}\\ \mathfrak{r}\end{matrix}\right\}$ Then $r : \mathfrak{z} :: a : a\frac{\mathfrak{z}}{\mathfrak{r}}$, and

$$\mathfrak{r} : \overline{1+\mathfrak{r}}|^{n} - 1 :: a\frac{\mathfrak{z}}{r} : \frac{\overline{1+\mathfrak{r}}^{n} - 1}{r} a \times \frac{\mathfrak{z}}{\mathfrak{r}} = m = \overline{1+\mathfrak{r}}|^{n} \times p.$$

To give a Specimen of this Method in Numbers, firſt the Quantities c and t are to be aſſum'd, which are not here to be accommodated to any particular Place or Eſtate, but to be taken in general: And firſt for c the Charge of Management; the uſual Rates of Collectors Fees in theſe Kingdoms, as I am inform'd, are 6*d.* 12*d.* and 18*d. per* Pound, but the moſt uſual 12*d.* which is 5 *per Cent.* However to be within compaſs, I ſhall take 4 *per Cent.* for the Medium, including what further trouble and charge may attend the Receipt of the Money, beſides Receivers Fees; and 2 *per Cent.* for all the other Charges before mention'd, in placing it out at Intereſt; both which make 6 *per Cent.* ſo that c is $= 0{,}06$, and $d\ (= 1 - c) = 0{,}94$. Next for t the loſs of time; ſince few Annuities are paid yearly, and a Diſcount being given for the loſs of time, we are to loſe no more than is diſcounted for; therefore I chooſe Half-yearly Payments for the Examples, being the moſt uſual, which with little Alteration may ſerve for quarterly; and conſidering the before-mention'd Circumſtances relating to the Time, I look upon two Months the leaſt, and ſeven or eight Months the moſt, that can well be ſuppos'd to be ſpent, one time with another, in receiving and putting out the Money, upon a moderate Management; between which the Medium is about four Months and half, which being $\frac{3}{4}$ of $\frac{1}{2}$ a Year, gives $\frac{3}{4} = t$: and if $\frac{dr}{e}$ be $= \mathrm{r}$, the yearly Rates of 4, 5, 6, 8 and 10 *per Cent.* will produce for half-yearly Rates reduc'd of 1 *l.* 0,018539, 0,023093, 0,027616, 0,036569 and 0,0454 each $= \mathrm{r}$. But if r be $= \frac{dr}{e} + \overline{q^2 - q^3\ \&c.} \times dr$, 'twill be, 0,01854219, 0,0230999155, 0,027627775, 0,036596289 and 0,04545235, each $= \mathrm{r}$, (ſo that each Rate loſes by this Eſtimate about $\frac{1}{12}$ Part.) Whence the following Amounts, and preſent values of 1 *l.* *per Annum* computed Half-yearly, are produc'd, and compared with thoſe of the uſual Method computed yearly, to agree with the common Tables.

Years

Years.	*Amounts of* 1 l. per An. at 5 per Cent. *by*		*Differences.*	*Amounts of* 1 l. per An. at 6 per Cent. *by*		*Differences.*
	Comp. Int.	*Cor. Int. cor.*		*Comp. Int.*	*Co. Int. cor.*	
5	5,52563	5,13104	,39459	5,63709	5,22138	,41571
10	12,57789	11,57846	,99943	13,18079	12,07854	1,10225
20	33,06595	29,85996	3,20599	36,78559	32,91054	3,87505
30	66,43885	58,72546	7,71339	79,05819	68,83976	10,21843
40	120,79977	104,30079	16,49898	154,76197	130,80733	23,95464
60	353,58372	289,88164	63,70208	533,12818	422,01429	111,11389
80	971,22882	752,53436	218,69446	1746, 5999	1288, 2484	458,3515
100	2610,02516	1905,92671	704,09845	5638, 3680	3864, 9753	1773,3927
	The ſame at 8 *per Cent.*			The ſame at 10 *per Cent.*		
5	5,86660	5,40633	,46027	6,10510	5,59705	,50805
10	14 48656	13,15092	1,33564	15,93743	14,32680	1,61063
20	45,76196	40,13759	5,62437	57,27500	49,17930	8,09570
30	113,28321	95,51623	17,76698	164,49402	133,96428	30,52974
40	259,05652	209,15737	49,89915	442,59257	340,21900	102,37357
60	1253,2133	920,90040	332, 3129	3034,81648	2062,57167	972,24481
80	5886.9354	3918,0578	1968, 8776	20474,0027	12255,3348	8218, 6679
100	27484,5157	16539,0989	10945,4168	137796,127	72575,3926	65220,7344

Years.	*Preſ. Values of* 1 l. per An. *at* 5 per Cent. *by*		*Differences.*	*Preſ. Values of* 1 l. per An. *at* 6 per Cent. *by*		*Differences.*
	Comp. Int.	*Co. Int. cor.*		*Comp. Int.*	*Co. Int. cor.*	
5	4,32948	4,08343	,24605	4,21236	3,97582	,23654
10	7,72174	7,33314	,38860	7,36009	7,00322	,35687
15	10,37966	9,91935	,46031	9,71225	9,30843	,40382
20	12,46221	11,97753	,48468	11,46992	11,06373	,40619
30	15,37245	14.91902	,45343	13,76483	13,41805	,34678
40	17.15909	16,78200	,37709	15,04630	14,78309	,26321
50	18,25593	17,96190	,29403	15,76186	15,57456	,18730
70	19,34268	19,18247	,16021	16,38454	16,29953	,08501
100	19 84791	19,79231	,05560	16,61755	16,59510	,02245
F.S.	20.00000	20,00000	,00000	16.66666	16 66666	.00000

Note, That this Correction is alſo applicable to the Valuations of Eſtates for Lives, in which the firſt Step being to find an Equivalent in Years of Continuance, brings them to the Caſe of Eſtates for Years.

Of Simple Intereſt.

2. The abſurdity of the Suppoſition, on which the uſual Method of computing preſent Values by Simple-Intereſt, is founded, *viz.* That the Rent or Annuity is conſtantly received, and put out again at Intereſt, as it becomes due; but that the Intereſt of the Purchaſe Money lies dead during the whole Term, is ſo apparent, and the Errors ariſing from it ſo groſs, that the Writers who have laid down this Method, have at the ſame time caution'd againſt the Uſe of it for any more than 6 or 7 Years, the Error for that time being not conſiderable.

The ſame Suppoſition does alſo occaſion the miſcomputation of Amounts, or rather the miſapplication of them to their proper Caſes.

Wherefore, ſince the Simple-Intereſt of Money is of equal Value *pro rata*, and of the ſame regard with Rents or Annuities, being each the Original Profits iſſuing alike from a principal Stock, Eſtate or Value, and equally improveable; This general Rule may ſerve for a juſt Correction of this Method, *viz.* That ſuppoſing in any Caſe an Intereſt ought to be, or not to be allowed to either of thoſe Profits, the ſame be done in the like Caſe to the other. Thus, in the Caſe of Debts, or Amounts of Sums of Money, Rents, or Annuities for the time paſt, its uſual in practiſe to allow no Intereſt to either: For though the Law, to curb the exorbitant Avarice of Uſurers, and for other Reaſons, does more expreſly diſallow Intereſt upon Intereſt for a Debt; our Courts both of Law and Equity, as I am inform'd, will be as far from allowing the Charge of Intereſt againſt a Tenant for Rent in Arrear; except on a *nomine pœnæ* (which is now become almoſt obſolete) ſo that in this Caſe (putting a and r for the Annuity and Rate yearly, half-yearly, or otherwiſe) as $prn+p$ is the Amount of a Sum of Money, ſo is an the Amount of an Annuity or Rent in Arrear, and not $\overline{\frac{n-1}{2}r+1}\times an$, as Arithmeticians commonly make it. But in the Computation of preſent Values, or Amounts for the time to come, the ſame being made on the Expectation of a conſtant regular Income of the Profits, without any extraordinary Interruption, an Intereſt ought to allow'd to both, eſpecially in preſent Values, which are found by ſetting the Amounts of both againſt each other: ſo that in theſe Caſes, putting $x=n-1$, if $\overline{\tfrac{1}{2}xr+1}\times an$, be made the Amount of a Rent or Annuity; then $\overline{\tfrac{1}{2}xr+1}\times\overline{prn+p}$, will be the proper Amount of a Sum of Money, and not $prn+p$: and conſequently $\frac{\overline{\frac{1}{2}xr+1}\times an}{\overline{\frac{1}{2}xr+1}\times\overline{rn+1}}=p$ will be the preſent Value of a Rent or Annuity, the ſubſequent Intereſt being remitted on both ſides in lieu of the loſs of Time and Charge of Management; which ſuch as are apt to depreciate long Futurities, may think the propereſt Method of approaching the true Value; but I rather look upon the former Method of Compound Intereſt corrected as more exact, as well as more general, the Intereſt remitted in this being in ſhort Terms leſs, and in long Terms more than an Equivalent for the Trouble, Charge and Delay in the Management. But it is however the moſt exact of any of the Methods, that have yet been deduc'd from Simple-Intereſt. The reduc'd Rate may alſo in ſome Caſes be perly made uſe of for Amounts, but not for preſent Values, except for ſhort Terms; and then, ſince $r:a::\overline{\tfrac{1}{2}x\mathfrak{r}+1}\times\mathfrak{r}n:\overline{\tfrac{1}{2}x\mathfrak{r}+1}\times an\frac{\mathfrak{r}}{r}$, 'twill be $\overline{\tfrac{1}{2}x\mathfrak{r}+1}\times an\frac{\mathfrak{r}}{r}=m=\overline{\tfrac{1}{2}x\mathfrak{r}+1}\times\overline{p\mathfrak{r}n+p}$, and $\frac{\overline{\frac{1}{2}x\mathfrak{r}+1}\times an}{\overline{\frac{1}{2}x\mathfrak{r}+1}\times\overline{\mathfrak{r}n+1}}\times\frac{\mathfrak{r}}{r}=p.$

Examples of this Method compar'd with thoſe of the former will ſtand as follows; in which all is computed Half-yearly, except the laſt Column of Compound Intereſt.

Amounts of 1 *l.* at 5 *per Cent.* computed 6 ſeveral Ways.

Years.	1 *Simple Int.*	2 *Id. for Bonds*	3 *Sim. Int. cor.*	4 *Id. by the red rat*	5 *Co. Int. cor.*	6 *Comp. Int.*
	$prn + p = m$	Id. till $prn = p$	$\overline{\tfrac{1}{2} \times r + 1} \times prn + p = m$	$\overline{\tfrac{1}{2} \times r + 1} \times p^n + p = m$	$\overline{r + rl}^n \times p = m$	$\overline{r + Rl}^N \times p = m$
5	1,25	1,25	1,27813	1,25501	1,25655	1,27628
10	1,5	1,5	1,61875	1,56338	1,57892	1,62889
20	2,	2,	2,48750	2,34021	2,49300	2,65330
30	2,5	2,	3,60625	3,33048	3,93625	4,32194
40	3,	2,	4,97500	4,53419	6,21504	7,04000
60	4,	2,	8,46250	7,58194	15,49408	18,67919
80	5,	2,	12,95000	11,48345	38,62672	49,56144
100	6,	2,	18,43750	16,23874	96,29634	131,50126
	Amounts of 1 l. *at* 6 per Cent. *by the ſame Theorems.*					
5	1,3	1,3	1,3405	1,31063	1,31328	1,33822
10	1,6	1,6	1,7710	1,69758	1,72471	1,79084
20	2,2	2,	2,9020	2,70048	2,97463	3,20713
30	2,8	2,	4,3930	4,00870	5,13039	5,74349
40	3,4	2,	6,2440	5,62223	8,84844	10,28572
60	4,6	2,	11,0260	9,76525	26,32086	32,98769
80	5,8	2,	17,2480	15,12954	78,29488	105,79599
100	7,	2,	24,9100	21,71510	232,89852	339,30208
	Amounts of 1 l. *at* 10 per Cent. *by the ſame Theorems.*					
5	1,5	1,5	1,6125	1,54749	1,55970	1,61051
10	2,	2,	2,4750	2,30157	2,43268	2,59374
20	3,	2,	4,9500	4,42951	5,91793	6,72750
30	4,	2,	8,4250	7,38381	14,39643	17,44940
40	5,	2,	12,9000	11,16449	35,02190	45,25925
60	7,	2,	24,8500	21,20493	207,25717	304,48165
80	9,	2,	40 8000	34,55084	1225,5335	2048,4003
100	11,	2,	60,7500	51,20222	7258,5398	13780,6127

Years.	*Preſ. Values of* 1 l. per An. *at* 5 per Cent. *by*			*Preſ. Values of* 1 l. per An. *at* 6 per Cent. *by*		
	Sim. In. cor.	*Co. Int. cor.*	*Comp. Int.*	*Sim. In. Cor.*	*Co. Int. Cor.*	*Comp. Int.*
5	4,35208	4,08343	4,32948	4,23349	3,97582	4,21236
10	7,64478	7,33314	7,72174	7,25579	7,00322	7,36009
15	10,10821	9,91935	10,37966	9,39341	9,30843	9,71225
20	11,95980	11,97753	12,46221	10,92350	11,06373	11,46992
30	14,45407	14,91902	15,37245	12,87275	13,41805	13,76483
40	15,97990	16,78200	17,15909	13,99744	14,78309	15,04630
50	16,96682	17,96190	18,25593	14,69544	15,57456	15,76186
70	18,10986	19,18247	19,34268	15,47252	16,29953	16,38454
100	18,91525	19,79231	19,84791	15,99759	16,59510	16,61755
F.S.	20,00000	20,00000	20,00000	16,66666	16,66666	16,66666

The Theorems to the preceding Columns of Amounts (of which the fourth and fifth are infinitely variable in the Reſult, by aſſuming *c* and *t* in the reduc'd Rates at Pleaſure) may ſerve to anſwer all the ſimple Caſes of Amounts that occur in Buſineſs: To inſtance in ſome.

1. The firſt Column contains the Amounts of ſuch Debts, or Sums of Money as carry a ſimple Intereſt till the Principal be paid.

2. The ſecond Column anſwers the common Caſe of Debts due by Bond, that by Law are allow'd not to exceed the Penalty, which is generally double the principal Debt.

3. The third Column anſwers the Caſe of a Security or joynt Obligor, that has duly and conſtantly paid the Intereſt, and at laſt the principal Sum of a Debt, from which he has a Counter-Bond from the principal Debtor to ſave him harmleſs, againſt whom he may make his Charge from this Column.

4. In caſe the Parties ſhall agree that the Debt ſhall lie for any time certain, or uncertain; and for the much greater eaſe, advantage and ſatisfaction of both of them, no Intereſt to be call'd for, till the Principal it ſelf is paid; but to carry Intereſt as it becomes due, the Lender allowing for the time and charge he muſt have been at, in receiving and putting out his Intereſt, the fourth Column will fit this Caſe, or elſe the fifth as a greater or leſs degree of Lenity is agreed upon in favour of the Borrower.

5. The fifth Column is alſo proper in the following Caſe, *viz.* if it be demanded, what Eſtate in reverſion, after a certain number of Years, any Sum in Hand will Purchaſe; the firſt Step being to find the Amount of that Sum to the time the Reverſion commences, its had in this Column.

6. The laſt Column gives the Amount of a Sum of Money, according to the common Method of Compound Intereſt, but being computed with that extraordinary rigor as has been ſaid, (except ſome ſmall allowance for the loſs of time, by being done Yearly) 'tis hardly ſuitable to any Caſe.

Of Intereſt Accounts.

3. The Inequality of the uſual Method of ſtating Intereſt Accounts, as practis'd in our Courts of Equity, will beſt appear by an Example; for which I ſhall take the common general Caſe, of an Intereſt Account to be ſtated on a Mortgage, *viz.* ſuppoſe 1000 *l.* to be let out at 6 *per Cent.* on a Mortgage of 120 *l. per anum*, payable Half-Yearly, and the Mortgagee after five Years end, to have Poſſeſſion till the Arrear of Intereſt, accruing Intereſt and Principal be diſcharged: *Quare*, How long that will be? ſuppoſing alſo, for the ſake of brevity in the Account, the Payments to be equally and punctually made as they become due. By the Chancery Method, the Rent is firſt apply'd to diſcharge the Arrear of Intereſt; and then the remainder of every Half-Years Rent, after deducting the ſame Half-Years Intereſt, is apply'd towards the Diſcharge of the Principal, and thereby the Principal and Intereſt continually leſſens, till the whole be paid off. Now by this means the Mortgagee, after the Arrear is diſcharged, pays Compound Intereſt, with the utmoſt rigour, for ſo much *per annum* of the Rent, as exceeds the Intereſt of the

whole Principal Money, and receives but Simple Intereſt for his Debt; which, however ſtrange it may ſeem, is eaſily prov'd, by applying the proper Theorems of Simple and Compound Intereſt to this Caſe, in which the Annuity, Principal Money, Rate and Arrear of Intereſt are given, and the time requir'd; the reſult being the ſame with that of the Chancery Method, except a very ſmall difference only when any part of the time is expreſs'd by a Fraction: *viz.* putting L for Logarithm, $\alpha = a - pr = 30$, $s = 1 + r = 1{,}03$, $t =$ time of contracting the Arrear $= 10$ Half-years, $n =$ any number of Half-years ſpent in diſcharging the whole or any part, $N =$ number of Years required; the Equation for the Arrear will be $prt + prn = an$; and for the Principal and accruing Intereſt $prn + p = prn + \frac{s^n - 1}{r} \times \alpha$. Whence $\frac{prt}{2\alpha} + \frac{La - L\alpha}{2Ls} = N = 16{,}7249$ Years $=$ the time demanded; *i. e.* $\frac{prt}{2\alpha} = 5$ Years $=$ the time of diſcharging the Arrear and $\frac{La - L\alpha}{2Ls} = 11{,}7249$ Years, $=$ the time in which the Principal and accruing Intereſt is diſcharged; during which its evident, the Mortgagee pays full Compound Intereſt for 60 *l. per annum* of the Rent. For the Correction of which inequality, in the firſt Place, to the end that neither Branch may exceed, or be depriv'd of its due Profits; This general Rule is propos'd as neceſſary to be always obſerv'd, *viz.* That Amounts of the Produce on each ſide be ſtated ſeparately, and ſet againſt each other in the Account, in order to a Ballance. And in the common Caſes of Mortgages, Government, and Stock-Securities, *&c.* where the Debt is paid off by a Rent, Annuity, Penſion, Dividend, or other Payments iſſuing in the ſame manner, and with the like trouble, charge, *&c.* as Intereſt Money does; I preſume this Rule will alſo be eaſily admitted, *viz.* That the ſame equitable Advantage be allowed on both ſides; for which the Method of Simple Intereſt, as corrected under the foregoing Head, ſeems truly adapted; whereby the Original Profits on each ſide are ſuppos'd to be deem'd, either as Intereſt, or elſe as Principal Money; and ſince the Amounts both of an Annuity and Sum of Money, for the time paſt, as there ſtated, on the firſt of theſe Suppoſitions (t being there $= 0$) are likewiſe vouch'd by our Laws, and the practice of our Courts, to be good when ſeparately us'd; I think its very evident, that the Account ought to be ſtated by ſetting thoſe Amounts againſt each other thus, $prt + prn + p = an$, (whence $\frac{1 + tr}{2\alpha} p = N, = 21{,}6666$ Years) and that this Method is moſt proper for general Uſe, in the Caſes mention'd: Unleſs it ſhould be thought fit, in conſideration of the various Ways found out for the ready improvement of Money, to allow a further Advantage on both ſides, by charging the Original Profits as Principal Money, and giving a Simple Intereſt thereto, which ſtill falls ſhort of the

Advantage allow'd to Rents by the Chancery Method. And this is to be done two ways, *viz.* either by applying an Amount of Rent to pay off the Arrear firſt, and afterwards another Amount of Rent to diſcharge the Principal, and accruing Intereſt; or elſe by letting the Profits with all Arrears and other Charges run on at Simple Intereſt on each ſide, till the end of the Term: *viz.* putting $x = n - 1$, $y = t - 1$, $\alpha = a - pr$, $f = \overline{2 - r} \times \alpha$, $g = f - 2ptr^2$, $\mu = \overline{\tfrac{1}{2} yr + 1} \times prt =$ Arrear of Intereſt; by the firſt of theſe 'twill be, for the Arrear, $\mu + ptr^2 n = \overline{\tfrac{1}{2} xr + 1} \times \alpha n$, and for the Principal and accruing Intereſt, $\overline{\tfrac{1}{2} xr + 1} \times \alpha n = p$; Whence $\frac{\sqrt{8\mu\alpha r + g^2} - \sqrt{8p\alpha r + f^2} - f + g}{4\alpha r} = N = 18,7653$ Years. By the other 'twill be, for the whole, $\mu + ptr^2 n + p = \overline{\tfrac{1}{2} xr + 1} \times \alpha n$. Whence $\frac{\sqrt{\overline{\mu + p} \times 8\alpha r + g^2} - g}{4\alpha r} = N = 18,1648$ Years. The Lender will have to alledge for the firſt of theſe two Ways, that as the Rent is not hindred by any other parallel Charge from making the utmoſt produce it can, ſo for that reaſon ought his Principal Money to have the Advantage of the Arrears being firſt diſcharged, which alſo agrees with the Chancery Method in this particular.

Laſtly, another way of ſtating this Account, may be taken from that Notion of Simple-Intereſt, whereby the Annuity only is charged as Principal Money: and then 'twill be, for the Arrear, $prt + prn = \alpha n$, and for the Principal and accruing Intereſt, $\overline{\tfrac{1}{2} xr + 1} \times \alpha n = p$; Whence $\frac{\sqrt{8p\alpha r + f^2} - g}{4\alpha r} \left(= \frac{prt}{2\alpha} + \frac{\sqrt{8p\alpha r + f^2} - f}{4\alpha r}\right) = N = 17,3072$ Years; which appears to be the ſame with the Chancery Method, only that the Compound Intereſt in that, is turned into Simple in this; and as it ſtill retains part of the ſame inequality, to the Advantage of the Borrower, it ſeems only fit to be obſerved in ſuch Caſes wherein the Borrower may be thought to merit favour, as when the Debt is paid out of the Profits of Trade, ariſing by extraordinary Riſque or Induſtry.

The following Specimen ſhews at ſight, how the Reſults of the ſeveral Methods differ, as the Rent, Arrear, or Rate of Intereſt, is greater or leſs, and conſequently of how much more or leſs concern it is to the Parties, as well as to the due Adminiſtration of Juſtice, to have regard thereto.

The time requir'd in	Years (computed Half-yearly)			
To diſcharge a Mort. of 1000 *l.* by a Rent of	120*l. per annum.*		90 *l. per annum.*	
At the Rate of Int. *per annum* of	5 *per* C	6 *per* C	5 *per* C.	6 *per* C.
No Arrear of Intereſt.				
By the Chancery Method	10,9141	11,7249	16,4205	18,5835
By the ſame turning the Comp. Int. into Sim. BySimp.Int.cor. Princ. and contin^d^ at Int. theOriginalPro- theArrear firſt diſcharg'd	11,2579	12,3072	17,5335	20,7400
fits charged as Intereſt	14,2857	16,6666	25,0000	33,3333
5 Years Arrear of Intereſt.				
By the Chancery Method	14,4855	16,7249	22,6705	28,5835
By the ſame turning the Comp. Int. into Sim.	14,8293	17,3072	23,7835	30,7400
BySimp.Int.cor. Princ. and continued at Int.	15,3191	18,1648	24,7148	32,7064
theOriginalPro- the Arrear firſt diſcharg'd	15,5897	18,7653	25,4899	34,8052
fits charged as Intereſt	17,8571	21,6666	31,2500	43,3333
10 Years Arrear of Intereſt.				
By the Chancery Method	18,0569	21,7249	28,9205	38,5835
By the ſame turning the Comp. Int. into Sim.	18,4007	22,3072	30,0335	40,7400
BySimp.Int.cor. Princ. and continued at Int.	20,3161	25,5998	33,6320	48,0874
theOriginalPro- theArrear firſt diſcharg'd	21,2895	27,5587	36,1932	53,8107
fits charged as Intereſt	21,4286	26,6666	37,5000	53,3333
15 Years Arrear of Int.				
By the Chancery Method	21,6283	26,7249	35,1705	48,5835
By the ſame turning the Comp. Int. into Sim.	21,9721	27,3072	36,2835	50,7400
BySimp.Int.cor. Princ.and continued at Int	26,1353	34,2913	43,9764	65,8386
theOriginalPro- theArrear firſt diſcharg'd	28,0209	37,7141	48,5159	74,5664
fits charged as Intereſt	25,0000	31,6666	43,7500	63,3333

I muſt alſo obſerve for the ſake of ſuch as are unacquainted with ſpecious Arithmetick, that though for brevities ſake, the foregoing Theorems and Examples are laid down, and wrought in Algebraic Terms: Yet the ſame Accounts may be ſtated after the Chancery manner it ſelf, according to the ſeveral Principles before deliver'd, and with the ſame Reſults, with this only caution, that (inſtead of a continual deduction of the Rent or Annuity out of the Principal and Intereſt of the Debt, which occaſions the Error before mention'd) the preceding General Rule of ſtating ſeparate Amounts be obſerv'd, which may be done by continually adding the profits together on each ſide, in the ſame manner, as if the Parties were to make a ſeparate Charge againſt each other, which is the rather to be noted, as being the only Courſe that can be taken, in caſe the Sums or Times of payment ſhould differ, but the reſpective Reſults will notwithſtanding be analogous to the above Examples.

Logarithmotechnia Generalis. by Mr. Craig. *n.* 328. *p.* 191.

XXVIII. Ad Logarithmorum inventum perficiendum, hoc tantum inveniendum ſupereſſe videtur; ut ſcil. omnes Series Logarithmicas inveniendi Methodum habeamus generalem; talis autem eſt hæc quæ ſequitur, facilis quidem illa & genuina, utpote ex ipſa Logarithmorum Natura deducta.

Per literam *l* numero cuilibet præfixam denotetur (ut vulgo ſolet) iſtius Numeri Logarithmus. Jam quoniam Numeri cujuſvis propoſiti Loga-

rithmus duobus modis inveſtigari poteſt, ideo Logarithmotechniæ hujus duas partes conſtituemus: In priori Logarithmum immediate ex ipſo numero deducimus; in poſteriori vero Numerorum aliquot antecedentium Logarithmi adhibentur, ut ex iis propoſiti Numeri Logarithmus inveniatur.

Pars Prior. Sit $a+1$ numerus quilibet propoſitus, & x ejus Logarithmus inveniendus. Jam ex hypotheſi $x = l.\,\overline{a+1}$, quæ æquatio vocetur Canon generalis. (1.) Fiat æquatio inter terminos ex a & y utcunque compoſitos, & cum aliis quibuſvis numeris quovis modo per Additionem, Subſtractionem Multiplicationem, Diviſionem aut Radicum extractionem combinatos. (2.) Ope æquationis ſic ad libitum aſſumptæ exterminetur a ex Canone generali, & habebitur æquatio exprimens relationem inter indeterminatos x, y. (3.) Hujus æquationis (per regulam *Bernoullianam*) inveniatur Differentialis, & hujus Integralis (per methodos notiſſimas) per Seriem Infinitam expreſſa dabit Logarithmi quæſiti x valorem cognitum.

Exemplum 1. Aſſumatur $a = y$, unde per Canonem generalem $x = l.\,\overline{1+y}$, cujus differentialis eſt $\dot{x} = \frac{\dot{y}}{1+y}$, & hujus integralis per Seriem infinitam expreſſa dat

$$x = y - \tfrac{1}{2}y^2 + \tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 + \tfrac{1}{5}y^5 - \tfrac{1}{6}y^6 + \tfrac{1}{7}y^7 \text{ \&c.}$$

Exemplum 2. Aſſumatur $y = \frac{a}{a+2}$, unde $a + 1 = \frac{1+y}{1-y}$, ideoque per Canonem generalem $x = l.\,\frac{1+y}{1-y}$, cujus Differentialis eſt $\dot{x} = \frac{2\dot{y}}{1-yy}$; & hujus Integralis in Seriem reſoluta dat.

$$x = 2 \times y + \tfrac{1}{5}y^3 + \tfrac{1}{5}y^5 + \tfrac{1}{7}y^7 + \tfrac{1}{9}y^9 \text{ \&c.}$$

Ubi numerus 2 Seriei præfixus multiplicari ſupponitur in ſingulos Seriei terminos. Nec plura addere exempla opus hic erit, cum ex his pateat Methodus inveniendi innumeras Series Logarithmicas, quæ, abſque ullo ad aliorum numerorum Logarithmos reſpectu, exhibent numeri propoſiti Logarithmum. *Q.E.I.*

Lemma 1. Sit z Logarithmus cujuſvis fractionis $\frac{b}{a+1}$, x Logarithmus denominatoris $a+1$; erit $lb - z = x$: Vel ſi ſit z Logarithmus fractionis $\frac{a+1}{b}$, erit $lb + z = x$.

Lemma 2. Sit e exponens cujuſvis poteſtatis numeri b, erit $l.\,b^e = e \times l.\,b$; ideoque datis Logarithmo numeri b^e & exponente e, datur ipſius b Logarithmus: Et ex Natura Logarithmorum conſtat utrumque Lemma.

Pars Poſterior. Sit (ut prius) $a+1$ Numerus cujus Logarithmus x eſt inveniendus, ſitque b Numerus productus ex Multiplicatione Numerorum, quorum maximus eſt minor quam $a+1$; & z Logarithmus fractionis $\frac{b}{a+1}$, id eſt $z = l.\,\frac{b}{a+1}$, quæ æquatio vocetur Canon generalis. Tum

(1.)

(1.) pro b ſumatur quantitas ex a & numeris quibuſvis determinatis utcunque compoſita, & hic valor numeri b ſic ad libitum ſumptus ſubſtituatur in fractione $\frac{b}{a+1}$, unde illa per a & numeros datos exprimetur. (2.) Fiat quælibet æquatio inter y & a cum numeris ad libitum ſumendis; & ope hujus exterminetur a ex Canone generali, unde habetur æquatio exprimens relationem inter indeterminatos z, y. (3.) Hujus æquationis inveniatur (per Regulam *Bernoullianam*) Differentialis, hujuſque Integralis (juxta Methodos notiſſimas) per Seriem infinitam expreſſa dabit fractionis $\frac{b}{a+1}$ Logarithmum z; & ex invento z habebitur (per *Lem.* 1.) numeri propoſiti $a+1$ Logarithmus $x = l.\, b - z$. Nam ex hypotheſi, b' producitur ex Multiplicatione Numerorum quorum maximus eſt minor quam $a+1$; & ex hypotheſi dantur Logarithmi omnium numerorum propoſito $a+1$ minorum, ergo & Logarithmus Numeri ex omnibus producti ſeu b', & proinde (per *Lem.* 2.) ipſius b Logarithmus datur.

Exemplum 1. Sumatur ſi placet $b = a$, unde $z = l.\, \frac{a}{a+1}$: Dein (per art. 2.) fiat ad libitum $y = 2a+1$, per hanc exterminetur a, & erit $z = l.\, \frac{y-1}{y+1}$, cujus Differentialis eſt $\dot{z} = \frac{2\dot{y}}{yy-1}$; cujus Integralis per Seriem expreſſa dat $z = -2 \times \frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \frac{1}{7y^7}$ &c. Unde per *Lemma* 1. $x = lb + 2 \times \frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + \frac{1}{7y^7} + \frac{1}{9y^9}$ &c.

Exemplum 2. Fiat $b = \sqrt{aa+2a}$, unde $z = l.\, \frac{\sqrt{aa+2a}}{a+1}$, ſumatur etiam ad libitum $y = 2a + 2a$, unde $z = l.\, \frac{1}{y} \sqrt{yy-4}$, cujus Differentialis eſt $\dot{z} = 4\dot{y} \times \overline{y^3-4y}|^{-1}$, & hujus Integralis eſt $z = -2 \times \frac{1}{y^2} + \frac{2^2}{2y^4} + \frac{2^4}{3y^6} + \frac{2^6}{4y}$ &c. Unde *Lemma* 1.

$x = l.\, b + 2 \times \frac{1}{y^2} + \frac{2^2}{2y^4} + \frac{2^4}{3y^6} + \frac{2^6}{4y^8} + \frac{2^8}{5y^{10}}$ &c.

Exemplum 3. Fiat $b = \sqrt{aa+2a}$, ut in præcedenti, ſed jam aſſumatur $y^2 = 2aa \quad 4aa + 1$; Si per has duas æquationes exterminentur b & a ex Canone generali, erit $z = l\, \frac{\sqrt{yy-1}}{\sqrt{yy+1}}$, cujus Differentialis eſt $\dot{z} = 2y\dot{y} \times \overline{y^4-1}|^{-1}$; & hujus Integralis per Seriem expreſſa eſt $z = -\frac{1}{y^2} - \frac{1}{3y^6} - \frac{1}{5y^{10}} - \frac{1}{7y^{14}}$ &c. Unde per *Lem.* 1.

$$x = l.\, b + \frac{1}{y^2} + \frac{1}{3y^6} + \frac{1}{5y^{10}} + \frac{1}{7y^{14}} + \frac{1}{9y^{18}} \text{ \&c.}$$

Notandum vero eſt quod numerus 2 Seriebus *Exemp.* 1 & 2. præfixus multiplicari ſupponitur in ſingulos ſerierum terminos. Simileſque Series deduci poſſunt eodem modo ex $z = l\,\frac{a+1}{b}$, atqui tum $x = l,\ b+z$, ut conſtat ex *Lem.* 1. *Part.* 2. Ex his igitur ſatis ſuperque conſtat Logarithmotechniam jam expoſitam eſſe facillimam & maxime genuinam, necnon adeo generalem, ut duobus modis innumeræ Series inveniri poſſunt, Numeri cujuſvis propoſiti Logarithmum exhibentes: Nam innumeras (ad libitum) aſſumere licet æquationes relationem inter y & a exprimentes, quarum unaquæque exhibet novam Seriem Logarithmicam. Summa tamen adhibenda eſt Cura, ut tales aſſumantur, quæ efficient ut Serierum termini quam celerrime convergant, *i. e.* ut Logarithmus quam minimo Calculi labore inveniatur. Ad hoc præſtandum, perquam apta eſt Series in exemplo poſtremo exhibita, & quæ eadem eſt cum illa quam exhibuit Celeberrimus *Halleius* * in eleganti ſua Logarithmos conſtruendi Methodo.

* *Abr.* Vol. I. p. 108.

Logometria, ſive de menſura Rationis. by Mr. Cotes. *n.* 338. *p.* 5.

XXIX. Agitur in hoc Tractatu de *Menſuris Rationum.* Hæ Menſuræ ſunt quantitates cujuſcunque generis, quarum magnitudines magnitudinibus rationum ſunt analogæ. In dato itaque Syſtemate, rationis ejuſdem eadem eſt menſura, duplicatæ dupla, triplicatæ tripla, ſubduplicatæ ſubdupla, ſeſquiplicatæ ſeſquialtera: denique quocunque modo per compoſitionem vel reſolutionem auctæ vel diminutæ rationis, ſimiliter aucta eſt vel diminuta menſura. Æqualitatis ratio nullam habet magnitudinem, quia nullam addita vel detracta mutationem inducit; rationes quæ dicuntur majoris & minoris inæqualitatis contrarias habent magnitudinum ſuarum affectiones, quoniam in compoſitione & reſolutione contraria ſemper efficiunt: itaque ſi menſura rationis quam habet terminus major ad minorem poſitiva cenſeatur, menſura rationis quam habet terminus minor ad majorem erit negativa, menſura vero rationis inter æquales terminos nullius erit magnitudinis. Porro diverſa menſurarum oriuntur *Syſtemata*, prout modis diverſis exponitur analogia illa determinata & immutabilis quæ eſt inter magnitudines rationum. Inde vero patet, exhiberi poſſe numero infinita Syſtemata, minuendo vel augendo Syſtematis cujuſvis dati menſuras omnes in eadem data quacunque proportione, aut etiam pro menſuris adhibendo quantitates diverſi generis. In tanta autem varietate confuſionem aliquam oboriri neceſſe eſt, ni probe conſtiterit ad quodnam Syſtema referendæ ſint menſuræ ſingulæ de quibus contingat ſermonem inſtitui. Huic malo remedium optime parari poteſt ſi menſura datæ alicujus rationis, quæ commodiſſima videbitur, pro *Modulo* habeatur ad quem conſtanter in omni Syſtemate menſuræ reliquarum rationum exigantur. Id enim ſi fiat, ſtatim ex dato illo Modulo determinabitur Syſtema totum: nam ex menſuris conſtabit quæ Modulo erunt homogeneæ, quæque eo majores habebunt magnitudines vel minores quo major ille fuerit vel minor, ut ita menſurandarum rationum invariata magnitudinum ſervetur analogia inter ipſas menſuras. Patebit igitur in ſequentibus rationem quandam dari, dupli inter & tripli rationes intermediam, ad rationem vero tripli aliquanto propius accedentem, quæ propoſito noſtro non immerito aptiſſima judicetur, ſiquidem ipſa rei natura hujus uſum ſuadere ac non incertis indiciis efflagitare quodammodo videatur. Hanc ego, ex officio ejus deſumpto nomine, *Modularem Rationem* appellabo; quo autem pacto ipſa ſit accuratius definienda, oſtendetur inferius, nunc enim de Logarithmis pauca ſunt addenda.

Logarithmi ſunt rationum menſuræ Numerales: ſolent autem in Canone ſic diſponi, ut ſingulis numeris naturali ordine creſcentibus, & in ſerie continua poſitis adſcribatur Logarithmus, non quidem ipſius numeri uti vulgo dicitur, ſed rationis quam habet numerus ad Unitatem. Exinde vero rationis per quoſcunque terminos deſignatæ facilis eſt inventio Logarithmi. Nam cum ratio antecedentis ad conſequentem ſit exceſſus rationis antecedentis ad Unitatem ſupra rationem conſequentis ad Unitatem: Logarithmus ejus ſimiliter erit exceſſus Logarithmi rationis quam habet antecedens ad Unitatem ſupra Logarithmum rationis quam conſequens habet ad Unitatem; hoc eſt, ut vulgari ſermone utamur, exceſſus Logarithmi antecedentis ſupra Logarithmum conſequentis; neutiquam enim diſplicet loquendi modus jam a multis annis receptus, ſi recte intelligatur. Exinde porro peregregium enaſcitur compendium ad operationes Arithmeticas. Datis enim duobus quibuſcunque numeris in ſe multiplicandis, ſi quæratur numerus ex multiplicatione productus; quoniam rationes numerorum datorum ad Unitatem, conficiunt ſimul additæ rationem producti ad Unitatem, & rationum componendarum menſuræ ſimul additæ conficiunt rationis compoſitæ menſuram: Logarithmus producti æquabitur Logarithmis numerorum datorum ſimul ſumptis. Ad eundem modum ſi quæratur numerus ex diviſione ortus; quoniam ratio diviſoris ad Unitatem e ratione dividendi ad Unitatem detracta relinquit rationem quoti ad Unitatem: habebitur quoti Logarithmus ſubducendo Logarithmum diviſoris e Logarithmo dividendi. Et eodem argumento, ſi quæratur dati cujuſvis numeri quælibet poteſtas; quoniam ratio dati numeri ad Unitatem per Indicem poteſtatis multiplicata rationem efficit quam habet numeri poteſtas ad Unitatem, & menſura prioris rationis multiplicata per eundem Indicem efficit pariter menſuram rationis poſterioris: Logarithmus poteſtatis æquabitur Logarithmo numeri dati per Indicem poteſtatis multiplicato. Et ſimiliter Logarithmus cujuſlibet radicis numeri dati æquabitur Logarithmo numeri dati per Indicem radicis diviſo. Igitur ope Canonis peragetur inventio poteſtatum & radicum per multiplicationem & diviſionem, multiplicatio autem & diviſio per additionem & ſubductionem. Cæterum de hiſce vulgo notis Logarithmorum uſibus non eſt mei inſtituti fuſius diſſerere: miſſis ergo ambagibus, ad alia nunc me confero, & rem ipſam protinus aggredior.

Propoſitio I. *Invenire Menſuram Rationis cujuſcunque propoſitæ.*

Proponatur Ratio inter AC & AB, cujus Menſuram oportet invenire. Terminorum differentia BC diviſa concipiatur in particulas innumeras quam minimas PQ, atque ratio inter AC & AB in totidem rationes quam minimas inter AQ & AP: & ſi detur magnitudo rationis inter AQ & AP, dividendo dabitur ratio quam habet PQ ad AP; atque adeo data illa magnitudo rationis inter AQ & AP, per datam quantitatem $\frac{PQ}{AP}$ exponi poteſt. Manente AP, augeri vel minui intelligatur particula PQ in proportione quavis; & in eadem proportione augebitur vel minuetur magnitudo rationis inter AQ & AP: capiatur particula dupla vel tripla, ſubdupla vel ſubtripla, & evadet ratio duplicata vel triplicata, ſubduplicata vel ſubtriplicata; etiamnum igitur exponetur per quantitatem $\frac{PQ}{AP}$. Sed &, aſſumpta determinata quavis quantitate M, exponi poteſt per $M \times \frac{PQ}{AP}$: erit ergo quantitas $M \times \frac{PQ}{AP}$ menſura rationis inter AQ & AP. Hæc vero menſura diverſam habebit magnitudinem, & ad Syſtema diverſum accommodabitur, pro diverſa magnitudine quantitatis aſſumptæ M, quæ adeo vocetur Syſtematis *Modulus*. Jam quemadmodum ſumma rationum omnium inter AQ & AP æqualis eſt propoſitæ rationi, quam utique habet AC ad AB: ita ſumma menſurarum omnium $M \times \frac{PQ}{AP}$ (per Methodos ſatis notas invenienda) æqualis erit ejuſdem propoſitæ rationis menſuræ quæſitæ. Q. E. I.

Plate 3. *Fig.* 3.

Corol. 1. Terminis AP, AQ ita ad æqualitatem accedentibus, ut quam minima ſit eorundem differentia PQ: erit $M \times \frac{PQ}{AP}$ vel $M \times \frac{PQ}{AQ}$ æqualis menſuræ rationis inter AQ & AP ad Modulum M.

Corol. 2. Unde Modulus ille M eſt ad menſuram rationis inter terminos AQ & AP, ut terminorum alteruter AP vel AQ ad terminorum differentiam PQ.

Corol. 3. Data ratione inter AC & AB, datur ſumma omnium $\frac{PQ}{AP}$, & ſumma omnium $M \times \frac{PQ}{AP}$ eſt ut M. Itaque menſura datæ cujuſcunque rationis eſt ut Modulus Syſtematis ex quo deſumitur.

Corol. 4. Modulus ergo, in omni menſurarum Syſtemate, ſemper æqualis fit menſuræ rationis cujuſdam determinatæ atque immutabilis: Quam proinde *Rationem Modularem* vocabo.

Scholium 1. Problematis ſolutio per Exemplum illuſtrabitur. Sit z quantitas determinata quævis & permanens, ſit vero x quantitas indeterminata fluxuque perpetuo variabilis, ejuſque fluxio ſit $\dot{x}$; & quæratur menſura rationis inter $z + x$ & $z - x$. Statuatur hæc ratio æqualis rationi inter y & 1, exponatur autem numerus y per AP, fluxio ejus $\dot{y}$ per PQ, 1 per AB: & ex Corollario primo colligetur fluxionem quæſitæ menſuræ rationis inter y & 1 eſſe $M \times \frac{\dot{y}}{y}$. Reponatur jam pro y valor ejus $\frac{z+x}{z-x}$, itemque pro $\dot{y}$ valoris fluxio $\frac{2z\dot{x}}{\overline{z-x}|^{2}}$: & fluxio menſuræ evadet $2M \times \frac{z\dot{x}}{zz-xx}$ vel $2M \times \frac{\dot{x}}{z-\frac{xx}{z}}$ ſive $2M$ in $\frac{\dot{x}}{z} + \frac{\dot{x}x^{2}}{z^{3}} + \frac{\dot{x}x^{4}}{z^{5}} +$ *&c.* Atque adeo menſura illa fiet $2M$ in $\frac{x}{z} + \frac{x^{3}}{3z^{3}} + \frac{x^{5}}{5z^{5}} +$ *&c.* Unde patet Corollarium ſequens.

Corol. 5. Si duarum quantitatum ſumma ſit z & differentia ſit x; & ſumatur $2M\frac{x}{z} = A$, $A\frac{xx}{zz} = B$, $B\frac{xx}{zz} = C$, $C\frac{xx}{zz} = D$, *&c.* Menſura rationis quam habet quantitas major ad quantitatem minorem, erit $A + \frac{1}{3}B + \frac{1}{5}C + \frac{1}{7}D +$ *&c.*

Scholium 2. Non abſimili computo menſura rationis inter $1+v$ & 1 erit M in $v - \frac{1}{2}v^{2} + \frac{1}{3}v^{3} - \frac{1}{4}v^{4} + \frac{1}{5}v^{5} -$ *&c.* Unde ſi menſura illa vocetur m, erit $\frac{m}{M} = v - \frac{1}{2}vv + \frac{1}{3}v^{3} - \frac{1}{4}v^{4} + \frac{1}{5}v^{5}$, *&c.* ac proinde $\frac{mm}{MM} = vv - v^{3} + \frac{11}{12}v^{4} - \frac{5}{6}v^{5}$, *&c.* ſimiliterque $\frac{m^{3}}{M^{3}} = v^{3} - \frac{3}{2}v^{4} + \frac{7}{4}v^{5}$, *&c.* quinetiam $\frac{m^{4}}{M^{4}} = v^{4} - 2v^{5}$, *&c.* ac denique $\frac{m^{5}}{M^{5}} = v^{5}$, *&c.* Ut igitur viciſſim, ex data menſura m, inveniatur ratio quam metitur; addendo æqualia æqualibus habebitur $\frac{m}{M} + \frac{mm}{2MM} = v * - \frac{1}{6}v^{3} + \frac{5}{24}v^{4} - \frac{13}{60}v^{5}$, *&c.* atque iterum $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^{3}}{6M^{3}} = v * * - \frac{1}{24}v^{4} + \frac{3}{40}v^{5}$, *&c.* rurſumque $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^{3}}{6M^{3}} + \frac{m^{4}}{24M^{4}} = v * * * - \frac{1}{120}v^{5}$, *&c.* atque tandem $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^{3}}{6M^{3}} + \frac{m^{4}}{24M^{4}} + \frac{m^{5}}{120M^{5}} = v * * * *$, *&c.* id eſt, $\frac{m}{M} + \frac{mm}{2MM} + \frac{m^{3}}{6M^{3}} + \frac{m^{4}}{24M^{4}} + \frac{m^{5}}{120M^{5}} +$ *&c.* $= v$. Itaque ratio quæſita inter $1+v$ & 1, eſt ea quam habet $1 + \frac{m}{M} + \frac{mm}{2MM} + \frac{m^{3}}{6M^{3}} + \frac{m^{4}}{24M^{4}} + \frac{m^{5}}{120M^{5}} +$ *&c.* ad 1. Ponatur $m = M$, ſive $\frac{m}{M} = 1$; & exinde ratio Modularis erit ea quam habet $1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} +$ *&c.* ad 1.

Eodem modo, ſi detur ratio inter 1 & $1-v$, menſura hujus rationis erit M in $v + \frac{1}{2}v^{2} + \frac{1}{3}v^{3} + \frac{1}{4}v^{4} + \frac{1}{5}v^{5}$, *&c.* Et viciſſim ſi detur rationis menſura m, ratio erit ea quam habet 1 ad $1 - \frac{m}{M} + \frac{mm}{2MM} - \frac{m^{3}}{6M^{3}} + \frac{m^{4}}{24M^{4}} - \frac{m^{5}}{120M^{5}} +$ *&c.* Ponatur $m = M$, ſive $\frac{m}{M} = 1$; & exinde

Ratio Modularis erit ea quam habet 1 ad $1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} +$ &c. Ex hiſce vero patet Corollarium ſequens.

Corol. 6. Expoſito termino R, ſi ſumatur $\frac{1}{1}R = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $S = R + A + B + C + D + E +$ &c: Ratio Modularis erit ea quæ eſt inter terminum minorem expoſitum R & majorem inventum S. Vel expoſito termino S, ſi ſumatur $\frac{1}{1}S = A$, $\frac{1}{2}A = B$, $\frac{1}{3}B = C$, $\frac{1}{4}C = D$, $\frac{1}{5}D = E$, &c. in infinitum; & capiatur $R = S - A + B - C + D - E +$ &c: Ratio Modularis erit ea quæ eſt inter terminum majorem expoſitum S & minorem inventum R. Porro eadem ratio eſt inter 2,718281828459 &c. & 1, vel inter 1 & 0,367879441171 &c.

Scholium 3. Si forte termini minores deſiderentur, qui eandem proxime Rationem Modularem ita exhibeant, ut nulli ipſis non majores propius: inſtituenda erit operatio ad modum ſequentem. Dividatur terminus major 2,71828 &c. per minorem 1, vel etiam major 1 per minorem 0,367879 &c. & rurſus minor per numerum qui reliquus eſt, & hic rurſus per ultimum reſiduum, atque ita porro pergatur: & prodibunt quotientes 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1, 16, 1, 1, &c. His inventis, perficiendæ ſunt binæ rationum columnæ, quarum altera terminos continet rationem habentes vera majorem, altera terminos quorum ratio eſt vera minor; ineundo computationem a rationibus 1 ad 0, 0 ad 1, quæ remotiſſimæ ſunt a vera; inde autem exorſam deducendo ad rationes reliquas, quæ continuæ ad veram propius accedunt. Multiplicentur itaque termini 1 & 0 per quotientem primum 2, & ſcribantur facti 2 & 0 infra

Rationes veræ Majores.		*Rationes veræ Minores.*	
1	0 × 2	0	1
2	1	2	0
3	1 × 2	2	1 × 1
8	3	6	2
11	4 × 1	8	3 × 1
76	28	11	4
87	32 × 1	19	7 × 4
106	39	87	32
193	71 × 6	106	39 × 1
1264	465	1158	426
1457	536 × 1	1264	465 × 1
21768	8008	1457	536
23225	8544 × 1	2721	1001 × 8
25946	9545	23225	8544
49171	18089 x 10	25946	9545 × 1
&c.	&c.	&c.	&c.

terminos 0 & 1; & addendo prodibit ratio 2 + 0 ad 0 + 1, ſive 2 ad 1. Hujus termini multiplicentur per quotientem ſecundum 1, factique 2 & 1 addantur terminis 1 & 0; & habebitur ratio 2 + 1 ad 1 + 0, ſive 3 ad 1. Hujus termini multiplicentur per quotientem tertium 2, factique 6 & 2 addantur terminis præcedentibus 2 & 1; & habebitur ratio 8 ad 3. Hujus termini multiplicentur per quotientem quartum 1, factique 8 & 3 addantur terminis præcedentibus 3 & 1; & habebitur ratio 11 ad 4. Hujus termini multiplicentur per quotientem quintum 1, factique 11 & 4 addantur præcedentibus 8 & 3; & habebitur ratio 19 ad 7. Hujus termini rurſus multiplicentur per quotientem ſextum 4, factique 76 & 28 addantur præcedentibus 11 & 4, ad inveniendam rationem 87 ad 32; & ſic porro pergendum quouſque libuerit, tranſitu alternis facto in alteram columnam. Hiſce peractis, habebuntur rationes vera majores 3 ad 1, 11 ad 4, 87 ad 32, 193 ad 71, 1457 ad 536, 23225 ad 8544, 49171 ad 18089, &c. Vera autem minores erunt 2 ad 1, 8 ad 3, 19 ad 7, 106 ad 39, 1264 ad 465, 2721 ad 1001, 25946 ad 9545, &c. Atque hæ quidem ſunt præcipuæ & primariæ rationes, quibus ad rationem propoſitam continue appropinquatur.

Quod ſi exquiratur integra ſeries rationum omnium vera majorum quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera major ad veram propius accedat; & ſimiliter ſeries integra rationum omnium vera minorem quæ ita dari poſſint, ut nulla minoribus terminis deſignata ratio vera minor ad veram propius accedat, inter primarias illas modo inventas inſerendæ ſunt aliæ ſecundariæ rationes. Hæ vero locum habent ubi quotiens unitatem ſuperat. Inveniuntur autem mutata multiplicatione, quæ ſupra per quotientem facta eſt, in continuam additionem terminorum tot vicibus quot ſunt unitates in quotiente. Sic quia quotiens primus erat 2, termini 1 & 0 bis addendi ſunt terminis 0 & 1; & ſummæ dabunt rationes 1 ad 1, 2 ad 1. Hi ultimi termini 2 & 1, quia quotiens ſecundus erat 1, ſemel addendi ſunt terminis 1 & 0; & ſummæ dabunt rationem 3 ad 1. Hi termini 3 & 1, quia quotiens tertius erat 2, bis addendi ſunt terminis 2 & 1; & ſummæ dabunt rationes 5 ad 2, 8 ad 3. Hi ultimi termini 8 & 3, quia quotiens quartus erat 1, ſemel addendi ſunt terminis 3 & 1; & ſummæ dabunt rationem 11 ad 4. Hi termini 11 & 4, quia quotiens quintus erat 1, ſemel addendi ſunt terminis 8 & 3; & ſummæ dabunt rationem 19 ad 7. Hi denique termini 19 & 7, quia quotiens ſextus erat 4, quater addendi ſunt terminis 11 & 4; & ſummæ dabunt rationes 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32. Et ſic porro procedere licebit quouſque commodum videbitur. Iſta tandem operatione peracta, ſeries integra rationum omnium vera majorum, erit 1 ad 0, 3 ad 1, 11 ad 4, 30 ad 11, 49 ad 18, 68 ad 25, 87 ad 32, *&c.* ſimiliterque ſeries integra rationum omnium vera minorum, erit 0 ad 1, 1 ad 1, 2 ad 1, 5 ad 2, 8 ad 3, 19 ad 7, *&c.*

Rationes Vera Majores.		*Rationes Vera Minores.*	
1	0 × 2	0	1
2	1	1	0
3	1 × 2	1	1
8	3	1	0
11	4 × 1	2	1 × 1
19	7	3	1
30	11	5	2
19	7	3	1
49	18	8	3 × 1
19	7	11	4
68	25	19	7 × 4
19	7	87	32
87	32 × 1	106	39 × 1.
&c.	&c.	&c.	&c.

Harum approximationum utilitas ad alia multa ſeſe diffundit: quapropter earum inventionem aliquanto prolixius expoſitam dedi, per Methodum quæ mihi ſimpliciſſima & facillima videtur. Idem argumentum paulo aliter pertractarunt Viri celeberrimi *Walliſius* & *Hugenius*.

Propoſitio II. *Logarithmorum Canonem* Briggianum *conſtruere.*

Numerorum Compoſitorum Logarithmi derivantur ex Logarithmis Primorum componentium, per additionem ſolam; horum autem inveſtigatio pluribus modis inſtitui poteſt: Exemplum unicum appono.

Per Corollarium quintum Propoſitionis ſuperioris, ſcribendo 1 pro *M*, inveniantur Logarithmi rationum inter 126 & 125, 225, & 224, 2401 & 2400, 4375 & 4374; qui vocentur reſpective p, q, r, s: & Logarithmus denarii ſeu rationis decupli erit $239p + 90q - 63r + 103s$, ſive 2,302585092994 &c. Itaque cum Logarithmus *Briggianus* denarii ſit 1; fiat (per *Coral.* 3. *Prop.* 1.) ut denarii Logarithmus modo inventus 2,302585092994 &c. ad Modulum ſuum 1, ita denarii Logarithmus *Briggianus* 1, ad Modulum *Briggianum*; qui adeo erit 0,434294481903 &c. Ponatur ergo deinceps iſte valor pro *M*, & erunt $M \times \overline{202p + 76q - 53r + 87s}$, $M \times \overline{167p + 63q - 44r + 72s}$, $M \times \overline{114p + 43q - 30r + 49s}$ Logarithmi *Briggiani* numerorum 7, 5, 3. Logarithmus numeri 2 habetur, ſubducendo Logarithmum numeri 5 a Logarithmo numeri 10. Atqua ita dantur & Modulus *Briggianus* & Logarithmi Primorum omnium qui ſunt minores denario.

Logarithmi numerorum ſequentium Primorum 11, 13, 17, 19, 23, &c. ita computari poſſunt. Quæratur tum factus a numeris Primo propoſito utrinque proxime adjacentibus, tum Primi ipſius quadratum, quod ſemper unitate factum illud ſuperabit. Logarithmo rationis quadrati ad factum per *Corol.* 5. *Prop.* 1. inveniendo) addatur ipſius facti Logarithmus, qui ſemper componetur ex datis Logarithmis Primorum qui propoſito Primo ſunt minores: & ſemiſumma erit Logarithmus Primi quæſitus.

Corol. Canonis *Briggiani* Modulus eſt 0,434294481903 *&c:* Hujus vero Reciprocus eſt 2,302585092994 *&c.*

Scholium. Ad hunc itaque modum perfici poſſet Logarithmorum Tabula ampliſſima, qualis edita eſt a *Briggio* vel *Vlacco.* Inventioni autem Numerorum & Logarithmorum ſibi invicem congruentium, qui intermedii ſunt & ultra Tabulæ limites excurrunt, abunde ſufficiet terminus primus Seriei quæ in Corollario quinto Propoſitionis præcedentis exhibetur.

Si dato Numero intermedio quæratur ejus Logarithmus; pone a & e pro Numero intermedio propoſito atque huic proximo tabulari, ita ut a deſignet majorem, e minorem; ſit eorum ſumma z, differentia x; pone λ pro Logarithmo rationis quam habet a ad e, hoc eſt, pro exceſſu Logarithmi Numeri a ſupra Logarithmum Numeri e: & erit $\lambda = 2M\frac{x}{z}$ quamproxime.

Si quæratur Numerus qui congruit Logarithmo intermedio; quoniam eſt $\lambda = \frac{2Mx}{z} = \frac{2Mx}{2a - x}$ vel $\frac{2Mx}{2e + x}$; erit $x = \frac{\lambda}{M + \frac{1}{2}\lambda}a$ vel $\frac{\lambda}{M - \frac{1}{2}\lambda}e$ quamproxime.

Propoſitio III. *Syſtematis cujuſvis Logometrici conſtructionem exponere per Canonem Logarithmorum.*

Caſ. 1. Si detur, e Syſtemate propoſito, menſura rationis alicujus determinatæ: rationis cujuſvis oblatæ menſura, erit ad menſuram illam datam determinatæ rationis, ut oblatæ rationis Logarithmus, ad Logarithmum rationis ejuſdem determinatæ.

Caſ. 2. Si non detur, e Syſtemate propoſito, menſura rationis alicujus determinatæ: inveniendus erit Modulus propoſiti Syſtematis, per Corollarium ſecundum Propoſitionis primæ. Et menſura cujuſvis oblatæ rationis, erit ad Modulum inventum, ut oblatæ rationis Logarithmus, ad Canonis Modulum.

Caſus hujus ultimi habentur Exempla in ſequentibus.

Propoſitio IV. *Spatium quodvis Hyperbolicum quadrare per Canonem Logarithmorum.*

Sit Hyperbola quævis $ERSF$ centro A, Aſymptotis ABC, AD, deſcripta; & quæratur area $BEFC$ quam claudunt rectæ BE, CF, ad Aſymptoton AD parallelæ. Compleatur parallelogrammum $ABED$, & ad hunc Modulum inveniatur (per Propoſitionem tertiam) menſura rationis inter AC & AB vel inter BE & CF: Dico menſuram inventam æqualem fore magnitudini areæ quæſitæ $BEFC$. Nam diviſa concipiatur hujus areæ baſis BC in particulas innumeras quam minimas PQ, ea lege, ut ubique detur ratio illa quæ eſt inter AQ & AP, & ducantur Aſymptoto AD parallelæ PR, QS. Quoniam itaque eſt AQ ut AP; erit diviſim

Fg. 4.

PQ ut AP, hoc eſt, ut PR reciproce. Unde data eſt area $PRSQ$, quæ proinde poteſt haberi pro menſura rationis datæ quæ eſt inter AQ & AP. Hujus autem menſuræ Modulus erit parallelogrammum $ABED$, per *Corol.* 2. *Prop.* 1. Nam ſi compleatur æquale parallelogrammum $APRT$; ſtatim intelligetur, ita illud ſe habere ad aream $PRSQ$, ut ſe habet AP ad PQ. Similes ergo ſummas arearum atque rationum utrinque colligendo; area tota $BEFC$ erit menſura rationis totius quæ eſt inter AC & AB, vel inter BE & CF, ad eundem Modulum $ABED$.

Fig. 5.

Aliter. Sit rurſus Hyperbola quævis AP, centro C atque Aſymptoto CB deſcripta; & quæratur area Sectoris cujuſlibet CAP, ſemidiametris CA, CP curvæque AP interjecti. Producta ſemidiametro utravis CAQ ultra verticem A, ducatur illius conjugata CR; & ad ipſas CQ, CR ordinatim applicentur a puncto P rectæ PQ, PR, quæ Aſymptoto CB occurrant in Z & X, deinde agatur AB quæ Hyperbolam tangat in A, Aſymptoton ſecet in B, rectamque CP in D; & Triangulo ABC exiſtente Modulo, area quæſita ſectoris CAP erit menſura rationis inter $QZ + QP$ & AB, ſive rationis inter AB & $QZ - QP$, ſive ſubduplicatæ rationis inter $QZ + QP$ & $QZ - QP$, ſive ſubduplicatæ rationis inter $AB + AD$ & $AB - AD$; vel erit menſura rationis inter $RP + RX$ & CA, vel rationis inter CA & $RP - RX$, vel ſubduplicatæ rationis inter $RP + RX$ & $RP - RX$. Nam ſi ducantur rectæ AE, PF quæ ſecent Aſymptoton CB in E & F, alterique Aſymptoto parallelæ ſint: æquales erunt hæ omnes rationes rationi quam habet AE ad PF, vel CF ad CE; erit & ſector CAP areæ $EAPF$ æqualis, ſimiliterque triangulum ABC duplicato triangulo AEC, ſive parallelogrammo Aſymptotis & Hyperbolæ inſcripto æquabitur. Quare patet propoſitum ex ſupra demonſtratis.

Data vero per modum priorem area $BEFC$, vel per modum poſteriorem area CAP; dabitur alia quævis area Hyperbolica ad arcum EF, vel ad arcum AP terminata: quippe quæ ſemper eſt areæ modo inventæ & areæ alicujus rectilineæ vel ſumma vel differentia. *Q. E. I.*

Fig. 6.

Scholium. Hinc facilem habent ſolutionem Problemata omnia, quæcunque pendent ab Hyperbolæ quadratura. Exemplum ſatis luculentum præbebit deſcenſus gravium in Mediis, quorum reſiſtentia eſt in duplicata ratione velocitatis corporis moti. Sit V velocitas maxima quam corpus in hujuſmodi Medio, infinite deſcendendo, poteſt acquirere; T dimidium temporis quo corpus idem in eodem Medio, vi ſola ponderis ſui relativi, abſque reſiſtentia cadendo velocitatem illam acquiret; S ſpatium hocce caſu deſcriptum; R pondus relativum corporis in Medio reſiſtente: & quæratur ſpatium s quod corpus deſcendens, tempore quovis t, deſcribet in Medio reſiſtente; & reſiſtentia r quam patitur in fine illius temporis; & velocitas v ex iſto deſcenſu acquiſita.

Centro D, vertice A deſcribatur Hyperbola æquilatera AT, cujus una Aſymptotorum eſt DC & ad verticem tangens AC ſemiaxi AD æqualis. Capiatur area DAT ad dimidium trianguli DAC ut t ad T, ſecetque DT tangentem AC in P: & erit v ad V ut AP ad AC. Sit AK ipſis AC, AP tertia proportionalis: & erit r ad R ut AK ad AC. Ad tangentem AC erigantur normales CZ, KN, AB; centroque C & Aſymptotis CA, CZ deſcribatur Hyperbola quævis BN: & erit s ad S ut area $ABNK$ ad rectangulum CKN. Patent hæc omnia per Propoſitiones octavam & nonam Libri ſecundi Philoſophiæ *Newtonianæ*.

Eſt itaque t ad T ut area Hyperbolica DAT ad dimidium trianguli DAC, hoc eſt, ut dimidiata menſura rationis inter $AC + AP$ & $AC - AP$ ad illius menſuræ dimidiatum Modulum. Ergo ſi recta quævis EF producatur ad f, ita ut t ſit menſura rationis inter Ef & EF ad Modulum T, & biſecetur Ff in G: erit GF ad GE ut AP ad AC, hoc eſt, ut v ad V. Sumantur GE, GF, GH continue proportionales: & erit GH ad GE ut AK ad AC, hoc eſt, ut r ad R. Erit inſuper EG ad EH ut CA ad CK; unde cum ſit s ad S ut area $ABNK$ ad rectangulum CKN, hoc eſt, ut menſura rationis inter CA & CK vel inter EG & EH ad menſuræ Modulum: erit s menſura rationis inter EG & EH ad Modulum S, atque inde dabitur.

Fig. 7.

Ex hiſce porro facillime ſe prodit, per unicam quamvis Hyperbolam, conſtructio non inconcinna; quam & adſcribere viſum eſt ob dignitatem Problematis. In recta quavis GE ſumatur utcunque punctum F inter E & G, & ab altera parte capiatur Gf ipſi GF æqualis, & ſint GE, GF, GH continue proportionales. Deinde per puncta E, F, H, G, f ducantur ſibi invicem parallelæ rectæ ER, FL, HM, GQ, fl, quas ſecet Hyperbola quævis $LMQl$ centro E, Aſymptotis ER, EG deſcripta, & compleatur parallelogrammum $EGQR$. Jam ſi ſit t ad T ut area Hyperbolica $LFfl$ ad parallelogrammum EQ: erit s ad S ut area $MHGQ$ ad EQ; v ad V ut GF ad GE; r ad R ut GH ad GE.

Libet & caſum alterum adjicere ubi corpus aſcendit; ne forte analogia illa, quæ inter utrumque ſervari debet, in allata conſtructione quodammodo perire videatur. Ergo eadem atque prius denotantibus V, R, T, S, ponantur v & r pro velocitate & reſiſtentia ſub aſcenſus initio, s pro ſpatio quod corpus aſcendendo deſcribere poſſit antequam tota velocitas amittatur, t pro tempore hujus aſcenſus. Ad EG erigatur perpendicularis GO ipſi EG æqualis, & ſumendo puncta F, f, ad eaſdem diſtantias hinc inde a puncto G, jungantur OF, Of, quibus occurrat in T & t circuli arcus TGt centro O deſcriptus, & ſint Gh, Gf, GE continue proportionales, & ducatur ipſi ER parallela hm Hyperbolæ occurrens in m. Deinde ſi t ſit menſura anguli FOf ad Modulum T, hoc eſt, ſi t ſit ad T ut arcus TGt ad radium OG: erit s menſura rationis inter Eh & EG ad Modulum S, vel erit s ad S ut area Hyperbolica $mhGQ$ ad EQ; & v erit ad V ut Gf ad GE; atque r ad R ut Gh ad GE.

Fig. 8.

Propoſitio V. *Logiſticam deſcribere per Canonem Logarithmorum.*

Si ad Logiſticæ $BQDG$ Aſymptoton $APCF$ ordinatim applicentur binæ quævis rectæ AB, FG intercludentes Aſymptoti portionem quamvis AF: erit illa portio menſura rationis quam ad invicem habent ordinatæ; hæc utique eſt natura Curvæ notiſſima. Integrum ergo & perfectum Syſtema Logometricum per hanc Lineam exhibetur: id quod etiam de Hyperbola dici poteſt per Propoſitionem præcedentem, de Spirali Æquiangula per ſubſequentem; nam omitto complures alias Figuras, quæ & ipſæ dudum ſunt in Geometriam receptæ. Itaque ſi detur Aſymptoti poſitio & ſimul duo puncta per quæ Curva tranſire debet, dabuntur puncta reliqua per caſum priorem Propoſitionis tertiæ. Quod ſi data poſitione Aſymptoti, detur inſuper Syſtematis Modulus atque unicum punctum per quod ducenda erit Curva; invenientur puncta reliqua per Caſum poſteriorem Propoſitionis ejuſdem. Iſte vero Modulus quo pacto definiendus ſit, & qualem habeat magnitudinem, jam oportet exponere.

Ducatur recta BC quæ Curvam tangat in B & Aſymptoton ſecet in C. Dico primo, magnitudinem ſubtangentis AC eandem permanere ubicunque ſumatur punctum B. Intelligatur enim Ordinata PQ viciniſſima Ordinatæ ARB, recta vero QR parallela Aſymptoto AC, ac detur Ordinatarum intervallum illud quam minimum AP. *Fig.* 9. Ob datam igitur lineolam AP, dabitur ratio quam habet AB ad PQ, & diviſim ratio quam habet AB ad RB, atque adeo (propter ſimilia triangula BAC, BRQ) ratio quam habet AC ad RQ ſive AP, atque inde magnitudo ipſius AC.

Dico ſecundo, determinatam hanc & immutabilem ſubtangentem AC, eſſe Modulum ad quem exigendæ ſunt menſuræ illæ interceptæ AF. Patet hoc per Corollarium ſecundum Propoſitionis primæ: nam dum termini AB & PQ ad æqualitatem proxime accedunt, erit AC ad AP, quæ metitur rationem inter AB & PQ, ut terminus AB ad terminorum differentiam BR. Unde data ſubtangente, facilis eſt deſcriptio Curvæ & ſolutio Problematum omnium quæ exhinc pendent.

Si Curva jam deſcripta habeatur, ſubtangentis magnitudo ſic determinabitur. Producatur Ordinata quævis CD ad E, ita ut CE ad CD rationem habeat Modularem, per Corollarium ſextum Propoſitionis primæ definitam; & recta EB quæ a puncto E parallela ducitur Aſymptoto, quæque Curvæ occurrit in puncto B, æqualis erit ſubtangenti quæſitæ.

Corol. 1. Area $ABIH$, quæ inter Curvam BDI & Aſymptoton ejus ACH infinite verſus HI extenditur, & ad alteram partem ab Ordinata AB terminatur, æqualis eſt parallelogrammo $ABEC$ ab Ordinata eadem AB & ſubtangente AC comprehenſo. Componuntur enim area & parallelogrammum ex elementis quæ ſunt ut $AP \times AB$ & $AC \times RB$, quæque adeo æquantur propter analogiam inter AP & RB, AC & AB.

Corol. 2. Atque hinc, ob datam ſubtangentis magnitudinem, area illa indefinita erit ut Ordinata ad quam terminatur.

Scholium. Hujus Propoſitionis uſus per Exemplum declarabitur. Proponatur ad quamlibet altitudinem a ſuperficie telluris, invenire denſitatem Atmoſphæræ. Sit AB telluris ſuperficies, & abinde ſurſum producatur perpendicularis AH, atque ad hujus puncta ſingula ductæ concipiantur Ordinatæ FG, quæ ſint ut Aeris denſitates in locis F; & Ordinatarum termini omnes G in Linea Logiſtica $BDGI$ ſiti erunt. Patet hoc per Corollarium ſecundum hujus Propoſitionis. Nam area indefinita $FGIH$ eſt ut quantitas ſeu pondus Atmoſphæræ ſupra locum F, & pondus illud eſt vis quæ comprimit Aerem in hoc loco, iſthæc vero vis (uti docet Experientia multiplex) eſt ut Aeris compreſſi denſitas FG.

Itaque ſi quotlibet altitudines ſumantur in Arithmetica progreſſione: denſitates Aeris in his altitudinibus erunt in progreſſione Geometrica; & differentia binarum quarumvis altitudinum, erit menſura rationis quæ eſt inter denſitates Aeris in iſtis altitudinibus.

Ceſſante vi gravitatis, ita jam per vim aliquam extraneam intelligatur Aeris facta compreſſio, ut eandem habeat ubique denſitatem quam ad terræ ſuperficiem; & quantitas ejus, quæ modo erat expoſita per aream indefinitam $HABI$, nunc per æquale rectangulum $ABEC$ exhibebitur. Atmoſphæræ hujus homogeneæ altitudo AC, eſt ad altitudinem Hydrargyri in tubo *Torricellii*, ut gravitas Hydrargyri ad gravitatem Aeris; atque inde datur. Huic autem datæ altitudini æquatur (per *Corol.* 1.) ſubtangens Curvæ $BDGI$, atque adeo Modulus Syſtematis menſurarum omnium AF. Eſt ergo Logarithmus rationis inter denſitates Aeris in binis quibuſvis altitudinibus, ad Modulum Canonis, ut altitudinum earundem differentia, ad Atmoſphæræ prædictæ homogeneæ altitudinem illam datam AC.

Hæc ita ſe habent ex Hypotheſi, quod vis gravitatis eadem ſit ad omnes altitudines. Cæterum ex Philoſophia *Newtoniana* conſtat eam diminui, in receſſu a centro telluris, in duplicata ratione diſtantiæ: concluſio itaque paulo aliter ſe habebit. Sit S centrum telluris, & AB ſuperficies ejuſdem; ſumatur ipſis SF, SA tertia proportionalis Sf, erigatur ordinata fg quæ ſit ut Aeris denſitas in F: & Curva Bgn quam punctum g perpetuo tangit, erit eadem atque prius Logiſtica, ſed inverſo ſitu. Augeatur enim altitudo AF particula quam minima FM, capiatur Sm ad SA ut SA ad SM, ducatur Ordinata mn quæ ſit ut Aeris denſitas in M; & erit Sm ad Sf ut SF ad SM, & diviſim fm ad FM ut Sf ad SM, ſive ut Sf ad SF, hoc eſt, ut SAq ad SFq. Unde fm eſt ut SFq inverſe & FM directe, id eſt, ut gravitatio & moles Aeris inter F & M conjunctim; adeoque $fm \times fg$ ſive area $fgnm$ eſt ut gravitatio, moles & denſitas ejuſdem Aeris conjunctim, hoc eſt, ut preſſio illius in Aerem inferiorem: & ſumma ſimilium omnium arearum infra fg eſt ut ſumma preſſionum omnium ſupra F, id eſt, ut Aeris in F denſitas fg: & ſummarum differentia $fgnm$ ut denſitatum differentia $fg - mn$. Detur lineola fm; & erit fg ut area $fgnm$, adeoque ut $fg - mn$, atque inde (componendo) ut mn. Ergo data lineola fm erit menſura datæ illius rationis quæ eſt inter fg & mn: atque hinc patet Curvam Bgn eſſe Logiſticam. Sed & eandem eſſe cum ſupra deſcripta Logiſtica, facile abinde colligitur, quod ordinatæ baſi AB viciniſſimæ & ad æqualia intervalla quam minima diſpoſitæ, reſpective ſint æquales in utraque Curva; ac proinde eadem curvatura, eadem inclinatio tangentis ad punctum B, eademque ſubtangentis magnitudo.

Fig. 10.

Ergo ſi diſtantiæ SF a centro telluris capiantur in Muſica progreſſione; harum reciprocæ, nempe diſtantiæ Sf, erunt in progreſſione Arithmetica; & Aeris denſitates fg erunt in progreſſione Geometrica.

Ad inveniendam itaque denſitatem in loco quovis F, minuenda eſt altitudo AF in ratione diſtantiæ SF ad telluris ſemidiametrum SA: & Logarithmus rationis inter denſitates Aeris in A & F, erit ad Modulum Canonis, ut altitudo illa diminuta Af, ad Atmoſphæræ homogeneæ altitudinem AC.

Quæ ſupra demonſtrata ſunt, accurate obtinebunt, ſi modo Atmoſphæra ex Aere pariter Elaſtico tota conſtet: rationes igitur allatas paululum conturbabunt admiſti vapores atque exhalationes, quibus etiam accedet Caloris Frigoriſque diverſa temperies ad altitudines diverſas.

Propoſitio VI. *Logarithmorum Canonem ad Spiralem Æquiangulam accommodare.*

Fig. 11. Æquiangula Spiralis appellatur Linea illa curva ADE, quæ polo P deſcripta, in eodem dato angulo ſecat exeuntes a polo radios PA, PD, PE, &c.

Si centro P & intervallo quovis PA deſcribatur circulus ABC, qui radiis PA, PD, PE occurrat in A, B, C: Dico interceptum arcum BC menſuram fore rationis quam habet PD ad PE, & interceptum arcum AB menſuram rationis quam habet PA ad PD. Dividatur enim arcus AB in particulas quam minimas & æquales QR, & jungantur PQ, PR ſecantes Spiralem ad S & T in angulis datis PST, PTS: & ob datam particulam QR, dabitur angulus QPR, atque adeo ſpecies Figuræ SPT, & ratio laterum PS, PT. Data ergo particula QR menſura erit rationis datæ quam habet PS ad PT; & ſumma particularum, nempe arcus AB, menſura erit ſummæ ſimilis rationum, hoc eſt, rationis quam habet PA ad PD. Et eodem argumento, erit arcus BC menſura rationis quam habet PD ad PE.

Ducatur AF Spiralem tangens ad Circuli & Spiralis interſectionem A, huic vero in F occurrat recta PC quæ ad radium PA normalis erigitur: & ſubtangens PF erit menſurarum Modulus, per *Corol.* 2. *Prop.* 1. Nam ſi in recta PS ſumatur PV ipſi PT æqualis, & jungantur puncta V, T; ſimilia erunt triangula PAF, VST. Unde PF eſt ad VT ut PA ad VS, ſed & VT eſt ad QR ut PT ad PA: ergo ex æquo perturbate, PF eſt ad QR quæ metitur rationem inter PS ad PT, ut terminus PT ad terminorum differentiam VS.

Scholium. Spiralem æquiangulam, ad Meridianæ Nauticæ diviſionem demonſtrandam, feliciter adhibuit Geometra clariſſimus *Edmundus Halleius.* Sit acp pars octava Sphæræ terreſtris, p Polus, ac quadrans Æquatoris, ap quadrans Meridiani; & quæratur magnitudo rectæ, quæ propoſitum quemlibet hujus arcum deſignet in Planiſphærio. Per Æquatoris & Meridiani interſectionem a, ducta intelligatur linea Helicoeides ade quæ ſecet omnes Meridianos ad angulum ſemirectum, huic occurrat in d parallelus Æquatori circulus gd, per idem punctum d agatur Meridianus pdb; & longitudo intercepti arcus Æquatoris ab, erit magnitudo Nautica quæſita arcus ag. Reſolvatur enim arcus ag in particulas innumeras quam minimas gk, ducatur parallelus kmn, ſecans Meridianum pdb in n, Lineam ade in m; & actus Meridianus pmb abſcindet Æquatoris particulam bb, quæ erit ad mn, ſive huic (ob angulum ſemirectum mdn) æqualem dn vel gk, ut peripheria Æquatoris ad peripheriam paralleli kmn. Eſt ergo particula bb magnitudo Nautica particulæ gk, & ſumma particularum omnium bb, nempe longitudo arcus ab, magnitudo Nautica ſummæ particularum omnium gk, id eſt, arcus ag. Manente jam Æquatore abc vel ABC, concipiatur Sphærica ſuperficies in plano ejus Stereographice depingi; & Polo p occupante centrum P, projicientur Meridiani pga, pdb, pec in totidem rectas PA, PDB, PEC a centro P exeuntes, ita ut diſtantia abinde puncti cujuſvis D vel A, tangens ſit arcus dimidiati pd vel pa quem diſtantia illa repræſentat. Linea vero Helicoeides ade convertet ſe in ſpiralem æquiangulam ADE, polo P deſcriptam, & ſecantem radios ſuos omnes ad angulum ſemirectum. Hoc ſiquidem poſtulat nota Lex hujuſce Projectionis, ut anguli omnes eandem in Plano ac in Sphærica ſuperficie magnitudinem ſervent. Arcus itaque propoſiti ag magnitudo Nautica ab vel AB, eſt ad ſubtangentem PF vel huic jam æqualem Sphæræ radium PC, ut Logarithmus rationis inter PA & PD, hoc eſt, inter tangentes dimidiatorum arcuum pa & pd, vel pa & pg, ad Modulum Canonis.

Fig. 12.

Hinc quoniam longitudo Radii eſt ad longitudinem arcus minuti unius primi, ut 3437,746770784939 &c. ad 1, & reciprocus Moduli Canonis eſt 2,302585092994 &c. atque hi numeri in ſe multiplicati efficiunt 7915,704467897819 &c: ſi magnitudo illa Nautica AB in minutis primis exhibenda ſit, uti mos exigit; ſubducta tangente artificiali dimidiati arcus pg a tangente artificiali dimidiati arcus pa, multiplicetur reſiduum per numerum 7915,704467897819 &c. & factus dabit partes Meridionales deſideratas. Perinde vero ſe habebit concluſio, ſive in Æquatore, ſive extra hunc alibi ad utramvis partem locetur punctum a.

Scholium Generale. In eum potiſſimum finem præcedentia conſcripſi, ut allatis aliquot Exemplis oſtenderem, qua commodiſſima ratione Logarithmorum uſus in Geometriam recipi, & ad reſolutionem Problematum difficiliorum adhiberi poſſit. Viſum eſt hoc loco nonnullas adjicere porro conſtructiones, eodem conſilio effectas, quæ mihi iſta tractanti ſubinde ſeſe obviam non invitæ dederunt: ut ita, ex uberiore ſpecimine, de præſtantia Methodi hujus Logometricæ judicium feratur.

Parabolæ *Apollonianæ* AP ſit A vertex, F focus, AQ axis, PQ ordinatim applicata ad axem. Ducatur AL quæ bifariam ſecet PQ in L, & productæ adjiciatur LM quæ ſit menſura rationis inter $LA + AQ$ & QL ad Modulum AF: & recta AM æqualis erit arcui Parabolico AP.

Fig. 13.

Spiralis *Archimedea* PQ ſimilem habet extenſionem in rectam. Sit Q polus ejus, QP radius a polo ductus ad Curvæ quodlibet punctum P, & ad eum radium normalis QA. Ducatur LA parallela tangenti Spiralem in P, quæ radium PQ bifariam ſecet in L; & ponendo AF ad QL ut QL ad QA, ipſi AL adjiciatur LM quæ ſit menſura rationis inter LA

Fig. 14.

$+ AQ$ & QL ad Modulum AF: & recta AM æquabitur Spiralis arcui PQ.

Fig. 15. Spiralis Reciprocæ AeE ſit A polus, AB radius primus & infinitus, CD aſymptotos radio primo parallela ad diſtantiam AC; & invenienda proponatur hujuſce Curvæ longitudo. Inter Spiralem illam vulgarem *Archimedis* atque hanc, quam Reciprocam appello, iſthæc intercedit differentia, quod cum illius radii ſint ut anguli quos faciunt cum radio ſuo primo, hujus radii e contrario ſunt reciproce ut iidem anguli: eandem utique proportionem habet radius AE ad radium Ae quam habet angulus eAB ad angulum EAB. Unde facile colligitur, ſi ad puncta E & e ducantur tangentes EF, ef, & ad radios AE, ae erigantur normales AF, Af, fore normales iſtas ſibi invicem & Aſymptoti intervallo AC æquales. Invenietur autem longitudo cujuſvis arcus Ee, ponendo LM menſuram rationis inter AE & $EF - AF$ ad Modulum AF, & ſimiliter lm menſuram rationis inter Ae & $ef - Af$ ad æqualem Modulum Af. Nam ſi tangentium differentiæ $EF - ef$ adjiciatur menſurarum differentia $lm - LM$, aggregatum æquabitur arcui Ee.

Fig. 16. Linea illa Logiſtica, cujus aliquas expoſuimus affectiones in Propoſitione quinta, non abſimilem habet longitudinis ſuæ determinationem; quam & hoc loco apponam in eorum gratiam qui hujuſmodi contemplationibus delectantur. Oblata ſit igitur Logiſtica $EMem$, cujus Aſymptotos $AFaf$: & quæratur longitudo cujuſvis arcus Ee. Demiſſis in Aſymptoton perpendiculis ELA, ela, & ductis tangentibus EF, ef, capiatur AL æqualis exceſſui quo tangens EF ſuperat ſubtangentem AF, & ſimiliter al æqualis exceſſui quo tangens ef ſuperat ſubtangentem af: & actis LM, lm Aſymptoto parallelis, ſi tangentium differentiæ $Ef - ef$ adjiciatur parallelarum differentia $lm - LM$, aggregatum æquabitur arcui Ee.

Fig. 17. Accedo ad Ciſſoidem *Diocleam*. Sit A vertex ejus, AB diameter Circuli genitoris, BC Aſymptotos, PQ perpendicularis in diametrum demiſſa, Ciſſoidi in P & diametro in Q occurrens. Agatur AC quæ ſecet Aſymptoton in C ac faciat angulum BAC qui ſit recti pars tertia, ſumptaque inter BQ & BA media proportionali BD jungatur CD; denique per medium perpendiculum PQ ducatur AE recta, quæ occurrat Aſymptoto in E: & Ciſſoidis arcus AP æquabitur duplicato exceſſui rectæ AE ſupra diametrum AB, & ſimul triplicatæ menſuræ rationis inter $BA + AC$ & $BD + DC$ ad Modulum BC.

Si Ciſſoidis area APQ convertatur circum axem AQ; generabitur ſolidum cujus dimenſio pendet a Logometria, & ſic conſtruitur. Sint AQ, AB, AR, AS, AT continue proportionales; deinde ad Modulum TS capiatur QX menſura rationis inter AB & BQ, & retro ponatur XZ æqualis ipſi SR una cum dimidio ipſius RB ac triente ſimul ipſius BQ: & ſolidum Ciſſoidale axem habens AQ baſiſque ſemidiametrum PQ, æquabitur Cylindro cujus eadem eſt baſis & cujus altitudo eſt QZ.

Fig. 18. Adjungam ſolidum ex Conchoide *Nicomedis* genitum. Sint AE, ae Curvæ conjugatæ, polo P, regula CD, intervallo CA vel Ca, axe $PaCA$

ad regulam normali, verticibuſque A & a deſcriptæ. Per polum P ducatur ad libitum recta $PeDE$, regulæ occurrens in D, Lineæ vero in E & e: & ex natura Conchoidis, erunt ſegmenta DE, De intervallo CA vel Ca æqualia. Eodem intervallo centroque P deſcribatur circuli arcus RS ſecans axem PC in R & rectam PD in S: & ſemiſumma ſolidorum Conchoidalium quæ generantur ex converſione Figurarum $AEDC$, $aeDC$ circum axem AaP, erit ad ſectorem Sphæræ genitum ex circuli ſectore PRS circum axem eundem converſo, ut $3PC \times PD + PRq$ ad PRq. Eorundem vero ſemidifferentia Cylindro æquatur, cujus baſis eſt circulus diametro Aa deſcriptus, & cujus altitudo eſt menſura duplicata rationis inter PD & PC ad Modulum PC.

Area vero Figuræ totius $AEea$ æquatur rectangulo cujus baſis eſt Aa, & cujus altitudo CM eſt menſura rationis inter $PD + DC$ & PC ad Modulum PC. Quod ſi deſideretur quadratura partium $AEDC$, $aeDC$; ductis ad axem normalibus AF, af, in regula CD ſumenda eſt CN quæ ſit anguli CPD menſura ad eundem Modulum PC: & acta per punctum M recta FMf quæ parallela ſit rectæ jungenti puncta P, N, quæque occurrat normalibus in F & f; erit area $AEDC$ æqualis Trapezio $AFMC$, & area $aeDC$ æqualis Trapezio $afMC$.

Fig. 19.

Hyperbolæ quadraturam in ſuperioribus expoſitam dedi, eo modo, qui mihi viſus eſt ad propoſitum quam maxime accommodatus. Libet aliam conſtructionem hoc loco apponere, & ſimul adjicere gravitatis centrum. Oblata ſit portio interior ADB, intercluſa curvæ ADB & rectæ cuivis AB ad diametrum PQ parallelæ. A Figuræ centro C producatur diameter CDE, quæ baſin AB bifariam ſecet in E; deinde ſi in diametro producta ſumantur CR ad CD, & CD ad CS, ut baſis AB ad diametrum PQ, & ad Modulum CS fiat CN menſura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ ADB.

Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2CR$ ad $3EN$.

Fig. 20.

Sit nunc oblata portio exterior $APQB$, intercluſa curvis oppoſitis AP, BQ, diametro PQ, & rectæ cuivis AB ad diametrum illam parallelæ. Eſto CD conjugatæ ſemidiametri longitudo extra portionem oblatam $APQB$ poſita, quæ producta in contrariam partem centri C bifariam ſecet baſim AB in E. Deinde in diametro producta ſi ſumantur CR ad CD, & CD ad CS, & CS ad CT, ut baſis AB ad diametrum PQ, ponantur vero CR & CT ad eandem centri partem cum baſi AB; & ad Modulum CS, in contrariam centri partem, ſumatur CN menſura rationis quam habet CD ad ER: triangulum rectilineum ANB æquabitur areæ curvilineæ $APQB$.

Hujus autem areæ centrum gravitatis Z invenietur, capiendo CZ ad CR ut $2TR$ ad $3EN$.

Fig. 21.

Pergo ad ſuperficies ab Hyperbola circum axes ſuos convoluta genitas. Sit AN Hyperbola deſcripta vertice A, centro C, Aſymptoto CB, foco F, ſemiaxe principali AC, ſemiaxe conjugato AB normali ad AC; & ad axis AC punctum quodvis X ſit XN ordinatim applicata, quæ Hyperbolæ occurrat ad N. In axe CA capiatur CE ad CA ut CA ad CF; & ad eundem axem erecta perpendiculari EZ, quæ Aſymptoto occurrat in G, angulo CEZ inſcribatur æqualis ipſi CX recta CZ, quæ porro producta ſecet ordinatim applicatam XN ad O. Tum ſumatur KL quæ ſit æqualis exceſſui quo XO ſuperat AB, atque LM quæ ſit menſura rationis inter $CZ + ZE$ & $CG + GE$ ad Modulum CE: & ſuperficies genita ex arcus AN converſione circum axem AX, erit ad Circulum ſemidiametro AB deſcriptum, ut exceſſus KM quo KL ſuperat LM, ad ſemidiametrum illam AB.

Fig. 22.

Sit rurſus BN Hyperbola deſcripta vertice B, centro C, foco F, ſemiaxe principali CB, ſemiaxe conjugato CA normali ad CB; & ad axis AC punctum quodvis X ſit XN ordinatim applicata, quæ Hyperbolæ occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum ſumatur KL quæ ſit ad XC ut XE ad CE, & LM quæ rationis inter $EX + XC$ & CE menſura ſit ad Modulum CE: & ſuperficies genita ex arcus BN converſione circum axem CX, erit ad Circulum ſemidiametro CB deſcriptum, ut linearum KL & LM aggregatum KM, ad ſemidiametrum illam CB.

Fig. 23.

His addere licebit ab Ellipſi genitas ſuperficies. Sit ANB Ellipſis deſcripta centro C, verticibus A & B, foco F, ſemiaxe principali CB, ſemiaxe conjugato CA; & ad axis CA punctum quodvis X ſit XN ordinatim applicata, quæ Ellipſi occurrat ad N. In axe CB capiatur CE ad CA ut CA ad CF, & jungatur EX. Tum ſumatur KL quæ ſit ad XC ut XE ad CE, & LM quæ rationis inter $EX + XC$ & CE menſura ſit ad Modulum CE: & ſuperficies genita ex arcus BN converſione circum axem CX, erit ad Circulum ſemidiametro CB deſcriptum, ut linearum KL & LM aggregatum KM, ad ſemidiametrum illam CB. Ut hæc ultima conſtructio locum habeat, oportet ſemiaxem CA circa quem converſio facta eſt, minorem eſſe altero ſemiaxe CB; aliter enim Moduli CE quantitas $\frac{CAq}{\sqrt{CBq - CAq}}$ evadet impoſſibilis, & conſtructio illa Logometrica (quod in hujuſmodi caſibus fieri ſolet) convertet ſe in Trigonometricam, qualis illa eſt quæ jam ſequitur.

Fig. 24.

Sit ANB Ellipſis deſcripta centro C, verticibus A & B, foco F, ſemiaxe principali CA, ſemiaxe conjugato CB; & ad axis CA punctum quodvis X ſit XN ordinatim applicata, quæ Ellipſi occurrat ad N. Angulo CXN inſcribatur recta CE, quæ ſit ad CA ut CA ad CF. Tum ſumatur KL quæ ſit ad XC ut XE ad CE, & LM quæ anguli XEC menſura ſit ad Modulum CE, hoc eſt, quæ ſit æqualis arcui cujus ſinus eſt XC ad radium CE: & ſuperficies genita ex arcus BN converſione circum axem CX, erit ad Circulum ſemidiametro CB deſcriptum, ut linearum KL & LM aggregatum KM, ad ſemidiametrum illam CB. Poſſet hujus etiam ſuperficiei dimenſio per Logometriam deſignari, ſed modo inexplicabili. Nam ſi quadrantis circuli quilibet arcus, radio CE deſcriptus, ſinum habeat CX ſinumque complementi ad quadrantem XE: ſumendo radium

CE pro Modulo, arcus erit rationis inter $EX + XC\sqrt{-1}$ & CE menſura ducta in $\sqrt{-1}$. Verum iſthæc aliis, quibus operæ pretium videbitur, diligentius excutienda relinquo. Ceterum ex præcedentibus intelligi poteſt, quanta ſit cognatio inter angulorum atque rationum menſuras, quamque levi mutatione in ſe invicem facillime convertantur pro variis ejuſdem Problematis caſibus. De Cubicarum æquationum radicibus dudum ab Analyſtis obſervatum eſt; vel eas exprimi poſſe per *Cardani* regulas, atque adeo per duarum mediarum proportionalium inventionem; vel per diviſionem arcus circularis in tres æquales partes, ſi forte fuerint inexplicabiles per memoratas regulas. * Hoc animadvertit *Carteſius*, ſed & ante *Carteſium* idem obſervabit *Franciſcus Vieta* ſub finem Supplementi Geometriæ. Exhinc autem aperte colligitur, qualis ſit ordo Naturæ tranſeuntis ad Anguli triſectionem a triſectione Rationis.

Fig. 26.

Mirabilem illam Harmoniam ulterius declarare lubet, Exemplo deſumpto ab eadem Figura circum axes ſuos convoluta. Sit igitur $APBQ$ Ellipſis, axis ejus major AB, minor PQ, centrum C, focus F. Hæc circum axem utrumvis convoluta Solidum generet, cujus particulæ conſtantes ex materia homogenea, vires attractivas habeant in duplicata diſtantiarum ratione decreſcentes: & quæratur vis qua Solidum illud attrahit corpuſculum quodvis, in ejus ſuperficie locatum ad axis illius terminum. Jungantur puncta P, F, ac ſumatur CD quæ ſit menſura rationis inter $PF + FC$ & CP ad Modulum CA, pariterque ſumatur CE quæ ſit anguli CPF menſura ad Modulum CP; ſitque FD exceſſus menſuræ CD ſupra CF, atque FE exceſſus ipſius CF ſupra menſuram CE: & Solidi convolutione circum axem majorem AB geniti vis in corpuſculum ad A locatum, erit ad Sphæræ homogeneæ & eodem axe deſcriptæ vim in idem corpuſculum, ut $3FD \times CPq$ ad CF *cub*; Solidi autem converſione circum axem minorem PQ geniti vis in corpuſculum ad P locatum, erit ad Sphæræ homogeneæ & eodem axe deſcriptæ vim in idem corpuſculum, ut $3FE \times CAq$ ad CF *cub*. Unde cum vis Sphæræ prioris in corpuſculum ad *A*, ſit ad vim Sphæræ poſterioris in corpuſculum ad P, ut *CA* ad *CP*: erit vis Solidi prioris in corpuſculum ad *A*, ad vim Solidi poſterioris in corpuſculum ad *P*, ut $FD \times CP$ ad $FE \times CA$.

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* Sublato etenim termino ſecundo, tres habentur Æquationum caſus. Hi vero reſolvuntur ope trianguli rectanguli ABC, rectum habentis angulum ad A, in quo inſuper triangulo ſemper data ſunt duo latera.

Fig. 25.

Caſ. 1. Nam ſi ſit $x^3 + 3aax = +2aab$: ponantur $AB = a$, $AC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica poſſibilis affirmativa, ſi habeatur $+ 2aab$; vel $N - M$ radix unica poſſibilis negativa, ſi habeatur $- 2aab$.

Caſ. 2. Si ſit $x^3 - 3aax = +2aab$, exiſtente a minore quam b: ponantur $AB = a$, $BC = b$; & ſumantur M & N binæ mediæ proportionales inter $BC + AC$ & $BC - AC$: & erit $M + N$ radix unica poſſibilis affirmativa, ſi habeatur $+ 2aab$; vel $- M - N$ radix unica poſſibilis negativa, ſi habeatur $- 2aab$.

Caſ. 3. Denique ſi ſit $x^3 - 3aax = \pm 2aab$, exiſtente a majore quam b: ponantur $AB = b$, $BC = a$; & ſumatur M ſinus trientis angulorum ſummæ $A + B$, atque N ſinus trientis angulorum differentiæ $A - B$, exiſtente radio $2BC$: & erunt $-M$, $-N$, & $M + N$ tres radices poſſibiles, ſi habeatur $+ 2aab$; vel M, N, & $-M - N$, tres radices poſſibiles, ſi habeatur $- 2aab$.

Atque ita Problemata omnia Solida ſolutionem facilem recipiunt, vel per Canonem Logarithmicum, vel per Canonem Trigonometricum.

Hinc quoniam Solidum poſterius medium eſt proportionale inter Solidum prius & Sphæram priorem: vis Solidi poſterioris in corpuſculum ad *A*, erit media proportionalis quamproxime inter vires Solidi prioris & Sphæræ prioris in idem corpuſculum ad *A*, ſi modo axes Ellipſeos ſint prope æquales. Itaque in hoc caſu, ponendo *CG* mediam proportionalem inter *CF* & $3FD$, & capiendo *CH* ad $3FE$ ut *CA* ad *CF*; poſterioris Solidi vires ad *A* & *P*, vel ad *B* & *Q*, erunt ad invicem quamproxime ut *CG* ad *CH*. Id quod non inutile præbet compendium ad inventionem Figuræ Telluris, qualem eam ſubtiliter inſtituit celeberrimus *Newtonus*, ſummus ille Philoſophiæ ſanioris Inſtaurator.

Conſideratio virium centripetarum aliud porro mihi ſuggerit Exemplum, in quo ſatis ampla ſe prodit mutationum varietas. Proponatur Trajectoriarum ſpecies enumerare, in quibus corpora moveri poſſunt, quæ a viribus centripetis in ratione diſtantiarum triplicata decreſcentibus agitantur, quæque de loco dato, cum data velocitate, ſecundum datam rectam egrediuntur.

Caſ. 1. Sit *S* centrum virium, exeatque corpus de loco *P* ſecundum rectam *PQ* vel *QP*, ea cum velocitate quam acquirere poſſet ab iiſdem viribus, libere cadendo verſus centrum *S* de loco *C*, & caſu ſuo deſcribendo altitudinem *CP*. In datam rectam *QPT* demittantur perpendicula *SQ*, *CT*, centroque *S* & intervallo $\sqrt{SQq + QTq}$ deſcribatur circulus *RTA*, rectæ *SPC* occurrens in *R*: deinde ad Modulum $\sqrt{SCq - SRq}$ ſit arcus *RA* menſura rationis inter $SR \pm \sqrt{SRq - SPq}$ & *SP*, jaceant autem arcus ille *RA* & punctum *Q* ad diverſas partes rectæ *SR*; & punctum *A* erit Apſis ſumma Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo *SM* æqualem ipſi $\sqrt{SCq - SRq}$, deinde in recta *SA* capiendo longitudinem quamvis *SD* quæ ſit minor quam *SA*, ad eandem erigendo perpendiculum *DE* ſecans circulum in *E*, & jungendo *SE*. Nam ſi ad utraſque partes puncti *A* ponatur arcus circularis *AR*, cujus longitudo ſit menſura rationis inter $SE + ED$ & *SD* ad Modulum *SM*, & in ſemidiametris *SR* capiantur diſtantiæ *SP* æquales ipſi *SD*: erunt puncta *P* ad Trajectoriam deſcribendam. Tempus autem quo radius *SP*, a centro ad corpus motum ductus, percurret aream quamvis *SAP*, erit ut recta *DE*: nam area percurſa æquatur ipſi *DE* in Modulum dimidiatum $\frac{1}{2} SM$ ductæ. Velocitas vero corporis in loco quovis *P*, erit ad velocitatem qua in Circulo, ad eandem diſtantiam *SP*, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq - SPq}$ ad *SC*. Ex ipſa conſtructione patet, hanc Spiralem primam infinitis gyris circa centrum virium contorqueri, quin & ſeipſam infinitis vicibus decuſſare, & ſiti erunt Nodi omnes ad Apſidis lineam *AS*. Fig. 27.

Caſ. 2. Recedat punctum *C* ad infinitam diſtantiam a centro *S*; & corporis de loco *P* ſecundum rectam *PM* vel *MP* exeuntis ea ſit velocitas, quam acquirere poſſet cadendo libere ad eundem locum P ab infinita diſtantia. Ad rectam SP ducatur normalis SM, quæ ſecet PM in M; deinde centro S & intervallo SP deſcribatur circulus, & in ejus circumferentia capiatur arcus PX, cujus longitudo ſit menſura rationis inter diſtantiam quamvis SD, & diſtantiam datam SP ad Modulum SM, jaceant autem arcus ille PX & punctum M ad diverſas partes rectæ SP ſi SD fuerit major quam SP, aliter ad eaſdem, inque ſemidiametro SX ponatur SZ æqualis ipſi SD; & punctum Z erit ad Trajectoriam deſcribendam. Tempus autem quo radius SZ, a centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia quadratorum ex SZ & SP: Nam area percurſa eſt ad illam differentiam, in data ratione Moduli dimidiati $\frac{1}{2}SM$ ad SP. Velocitas vero corporis in loco quovis P, æqualis erit velocitati qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet. Ex conſtructione patet hanc ſecundam Spiralem eſſe Æquiangulam illam Propoſitionis ſextæ; ea vero migrabit in Circulum ubi angulus SPM fit rectus. Fig. 28.

Caſ. 3. Ut velocitas ſit adhuc major, abeat jam punctum C ad diſtantiam pluſquam infinitam a centro S, vel (quod perinde eſt) accedat a parte contraria eidem centro, ad finitam diſtantiam; & corporis de loco P ſecundum rectam PQ vel QP exeuntis, ea ſit velocitas, quam acquirere poſſet aſcendendo libere de loco C ad infinitam diſtantiam, & deinde ab infinita diſtantia ex altera centri parte deſcendendo ad locum P, viribus centripetis inter aſcendendum in æquales vires centrifugas converſis. In datam rectam PQT demittantur perpendicula SQ, CT; & erit TQ vel major, vel æqualis, vel minor quam SQ. Si TQ fuerit major quam SQ; centro S & intervallo $\sqrt{TQq - SQq}$ deſcribatur circulus RBE rectæ SP occurrens in R, deinde ad Modulum $\sqrt{SCq - SRq}$ ſit arcus RB menſura rationis inter $SR \pm \sqrt{SRq + SPq}$ & SP, jaceant autem arcus ille RB & punctum Q ad partes diverſas rectæ SP. Exhinc Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq - SRq}$, in recta SB capiendo longitudinem quamvis SD, ad eandem erigendo perpendiculum SE circulum ſecans in E, & jungendo DE. Nam ſi retro ponatur a puncto B circularis arcus BR, cujus longitudo menſura ſit rationis inter $SE + ED$ & SD ad Modulum SM, & in ſemidiametro SR capiatur diſtantia SP æqualis ipſi SD: erit punctum P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis hujus Trajectoriæ, erit ut incrementum vel decrementum rectæ DE per tempus illud factum: nam area percurſa æquatur huic incremento vel decremento in Modulum dimidiatum $\frac{1}{2}SM$ ducto. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq + SPq}$ ad SC. Ex conſtructione patet, hanc Spiralem tertiam infinitis gyris centrum cingere infra punctum datum P; at ſupra idem punctum vel non undique cinget, ſi arcus RB minor fuerit quam circumferentia tota $RBER$; vel toties cinget, quoties arcus ille circumferentiam excedit. Fig. 29.

Caſ. 4. Reliquis manentibus, ſint jam TQ & SQ æquales. Centro S & intervallo SP deſcribatur circulus PXB, & ſit arcus PB æqualis ipſi SC, jaceant autem arcus PB & punctum Q ad partes diverſas rectæ SP. Exhinc Trajectoria dabitur, ſumendo in recta SB longitudinem quamvis SD, centroque S & intervallo SD deſcribendo circuli arcum DZ æqualem ipſi SC. Nam ſi ordine circulari contrario ponantur arcus PB a puncto
Fig. 30. P & arcus DZ a puncto D: erit punctum Z ad Trajectoriam deſcribendam. Tempus autem quo radius SZ, a centro ad corpus motum ductus, percurret aream quamvis SPZ, erit ut differentia radiorum SZ & SP: nam area percurſa æquatur huic differentiæ ductæ in ſemiſſem diſtantiæ SC. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq+SPq}$ ad SC. Ex conſtructione patet, hanc Spiralem quartam eſſe Reciprocam illam, cujus longitudinem ſupra dimenſam dedimus.

Caſ. 5. Reliquis adhuc manentibus, ſit jam TQ minor quam SQ. Centro S & intervallo $\sqrt{SQq-TQq}$ deſcribatur circulus RAE rectæ SP occurrens in R; deinde ſit arcus RA, ad ejuſdem circuli arcum cujus ſecans eſt SP, ut $\sqrt{SCq+SRq}$ ad SR; ponatur autem arcus ille RA ad eaſdem partes rectæ SP cum puncto Q: & A erit Apſis ima Trajectoriæ. Exhinc vero Trajectoria dabitur, ſumendo SM æqualem ipſi $\sqrt{SCq-SRq}$, in recta SA capiendo longitudinem quamvis SD quæ ſit major quam SA, ducendo DE quæ circulum tangat in E, & jungendo SE. Nam ſi ad utraſque partes puncti A ponatur arcus circularis AR, cujus longitudo menſura ſit anguli DSE ad Modulum SM, & in ſemidiametris SR capiantur diſtantiæ SP æquales ipſis SD: erunt puncta P ad Trajectoriam deſcribendam. Tempus autem quo radius SP, a centro ad corpus motum ductus, percurret aream quamvis SAP, erit ut recta DE: nam area percurſa æquatur ipſi DE in Modulum dimidiatum $\frac{1}{2}SM$ ductæ.
Fig. 31. Velocitas vero corporis in loco quovis P, erit ad velocitatem qua in Circulo, ad eandem diſtantiam SP, cum iiſdem viribus revolvi poſſet, ut $\sqrt{SCq+SPq}$ ad SC. Ex conſtructione patet, hanc quintam Spiralem vel nullum habere Nodum, vel unicum, vel plures, pro varia proportione rectæ SM ad diametrum circuli EAR: toties enim Trajectoria ſeſe decuſſabit, quoties illa recta diametrum excedit, & Nodi omnes ſiti erunt ad Apſidis lineam AS.

Sunt itaque Trajectoriarum quinque Species. Harum primam atque ultimam deſcripſit olim *Newtonus* per Hyperbolæ & Ellipſeos quadraturam.

Geometris integrum erit, ex adductis hactenus Exemplis de Methodo noſtra judicare; quam quidem, ſi proba fuerit, ulterius excolere pergent & excolendo latius promovebunt. Patet utique campus ampliſſimus in quo vires ſuas experiri poterunt, præſertim ſi Logometriæ Trigonometriam inſuper adjungant, quibus miram quandam affinitatem in ſe invicem euntibus intercedere notabam. Hiſce quidem Principiis haud facile crediderim generaliora dari poſſe; cum tota Matheſis vix quicquam in univerſo ſuo ambitu complectatur, præter angulorum & rationum Theoriam.

Neque ſane commodiora ſperabit, qui animadverterit Effectionis facilitatem per ampliſſimas illas, omnibuſque ſuis numeris abſolutas, tum Logarithmorum tum Sinuum & Tangentium Tabulas, quas anteceſſorum noſtrorum laudatiſſimæ ſolertiæ debemus acceptas. Ut vero tanti beneficii uberior nobis exſurgat fructus, id nunc exponendum reſtat, quibus artibus ad iſtiuſmodi concluſiones rectiſſime perveniatur. In hunc finem Theoremata quædam, tum Logometrica tum Trigonometrica adjeciſſem, quæ parata ad uſum aſſervo; ni conſultius viſum eſſet, quum abſque nimiis ambagibus ea tradi non poſſent, intacta potius præterire atque aliis denuo inveſtiganda relinquere. Cæterum iſthoc apparatu non ſemper eſt opus; nam in Methodo Fluxionum ſæpe evenit ut ipſæ Fluentes, omiſſis hujuſmodi ſubſidiis, ad Logometriam ſatis commode revocentur: id quod uno atque altero Exemplo oſtendam.

Egimus in præcedentibus de rectilineo Gravium deſcenſu, per Medii reſiſtentiam continuam retardato, ex Hypotheſi quod illa reſiſtentia eſſet in duplicata ratione velocitatis. Ex eadem Hypotheſi reſiſtentiam corporis penduli, in Cycloide oſcillantis, jam ſit propoſitum invenire. Cycloidis itaque in rectam explicatæ ſit AC dimidium, C punctum infimum, B punctum a quo cadere incipit corpus pendulum, BC, CD arcus deſcenſu ejus & ſubſequente aſcenſu deſcripti. Hiſce poſitis, exquirenda eſt ratio quam habet reſiſtentia corporis in loco quovis E, ad pondus ejus relativum in Medio reſiſtente. Exponatur pondus illud per AC; & vis ab eodem oriunda, qua pendulum acceleratur ad E, exponetur per CE: quæ ſi dicatur x, & momentum ejus $+\dot{x}$; momentum arcus jam deſcripti BE erit $-\dot{x}$. Exponatur vis reſiſtentiæ per z; & vis qua pendulum vere acceleratur, erit ut exceſſus vis prioris ſupra reſiſtentiam, hoc eſt, ut $x-z$. Itaque cum reſiſtentia ſit ut quadratum velocitatis, reſiſtentiæ momentum $\dot{z}$ erit ut velocitas & velocitatis momentum, hoc eſt, ut $-\dot{x}$ & $x-z$, ſive ut $z\dot{x}-x\dot{x}$. Nam ſi tempus in particulas æquales dividatur, erit velocitas ut arcus deſcripti momentum $-\dot{x}$, & velocitatis momentum ut vis acceleratrix $x-z$ quæ momentum illud generat. Quoniam ergo $\dot{z}$ eſt ut $z\dot{x}-x\dot{x}$, ſi capiatur quantitas invariabilis a, quæ ſit idoneæ magnitudinis: erit $a\dot{z}=z\dot{x}-x\dot{x}$. Fig. 32.

Ad hanc æquationem conſtruendam, aſſumatur quantitas v quæ ſit variabilis, & fingatur æquatio $z=p+qx+rv$, in qua notæ p, q, r deſignent alias novas quantitates invariabiles; & erit $\dot{z}=q\dot{x}+r\dot{v}$. Hiſce porro valoribus ipſarum z & $\dot{z}$ ſubſtitutis in æquatione prima $a\dot{z}=z\dot{x}-x\dot{x}$, habebitur $\overline{aq-p},\ \dot{x}+ar\dot{v}=\overline{q-1},\ x\dot{x}+rv\dot{x}$. Ut hæc æquatio ſimplicior evadat, ponatur $q-1=0$, & $aq-p=0$; ſive $q=1$, & $p=a$: & fiet $a\frac{\dot{v}}{v}=\dot{x}$, ac præterea $z=a+x+rv$. Jacentibus punctis D & S ad eandem partem puncti C, intelligatur CS æqualis ipſi a: & erit $z=SE+rv$, atque $CS\frac{\dot{v}}{v}=\dot{x}$. Sit c valor quantitatis v, dum incidit punctum E in punctum C: & quantitas x, ſive CE, æquabitur menſuræ rationis quam habet v ad c pro Modulo CS, per Propoſitionem primam: quam æqualitatem ſic deſignare ſoleo, $CE = CS\left|\frac{v}{c}\right.$. Tota ergo Problematis difficultas jam revocatur ad binas illas æquationes $CE = CS\left|\frac{v}{c}\right.$, atque $z = SE + rv$: hæ vero deduci non poterunt in uſum priuſquam determinatæ fuerint quantitates r & CS. Ad hoc efficiendum, duæ reſtant conditiones nondum adimpletæ; oportet enim reſiſtentiam eſſe nullam, atque adeo quantitatem z ſive $SE + rv$ evaneſcere, ubi punctum E in puncta B & D inciderit.

Sint ergo b & d valores ipſius v, dum incidit punctum E in puncta B & D reſpective: & in his caſibus habebuntur $SB + rb = 0$, $SD + rd = 0$. Unde $r = -\frac{SB}{b}$, $r = -\frac{SD}{d}$, atque $z = SE + rv = SE - \frac{v}{b}SB = SE - \frac{v}{d}SD$. Porro erit $\frac{SB}{SD} = \frac{b}{d}$; atque adeo $CS\left|\frac{SB}{SD}\right. = (CS\left|\frac{b}{d}\right. = CS\left|\frac{b}{c}\right. - CS\left|\frac{d}{c}\right. = CB + CD =)\ BD$: unde dabitur punctum S.

Componetur itaque Problema hunc in modum. Producatur BD verſus D ad S, eo uſque donec BD fuerit menſura rationis inter SB & SD ad Modulum CS. Deinde ad arbitrium poſita quantitate c, ita capiantur quantitates b & v, ut eodem Modulo CS; fiat CB menſura rationis quam habet b ad c, fiat quoque CE menſura rationis quam habet v ad c: & erit vis reſiſtentiæ in loco E, ad pondus relativum corporis penduli, ut $SE - \frac{v}{b}SB$ ad CA.

Hujus Problematis ſolutio utilitatem habet in Phyſica non contemnendam: quapropter conſtructionem ejuſdem Linearem, ex eadem Analyſi deductam, ſubjungere viſum eſt. Invento uti ſupra puncto S; ad rectam SA erigantur perpendicula DH, Cc, EK, BF, AN, rectæ SN utcunque per S ductæ occurrentia in H, c, K, F, N. Per punctum c ducatur recta da parallela rectæ DA, quæ iiſdem perpendiculis occurrat in d, c, e, b, a; & ad Aſymptoton SA ducatur Logiſtica $HGIF$, quæ tranſeat per puncta H & F, ſecetque perpendicula Cc, EK in G & I, ac parallelam da in m: namque his poſitis, erit pondus relativum corporis penduli, ad vim illam qua pendulum acceleratur ad punctum E in Medio non reſiſtente, ut aN ad eK; erit autem ad vim reſiſtentiæ in loco E, ut aN ad KI; atque adeo ad vim qua pendulum acceleratur ad punctum E in Medio reſiſtente, ut aN ad eI. Porro, ſi per punctum m ducatur ad rectam SMA perpendicularis LmM, quæ ſecet SN in L: erit M locus ubi reſiſtentia fit maxima: atque adeo reſiſtentia illa maxima, erit ad pondus relativum penduli, ut Lm ad Na, hoc eſt, ut CM ad CA. Fig. 33.

Ceterum ſi ita ducatur recta SN, ut abſcindat rectam DH quæ ſit dupla ipſius SD, centroque C & intervallo CB deſcribatur Circulus BOP, qui occurrat perpendiculo KE in O: erit penduli in Medio reſiſtente oſcillantis velocitas in loco *E*, ad velocitatem penduli ejuſdem ad eundem locum *E* delati per idem pondus relativum in Medio non reſiſtente, ut media proportionalis inter *CS* & *KI*, ad *EO*.

Adhæc ſi jungatur *CO*, & in perpendiculo *KE* ſumatur *ER*, quæ ſit ad *CB* ut *CB* ad mediam proportionalem inter 4 *CS*, & *KI*; continuoque ductu rectæ *ER* in baſin *BE* generetur area *BQRE*: erit tempus quo Cycloidis arcus *BE* deſcribitur in Medio reſiſtente, ad tempus quo idem arcus deſcriberetur in Medio non reſiſtente, ut area illa *BQRE*, ad Circuli ſectorem *BOC*. Pergo nunc ad alia.

Denſitatem Aeris invenimus ad quamvis altitudinem, ubi vis Gravitatis vel erat uniformis, vel decreſcebat in receſſu a centro telluris in duplicata ratione diſtantiæ: libet eandem exquirere denuo, ubi gravitatio vel augetur vel diminuitur in ratione datæ cujuſvis dignitatis diſtantiæ. Sit *S* centrum telluris, *A* punctum in ejus ſuperficie vel alibi utcunque ſitum, *SAF*2 recta a centro ad ſummitatem Atmoſphæræ producta: & quærenda ſit ratio denſitatis in loco *A*, ad denſitatem in loco quovis *F*, ex Hypotheſi quod vis gravitatis in *F* ſit ut diſtantiæ *SF* dignitas quæcunque SF^n, cujus index eſt n. Pro *SF* ſcribatur x, ac deſignent d & v denſitates Aeris ad *A* & *F*; & cum denſitas ſit ubique ut preſſura totius Aeris incumbentis, erit denſitatis momentum ut momentum preſſuræ, hoc eſt, $\dot{v}$ ut $v\dot{x}x^n$, atque adeo $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$. Sit *AC* altitudo Atmoſphæræ, cujus uniformis denſitas eadem eſſet ac denſitas loci *A*; vel ſit *AC* ad altitudinem Hydrargyri barometrici in loco *A*, ut denſitas Hydrargyri ad denſitatem Aeris in eodem loco *A*: & ſi punctum *F* accedere intelligatur ad punctum *A*; erit altitudo Hydrargyri barometrici in loco *A*, ad altitudinem Hydrargyri barometrici in loco *F*, ut *AC* ad *FC*. Aeris ergo in loco *A* denſitas d, eſt ad Aeris in loco *F* denſitatem v, ut *AC* ad *FC*: unde conſequitur ut ſit $d - v$ ſive $\dot{v}$ ad d ſive v, ut *AF* ſive $\dot{x}$, ad *AC*. Erit itaque, in hoc caſu, $AC\frac{\dot{v}}{v} = \dot{x} = \frac{\dot{x}x^n}{SA^n}$. Quoniam ergo, ubicunque ſumeretur punctum *F*, erat $\frac{\dot{v}}{v}$ ut $\dot{x}x^n$: erit porro $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$, ubicunque ſumatur punctum *F*.

Fig. 34.

Jam ſi gravitatio ſit reciproce ut diſtantia a centro, ſive ut $\frac{1}{x}$ vel x^{-1}; erit $n = -1$, atque inde $AC\frac{\dot{v}}{v} = SA\frac{\dot{x}}{x}$; unde ſi Fluentes ſtatuantur æquales, menſura rationis inter denſitates d & v ad Modulum *AC*, æquabitur menſuræ rationis inter diſtantias *SF* & *SA* ad Modulum *SA*.

Si gravitationis ſit alia quævis Lex: quoniam eſt $AC\frac{\dot{v}}{v} = \frac{\dot{x}x^n}{SA^n}$; ſi Fluentes ſtatuantur æquales, erit $\frac{1}{n+1}$ in $\frac{SF^{n+1}}{SA^n} - SA$ menſura rationis inter denſitates d & v ad Modulum *AC*. Itaque ſi ſumantur in progreſſione Geometrica termini creſcentes SA, SF, $SF1$, $SF2$, &c. decreſcentes SF, SA, $Sf2$, $Sf3$, &c. menſura rationis inter denſitates Aeris in A & F ad Modulum AC, erit $\frac{1}{2}Af3$, ſi gravitatio ſit reciproce in triplicata ratione diſtantiæ; erit $Af2$, ſi gravitatio ſit reciproce in duplicata ratione diſtantiæ; erit AF, ſi gravitatio uniformis ſtatuatur; erit $\frac{1}{2}AF1$, ſi gravitatio ſit ut diſtantia; erit $\frac{1}{3}AF2$, ſi gravitatio ſit in duplicata ratione diſtantiæ. Et ſic proceditur in infinitum.

Denique ut plenius conſtet, Syntheticas etiam demonſtrationes ex elementis præmiſſis levi negotio concinnari poſſe; ſufficiet unicum inſuper addidiſſe Exemplum, tædet utique plura jam proferre. Repetatur itaque diviſio illa Nautica Meridianæ quam ſupra attigimus, & videamus etiam abſque ope Curvæ cujuſpiam Logometricæ, annon ſimplicior aliquanto ſit futura demonſtratio ad modum ſequentem. Sit $PXCQ$ Telluris axis, CO ſemidiameter Æquatoris, $PAOBQ$ Meridianus; & invenienda ſit in planiſphærio Nautico magnitudo cujuſvis arcus AB. Ad arcus illius terminos A & B ducantur ab alterutro Polorum P vel Q rectæ QA, QB, ſemidiametro CO occurrentes in D & E: Dico magnitudinem Nauticam arcus AB æqualem eſſe menſuræ rationis inter EC & DC ad Modulum OC. Nam diviſus intelligatur arcus AB in particulas quam minimas RS, & jungantur QR, QS quæ ſecent CO in T & V; & demiſſo in axem perpendiculo SX quod rectæ QR occurrat in Z, erit lineola SZ æqualis particulæ RS. Itaque magnitudo Nautica naſcentis arcus RS, erit ad Sphæræ ſemidiametrum OC, ut arcus ille RS ſive lineola SZ ad SX, hoc eſt, ut VT ad VC. Unde per *Corol.* 2. *Prop.* 1.) magnitudo illa Nautica æquatur menſuræ rationis inter VC & TC ad Modulum OC: & ſimiles utrobique ſummas colligendo, magnitudo Nautica totius arcus AB æquabitur menſuræ totius rationis inter EC & DC ad eundem Modulum OC. *Fig.* 35.

XXX. *A new Method and Table for Logarithms. Com. by Mr.* Long. *n.* 339. *p.* 52.

Log.	*Nat. Num.*
0,9	7.943282347
0,8	6.309573445
0,7	5.011872336
0,6	3.981071706
0,5	3.162277660
0,4	2.511886432
0,3	1.995262315
[illegible],2	1.584893193
0,1	1.258925412

Log.	*Nat. Num.*
0,09	1.230268771
0,08	1.202264435
0,07	1.174897555
0,06	1.148153621
0,05	1.122018454
0,04	1.096478196
0,03	1.071519305
0,02	1.047128548
0,01	1.023292992

Fig: 1

Fig: 2

Fig: 3

Fig: 7.

Fig: 4.

Fig: 5.

Fig: 6.

Fig. 8.

Fig: 9.

Fig: 18.

Fig: 10.

Fig: 11.

Fig: 12.

Fig: 13.

Fig: 14.

Fig: 15.

Fig: 16

Fig: 17.

Fig: 19.

Fig: 20.

Fig: 21.

Fig: 22.

Fig: 23

Fig: 24.

Fig: 25.

Fig: 26.

Fig: 27.

Fig. 28.

Fig: 29.

Fig: 30.

Fig: 31.

Fig: 31.

Fig: 31.

Fig: 32.

Fig: 33.

Fig: 34.

Fig: 35.

Fig: 30.

Fig: 36.

Fig: 37.

Fig: 38.

Fig: 4

Fig: 41.

Fig: 43.

Fig: 45

47

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50

Fig: 51.

Fig: 53.
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June 17 18
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Part I. Plate 3. *Page* 182

E
a
b
c
d
e
f
q
Fig. 64.
L
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R
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Fig. 39.
H
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Fig. 40.
L
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Fig: 54.
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Fig: 60.
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Fig: 62.
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14 Feb.
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16 Nov.
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6 March
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28 Nov
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1 Dec.
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17 Sept.
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4 Sept.
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Fig. 42.
Fig. 55.
56.
Fig. 57.
Fig. 44.
Fig: 61.
44
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Fig: 59.
46
c
a
b
48
a
b
Fig: 52.
June
July
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Fig. 63.
15 Jan.
16
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Nov.
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30 July
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23
24 June
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23
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Fig. A.
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Log.	*Nat Num.*	*Log.*	*Nat Num.*
0,009	1.020939484	0,000009	1.000020724
0,008	1.018591388	0,000008	1.000018421
0,007	1.016248694	0,000007	1.000016118
0,006	1.013911386	0,000006	1.000013816
0,005	1.011579454	0,000005	1.000011513
0,004	1.009252886	0,000004	1.000009210
0,003	1.006931669	0,000003	1.000006908
0,002	1.004615794	0,000002	1.000004605
0,001	1.002305238	0,000001	1.000002302
0,0009	1.002074475	0,0000009	1.000002072
0,0008	1.001843766	0,0000008	1.000001842
0,0007	1.001613109	0,0000007	1.000001611
0,0006	1.001382506	0,0000006	1.000001381
0,0005	1.001151956	0,0000005	1.000001151
0,0004	1.000921459	0,0000004	1.000000921
0,0003	1.000691015	0,0000003	1.000000690
0,0002	1.000460623	0,0000002	1.000000460
0,0001	1.000230285	0,0000001	1.000000230
0,00009	1.000207254	0,00000009	1.000000207
0,00008	1.000184224	0,00000008	1.000000184
0,00007	1.000161194	0,00000007	1.000000161
0,00006	1.000138165	0,00000006	1.000000138
0,00005	1.000115136	0,00000005	1.000000115
0,00004	1.000092106	0,00000004	1.000000092
0,00003	1.000069080	0,00000003	1.000000069
0,00002	1.000046053	0,00000002	1.000000046
0,00001	1.000023026	0,00000001	1.000000023

This Table I ſometimes make uſe of for finding the Logarithm of any Number propoſ'd, and *vice verſa*. Suppoſe I had occaſion to find the Logarithm of 2000. I look in the firſt Claſs of my Table (the whole Table conſiſts of 8 Claſſes) for the next leſs to 2, which is 1.995262315, and againſt it is 3, which conſequently is the firſt Figure of the Logarithm ſought. Again, dividing the Number propoſ'd 2, by 1.995262315 the Number found in the Table, the Quotient is 1.002374467; which being look'd for in the ſecond Claſs of the Table, and finding neither its equal, nor a leſſer, I add 0 to the part of the Logarithm before found, and look for the ſaid Quotient 1.002374467 in the third Claſs, where the next leſs is 1,002305238, and againſt it is 1, to be added to the part of the Logarithm already found; and dividing the Quotient 1.002374467, by 1.002305238, laſt found in the Table, the Quotient is 1,000069070; which being ſought in the fourth Claſs gives 0, but being ſought in the fifth Claſs gives 2, to be added to the part of the Logarithm already found; and dividing the laſt Quotient by the Number laſt found in the Table, *viz.* 1.000046053, the Quotient is 1.000023015, which being ſought in the ſixth Claſs, gives 9 to the part of the Logarithm already found: And dividing the laſt Quotient by the new Diviſor, *viz.* 1.000002072, the Quotient is 1.000000219, which being greater than 1.000000115, ſhews that the Logarithm already found, *viz.* 3,3010299 is leſs than the Truth by more than half an Unit; wherefore adding 1, you have *Briggs*'s Logarithm of 2000, *viz.* 3,3010300.

If any Logarithm be given, ſuppoſe 3,3010300, throw away the Characteriſtic, then over againſt theſe Figures 3 ... 0 .. 1 ... 0 .. 3 .. 0 .. 0, you have in their reſpective Claſſes 1,995262315 ..... 0 ..... 1,002305238 .... 0 ..... 1,000069080 .... 0 ... 0 which multiplied continually into one another, the Product is 2.00000001*9966*, which by reaſon the Characteriſtic is 3, becomes 2.000,00001*9966*, *&c.* that is, 2000, the natural Number deſired. I ſhall not mention the Method by which this Table is fram'd, becauſe you will eaſily ſee that from the uſe of it.

It is obvious to the intelligent Reader, that theſe Claſſes of Numbers are no other than ſo many Scales of mean Proportionals: In the firſt Claſs, between 1 and 10; ſo that the laſt Number thereof, *viz.* 1,258925412 is the tenth Root of 10, and the reſt in order aſcending are the Powers thereof. So in the ſecond Claſs, the laſt Number 1,023292992 is the hundredth Root of 10, and the reſt in the ſame manner are Powers thereof. So 1,002305238 in the third Claſs, is the tenth Root of the laſt of the ſecond, and the reſt its Powers, *&c.* Or, which is all one, each Number in the preceeding Claſs, is the tenth Power of the correſponding Number in the next following Claſs: Whence 'tis plain, that to conſtruct theſe Tables requires no more than one Extraction of the fifth or ſurſolid Root for each Claſs, the reſt of the Work being done by the common Rules of Arithmetick; and for extracting the fifth Root, you will find more than one very compendious Rule in *Num.* * 210 of theſe *Tranſactions*, if any one ſhall deſire to examin the *computus* of theſe Tables.

* *Abr.* Vol. I. *p.* 81.

The Doctrine of Combinations and Alternations improv'd, and compleated by Maj. Thornycroft. *n.* 299. *p.* 19[illegible]1.

XXXI. In order to underſtand what follows, it muſt be obſerved,

1ſt, That as in the notation of Powers, *aaaabbbcc* is deſign'd by $a^4 b^3 c^2$, and univerſally p times the poſition of a, q times the poſition of b, r times the poſition of c, by $a^p b^q c^r$, ſo in things expos'd likewiſe: (unleſs where 'tis propos'd they ſhould be all different) which Indices, as they have here no relation to Powers, but expreſs only the Occurrences of thoſe things to which they reſpectively belong; I therefore call Indices of Occurrences.

2dly, That as often as I ſhall hereafter mention the Combinations or Alternations of the p' q' r' or s', (which conſider'd by themſelves are capable of no variation) I mean of thoſe things whoſe Indices they are.

3dly, That m is generally put for the whole number of things expos'd, whether all different or not, *i. e.* equal to the ſum of their Indices; and n, for ſuch a number of them, as each Combination and Alternation muſt conſiſt of; (unleſs preſuppoſed equal) which explains what is hereafter meant by the Combinatious and Alternations of m things taken n and n; or of m things taken m and m; and the like Expreſſion, by whatever Symbols the number of things out of which the Combinations and Alternations are to be made, or of which they are to conſiſt, may be deſign'd.

Lemma 1ſt. If in a right Line, at any diſtances, be plac'd any number of things, $a\,b\,c\,d$, &c. the number of the Intervals ab, bc, cd, &c. terminated each by two adjacent things, is one leſs than the number of things.

For, whereas every Interval is terminated by two adjacent things, if to any number of things, be added one thing more, one Interval only is thereby added. *Q. E. D.*

Lemma 2. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken m and m, is m times the number of the Alternations of $m-1$ things $a\,b\,c$, taken $m-1$ and $m-1$.

For, (by *Lem.* 1ſt) the laſt Letter d, beſides the poſition it hath, may have $m-2$ poſitions, *viz.* in the Intervals which are between $m-1$ things $a\,b\,c$; but it may alſo have one more, for it may be put firſt of all, it may therefore have m poſitions; and thoſe in all the different Orders, whereof $m-1$ things are capable; which being all the poſſible poſitions of d, in all the varieties of $a\,b\,c$, is all the variety whereof the whole number of things expoſed $a\,b\,c\,d$, &c. is capable. *Q. E. D.*

Lemma 3d. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken m and m, is equal to $m\times m-1\times mm-2\times m-3\times m-4$, &c. continu'd to m places.

For let mO, expreſs the number of the Alternations of m things different each from other; $\overline{m-1}\,O$, of $m-1$ things and the like.

'Tis evident that if $m=1$, it will be $mO=m$; for there can be but one order of one thing.

And if m be greater than unity, then will it be (by *Lem.* 2.) $mO=m\times\overline{m-1}\,O=m\times m-1\times\overline{m-2}\,O=m\times m-1\times m-2\times\overline{m-3}\,O=$, &c. till we have an Equation conſiſting of m places; *i. e.* $=m\times m-1\times m-2\times m-3\times$ &c. continu'd to m places. *Q. E. D.*

Lemma 4th. If $m\omega$ expreſs the number of the Alternations of m things $a^p\,b^p\,c^p\,d^q\,e^r\,f^r$ &c. taken m and m, and α the number of p's, β the number of q's, γ the number of r's, it will be $m\omega=\dfrac{m\times m-1\times m-2\times m-3\times m-4\times m-5\times \&c.}{\overline{p\times p-1\times p-2\times \&c.}^{\,\alpha}\times\overline{q\times q-1\times \&c.}^{\,\beta}\times\overline{r\times r-1\times \&c.}^{\,\gamma}}$ continu'd to m places, each Series continued to p, q, r, &c. places reſpectively.

For the number of the Alternations of any number of things, however divided into parts, is produc'd by a continual Multiplication of the Alternations of thoſe things amongſt themſelves reſpectively, which compoſe each part, into the number of their Alternations one amongſt the other; *i. e.* in the preſent Caſe (the ſeveral occurrences being ſuppoſed to compoſe the ſeveral Parts, and conſequently the number of the Alternations of the things compoſing each part equal to unity) $m\omega=$ to the number of the Alternations of the things compoſing the parts one amongſt the other; but the number of their Alternations one amongſt the other, is the ſame in this Caſe, as if the things expos'd, being all different, were divided into the ſame parts; for the things which compoſe each part in both Caſes, are different from the reſt of the things expos'd; *i. e.* by *Lem.* 3*d*.

$$m\,\omega = \frac{m\times m-1\times m-2\times m-3\times m-4\times m-5\times \&c. \text{ continu'd to } m \text{ places}}{\overline{p\times p-1\times p-2\times \&c}|^{\alpha}\times\overline{q\times q-1\times \&c.}|^{\beta}\times\overline{r\times r-1\times \&c.}|^{\gamma}}$$ each Series continu'd to p, q, r places reſpectively. *Q. E. D.*

Lemma 5th. The number of the Combinations of m things $a\,b\,c\,d$, &c. different each from other, taken n and n, is equal to $\frac{m\times m-1\times m-2\times m-3\times \&c.}{n\times n-1\times n-2\times n-3\times \&c.}$ each Series continued to n places.

For if the things expos'd be divided in two parts, *viz.* in the *Ratio* of n and $m-n$, 'tis evident that their different Combinations taken n and n, are produc'd by the Alternations of the things compoſing the Parts one amongſt the other: And therefore the number of thoſe = to the number of theſe = to the number of the Alternations of m things taken m and m, the Indices of whoſe occurrences are n and $m-n =$ $\frac{m\times m-1\times m-2\times m-3\times \&c. \text{ continued to } m \text{ places.}}{n\times n-1\times \&c.\times m-n\times m-n-1\times \&c.}$ each Series continued to n and $m-n$ places reſpectively (by *Lem.* 4th) *i. e.* becauſe $n+m-n=m=$ $\frac{m\times m-1\times m-2\times m-3\,\&c.}{n\times n-1\times n-2\times n-3\,\&c.}$ each Series continued to n places. *Q. E. D.*

But the number of the Alternations in every Combination is $= n\times n-1\times n-2\times n-3\times$, &c. continued to n places, by *Lem.* 3d therefore

Lemma 6th. The number of the Alternations of m things $a\,b\,c\,d$, &c. different each from other, taken n and n, is $= m\times m-1\times m-2\times m-3\times$, &c. continued to n places. *Q. E. D.*

Scholium. Since in the things expos'd the ſame things may occur more than once, and alſo n be leſs than m, the Indices of the occurrences which are in ſome of the Combinations of m things taken n and n, may differ from thoſe which are in others; but thoſe Combinations, the Indices of whoſe occurrences are the ſame, are ſaid to be in the ſame Form: Therefore, whereas n is equal to the Sum of the Indices, which are in each Combination taken n and n, if n be expreſs'd by all the different Combinations of ſuch Indices only (being integer Numbers) whereof no one may exceed the higheſt Index of the things expos'd, and being more than one in a Combination, are each of them, which are in the ſame Combination, comprehended in a diſtinct Index thereof; theſe Expreſſions of n will neceſſarily be the ſeveral Forms of the Combinations taken n and n, whereof m things are capable: Whence is deriv'd a General Theorem for finding the Combinations and Alternations of m things taken n and n univerſally: *i. e.* Whether m conſiſt of things all different or not, and whether n be equal to, or leſs than m.

Theorem. If n be expreſs'd, according to all the different Forms of Combination which the things expos'd are capable of,

$$\text{and}\left\{\begin{array}{l} p = \text{the higheſt Index} \\ q = \text{the next higheſt} \\ r = \text{the next higheſt} \\ ſ = \text{the next higheſt} \\ \quad \&c. \end{array}\right. \left\{\begin{array}{l} \alpha = \text{the number of } p\text{'s} \\ \beta = \text{the number of } q\text{'s} \\ \gamma = \text{the number of } r\text{'s} \\ \delta = \text{the number of } ſ\text{'s} \end{array}\right\} \begin{array}{l} \text{In every form of} \\ \text{Combination.} \end{array}$$

$$\left\{\begin{array}{l} A = \text{the number of all Indices not leſs than } p \\ B = \text{the number of all Indices not leſs than } q \\ C = \text{the number of all Indices not leſs than } r \\ D = \text{the number of all Indices not leſs than } ſ \\ \quad \&c. \end{array}\right\} \begin{array}{l} \text{Which are in} \\ \text{the things Expos'd} \end{array}$$

and $b = \alpha + \beta$, $c = b + \gamma$, $d = c + \delta$, &c.

I ſay the number of the Combinations of m things taken n and n, in any one Form of Combination, ſhall be $\frac{A \times A-1 \times A-2}{\alpha \times \alpha-1 \times \alpha-2}$ &c. $\times$ $\frac{B-\alpha \times B-\alpha-1}{\beta \times \beta-1} \times \&c. \times \frac{C-b \times C-b-1}{\gamma \times \gamma-1} \times \&c. \times \frac{D-c \times D-c-1}{\delta \times \delta-1}$ &c. continued to ſo many Terms as there are different Indices in the form of Combination, and each Term to $\alpha, \beta, \gamma, \delta$, &c. places reſpectively, and this number multiply'd into

$$\frac{\times n-1 \times n-2 \times n-3 \times n-4 \times n-5 \times n-6\ \&c.\ \text{continu'd to } n \text{ places}}{\overline{p \times p-1 \times p-2 \times \&c.}|^{\alpha} \times \overline{q \times q-1 \times \&c}|^{\beta} \times 1 \times r-1 \times \&c|^{\gamma} \times \&c.}$$

each Series continued to p, q, r, &c. places reſpectively, ſhall be the number of their Alternations.

But the Sum of all the Combinations and Alternations which are in every Form of $\bar{n}$, ſhall be the whole number of Combinations and Alternations of m things taken n and n.

Demonſtration. Firſt, Then 'tis evident, that thoſe Combinations, which are in different Forms, differ from each other.

Again, 'Tis evident that the Combinations of m things, as $a^p\ b^p\ c^p\ d^p\ e^q\ f^q\ g^q\ h^r\ i^r$, &c. (the Indices ſimply conſider'd) taken n and n, in a Form wherein are p's q's and r's, ſhall be equal to the number of the Combinations of the p's, which are in the things expos'd, taken α and α, multiply'd into the number of the Combinations of the q's taken β and β multiply'd into the number of the Combinations of the r's taken γ and γ.

But becauſe p and all leſſer Indices are comprehended in every Index which is greater than themſelves; therefore is $A =$ to the number of p's which are in the things expos'd, and for the ſame reaſon, would $B =$ the number of the q's, and C the number of r's: But the number of the p's, which are in every Form of Combination, is $= \alpha$; therefore is $B - \alpha =$ to the number of q's; alſo becauſe the number of p's and q's together, which are in every Form of Combination, wherein there are q's, is $= \alpha + \beta = b$; therefore is $C - b =$ to the number of r's, and ſo on, how many ſoever were the different Indices in any form of Combination.

But (by *Lem.* 5th) the number of the Combinations of the p', which are in the things expos'd, whoſe number is A, taken α and α, is $= \frac{A \times A - 1 \times A - 2}{\alpha \times \alpha - 1 \times \alpha - 2}$ &c. continu'd to α places, and the number of the Combinations of the q', whoſe number is $B - \alpha$, taken β and β, is $= \frac{B - \alpha \times B - \alpha - 1}{\beta \times \beta - 1} \frac{\times B - \alpha - 2}{\times \beta - 2}$ &c. continued to β places, and the number of the Combinations of the r', whoſe number is $C - b$, taken γ and γ, is $= \frac{C - b \times C - b - 1}{\gamma \times \gamma - 1}$ &c. continued to γ places. *Q. E. D.*

But every Combination, in one and the ſame Form, affords the ſame number of Alternations: Therefore the number of Alternations, in any one Form, is ſo many times the number of Combinations, as is the number of Alternations in any one of theſe Combinations.

But (by *Lem.* 4th) the number of Alternations in any of thoſe Combinations ſhall be

$$\frac{n \times n - 1 \times n - 2 \times n - 3 \times n - 4 \times n - 5 \times n - 6 \times \&c. \text{ continued to } n \text{ places}}{\left.\overline{p \times p - 1 \times p - 2 \times \&c.}\right|^{\alpha} \times \left.\overline{q \times q - 2 \times \&c}\right|^{\beta} \left.\overline{r \times r - 1 \times \&c.}\right|^{\gamma}}$$

× each Series continued to $p\ q\ r$ &c. places reſpectively. *Q. E. D.*

Now to make an application of this general Rule to thoſe particular Caſes which have already been conſider'd by others, and which are contain'd in our *3d*, *4th*, *5th*, and *6th Lemma's*, and by us more generally demonſtrated; I ſay

If $n = m$, there can be but one form of Combination, and but one Combination in that form; and therefore the number of Alternations =

$$\frac{m \times m - 1 \times m - 2 \times m - 3 \times m - 4 \times \&c. \text{ continu'd to } m \text{ places}}{\left.\overline{p \times p - 1 \times \&c.}\right|^{\alpha} \times \left.\overline{q \times q - 1 \times \&c.}\right|^{\beta} \times \left.\overline{r \times \&c.}\right|^{\gamma} \times \&c.}$$

each Series to $p\ q\ r$, &c. places reſpectively, *i. e.* (if $p = 1$) $= m \times m - 1 \times m - 2 \times m - 3 \times m - 4 \times$ &c. continu'd to m places, which are the Caſes of the 4th and 3d *Lemma's.*

But if the things expos'd are all different, and n be leſs than m, which is the caſe of the 5th and 6th *Lemma*'s, then alſo can there be but one form of Combination, and it will be $A = m$ and $\alpha = n$, and the whole number of Combinations $= \frac{A \times A - 1 \times A - 2 \times \&c.}{\alpha \times \alpha - 1 \times \alpha - 2 \times \&c.}$ *i. e.* $= \frac{m \times m - 1 \times m - 2 \times \&c.}{n \times n - 1 \times n - 2 \times \&c.}$ each Series continued to n places, and therefore the number of Alternations $= m \times m - 1 \times m - 2 \times$ &c. continu'd to n places.

To illuſtrate this *Theorem*, which, as delivered in general, may ſeem ſomewhat too Abſtracted to be commonly underſtood, I ſhall ſubjoyn one ſhort Example.

Example. Let the things expos'd be $aaabbbcc$, or according to our way of notation $a^3\,b^3\,c^2$; 'Tis requir'd to find the number of their Combinations and Alternations taken 4 and 4.

Then (becauſe in the things expos'd, there is no one thing occurs more than thrice, nor more than three things different from each other) will all the forms of Combination, which the things expos'd are capable of,

$$\text{be theſe, } viz. \left\{\begin{matrix} 3\,.\,1 \\ 2\,.\,2 \\ 2\,.\,1\,.\,1 \end{matrix}\right\}\text{Then.}$$

In the firſt form will $p=3,\ q=1,\ \alpha=1,\ \beta=1,\ A=2,\ B=3$

In the ſecond form will $p=2$, ——, $\alpha=2$, ——, $A=3$, ——.

In the third form will $p=2;\ q=1;\ \alpha=1,\ \beta=2,\ A=3,\ B=3$

The number of Combinations in the firſt Form $=\dfrac{A}{\alpha}\times\dfrac{B-\alpha}{\beta}=\dfrac{2}{1}\times\dfrac{2}{1}=4$

The number of Combinations in the 2d Form $=\dfrac{A\times\overline{A-1}}{\alpha\times\overline{\alpha-1}}=\dfrac{3\times2}{2\times1}=3$

The number of Combinations in the third Form $=\dfrac{A}{\alpha}\times\dfrac{\overline{B-\alpha}\times\overline{B-\alpha-1}}{\beta\times\overline{\beta-1}}=\dfrac{2\times1}{2\times1}=3$

And the whole number of Combinations $=10$

Alſo the number of Alternations.

In the firſt Form $=4\times\dfrac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}\times\overline{p-2}}|^{\alpha}\times q^{\beta}}=4\times\dfrac{4\times3\times2\times1}{\overline{3\times2\times1}|^{1}}=4\times4=16$

In the 2d Form $=3\times\dfrac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}}|^{\alpha}}=4\times\dfrac{4\times3\times2\times1}{\overline{2\times1}|^{2}}=3\times6=18$

In the 3d Form $=3\times\dfrac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}{\overline{p\times\overline{p-1}}|^{\alpha}\times q^{\beta}}=3\times\dfrac{4\times3\times2\times1}{\overline{2\times1}|^{1}\times1}=3\times12=36$

And the whole number of Alternations $=70$

Many are the Properties of this *Theorem*, in common with others, as, to find the *Unciæ* of a Multinomial rais'd to any integer Power. To raiſe an infinite Series to an integer Power, though of an interrupted Order, without introducing any thing immaterial, or which muſt afterwards be expung'd; and many others. But then ſo many Terms of the Series muſt be taken in at firſt as ſhall ſerve to the purpoſes of the intended Approximation, otherwiſe as often as it ſhall fall ſhort of that, the Operation muſt be begun *de novo*.

Many likewiſe are the Properties peculiar to this *Theorem*, and great variety of Problems might be fram'd; and I ſcruple not to ſay, many may occur in Practice, which are ſolvable by this, and no other Method whatever.

Hence may be found the number of Words whereof the 24 Letters are capable, from one Letter in each Word, to any number of Letters given.

Hence may be found the number of all Numbers, to any given number of Places, which may be produc'd from any number of Figures given.

Hence alſo the Compaſs of a Muſical Inſtrument being given, the Time and number of the Bars, whereof each Tune ſhall conſiſt, the number of Tunes may be found which that Inſtrument is capable of.

To give an inſtance of the prodigious variety that there is in Muſick, I have calculated the number of Tunes in Common Time, conſiſting of eight Bars each, which may be play'd on an Inſtrument of one Note Compaſs only, and it is this, *viz*, 27584 270157. 013570. 368586. 999728. 299176. whereas the Changes on 24 Bells is but 620448. 401733. 239439. 360000, which is but $\frac{1}{444588.\ 604583}$ of the number of Tunes, and yet Dr. *Wallis* in his *Algebra* demonſtrates, could not be diſpatch'd in 31557. 600000. 000000 years.

If then the Inſtrument were of as many Notes Compaſs as any Inſtrument now in uſe, how prodigiouſly muſt the number of Tunes be encreas'd; the Calculation of which (tho' much more intricate and operoſe) would be equally attainable by our *Theorem.*

Of the Laws of Chance. by Mr. De Moivre. *n.* 329. *p.*215.

XXXII. 1. Si p ſit numerus caſuum quibus eventus aliquis contingere poſſit, & q numerus caſuum quibus poſſit non-contingere; tam contingentia quam non-contingentia eventus ſuum habent probabilitatis gradum: Quod ſi caſus omnes quibus eventus contingere vel non-contingere poteſt, ſint æque faciles; probabilitas contingentiæ erit ad probabilitatem non-contingentiæ ut p ad q.

Si A & B, colluſores duo ita de eventibus certent, ut ſi caſus p contingant, A vicerit; ſin caſus q contingant, B vicerit; atque ſit a ſumma depoſita, ſors ſeu expectatio ipſius A erit $\frac{pa}{p+q}$, ſors vero ſeu expectatio ipſius B erit $\frac{qa}{p+q}$, adeoque ſi A vel B expectationes ſuas vendant, æquum eſt ut pro illis recipiant $\frac{pa}{p+q}$ & $\frac{qa}{p+q}$ reſpective.

Si præmium aliquod a proponatur victori concedendum, ita ut ſi caſus p contigerint, præmium concedatur ipſi A, ſin vero caſus q contigerint, præmium ipſi B concedatur, atque A & B hoc pactum ineant, ut ante eventum, præmium dividatur pro ratione ſortium, A debebit ſumere partem $\frac{pa}{p+q}$, B vero partem $\frac{qa}{p+q}$.

Si eventus duo nullo modo ex ſe invicem pendeant, ita ut p ſit numerus caſuum quibus eventus primus contingere poſſit, & q numerus caſuum quibus poſſit non-contingere; & ſit r numerus caſuum quibus eventus ſe-

cundus contingere poſſit, & s numerus caſuum quibus poſſit non-contingere: Multiplicetur $p+q$ per $r+s$, & Productum Multiplicationis, *viz.* $pr+qr+ps+qs$ erit numerus caſuum omnium quibus contingentia & non-contingentia eventuum inter ſe variari poſſunt.

Ergo ſi *A* & *B* inter ſe ita de his eventibus certent, ut *A* contendat fore ut uterque contingat, ratio ſortium erit ut pr ad $qr+ps+qs$.

Sed ſi *A* contendat fore ut alteruter contingat, ratio ſortium erit ut $pr+qr+ps$ ad qs.

Si vero *A* contendat fore ut eventus primus contingat, ſecundus autem non contingat, ratio ſortium erit ut ps ad $pr+qr+qs$.

Et eodem argumentandi modo, ſi tres vel plures ſint eventus de quibus *A* & *B* certent, ratio ſortium invenietur Multiplicatione ſola.

Si eventus omnes habeant datum numerum caſuum quibus contingere poſſint, & datum itidem numerum caſuum quibus poſſint non-contingere, & ſit a numerus caſuum quibus eventus aliquis poſſit contingere, & b numerus caſuum quibus poſſit non-contingere, & ſit n numerus eventuum omnium; elevetur $a+b$ ad poteſtatem n.

Et ſi *A* cum *B* certet ea conditione ut ſi eventus unus vel plures contigerint, ipſe *A* vicerit; ſin nullus, tum *B* vicerit; ratio ſortium erit ut $\overline{a+b}|^n - b^n$ ad b^n; etenim terminus unicus, in quo a non reperitur eſt b^n.

Si *A* cum *B* certet ea conditione, ut ſi eventus duo vel plures contigerint, *A* vicerit; ſin nullus vel unus, tum *B* vicerit; ratio ſortium erit ut $\overline{a+b}|^n - b^n - nab^{n-1}$, ad $b^n + nab^{n-1}$: Etenim termini duo in quibus aa non reperitur, ſunt b^n & nab^{n-1}; & ſic deinceps de cæteris.

Prob. I. A & B *una teſſera ludunt, ea conditione, ut ſi* A *bis vel pluries, octo jactibus teſſeræ monada jecerit, ipſe* A *vincat; ſin ſemel tantum, vel non omnino,* B *vincat; quænam erit ratio ſortium?*

Solutio. Quoniam eſt caſus unicus quo monas contingere poteſt, & quinque caſus quibus poteſt non-contingere, fiat $a=1$, & $b=5$. Rurſus quoniam ſunt octo jactus teſſeræ, fiat $n=8$, & erit $\overline{a+b}|^n - b^n - nab^{n-1}$ ad $b^n + nab^{n-1}$ ut 663991 ad 1015625. hoc eſt, ut 2 ad 3 circiter.

Prob. II. A & B *ſingulis globis ea conditione certant, ut qui globum propius ad metam miſerit, unum ludum vincat; jam poſt ludos aliquot peractos, ipſi* A *deſunt ludi* 4 *quo minus victor abeat, ipſi vero* B, 6; *at ea eſt ipſius* A *in mittendis globis dexteritas, ut ſors illius foret ad ſortem ipſius* B *ut* 3 *ad* 2, *ſi de unico ludo contenderent; quænam eſt ratio ſortium in caſu propoſito?*

Solutio. Quoniam ipſi *A* deſunt 4 ludi quominus victor abeat, ipſi vero *B* 6, ſequitur fore ut certamen futuris concludatur ludis ad plurimum 9 videlicet ſumma deficientium ludorum minus unitate, ergo elevetur $a+b$ ad poteſtatem nonam, hæc erit, $a^9a + 9a^8b + 36a^7bb + 84a^6b^3 + 126a^5b^4 + 126a^4b^5 + 84a^3b^6 + 36aab^7 + 9ab^8 + b^9$. Et ſumantur pro *A* termini omnes in quibus a habet 4 vel plures dimenſiones, & pro *B* termini omnes in quibus *B* habet 6 vel plures dimenſiones, ergo ratio ſortium erit ut $a^9 + 9a^8b + 36a^7bb + 84a^6b^3 + 126a^5b^4 + 126a^4b^5$ ad $84a^3b^6 + 36aab^7 + 9ab^8 + b^9$. Exponatur a per 3, & b per 2, & habebitur ratio ſortium in numeris, videlicet 1759077 ad 194048.

Et generaliter, poſito quod p & q ſint numeri deficientium ludorum reſpective; elevetur $a + b$ ad poteſtatem $p + q - 1$, & ſumantur pro A & B reſpective tot termini quot ipſis deſunt ludi reciproce, hoc eſt, pro A ſumantur tot termini quot ſunt unitates in q, pro B vero tot termini quot ſunt unitates in p.

Prob. III. Si A & B *ſingulis globis ludant, & ea ſit ipſius* A *in mittendis globis dexteritas, ut poſſit ipſi* B *duos ludos ex tribus largiri; quæritur quænam foret ratio ſortium ſi de de ludo uno contenderent.*

Solutio. Sint ſortes quæſitæ ut z ad 1, & elevetur $z + 1$ ad Cubum; hic erit, $z^3 + 3zz + 3z + 1$. Jam cum A poſſit duos ludos ex tribus ipſi B largiri, A in ſe id ſuſcipere poterit, ut tres ludos continuos vincat, adeoque ſortes hoc in caſu erunt ut z^3 ad $3zz + 3z + 1$. Ergo $z^3 = 3zz + 3z + 1$. Sive $2z^3 = z^3 + 3zz + 3z + 1$. Ergo $z\sqrt[3]{2} = z + 1$, adeoque $z = \frac{1}{\sqrt[3]{2} - 1}$: Igitur ſortes quæſitæ erunt $\frac{1}{\sqrt[3]{2} - 1}$ & 1 reſpective.

Et generaliter, ſi ea ſit ipſius A dexteritas, ut poſſit æquali ſorte in ſe ſuſcipere ut n vices continuas vincat, A poterit deponere $\frac{1}{\sqrt[n]{2} - 1}$ contra 1, fore ut prima vice vincat.

Prob. IV. *Si* A *poſſit æqua ſorte unum ex tribus ludis ipſi* B *largiri, quæritur ratio ſortium ipſorum* A & B *cum de ludo unico contendunt, hoc eſt requiritur ratio dexteritatum.*

Solutio. Sit ratio dexteritatum ut z ad 1. Si autem A unum ludum ex tribus ipſi B largiatur, ergo ſuſcipit A ſe ter victurum, priuſquam B bis vicerit; elevetur itaque $z + 1$ ad poteſtatem quartam, videlicet, $z^4 + 4z^3 + 6zz + 4z + 1$, ergo ratio ſortium erit ut $z^4 + 4z^3$ ad $6zz + 4z + 1$; Ergo cum æqua ſorte contendant, fiat $z^4 + 4z^3 = 6zz + 4z + 1$; Qua æquatione ſoluta, obtinebitur $z = 1.6$ prope. Ergo ratio dexteritatum erit circiter ut 8 ad 5.

Prob. V. *Invenire quotenis tentaminibus futurum ſit probabile ut eventus aliquis contingat, poſito quod ſint caſus* a *quibus primo tentamine contingere poſſit, & caſus* b *quibus poſſit non-contingere, ita ut ſi* A & B *de eventu contendant, poſſint* A & B *æqua ſorte eventum affirmare & negare.*

Solutio. Sit x numerus tentaminum quibus eventus aliquis poſſit æquali expectatione contingere vel non-contingere, ergo per jam demonſtrata erit $\overline{a+b}|^x - b^x = b^x$, ſive $\overline{a+b}|^x = 2b^x$, ergo $x = \frac{\text{Log. } 2.}{\text{Log. } \overline{a+b} - \text{Log. } b}$

Inſuper reſumatur æquatio $\overline{a+b}|^x = 2b^x$, & ſit $a : b :: 1 : q$, & æquatio migrat in iſtam, $\overline{1 + \frac{1}{q}}|^x = 2$. Elevetur $1 + \frac{1}{q}$ ad poteſtatem x, ope

Theorematis *Newtoniani*, & fiet $1 + \frac{x}{q} + \frac{x}{1} \times \frac{x-1}{2qq} + \frac{x}{1} \times \frac{x-1}{2} \times \frac{x-2}{3q^3}$ *&c.* $= 2$. In hac æquatione ſi ſit $q = 1$, erit $x = 1$; ſi q ſit infinita, erit x infinita. Sit x infinita, ergo æquatio ſuperior fiet, $1 + \frac{x}{q} + \frac{xx}{2qq} + \frac{x^3}{6q^3}$ *&c.* $= 2$. Iterum ſit $\frac{x}{q} = z$, & erit $1 + z + \frac{1}{2}zz + \frac{1}{6}z^3$ *&c.* $= 2$. Sed $1 + z + \frac{1}{2}zz + \frac{1}{6}z^3$ *&c.* eſt numerus cujus Logarithmus Hyperbolicus eſt z, ergo $z = Log.\ 2$. Sed Logarithmus Hyperbolicus ipſius 2 eſt 7 proxime, ergo $z = 7$ proxime.

Igitur ubi q eſt 1, erit $x = 1q$; & ubi q eſt infinita, erit $x = 7q$ proxime.

Jam ergo definivimus limites arctiſſimos intra quos ratio x ad q conſiſtet, etenim ratio illa orditur ab æqualitate, & cum ad infinitum eſt provecta, deſinit tandem in ratione 7 ad 10 proxime.

Exemp. I. *Inveniendum ſit quotenis jactibus* A *ſuſcipere in ſe poſſit, ut duas monadas duabus teſſeris jaciat.*

Solutio. Quoniam *A* habet caſum unicum quo duas monadas jacere poſſit, & 35 quibus illas non jaciat, erit $q = 35$; Multiplicetur igitur 35 per .7, & productum 24.5 indicabit numerum jactuum quæſitum fore inter 24 & 25.

Exemp. II. *Inveniendum ſit quotenis jactibus* A *ſuſcipere in ſe poſſit, ut tres monadas tribus teſſeris jaciat.*

Solutio. Quoniam *A* habet caſum unicum quo monadas tres tribus teſſeris jacere poſſit, & caſus 215 quibus illas non jaciat; Multiplicetur 215 per .7, & productum 150.5 indicabit numerum jactuum quæſitum fore inter 150 & 151.

Lemma. *Invenire numerum caſuum quibus datus punctorum numerus dato teſſerarum numero jaci poſſit.*

Solutio. Sit $p + 1$ datus punctorum numerus, n numerus teſſerarum, f numerus facierum in teſſera: fiat $p - f = q$, $q - f = r$, $r - f = s$, $s - f = t$, *&c.* Numerus caſuum quæſitus erit,

$$+ \frac{p}{1} \times \frac{p-1}{2} \times \frac{p-2}{3} \text{ \&c.}$$

$$- \frac{q}{1} \times \frac{q-1}{2} \times \frac{q-2}{3} \text{ \&c.} \times \frac{n}{1}.$$

$$+ \frac{r}{1} \times \frac{r-1}{2} \times \frac{r-2}{3} \text{ \&c.} \times \frac{n}{1} \times \frac{n-1}{2}.$$

$$- \frac{s}{1} \times \frac{s-1}{2} \times \frac{s-2}{3} \text{ \&c.} \times \frac{n}{1} \times \frac{n-1}{2} \times \frac{n-2}{3},$$

Quam ſeriem continuari oportebit, donec aliqui factorum fiant vel æquales nihilo, vel negativi.

N.B. Tot factores ſingulorum productorum, $\frac{q}{1} \times \frac{p-1}{2} \times \frac{p-2}{3}$ &c. $\frac{r}{1} \times \frac{r-1}{2} \times \frac{r-2}{3}$ &c. $\frac{s}{1} \times \frac{s-1}{2} \times \frac{s-2}{3}$ &c. ſumendi ſunt, quot ſunt unitates in $n-1$.

Praxis. Requiratur *v.g.* numerus caſuum, quibus 16 puncta 4 teſſeris jaci poſſint.

$$+\frac{15}{1} \times \frac{14}{2} \times \frac{13}{3} = +455$$

$$-\frac{9}{1} \times \frac{8}{2} \times \frac{7}{3} \times \frac{4}{1} = -336$$

$$+\frac{3}{1} \times \frac{2}{2} \times \frac{1}{3} \times \frac{4}{1} \times \frac{3}{2} = +\ 6$$

Jam $455 - 336 + 6 = 125$. Ergo 125 eſt numerus caſuum quæſitus.

Requiratur numerus caſuum quibus 15 puncta 6 teſſeris jaci poſſint.

$$+\frac{14}{1} \times \frac{13}{2} \times \frac{12}{3} \times \frac{11}{4} \times \frac{10}{5} = +2002$$

$$-\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} \times \frac{4}{5} \times \frac{6}{1} = -\ 336$$

Jam $2002 - 336 = 1666$ numerus caſuum quæſitus.

Requiratur numerus caſuum quibus 27 puncta 6 teſſeris jaci poſſint.

$$+\frac{26}{1} \times \frac{25}{2} \times \frac{24}{3} \times \frac{23}{4} \times \frac{22}{5} = +65780$$

$$-\frac{20}{1} \times \frac{19}{2} \times \frac{18}{3} \times \frac{17}{4} \times \frac{16}{5} \times \frac{6}{1} = -93024$$

$$+\frac{14}{1} \times \frac{13}{2} \times \frac{12}{3} \times \frac{11}{4} \times \frac{10}{5} \times \frac{6}{1} \times \frac{5}{2} = +30030$$

$$-\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} \times \frac{4}{5} \times \frac{6}{1} \times \frac{5}{2} \times \frac{4}{3} = -\ 1120.$$

Jam $65780 - 93024 + 30030 - 1120 = 1666$ numerus caſuum quæſitus.

Corollarium. Puncta omnia æqualiter ab extremis diſtantia habent eundem numerum caſuum quibus producantur, adeoque ſi numerus punctorum datus vicinior ſit majori extremo quam minori, ſubtrahatur numerus iſte ex ſumma extremorum, & inveniatur numerus caſuum quibus reſiduus numerus producatur, & fiet operatio brevior.

Exem. III. *Invenire quotenis jactibus A ſuſcipere in ſe poſſit ut 15 puncta 6 teſſeris jaciat.*

Solutio. Quoniam *A* habet caſus 1666 quibus jacere poſſit 15 puncta, & 44990 quibus illa non jaciat, dividatur 44990 per 1666, & quotus 27 erit $= q$. Ergo multiplicetur 27 per 7, & productum multiplicationis, 18,9 indicabit numerum jactuum quæſitum eſſe 19 fere.

Prob. IV. *Invenire quotenis tentaminibus futurum ſit probabile, ut eventus aliquis bis contingat, poſito quod ſint caſus* a *quibus primo tentamine contingere poſſit, & caſus* b *quibus poſſit non-contingere; ita ut ſi* A *&* B *de eventu contendant, poſſint* A *&* B *æqua ſorte eventum affirmare & negare.*

Solutio. Sit x numerus tentaminum, ergo per jam demonſtrata patebit fore $\overline{a+b}|^{x} = 2b^{x} + 2axb^{x-1}$. Sive faciendo $a:b::1:q$, $\overline{1+\frac{1}{q}}|^{x} = 2 + \frac{2x}{q}$. 1°. Sit $q = 1$, & erit $x = 3$. 2°. Sit q infinita, & erit x infinita: Pone x infinitam, & $\frac{x}{q} = z$, & erit $1 + z + \frac{1}{2}z^{2} + \frac{1}{6}z^{3}$, &c. $= 2 + 2z$, adeoque $z = Log.\ 2. + Log.\ \overline{1+z}$; jam ſi $Log.\ 2.$ vocetur y, æquatio iſta in hanc Fluxionalem transformabitur $\frac{\dot{z}z}{1+z} = \dot{y}$. Si autem valor ipſius z, inveſtigetur per Poteſtates ipſius y, invenietur $z = 1.678$ proxime, ergo x ſemper conſiſtet intra limites $3q$ & $1.678q$; ſed x citiſſime converget ad $1.678q$, adeoque ſi q ad 1 habuerit rationem non adeo parvam, poterit aſſumi $x = 1.678q$. Si vero ſit aliqua ſuſpicio ne x ſit juſto minor, ſubſtituatur ipſius valor in æquatione $\overline{1+\frac{1}{q}}|^{x} = 2 + \frac{2x}{q}$, & notetur error, ſi quis ſit notatu dignus, tunc augeatur x aliquantulum, & ſubſtituatur valor ſic auctus pro x in prædicta æquatione, & notetur novus error, & ope duorum errorum, valor ipſius x poterit ſatis accurate corrigi.

Exemp. I. *Inveniendum ſit quotenis vicibus,* A *in ſe ſuſcipere poſſit, ut tres monadas tribus teſſeris bis jaciat.*

Solutio. Quoniam *A* caſum habet unicum quo tres monadas jaciat, & 215 quibus illas non jaciat, erit $q = 215$: Ergo multiplicetur 215 per 1.678, & productum multiplicationis 360.7 indicabit numerum jactuum quæſitum, fore inter 360 & 361.

Exemp. II. *Inveniendum ſit quotenis vicibus,* A *in ſe ſuſcipere poſſit ut* 15 *puncta,* 6 *teſſeris bis jaciat.*

Solutio. Quoniam *A* habet caſus 1666 quibus jacere poſſit 15 puncta, & 44990 quibus illa non jaciat; dividatur 44990 per 1666, & quotus 27 erit $= q$: Ergo multiplicetur 27 per 1.678, & productum multiplicationis 45.3, indicabit numerum jactuum quæſitum, fore inter 45 & 46.

Prob. VII. *Invenire quotenis temtaminibus futurum ſit probabile, ut eventus aliquis, ter, quater, quinquies, &c. contingat, poſito quod ſint caſus* a *quibus primo tentamine contingere poſſit, & caſus* b *quibus poſſit non contingere.*

Solutio. Sit x numerus tentaminum quæſitus; & ex jam demonſtratis ſi de triplici eventu contendatur, facto $a : b :: 1 : q$, erit $\overline{1+\frac{1}{q}}\Big|^{x} = 2 \times \overline{1+\frac{x}{q}+\frac{x}{1}\times\frac{x-1}{2qq}}$. Si de quadruplici, $\overline{1+\frac{1}{q}}\Big|^{x} = 2 \times \overline{1+\frac{x}{q}+\frac{x}{1}\times\frac{x-1}{2qq}+\frac{x}{1}\times\frac{x-1}{2}\times\frac{x-2}{3q^3}}$. Et continuatio iſtarum æquationum eſt manifeſta. Jam in priori æquatione, ſi ſit $q=1$, erit $x=5q$; ſi vero q ſit infinita, vel ad unitatem habuerit rationem ſatis magnam, æquatio prædicta, ponendo $\frac{x}{q}=z$, migrabit in iſtam $z = Log.\,2 + Log.\,1+z+\frac{1}{2}z^2$, vel in iſtam Fluxionalem poſito $Log.\,2=y$, $\frac{\frac{1}{2}z^2\dot{z}}{1+z+\frac{1}{2}zz}=\dot{y}$; ubi reperietur $z=2.675$ proxime; ergo x ſemper conſiſtet intra $5q$ & $2.675q$.

In æquatione poſteriori, ſi q ſit $=1$, erit $x=7q$; ſi vero x ſit infinita, vel ad unitatem habuerit rationem ſatis magnam, erit $z = Log.\,2 + Log.\,1+z+\frac{1}{2}z^2+\frac{1}{6}z^3$, vel $\frac{\frac{1}{6}z^3\dot{z}}{1+z+\frac{1}{2}zz+\frac{1}{6}z^3}=\dot{y}$, ubi reperietur $z=3.67199q$ proxime; & par eſt ratio omnium ſequentium, & limites ſemper approximant ad rationem numeri binarii ad unitatem.

Tabella Limitum. Si de eventu ſimplici contendatur, numerus tentaminum erit intra	$1q$ & $0.693q$
Si de duplici, intra	$3q$ & $1.678q$
Si de triplici, intra	$5q$ & $2.675q$
Si de quadruplici, intra	$7q$ & $3.67199q$
Si de quintuplici, intra	$9q$ & $4.679q$
Si de ſextuplici, intra	$11q$ & $5.668q$

Si de pluribus, quorum numerus ſit n, contendatur; modo n & q ad unitatem habuerint rationem ſatis magnam, conjectura de numero tentaminum non multum a vero aberrans facile fiet, ponendo numerum tentaminum $=\frac{2n-1}{2}q$. Etenim x cito converget ad limitem minorem.

Prob. VIII. *Tres colluſores* A, B, C, *ſingulis globis certant, ea conditione ut qui primus datum ludorum numerum vicerit depoſitum lucretur; jam poſt ludos aliquot peractos, deſunt ipſi* A, 1; *ipſi* B, 2; *ipſi* C, 3 *ludis; rationes vero dexteritatum ſunt ut* a, b, c *reſpective, quæritur ratio expectationum.*

Solutio. Elevetur $a+b+c$ ad poteſtatem quartam (etenim 4 ad plurimum ludis certamen neceſſario concludetur) hæc erit $a^4+4a^3b+6aabb+4ab^3+b^4+4a^3c+12aabc+6aacc+12abcc+6bbcc+4ac^3+c^4$.

Termini $a^4 + 4a^3b + 12aabc + 4a^3c + 12abcc$, ubi a ad dimenſionem æque altam ac eſt numerus ludorum ipſi A deſideratus, vel altiorem aſcendit; & ubi b & c ad pauciores dimenſiones, quam ſunt numeri ludorum ipſis B & C deſiderati, aſcendunt; componunt partem expectationis ipſius A. Eodem modo termini $b^4 + 4b^3c + 6bbcc$ componunt partem expectationis ipſius B. Et termini $4bc^3 + c^4$ componunt partem expectationis ipſius C: Reliqui omnes termini ſunt communes, & ita dividi debent, ut partes illæ omnes quæ favent uni colluſorum illi ipſi tribuantur.

Jam cum ipſi A deſit 1 ludus, ipſi B 2, ipſi C 3, partes illæ omnes in quibus a dimenſionem 1^{am} vel altiorem aſſecutus fuerit, priuſquam b 2^{am} & c 3^{m} aſſecuti fuerint, ipſi A favent; & eadem eſt ratio partium quæ ipſis B & C favent, adeoque ſi terminus $6aabb$ in partes ſuas $aabb$, $abab$, $abba$, $baab$, $baba$, $bbaa$, ſit diviſus, partes 5 priores ipſi A ſunt tribuendæ, pars unica poſterior ipſi B; ergo jam $5aabb$ addi debebit expectationi ipſius A, & $1aabb$ expectationi ipſius B. Si terminus $4ab^3$ in partes ſuas $abbb$, $babb$, $bbab$, $bbba$, ſit diviſus, pars prima & ſecunda favent ipſi A, pars tertia & quarta favent ipſi B, adeoque $2ab^3$ utrique eſt tribuenda. Si terminus $12abbc$ in partes ſuas ſit diviſus, partes 8 ipſi A, partes vero 4 ipſi B ſunt tribuendæ, ſi terminus $4ac^3$ in partes ſuas ſit diviſus, partes 3 ipſi A ſunt tribuendæ, pars vero unica ipſi C, adeoque expectationes totales jam erunt

1^a. $a^4 + 4a^3b + 5aabb + 2ab^3 + 12aabc + 4a^3c + 6aacc + 8abbc + 3ac^3$.

2^a. $b^4 + 4b^3c + 6bbcc + aabb + 2ab^3 + 4abbc$.

3^a. $4bc^3 + ac^3 + c^4$.

Sit n numerus deficientium ludorum, p numerus colluſorum, rationes expectationum ut a, b, c, d, *&c.* elevetur $a + b + c + d$, *&c.* ad poteſtatem $n + 1 - p$, & eodem modo procedatur.

Prob. IX. A & B *aſſumentes uterque* 12 *nummos, ludunt tribus teſſeris, hac conditione, ut ſi* 11 *puncta jaciantur,* A *tradat unum nummum ipſi* B, *at ſi* 14 *puncta jaciantur,* B *tradat unum nummum ipſi* A, *& ut ille ludum victurus ſit qui primus nummos habuerit omnes: Quæritur ratio ſortis ipſius* A *ad ſortem ipſius* B.

Solutio. Sit p numerus nummorum quos uterque ſingulatim aſſumit, ſint a & b numerus caſuum quibus A & B reſpective nummum unum obtinere poſſunt, & ratio ſortium erit ut a^p ad b^p; hoc in caſu eſt $p = 12$, $a = 27$, $b = 15$; ſive cum ſit $27 : 15 :: 9 : 5$, fiat $a = 9$, $b = 5$, adeoque ratio expectationum erit ut 9^{12} ad 5^{12}, ſive ut 244140625 ad 282429536481 qualem *Hugenius* fore aſſeruit.

Solutio Generalior. Sit p numerus nummorum ipſius A, q vero numerus nummorum ipſius B; & A in ſe ſuſcipiat ut prius nummos q, quam B nummos p lucretur, erunt ſortes ut $a^q \times \overline{a^p - b^p}$, ab $b^p \times \overline{a^q - b^q}$. Fingatur enim A nummos habere E, F, G, H, *&c.* quorum numerus p; & B nummos habere I, K, L, *&c.* quorum numerus q; fingatur inſuper, valorem cujuſlibet nummi eſſe ad valorem ſequentis ut a ad b, ita ut E, F, G, H, I, K, L, ſint in progreſſione Geometrica; his ita poſitis, poterunt A & B qualibet vice deponere nummos quorum valor ſit proportionalis numero caſuum quibus alter alterum vincere poſſit; etenim prima vice poterit A deponere H, B vero I; at H ad I ex hypotheſi eſt ut a ad b; ergo jam A & B æquali conditione certant; ſi vicerit A, poterit ille deponere I, B vero K; ſed I ad K ex Hypotheſi eſt ut a ad b; ſin B vicerit, poterit A deponere G, B vero H, quorum ipſorum G & H ratio eſt ut a ad b, & ſic deinceps. Ergo quamdiu A & B certant, ſemper certant æquali conditione: Igitur eorum expectationes ſunt inter ſe ut ſumma terminorum E, F, G, H, &c. quorum numerus eſt p; ad ſummam terminorum I, K, L, quorum numerus eſt q; hoc eſt, ut $a^q \times \overline{a^p - b^p}$ ad $b^p \times \overline{a^q - b^q}$, quod facile conſtabit, ſi ſummentur progreſſiones iſtæ Geometricæ: Jam poſito, quemlibet nummum eſſe ad ſequentem ut a ad b, non exinde mutantur probabilitates vincendi, ergo poſito, valorem nummorum eſſe æqualem, probabilitates vincendi, ſeu ſortes ipſorum A & B etiamnum erunt in illa ipſa ratione quam determinavimus.

Maxime cavendum eſt ne Problemata propter ſpeciem aliquam affinitatis inter ſe confundantur. Problema ſequens videtur affine ſuperiori.

Prob. X. C *aſſumptis* 24 *calculis, tres teſſeras jaciat; jam quoties* 27 *puncta jecerit, tradat calculum unum ipſi* A, *quoties vero* 14 *puncta jecerit, tradat calculum unum ipſi* B, *at* A *&* B *hoc pacto certent, ut qui prior calculos* 12 *habuerit, depoſitum obtineat; quæritur ratio expectationum.*

Problema iſtud a ſuperiore in hoc differt, quod 23 ad plurimum teſſerarum jactibus, ludus neceſſario finietur; cum ludus ex lege ſuperioris problematis, poſſet in æternum continuari, propter reciprocationem lucri & jacturæ ſe invicem perpetuo deſtruentium.

Solutio. Elevetur $a + b$ ad poteſtatem 23^{am}, & termini 12 priores erunt ad 12 poſteriores, ut expectatio ipſius A ad expectationem ipſius B.

Prob. XI. *Tres colluſores* A, B, C, *aſſumentes duodecim calculos, quorum* 4 *albi, &* 8 *nigri ſint, ludant hac conditione, ut qui primus ipſorum, velatis oculis, album calculum elegerit, vincat; & ut prima electio ſit penes* A, *ſecunda penes* B, *tertia penes* C; *& tum ſequens rurſus penes* A, *& ſic deinceps ordine: Quæritur quænam futura ſit ratio ſortium ipſorum* A, B, C.

Solutio. Sit n numerus calculorum omnium, a numerus alborum, b numerus nigrorum, 1 ſumma depoſita, ſeu præmium victori concedendum.

1°. A habet caſus a quibus album, & caſus b quibus nigrum eligat adeoque ejus expectatio ex prima electione oriunda eſt $\frac{a}{a+b}$ ſive $\frac{a}{n}$. Igitur ſi $\frac{a}{n}$ ex 1 ſubtrahatur, valor reſiduarum expectationum erit $1 - \frac{a}{n} = \frac{n-a}{n} = \frac{b}{n}$.

2°. B habet caſus a quibus album, & caſus $b - 1$ quibus nigrum eligat; ſed prima electio eſt penes A, & incertum eſt utrum ille victurus ſit necne, ne, adeoque præmium reſpectu ipſius *B* non eſt 1, ſed tantummodo $\frac{b}{n}$, igitur illius expectatio ex ſecunda electione oriunda eſt $\frac{a}{a+\overline{b-1}} \times \frac{b}{n} = \frac{ab}{n\times\overline{n-1}}$. Subtrahatur $\frac{ab}{n\times\overline{n-1}}$ ex $\frac{b}{n}$, & valor reſiduarum expectationum erit $\frac{nb-b-ab}{n\times\overline{n-1}} = \frac{b\times\overline{b-1}}{n\times\overline{n-1}}$.

3°. *C* habet caſus a quibus album, & caſus $b-2$, quibus nigrum eligat, adeoque ejus expectatio ex tertia electione eſt $\frac{a\times b\times\overline{b-1}}{n\times\overline{n-1}\times\overline{n-2}}$.

4°. Eodem modo *A* habet caſus a quibus album, & caſus $b-3$ quibus nigrum eligat, adeoque ejus expectatio ex quarta electione erit $\frac{a\times b\times\overline{b-1}\times\overline{b-2}}{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}}$. Et ſic deinceps de cæteris.

Scribatur ergo ſeries $\frac{a}{n} + \frac{b}{n-1}P + \frac{b-1}{n-2}Q + \frac{b-2}{n-3}R + \frac{b-3}{n-4}S$, &c. ubi P, Q, R, S, &c. denotant terminos præcedentes cum ſuis ſignis; & ſumantur tot termini hujus ſeriei quot ſunt unitates in $b+1$ (etenim non plures erunt electiones quam ſunt unitates in $b+1$) Et ſumma tertiorum omnium, intermiſſis binis, terminorum, incipiendo ab $\frac{a}{n}$, erit tota expectatio ipſius *A*, ſumma tertiorum itidem omnium incipiendo a $\frac{b}{n-1}P$, erit tota expectatio ipſius *B*; ſumma tertiorum omnium incipiendo a $\frac{b-1}{n-2}Q$, erit tota expectatio ipſius *C*.

Si plures ſint colluſores, *A*, *B*, *C*, *D*, &c. ſive calculum unum, ſive plures, ſive eundem calculorum numerum, ſive diverſum unaquaque vice elegerint, illorum expectationes, ope præcedentis ſeriei, facili negotio itidem determinabuntur.

Sed ut ad caſum in Problemate propoſitum revertamur, fiat $a=4$, $b=8$, $n=12$, & ſeries generalis jam in iſtam migrabit, $\frac{4}{12} + \frac{8}{11}P + \frac{7}{10}Q + \frac{6}{9}R + \frac{5}{8}S + \frac{4}{7}T + \frac{3}{6}V + \frac{2}{5}X + \frac{1}{4}Y$.

Sive in alteram iſtam (multiplicando terminos omnes per numerum iſtum qui tollendis fractionibus magis idoneus judicabitur, nempe hoc in caſu per 495)

$$165 + 120 + 84 + 56 + 35 + 20 + 10 + 4 + 1.$$

adeoque tribuantur ipſi *A*, $165 + 56 + 10 = 231$; ipſi *B*, $120 + 35 + 4 = 159$; ipſi *C*, $84 + 20 + 1 = 105$. Adeoque expectationes erunt ut 231, 159, 105; ſive ut 77, 53, 35.

Corollarium. Si numerus caſuum quibus *A*, *B*, *C*, vel colluſores quotcunque vincere poſſunt, tandem aliquando exhauriatur, expreſſiones fortium erunt finitæ.

Prob. XII. *Si colluſores tres,* A, B, C, *vicibus ſuis Dodecaedron* 4 *albis faciebus,* & 8 *nigris, jaciant, ea conditione ut qui primus faciem albam jecerit, vincat; quæritur ratio expectationum.*

Solutio. Ratiocinia circa hanc Propoſitionem eadem ſunt atque illa quibus uti ſumus in præcedenti, ſed cum jactus Dodecaedri nihil detrahant de numero facierum, pro $b-1$, $b-2$, $b-3$, $b-4$, *&c.* $n-1$, $n-2$, $n-3$, $n-4$, *&c* ſubſtituantur b & n reſpective, & ſeries præcedentis Problematis evadet $\frac{a}{n}+\frac{ab}{nn}+\frac{abb}{n^3}+\frac{ab^3}{n^4}+\frac{ab^4}{n^5}+\frac{ab^5}{n^6}$ *&c* quæ ſeries in infinitum eſt continuanda. Et ſumendo tertios quoſque terminos, expectationes erunt

$$\frac{a}{n}+\frac{ab^3}{n^4}+\frac{ab^6}{n^7}\ \&c.$$

$$\frac{ab}{nn}+\frac{ab^4}{n^5}+\frac{ab^7}{n^8}\ \&c.$$

$$\frac{abb}{n^3}+\frac{ab^5}{n^6}+\frac{ab^8}{n^9}\ \&c.$$

Sed termini ex quibus expectationes ſingulæ componuntur ſunt in progreſſione geometrica, & ratio cujuſlibet termini ad ſequentem eadem eſt in ſingulis ſeriebus, nempe ut n^3 ad b^3; ergo ſummæ ſerierum ſunt ut primi ſerierum termini, nempe ut $\frac{a}{n}$, $\frac{ab}{nn}$, $\frac{abb}{n^3}$, ſive ut nn, bn, bb. Hoc eſt, in caſu iſtius Problematis, ut 9, 6, 4.

Corollarium. Si plures ſint colluſores, *A, B, C, D, &c.* iiſdem conditionibus ac ſupra certantes, ſumantur tot termini in ratione n ad b, quot ſunt colluſores, & termini illi denotabunt expectationes colluſorum reſpective.

Prob. XIII. A & B *ludant binis teſſeris, hac conditione, ut* A *vincat ſi punctum ſenarium jecerit;* B, *ſi ſeptenarium.* A *primo jactum unum inſtituat, deinde* B *duos jactus ſimul; tum rurſus* A *duos jactus, atque ſic deinceps, donec hic vel ille victor evadat: Quæritur ratio ſortis ipſius* A, *ad ſortem ipſius* B.

Solutio. Ponatur a numerus caſuum quibus *A* vincere poſſit, & b numerus caſuum quibus *B* vincere poſſit, n numerus variationum in teſſeris datis; ſit inſuper $n-a=d$, & $n-b=e$; ſit etiam 1 præmium victori concedendum.

1°. A habet caſus a quibus vincere poſſit, & caſus $n-a$ quibus non vincat, adeoque illius expectatio ex primo jactu oriunda eſt $\frac{a}{n}$; igitur ſi $\frac{a}{n}$ ex 1 ſubtrahatur, valor reſiduarum expectationum erit $1-\frac{a}{n}=\frac{n-a}{n}=\frac{d}{n}$.

2°. Si B ad jactum ſuum perveniat, ejus expectatio ex jactu ipſius oriunda, erit $\frac{b}{n}$; ſed quoniam incertum eſt utrum ille ad jactum ſuum ſit perventurus nec ne, expectatio $\frac{b}{n}$ minuenda eſt in ratione d ad n; Etenim præmium illius reſpectu, non 1, ſed tantummodo $\frac{d}{n}$ cenſendum eſt, adeoque expectatio ipſius B priuſquam A jactum ſuum inſtituat, erit $\frac{bd}{nn}$; ſubtrahatur $\frac{bd}{nn}$ ex $\frac{d}{n}$, & valor reſiduarum expectationum erit $\frac{d}{n} - \frac{bd}{nn} = \frac{nd - bd}{nn} = \frac{ed}{nn}$.

3°. Eodem argumentandi modo, expectatio ipſius B huic noviſſimæ deinceps ſubſequens, eſt $\frac{bed}{n^3}$.

4°. Et expectatio ipſius A huic ſubſequens, eſt $\frac{aeed}{n^4}$.

5°. Et expectatio ipſius A huic demum ſubſequens eſt $\frac{aeedd}{n^5}$. Et ſic deinceps de cæteris; adeoque erunt

Expectationes omnes ipſius A.

$$\begin{array}{llll} & \frac{a}{n} & & \\ + & \frac{aeed}{n^4} & + & \frac{aeedd}{n^5} \\ + & \frac{ae^4d^3}{n^8} & + & \frac{ae^4d^4}{n^9} \\ + & \frac{ae^6d^5}{n^{12}} & + & \frac{ae^6d^6}{n^{13}} \\ & \&c. & & \end{array}$$

Jam ſepoſito parumper primo termino $\frac{a}{n}$, columna prima perpendicularis conſtituit progreſſionem geometricam infinite decreſcentem, cujus ſumma eſt $\frac{aeed}{n^4 - eedd}$. Reſumatur primus terminus $\frac{a}{n}$, iſque addatur ſummæ progreſſionis, & aggregatum erit $\frac{naeed + an^4 - aeedd}{n \times \overline{n^4 - eedd}}$.

Columna ſecunda conſtituit progreſſionem alteram Geometricam, cujus ſumma eſt $\frac{aeedd}{n \times \overline{n^4 - eedd}}$.

Summa igitur expectationum ipſius *A* eſt $\frac{aeed^4 + an^3}{n^4 - eedd}$.

Expectationes omnes ipſius B

$$\frac{bd}{nn} + \frac{bed}{n^3}$$

$$\frac{beed^3}{n^6} + \frac{be^3d^3}{n^7}$$

$$\frac{be^4d^5}{n^{10}} + \frac{be^5d^5}{n^{11}}$$

&c.

Summa primæ columnæ eſt $\frac{bdnn}{n^4 - eedd}$:

Summa ſecundæ columnæ eſt $\frac{bden}{n^4 - eedd}$:

Adeoque ſumma expectationum ipſius *B* erit $\frac{bdnn + bden}{n^4 - eedd}$.

Ergo ratio expectationum erit, ut $aeed + an^3$ ad $bdnn + bden$.

Si pro *a*, *b*, *n*, *d*, *e*, ſcribantur 5, 6, 36, 31, 30, reſpective, exprimetur ratio quæſita in numeris, nempe ut 10355 ad 12276.

Corollarium. Si numerus caſuum, quibus colluſores vincere poſſunt, numquam exhauriatur, adeo ut ludus poſſit in infinitum continuari, ita tamen ut colluſores, propter iſtam continuationem, ponantur aliquando in iiſdem circumſtantiis in quibus antea fuerunt; expreſſiones ſortium finitæ erunt.

Prob. XIV. *Aſſumptis* 12 *calculis* 4 *albis,* & 8 *nigris, certet* A *cum* B *fore ut velatis oculis, ſi* 7 *calculos exemerit, eorum* 3 *albi ſint futuri: Quæritur ratio expectationis ipſius* A *ad expectationem ipſius* B.

Solutio. 1°. Inveniantur caſus omnes quibus 7 calculi ex 12 eximi poſſint; caſus erunt 792, ut patet ex Doctrina combinationum.

$$\frac{12}{1} \times \frac{11}{2} \times \frac{10}{3} \times \frac{9}{4} \times \frac{8}{5} \times \frac{7}{6} \times \frac{6}{7} = 792.$$

2°. Seponantur 3 albi, & inveniantur caſus omnes quibus 4 nigri ex 8 iis adjungi poſſint; caſus illi erunt 70.

$$\frac{8}{1} \times \frac{7}{2} \times \frac{6}{3} \times \frac{5}{4} = 70.$$

Quoniam autem 4 ſunt caſus quibus 3 albi ex 4 poſſint eligi, multiplicetur 70 per 4, adeoque caſus erunt 280, quibus 3 albi cum 4 nigris poſſint eximi.

3°. Ex lege ludorum, ille qui in ſe ſuſcipit ut effectum aliquem producat, etiamnum victor cenſetur, ſi effectum pluries produxerit quam in ſe ſuſceperit, niſi contrarium expreſſe ſit cautum, adeoque ſi 4 albi cum 3 nigris eximantur, A victor cenſendus erit; Igitur ſeponantur 4 albi, & inveniantur caſus omnes quibus 3 nigri, ex 8, 4 albis adjungi poſſint; caſus illi erunt 56.

$$\frac{8}{1}\times\frac{7}{2}\times\frac{6}{3}=56.$$

4°. Igitur A caſus habet $280+56=336$, quibus victor evadat: Subtrahantur caſus illi ex 792, & caſus reſidui erunt 456 quibus B victor evadere poſſit: Ergo ratio ſortis ipſius A, ad ſortem ipſius B, erit ut 336 ad 456, ſive ut 14 ad 19.

Generaliter. Sit n numerus calculorum omnium, a numerus alborum, b numerus nigrorum, c numerus quem A eximat; & erit

Numerus Caſuum omnium $\frac{n}{1}\times\frac{n-1}{2}\times\frac{n-2}{3}\times\frac{n-3}{4}\times\frac{n-4}{5}\times\frac{n-5}{6}$ *&c.* quæ ſeries continuari debet ad tot terminos quot ſunt unitates in c.

Numerus caſuum quibus A calculos c eximere poteſt abſque ullo albo

$$\frac{b}{1}\times\frac{b-1}{2}\times\frac{b-2}{3}\times\frac{b-3}{4}\times\frac{b-4}{5}\times\frac{b-5}{6}\ \&c.$$

Numerus caſuum quibus A calculum unum album eximere poteſt.

$$\frac{b}{1}\times\frac{b-1}{2}\times\frac{b-2}{3}\times\frac{b-3}{4}\times\ \&c.\times\frac{a}{1}.$$

Numerus caſuum quibus A calculos duos albos eximere poteſt

$$\frac{b}{1}\times\frac{b-1}{2}\times\frac{b-2}{3}\times\frac{b-3}{4}\ \&c.\times\frac{a}{1}\times\frac{a-1}{2}.$$

Numerus caſuum quibus A calculos tres albos eximere poteſt

$$\frac{b}{1}\times\frac{b-1}{2}\times\frac{b-2}{3}\ \&c.\times\frac{a}{1}\times\frac{a-1}{2}\times\frac{a-2}{3}.$$

Numerus caſuum quibus A calculos quatuor albos eximere poteſt

$$\frac{b}{1}\times\frac{b-1}{2}\ \&c.\times\frac{a}{1}\times\frac{a-1}{2}\times\frac{a-2}{3}\times\frac{a-3}{4}.$$

Et ſic deinceps.

Prob. XV. A, B, C, *tres colluſores, quorum dexteritates ſint æquales, deponant ſinguli* 1, *& iſtis conditionibus certent*; 1°. *Ut illorum duo ludum incipiant*; 2°. *Ut victus locum ſuum tertio cedat, ita ut ille tertius jam cum victore contendat, quæ conditio in poſterum ſemper ſit obſervanda*; 3°. *Ut victus ſemper multetur ſumma* q *quæ depoſito augendo inſerviat*; 4°. *Ut ille depoſitum ſic gradatim auctum, totum obtineat, qui alteros duos* *ſucceſſive vicerit. Quæritur quanto melior vel deterior ſit ſors ipſorum* A *&* B, *quos ludum incipere ponimus, quam ipſius* C.

Solutio. Ponatur ludum in infinitum continuari poſſe, hoc pacto.

$$\left.\begin{array}{l} A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \\ A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \\ A \text{ vincit } B \\ C \text{ vincit } A \\ B \text{ vincit } C \\ \quad \&c. \end{array}\right\} \text{Depoſitum} \left\{\begin{array}{l} 3+p \\ 3+2p \\ 3+3p \\ \\ 3+4p \\ 3+5p \\ 3+6p \\ \\ 3+7p \\ 3+8p \\ 3+9p \\ \quad \&c. \end{array}\right.$$

Sit *R* ſpectator aliquis, qui poſtquam *A* vicerit *B* ſemel, roget *A* an velit ſummas quas ſe obtenturum ſperat ipſi vendere, & quanti illas æſtimet cui *A* annuens reſpondeat.

Cum jam vicerim *B*, eſt mihi æqua ſors utrum obtineam vel non obtineam $3+2p$, adeoque ſumma iſta valet $\frac{3+2p}{2}$.

Si jam acciderit ut *C* me vincat, ſed tamen vices meæ certandi cum *C* revertantur, erit tunc mihi ſors æqua utrum obtineam, vel non obtineam $3+5p$, adeoque expectatio vincendi ipſum *C* tunc temporis valebit $\frac{3+5p}{2}$. Sed cum ſint 7 adverſus 1 fore ut vices illæ non revertantur (etenim *C* vincere me debet, *B* vincere *C*, ego *B* rurſus,) ſumma iſta quam me obtenturum ſpero valet $\frac{3+5p}{2 \times 8}$.

Ad eundem modum, *A* computatione rurſus inita deprehendet, valorem deinceps ſummæ quam ſe obtenturum ſperat, eſſe $\frac{3+8p}{2 \times 8 \times 8}$.

Et ſequentis $\frac{3+11p}{2 \times 8 \times 8 \times 8}$. Et ſic in infinitum.

R computationem hanc juſtam eſſe comperiens, pendat ipſi *A* ſummas, $\frac{3+2p}{2}$, $\frac{3+5p}{2 \times 8}$, $\frac{3+8p}{2 \times 8 \times 8}$, $\frac{3+11p}{2 \times 8 \times 8 \times 8}$, *&c.* quæ ope ſequentis Theorematis in ſummam unam redigantur.

Theorema. $\frac{n}{b} + \frac{n+d}{bb} + \frac{n+2d}{b^3} + \frac{n+3d}{b^4}$ *&c.* ad inf. $= \frac{n}{b-1} + \frac{d}{\overline{b-1}|^2}$

Diſtinguatur ſeries $\frac{3+2p}{2} + \frac{3+5p}{2\times8}$ &c. in partes duas

$$\overline{\frac{3}{2}\times1+\frac{1}{8}+\frac{1}{8\times8}+\frac{1}{8\times8\times8}+\frac{1}{8\times8\times8\times8}\ \&c.}$$

$$\overline{+\frac{p}{2}\times2+\frac{5}{8}}\ \overline{+\frac{8}{8\times8}+\frac{11}{8\times8\times8}}+\frac{14}{8\times8\times8\times8}\ \&c.$$

Pars 1ª conſtituit progreſſionem geometricam, cujus ſumma eſt $\frac{12}{7}$.

Pars 2ª ſepoſitis communi multiplicatore $\frac{p}{2}$, & termino primo 2, ſummatur per Theorema præmiſſum, & fit $\frac{5}{7}+\frac{3}{49}=\frac{38}{49}$, cui jam addito primo 2, ſumma erit $\frac{136}{49}$, qua multiplicata per $\frac{p}{2}$, productum $\frac{68}{49}p$, exhibebit ſummam ſecundæ ſeriei. Ergo R pendet ipſi A $\frac{12}{7}+\frac{68}{49}p$.

Eodem modo R ad B ſe convertens, illum roget utrum velit ſummas quas ille ſe obtenturum ſperat, ipſi vendere, cui *B* aſſentiens, & eadem innixus ratione qua ipſe A, requirat ſummam $\frac{3}{7}+\frac{31}{49}p$, quam R juſtam eſſe deprehendens, ipſi B pendat.

Denique R eodem cum C pacto inito, pendat ipſi pro ſummis quas ille ſe obtenturum ſperat, $\frac{6}{7}+\frac{48}{49}p$.

Sit S ſpectator alius, quem A roget (poſtquam vicerit B ſemel) utrum velit ipſius jacturas ſuſtinere, hoc eſt utrum velit multari ſummis *p*, pro ipſo A, quoties acciderit ut ipſe ſit multandus, & quanto pretio velit hanc in ſe ſortem ſuſcipere, cui S reſpondeat.

Quoniam tibi ſors eſt æqua utrum vincas C vel non, adeoque utrum multeris ſumma *p*, vel non, hujus multæ ſortem, ſi in manum mihi dederis $\frac{1}{2}p$, ſuſtinebo.

Quod ſi illud evenerit ut C te vincat, & B vincat C, adeo ut ſecunda vice tibi cum C certandum ſit, tunc multæ ejuſdem ſortem ſi dederis mihi $\frac{1}{2}p$, pariter ſuſtinebo. Verum cum ſint 3 adverſus 1 fore ut illud non eveniat, hujus multæ ſortem, nunc ſi mihi in manum dederis $\frac{1}{8}p$, ſuſtinebo.

Et eodem argumentandi modo, huic proximam ſortem ſi mihi dederis $\frac{1}{16}p$.

Et huic deinceps proximam, ſi dederis $\frac{1}{64}p$, &c.

Jam A ipſi S aſſentiens, tradat ipſi S ſummas, $\frac{1}{2}p * + \frac{1}{8}p + \frac{1}{16}p * + \frac{1}{64}p + \frac{1}{128}p * + \frac{1}{512}p + \frac{1}{1024}p$, &c. quæ ſummæ in unam redactæ fiunt $\frac{5}{7}p$.

Et eodem modo B & C pacto inito cum S, ipſi tradant $\frac{3}{7}p$ & $\frac{6}{7}p$, reſpective, ut ſuas multarum ſortes ſuſtineat.

A recipit ab R $\frac{12}{7}+\frac{68}{49}p.$

A tradit ipſi S $\frac{35}{49}p.$

Ipſi A ſupereſt $\frac{12}{7}+\frac{33}{49}p.$

Sed A depoſuerat 1, priuſquam ludus inciperetur: Ergo lucratur A $\frac{5}{7}+\frac{33}{49}p.$

B recipit ab R $\frac{3}{7}+\frac{31}{49}p.$

B tradit ipſi S $\frac{21}{49}p=\frac{3}{7}p.$

Ipſi B ſupereſt $\frac{3}{7}+\frac{10}{49}p.$

Sed B depoſuerat $1+p$, (videlicet 1 priuſquam ludus inciperetur, & p poſtquam ſemel victus fuerat ab A,) ergo B lucratur $-\frac{4}{7}-\frac{39}{49}p.$

Summa igitur lucrorum ipſorum A & B eſt $\frac{1}{7}-\frac{6}{49}p.$

Jam poſueramus A viciſſe ipſum B ſemel priuſquam colluſores pacta inirent cum R & S; ſed priuſquam ludus inchoaretur, B poterat æqua ſorte expectare ut vinceret ipſum A; adeoque ſumma lucrorum $\frac{1}{7}-\frac{6}{49}p$ in duas partes æquales dividenda, adeo ut utriuſque lucrum cenſendum ſit $\frac{1}{14}-\frac{3}{49}p.$

Ergo concludere jam licet, jacturam ipſius C, eſſe $\frac{1}{7}-\frac{6}{49}p$, ſive lucrum $-\frac{1}{7}+\frac{6}{49}p.$

Sed ut corroboretur computatio noſtra, videamus quale futurum ſit lucrum ipſius C, eadem methodo qua uſi fuimus pro inveniendis lucris ipſorum A & B.

C recipit ab R $\frac{6}{7}+\frac{48}{49}p.$

C tradit ipſi B $\frac{42}{49}p.$

Ipſi C ſupereſt $\frac{6}{7}+\frac{6}{49}p.$

Sed C depoſuerat $\frac{7}{7}$

Ergo C lucratur $-\frac{1}{7}+\frac{6}{49}p.$

Jam fiat $\frac{1}{7} - \frac{6}{49}p = 0$, & invenietur $p = \frac{7}{6}$, ergo ſi multa ad ſummam quam ſinguli deponunt ſit ut 7 ad 6, colluſores æquali conditione certant.

Si multa ſit ad ſummam quam ſinguli deponunt in minori ratione quam 7 ad 6, *A* & *B* potiori conditione certabunt, *C* deteriori.

Si multa ſit ad ſummam quam ſinguli deponunt in majori ratione quam 7 ad 6, *A* & *B* deteriori conditione certant, *C* potiori.

Corol. I. Poſtquam *A* vicerit *B* ſemel, probabilitates vincendi erunt ut $\frac{12}{7}$, $\frac{6}{7}$, $\frac{3}{7}$, ſive ut 4, 2, 1; ita ut maxima probabilitas ſit ipſius *A*, proxima ipſius *C*, minima ipſius *B*.

Corol. II. Spectator *R* priuſquam ludus inchoetur, id ſuſcipere in ſe poterit, ut ſummam 3 de qua colluſores contendunt, & multas omnes pendat, ſi ſibi initio in manus datum ſit $3 + 3p$.

Corol. III. Si dexteritates colluſorum ſint in ratione data, ſortes colluſorum eadem ratiocinatione determinabuntur.

Corol. IV. Si multa ſit negativa, ita ut victus portiunculam depoſiti 3 ſumat, v. g. $\frac{3}{10}$, & ludus ſit finiendus ſtatim atque depoſitum exhauſtum fuerit, ſortes colluſorum eadem ratiocinatione determinabuntur.

Corol. V. Si plures ſint colluſores, *A*, *B*, *C*, *D*, *&c.* & non prius ludo deſiſtant quam illorum unus alios omnes ſucceſſive vicerit, ratio ſortium etiam invenietur.

Corol. VI. Si multa non ſit definita, ſed continuo creſcat vel decreſcat, qua libuerit lege, ratio ſortium etiam determinabitur, ſi non per expreſſiones finitas, at ſaltem per ſeries ad verum perpetuo convergentes.

Prob. XVI. A & B, *quorum dexteritates ſint æquales inter ſe, dato Globorum numero certent; jam poſt ludos aliquot peractos, deſit ipſi* A *ludus* 1 *quominus victor evadat, ipſi* B *vero* 2: *Quæritur ratio illorum ſortium.*

Solutio. Sit m numerus globorum omnium, ita ut uterque habeat $\frac{1}{2}m$; ſit p numerus caſuum quibus duo vel plures ex globis ipſius B propius ad metam accidere poſſint; ſit q numerus caſuum quibus unus vel plures ex globis ipſius B propius ad metam accidere poſſint, adeo ut $q-p$ ſit numerus caſuum quibus unus ex globis ipſius B (excluſive pluribus) poſſit ad metam propius accidere; ſit s numerus variationum omnium quas globi omnes ſubire poſſint; ſit 1 depoſitum totum.

Patet B habere caſus p quibus obtineat 1, & caſus $q-p$ quibus obtineat $\frac{1}{2}$, adeoque illius expectationem eſſe $\frac{p+\frac{1}{2}q-\frac{1}{2}p}{S} = \frac{\frac{1}{2}p+\frac{1}{2}q}{S}$.

Jam conſtat ex Doctrina combinationum, globos omnes m variari poſſe vicibus, $m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$, &c. quæ ſeries continuari debet, donec

nec ultimus terminus fiat æqualis unitati, adeoque eſſe $s = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3}$ &c.

Conſtat ex eadem Doctrina globos numero $\frac{1}{2}m$, poſſe permutari binos, vicibus $\frac{1}{2}m \times \overline{\frac{1}{2}m-1}$, dum globi reliqui omnes ipſorum A & B, quorum numerus $m-2$ poſſunt variari vicibus $\overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. adeoque eſſe $p = \frac{1}{2}m \times \overline{\frac{1}{2}m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. Igitur $s : p :: m \times \overline{m-1} : \frac{1}{2}m \times \overline{\frac{1}{2}m-1} :: m-1 : \frac{1}{4}m - \frac{1}{2}$, & $p = \frac{\frac{1}{4}ms - \frac{1}{2}s}{m-1}$.

Liquet globos numero $\frac{1}{2}m$, poſſe ſumi ſigillatim vicibus $\frac{1}{2}m$, dum globi reliqui omnes ipſorum A & B quorum numerus $m-1$, variari poſſunt vicibus $\overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. adeoque eſſe $s : q :: m : \frac{1}{2}m :: 1 : \frac{1}{2}$; Eſt igitur $q = \frac{1}{2}s = \frac{\frac{1}{2}ms - \frac{1}{2}s}{m-1}$.

Ergo $\frac{\frac{1}{2}p + \frac{1}{2}q}{s} = \frac{\frac{3}{8}m - \frac{1}{2}}{m-1}$, ſubtrahatur hoc ex 1, & reſiduum $\frac{\frac{5}{8}m - \frac{1}{2}}{m-1}$ erit expectatio ipſius A, adeoque ratio ſortium ipſorum A & B erit ut $\frac{5}{8}m - \frac{1}{2}$ ad $\frac{3}{8}m - \frac{1}{2}$, ſive ut $5m-4$ ad $3m-4$.

Corol. I. Si numerus globorum eſſet infinitus, ratio ſortium fieret tandem ut 5 ad 3.

Corol. II. Si dexteritates ſint in ratione data, ratio ſortium eadem ratiocinatione invenietur.

Prob. XVII. A & B *quorum dexteritates ſint æquales inter ſe, dato globorum numero certent; jam poſt ludos aliquot peractos, deſit ipſi* A *ludus* 1 *quominus victor evadat, ipſi vero* B 3: *Requiritur ratio ſortium ipſorum* A & B.

Solutio. Sit ut in præcedenti Problemate m numerus globorum omnium; ſit r numerus caſuum quibus 3 vel plures ex globis ipſius B ad metam propius accidere poſſint, p numerus caſuum quibus 2 vel plures, q numerus caſuum quibus 1 vel plures propius ad metam poſſint accidere; ſit s numerus variationum omnium quas globi omnes poſſint ſubire.

Ergo B caſus habet r quibus obtineat 1, caſus $p-r$ quibus obtineat $\frac{1}{2}$, & caſus $q-p$ quibus obtineat $\frac{3m-4}{8m-8}$, ut patet ex præcedenti, adeoque ſumma illius expectationum erit $\frac{r \times 1 + \overline{p-r} \times \frac{1}{2} + \overline{q-p} \times \frac{3m-4}{8m-8}}{s}$

$$= \frac{\frac{1}{2}r + \frac{1}{2}p + \overline{q-p} \times \frac{3m-4}{8m-8}}{s}.$$

Jam globi numero $\frac{1}{2}m$ poſſunt permutari terni, vicibus $\frac{1}{2}m \times \overline{\frac{1}{2}m-1} \times \overline{\frac{1}{2}m-2}$, dum globi omnes reliqui ipſorum *A & B* quorum numerus $m-3$, poſſunt variari vicibus $\overline{m-3} \times \overline{m-4}$, *&c.* Igitur eſt $r = \frac{1}{2}m \times \overline{\frac{1}{2}m-1}$

$\frac{1}{2}\overline{m-1} \times \frac{1}{2}\overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. Sed eſt $s = m \times \overline{m-1} \times \overline{m-2} \times \overline{m-3} \times \overline{m-4}$, &c. ergo $r = \frac{\frac{1}{8}ms - \frac{1}{2}s}{m-1}$.

Sed ex præcedenti Problemate eſt $p = \frac{\frac{1}{4}ms - \frac{1}{2}s}{m-1}$, & $q = \frac{\frac{1}{2}ms - \frac{1}{2}s}{m-1}$.

Subſtitutis igitur valoribus iſtis pro r, p, q, fiet expectatio ipſius $B = \frac{9mm - 26m + 16}{32mm - 64m + 32}$. Subtrahatur hæc ab 1, & erit expectatio ipſius $A = \frac{23mm - 38m + 16}{32mm - 64m + 32}$; adeoque ratio ſortium ipſorum *A* & *B*, erit ut $23mm - 38m + 16$ ad $9mm - 26m + 16$, quæ convenit numero globorum cuicunque, binario excepto.

Verum ut ratio ſortium ipſorum *A* & *B* quum ſingulis globis certant, ſive quum numerus globorum eſt 2, inveniatur; reſumatur expreſſio generalis expectationis ipſius *B*, videlicet $\frac{\frac{1}{2}r + \frac{1}{2}p + \overline{q-p} \times \frac{3m-4}{8m-8}}{s}$, & ponantur r & $p = 0$, & erit expectatio ipſius $B = \frac{q \times \frac{3m-4}{8m-8}}{s} = \frac{\frac{1}{2}m - \frac{1}{2}}{m-1} \times \frac{3m-4}{8m-8} = \frac{1}{2} \times \frac{2}{8} = \frac{1}{8}$, qua ſubſtracta ex 1, erit expectatio ipſius $A = \frac{7}{8}$, ergo ratio ſortium ipſorum *A* & *B* hoc in caſu erit ut 7 ad 1, quod aliunde conſtat ex principiis jamdudum expoſitis.

Corol. I. Si numerus globorum eſſet infinitus, ratio ſortium fieret tandem ut 23 ad 9.

Corol. II. Si deſint ipſi *A* ludi quotvis quominus victor evadat, & ipſi *B* ludi itidem quotvis, ratio ſortium eadem ratiocinatione invenietur.

Corol. III. Si dexteritates ſint in ratione data, ratio ſortium etiam invenietur.

Prob. **XVIII.** *Certet* A *cum* B, *fore ut ipſe, dato tentaminum numero, teſſera dato facierum numero conſtante, facies quaſcunque datas jecerit: Quæritur expectatio ipſius* A.

Solutio. Sit $p + 1$ numerus facierum in teſſera, n numerus tentaminum datus, f numerus facierum quas jaci oporteat.

Numerus caſuum quibus *A* monada ſemel vel pluries, tentaminibus numero n, jacere poſſit, eſt $\overline{p+1}|^n - p^n$, ut patet ex jam demonſtratis.

Expungatur binarius e numero facierum, ita ut numerus facierum reducatur ad p; & erit numerus caſuum quibus *A* monada ſemel vel pluries tentaminibus numero n, jacere poſſit $p^n - \overline{p-1}|^n$.

Ergo jam reſtituto binario, numerus caſuum quibus *A* monada & binarium jacere poſſit, eſt differentia iſtorum caſuum, videlicet $\overline{p+1}|^n - 2p^n + \overline{p-1}|^n$.

Expungatur nunc ternarius, & erit numerus caſuum quibus *A* monada & binarium jacere poſſit, $p^n - 2 \times \overline{p-1}|^n + \overline{p-2}|^n$.

Ergo, jam reſtituto ternario, numerus caſuum quibus *A* monada, binarium, & ternarium jacere poſſit, eſt $\overline{p+1}|^n - 3 \times p^n + 3 \times \overline{p-1}|^n - \overline{p-2}|^n$. Et ſic deinceps de cæteris.

Scribantur ergo ordine poteſtates omnes, (mutatis alternatim ſignis) $\overline{p+1}|^n - p^n + \overline{p-1}|^n - \overline{p-2}|^n + \overline{p-3}|^n$ *&c.* Et præfigantur illis coefficientes poteſtatis deſignatæ per *f*, & ſumma terminorum erit numerator expectationis ipſius *A*, cujus denominator erit $\overline{p+1}|^n$.

Exemp. I. Sit 6 numerus facierum in teſſera, & 2 numerus facierum datarum quas jaci oporteat tentaminibus 8, & erit expectatio ipſius *A*, $\frac{6^8 - 2 \times 5^8 + 4^8}{6^8}$.

Exemp. II. Sit 6 numerus facierum in teſſera, & 6 numerus facierum quas jaci oporteat tentaminibus 12, & erit expectatio ipſius *A*,

$$\frac{6^{12} - 6 \times 5^{12} + 15 \times 4^{12} - 20 \times 3^{12} + 15 \times 2^{12} - 6 \times 1^{12}}{6^{12}}.$$

Exemp. III. Contendat *A* cum *B* fore ut ipſe, tentaminibus numero 43, teſſera faciebus 36 conſtante, facies duas datas jecerit, ſive ut binis teſſeris vulgaribus jecerit duas monadas ſimul, atque etiam duos binarios ſimul, & erit expectatio ipſius *A* $\frac{36^{43} - 2 \times 35^{43} + 34^{43}}{36^{43}}$.

N. B. Facilis erit additio & ſubtractio partium ex quibus expectationes iſtæ componuntur, ope Tabulæ Logarithmorum.

Prob. XIX. *Invenire quotenis tentaminibus futurum ſit probabile ut colluſorum alter* A *facies quaſcunque datas jaciat, teſſera conſtante dato facierum numero.*

Solutio. Sit ut prius $p + 1$ numerus facierum in teſſera, n numerus tentaminum datus, f numerus facierum quæſitus. Ponatur Log. $\frac{1}{1 - \sqrt[f]{\frac{1}{2}}} = \alpha$, & Log. $\frac{p+1}{p} = \beta$; & erit $n = \frac{\alpha}{\beta}$ prope.

Demonſtratio. Si numerus facierum quas jaci oporteat ſit 6, expectatio ipſius *A* erit

$$\frac{\overline{p+1}|^n - 6p^n + 15 \times \overline{p-1}|^n - 20 \times \overline{p-2}|^n + 15 \times \overline{p-3}|^n - 6 \times \overline{p-4}|^n + \overline{p-5}|^n}{\overline{p+1}|^n}$$

Fingatur terminos $p+1$, p, $p-1$, $p-2$, *&c.* eſſe in progreſſione Geometrica, quæ ſuppoſitio non multum a vero aberrabit, ſi præſertim p ad 1 habuerit rationem ſatis magnam, & ponatur $\frac{p^n}{\overline{p+1}|^n} = \frac{1}{r}$; ergo

Expectatio ipſius *A* erit $1 - \frac{6}{r^n} + \frac{15}{r^{2n}} - \frac{20}{r^{3n}} + \frac{15}{r^{4n}} - \frac{6}{r^{5n}} + \frac{1}{r^{6n}} = = \frac{1}{2}$.

Extrahatur utrinque radix ſexta, & fiet $1 - \frac{1}{r^n} = \sqrt[6]{\frac{1}{2}}$, ergo $r^n = \frac{1}{1 - \sqrt[6]{\frac{1}{2}}}$

ponatur jam Log. $r = \beta$, & Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = a$, & erit $n\beta = a$, adeoque $n = \frac{a}{\beta}$, & eadem erit demonſtratio de cæteris caſibus.

Si ſit aliqua ſuſpicio ne valor indicis *n* ſic inventus non ſit ſatis accuratus, tunc ſubſtituatur valor iſte pro *n*, & notetur error, tunc mutetur aliquantulum valor iſte, & notetur novus error, & ope duorum errorum valor indicis *n* ſatis accurate corrigetur, ſi Regula falſi adhibeatur.

Poteſt valor indicis *n* ſic inventus corrigi per ſeriem infinitam, ex natura Problematis depromptam, talem ut primus terminus hujus ſeriei ſit valor iſte quem aſſignavimus; ſed correctio per differentiam errorum ſufficit ad uſus practicos.

Exemp. I. Invenire quotenis jactibus vulgaris teſſeræ, probabile ſit ut *A* facies omnes jaciat.

Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = 0.9621753$, Log. $\frac{6}{5} = 0.0791812$, ergo $n = \frac{0.9621753}{0.0791812}$ $= 12 +$. Ergo concludere jam licet numerum jactuum quæſitum fore 12 circiter, ſi vero 12 ſubſtituatur pro *n* in æquatione caſui huic competente, invenietur expectatio ipſius *A* .437 prope, quæ aliquanto debita nempe .5 minor eſt; ergo ponatur 13 pro *n*, & invenietur expectatio ipſius *A* .513, quæ eſt debita major; ergo poterit *A* in ſe ſuſcipere ut facies omnes tentaminibus 13 jaciat, idque potiori conditione.

Exemp. II. Invenire quotenis tentaminibus futurum ſit probabile ut *A* teſſera faciebus 216 conſtante, facies ſex datas jaciat, ſive ut tribus teſſeris vulgaribus * *Triadas* omnes jaciat.

* *Raffles.*

Log. $\frac{1}{1 - \sqrt[6]{\frac{1}{2}}} = 0.9621753$, Log. $\frac{216}{215} = 0.0020152$, ergo $n = \frac{0.9621753}{0.0020152}$ $= 477$ prope.

De Duratione Ludorum.

Prob. XX. A & B *quorum dexteritates ſint in ratione data, videlicet, ut* a *ad* b, *ea conditione ludant, ut quoties* A *ludum unum vicerit,* B *tradat ipſi nummum unum; quoties vero* B *vicerit,* A *ipſi tradat nummum unum: & non prius ludo deſiſtant, quam eorum alter nummos omnes alterius lucratus fuerit. Adſtent vero ſpectatores duo* R & S, *quorum* R *affirmet certamen finitum iri intra datum ludorum numerum,* S *neget: Quæritur expectatio ipſius* S.

Solutio. *Caſus* I. Sit 2 numerus nummorum quos uterque colluſorum habeat; ſit etiam 2, numerus de quo *R* & *S* contendant: Jam propter 2, numerum ludorum de quo contenditur, elevetur $a+b$ ad poteſtatem 2, quæ erit $aa+2ab+bb$: terminus $2ab$ ipſi S favet, reliqui adverſantur, adeoque illius expectatio erit $\frac{2ab}{\overline{a+b}|^{2}}$.

Caſus II. Sit 2 numerus nummorum quos uterque colluſorum habeat, & ſit 3 numerus ludorum de quo R & S contendant; elevetur itaque $a+b$ ad poteſtatem 3^{am}, quæ erit $a^3+3aab+3abb+b^3$. Jam termini duo $+a^3+b^3$, omnino ipſi S adverſantur, reliqui duo $3aab+3abb$, partim favent, partim adverſantur; dividantur ergo termini iſti in partes ſuas, videlicet $3aab$ in aab, aba, baa, atque $3abb$ in abb, bab, bba, & partes $aba+baa+abb+bab$, ſive $2aab+2abb$ ipſi S favent, reliquæ adverſantur. Adeoque expectatio ipſius S erit $\frac{2aab+2abb}{\overline{a+b}|^{3}}$, ſive (diviſis numeratore & denominatore per $a+b$) $\frac{2ab}{\overline{a+b}|^{2}}$, quæ eadem eſt ac in caſu præcedenti.

Caſus III. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 4 numerus ludorum de quo ſpectatores contendant; elevetur itaque $a+b$ ad poteſtatem 4^{am}, quæ erit $a^4+4a^3b+6aabb+4ab^3+b^4$; termini $a^4+4a^3b+4ab^3+b^4$ omnino ipſi S adverſantur, terminus unicus $6aabb$ partim favet, partim adverſatur: dividatur ergo terminus iſte in partes ſuas, $aabb$, $abab$, $abba$, $baab$, $baba$, $bbaa$, & partes quatuor, $abab$, $abba$, $baab$, $baba$, ſive $4aabb$, ipſi S favent; adeoque illius expectatio erit $\frac{4aabb}{\overline{a+b}|^{2}}$.

Caſus IV. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 5 numerus ludorum de quo ſpectatores contendant, & expectatio ipſius S invenietur eadem ac in præcedenti caſu.

Caſus V. Sit 2 numerus nummorum quos uterque colluſorum habeat, & 6 numerus ludorum de quo ſpectatores contendant, & expectatio ipſius S invenietur $\frac{8a^3b^3}{\overline{a+b}|^{6}}$.

Generalius. Sit 2 numerus nummorum quos uterque colluſorum habeat, & $2+d$ numerus ludorum de quo ſpectatores contendant, erit

$$\frac{\overline{2ab}|^{1+\frac{1}{2}d}}{\overline{a+b}|^{2+d}}$$

expectatio ipſius S.

Ubi nota d numerum eſſe parem; quod ſi d ſit numerus impar, expectatio ipſius S eadem erit ac ſi numerus ille unitate eſſet diminutus.

Caſus VI. Sit 3 numerus nummorum quos uterque colluſorum habeat, & $3+d$ numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius

$$S=\frac{\overline{3ab}|^{1+\frac{1}{2}d}}{\overline{a+b}|^{2+d}}.$$

Ubi nota d numerum eſſe parem; quod ſi d ſit numerus impar, expectatio ipſius S eadem erit ac ſi numerus ille unitate eſſet diminutus.

Caſus VII. Sit 4 numerus nummorum quos uterque colluſorum habeat, & 4 numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius $S\ \frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$.

Caſus VIII. Sit 4 numerus nummorum quos uterque colluſorum habeat, & 6 numerus ludorum de quo ſpectatores contendant, & invenietur expectatio ipſius $S\ \frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6}$.

Tabula expectationum ipſius *S*, pro numero nummorum 4.

$$
\begin{array}{r|l}
4. & \frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4} \\
6. & \frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6} \\
8. & \frac{48a^5b^3 + 68a^4b^4 + 48a^3b^4}{\overline{a+b}|^8} \\
10. & \frac{164a^6b^4 + 232a^5b^5 + 164a^4b^6}{\overline{a+b}|^{10}} \\
12. & \frac{560a^7b^5 + 792a^6b^6 + 560a^5b^5}{\overline{a+b}|^{12}}
\end{array}
$$

&c.

Tabula iſta facile continuabitur, ſi ſequentia adnotentur.

1°. Coefficientem termini primi in quolibet numeratore eſſe ſummam coefficientem terminorum omnium in numeratore præcedenti. 2°. Coefficientem termini ſecundi eſſe aggregatum ſummæ iſtius, & coefficientis termini ſecundi præcedentis. 3°. Coefficientem termini tertii eundem eſſe, ac coefficientem termini primi. 4°. Producta literalia, ex præcedentibus, prima ex primis, ſecunda ex ſecundis, formari, multiplicatis præcedentibus per *ab*. 5°. Denominatores omnes eſſe poteſtatem illam binomii $a + b$, quæ deſignatur per numerum ludorum de quo *R* & *S* contendunt.

Hic obiter venit obſervandum coefficientes omnes, primi ex primis, ſecundi ex ſecundis, generari poſſe. Etenim ſi ex ultimo præcedente, quadruplicato, ſubtrahatur penultimus duplicatus, orietur coefficiens quæſitus.

Regula Generalis. Sit *n* numerus nummorum quos uterque colluſorum habeat, $n + d$ numerus ludorum de quo ſpectatores contendant.

Elevetur $a + b$ ad poteſtatem *n*, & reſecentur termini duo extremi; multiplicetur reſiduum per $aa + 2ab + bb$, & rejiciantur termini extremi; fiat rurſus multiplicatio reſidui per $aa + 2ab + bb$, & rejiciantur extremi, & ſic deinceps fiant tot multiplicationes quot ſunt unitates in $\frac{1}{2}d$; & productum ultimum erit numerator expectationis ipſius *S*; denominator vero ſemper erit $\overline{a+b}|^{n+d}$.

N. B. Si *d* ſit numerus impar, ſubſtituatur $d - 1$ pro *d*.

Sit

Si n ſit numerus impar, dividi poterunt numerator & denominator expectationis per $a + b$, & fiet expectatio ſimplicior.

Exemp. I. Sit 4 numerus nummorum quos uterque colluſorum habeat, & 10 numerus ludorum de quo ſpectatores contendant, ſint autem dexteritates in ratione æqualitatis; quæritur expectatio ipſius S.

Eſt $n = 4$, & $n + d = 10$; igitur eſt $d = 6$, & $\frac{1}{2}d = 3$. Elevetur itaque $a + b$ ad poteſtatem 4^{am}, & reſectis ſemper extremis, fiant 3 multiplicationes per $aa + 2ab + bb$.

$$\begin{array}{l}
a^4 \mid + 4a^3b + 6aabb + 4ab^3 \mid + b^4 \\
\qquad\quad aa + 2ab + bb \\
\hline
4a^5b \mid + 6a^3bb + 4a^3b^3 \\
\qquad\quad + 8a^4bb + 12a^3b^3 + 8aab^4 \\
\qquad\qquad\qquad + 4a^3b^3 + 6aab^4 \mid + 4ab^5 \\
\hline
\qquad 14a^4bb + 20a^3b^3 + 14aab^4 \\
\qquad\qquad aa + 2ab + bb \\
\hline
14a^6bb \mid + 20a^5b^3 + 14a^4b^4 \\
\qquad\quad + 28a^5b^3 + 40a^4b^4 + 28a^3b^5 \\
\qquad\qquad\qquad + 14a^4b^4 + 20a^3b^5 \mid 14aab^6 \\
\hline
\qquad 48a^5b^3 + 68a^4b^4 + 48a^3h^5 \\
\qquad\qquad aa + 2ab + bb \\
\hline
48a^7b^5 \mid + 68a^6b^4 + 48a^5b^5 \\
\qquad\quad + 96a^6b^4 + 136a^5b^5 + 96a^4b^6 \\
\qquad\qquad\qquad + 48a^5b^5 + 68a^4b^6 \mid + 48a^3b^7 \\
\hline
\qquad 164a^6b^4 + 232a^5b^5 + 164a^4b^6
\end{array}$$

Et erit expectatio ipſius $S = \frac{164a^6b^4 + 232a^5b^5 + 164a^4b^6}{\overline{a+b}|^{10}}$, & propter a & b æquales erit iſta expectatio $\frac{164 + 232 + 164}{2^{10}} = \frac{560}{1024} = \frac{35}{64}$.

Exemp. **II.** Sit 5 numerus nummorum quos uterque colluſorum habeat, & 10 numerus ludorum de quo ſpectatores contendant, ita ut S neget certamen finitum iri intra ludos 10; ſit autem dexteritas ipſius A ad dexteritatem ipſius B ut 2 ad 1.

Eſt $n = 5$, & $n + d = 10$; eſt igitur $d = 5$. Et propter d imparem, fingatur $d = 4$, ergo $\frac{1}{2}d = 2$. Elevetur itaque $a + b$ ad poteſtatem 5^{am}, & reſectis ſemper extremis, fiant 2 multiplicationes per $aa + 2ab + bb$.

$$\begin{array}{l}
a^5 \mid + 5a^4b + 10a^3bb + 10a^2b^3 + 5ab^4 \mid + b^5 \\
\qquad aa + 2ab + bb \\
\hline
5a^6b \mid + 10a^5bb + 10a^4b^3 + 5a^3b^4 \\
\qquad + 10a^5bb + 20a^4b^3 + 20a^3b^4 + 10aab^5 \\
\qquad\qquad + 5a^4b^3 + 10a^3b^4 + 10aab^5 \mid + 5ab^6 \\
\hline
20a^5bb + 35a^4b^3 + 35a^3b^4 + 20aab^5 \\
\qquad aa + 2ab + bb \\
\hline
20a^7bb \mid + 35a^6b^3 + 35a^5b^4 + 20a^4b^5 \\
\qquad + 40a^6b^3 + 70a^5b^4 + 70a^4b^5 + 40a^3b^6 \\
\qquad\qquad + 20a^5b^4 + 35a^4b^5 + 35a^3b^6 \mid + 20a^2b^7 \\
\hline
\qquad 75a^6b^3 + 125a^5b^4 + 125a^4b^5 + 75a^3b^6
\end{array}$$

Ergo expectatio ipſius S erit $\frac{75a^6b^3 + 125a^5b^4 + 125a^4b^5 + 75a^3b^6}{\overline{a+b}|^9}$.

Sive diviſis numeratore & denominatore per $a + b$, propter numerum n imparem, fiet expectatio $= \frac{75a^5b^3 + 50a^4b^4 + 75a^3b^5}{\overline{a+b}|^8} = 25a^3b^3 \times \frac{3aa + 2ab + 3bb}{\overline{a+b}|^8}$.

Et poſitis 2 & 1 pro a & b reſpective, fiet expectatio $= \frac{8 \times 25 \times 19}{6561} = \frac{3800}{6561}$.

Prob. XXI. *Sit* 4 *numerus nummorum quos uterque colluſorum habeat; Requiritur ratio dexteritatum quæ faciat ut* R *poſſit æqua ſorte affirmare certamen finitum iri intra ludos* 4, S *negare.*

Solutio. Expectatio ipſius S, jam inventa, eſt $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$, & quoniam, ex Hypotheſi, R & S æqua ſorte contendunt, ponatur $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4} = \frac{1}{2}$, ſive $a^4 - 4a^3b - 6aabb - 4ab^3 + b^4 = 0$. Addatur $12aabb$ utrobique, & fiet $a^4 - 4a^3b + 6aabb - 4ab^3 + b^4 = 12aabb$. Extrahatur hinc inde radix quadratica, & erit $aa - 2ab + bb = ab\sqrt{12}$, ſive facto $a : b :: z : 1$, $zz - 2z + 1 = z\sqrt{12}$, ubi invenietur radix duplex $z = 5.274$, & $\frac{1}{5.274}$. Ergo ſive ratio dexteritatis ipſius A ad dexteritatem ipſius B ſit ut 5.274 ad 1, vel ut 1 ad 5.274, R & S æqua ſorte contendent.

Prob. XXII. *Sit* 4 *numerus nummorum quos uterque colluſorum habeat; Requiritur ratio dexteritatum talis, ut poſſit* R *affirmare finitum iri certamen*

men intra 4 ludos, S *negare, atque ſint ſortes ipſorum* R & S *in ratione data, videlicet ut 3 ad 1.*

Solutio. Expectatio ipſius *S* ex numero ludorum 4, & ratione dexteritatum oriunda eſt $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4}$. Eadem expectatio propter datam rationem ſortium eſt $\frac{1}{4}$. Ergo ſit $\frac{4a^3b + 6aabb + 4ab^3}{\overline{a+b}|^4} = \frac{1}{4}$; ſive $a^4 - 12a^3b - 18aabb - 12ab^3 + b^4 = 0$. Jam facto $a : b :: z : 1$, erit $z^4 - 12z^3 - 18zz - 12z + 1 = 0$. Supponatur hæc æquatio ex binis iſtis quadraticis formari, $zz + yz + 1 = 0$. Et $z^2 + pz + 1 = 0$.

$$\text{Ergo } z^4 \begin{smallmatrix}+y\\+p\end{smallmatrix} z^3 \begin{smallmatrix}+py\\+2\end{smallmatrix} zz \begin{smallmatrix}+y\\+p\end{smallmatrix} z + 1 = 0.$$

Comparentur coefficientes terminorum Homologorum, & erit $y + p = -12$, & $py + 2 = -18$, ſive $py = -20$; unde orietur æquatio $yy + 12y = 20$, cujus radix negativa erit $= -13.483$. Subſtituatur valor iſte in locum ipſius *y*, & erit $zz - 13.483z + 1 = 0$, cujus æquationis radix duplex invenietur 13.407, & $\frac{1}{13.407}$ prope, ergo ſive *a* ad *b* ſit ut 13.407 ad 1, ſive ut 1 ad 13.407, ratio ſortium ipſorum *R* & *S* erit ut 3 ad 1,

Prob. XXIII. *Sit 4 numerus nummorum quos uterque colluſorum habeat; Requiritur ratio dexteritatum quæ faciat ut* R *poſſit æqua ſorte affirmare certamen finitum iri intra ludos 6,* S *negare.*

Solutio. Expectatio ipſius *S* ex numero ludorum, & ratione dexteritatum oriunda, erit $\frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6}$. Ejuſdem expectatio propter datam ſortium æqualitatem erit $= \frac{1}{2}$. Ergo erit $\frac{14a^4bb + 20a^3b^3 + 14aab^4}{\overline{a+b}|^6} = \frac{1}{2}$, ſive $a^6 + 6a^5b - 13a^4bb - 20a^3b^3 - 13aab^4 + 6ab^5 + b^6 = 0$, & facto $a : b :: z : 1$.

$$z^6 + 6z^5 - 13z^4 - 20z^3 - 13zz + 6z + 1 = 0.$$

Ponatur hæc æquatio ex binis iſtis formari.

$$z^2 + yz + 1 = 0.$$
$$\& \; z^4 + pz^3 + qz^2 + pz + 1 = 0.$$

$$\begin{array}{lllllll}
\text{Ergo } z^6 & + yz^5 & + z^4 & & & & \\
 & + pz^5 & + pyz^4 & + pz^3 & & & \\
 & & + qz^4 & + qyz^3 & + qzz & & \\
 & & & + pz^3 & + pyzz & + pz & \\
 & & & & + zz & + yz & + 1. \\
\hline
\end{array}$$

$$\text{Sive } z^6 \begin{smallmatrix}+y\\+p\end{smallmatrix} z^5 \begin{smallmatrix}+1\\+py\\+q\end{smallmatrix} z^4 \begin{smallmatrix}+2p\\+qy\end{smallmatrix} z^3 \begin{smallmatrix}+1\\+py\\+q\end{smallmatrix} zz \begin{smallmatrix}+p\\+y\end{smallmatrix} z + 1 = 0.$$

Et comparatis coefficientibus erit $y+p=6$, $1+py+q=-13$, ſeu $py+q=-14$, $2p+qy=-20$. Unde orietur æquatio $y^3-6yy-16y+32=0$, cujus una radicum erit -2.9644, qua ſubſtituta in locum ipſius y, in æquatione $z^2+yz+1=0$, habebitur æquatio nova $z^2-2.9644z+1=0$. Ubi invenietur radix duplex 2.576, & $\frac{1}{2.576}$; ergo ſive dexteritas ipſius *A* ad dexteritatem ipſius *B* ſit ut 2.576 ad 1, ſeu ut 1 ad 2.576, *R* & *S* æqua ſorte contendent.

Corollarium. Omnes hujus generis æquationes, in quibus ratio dexteritatum determinanda venit ex datis numero nummorum & numero ludorum, ad dimenſiones dimidio ſaltem pauciores, quam ſit numerus ludorum datus, ſemper reducentur; etenim coefficientes terminorum hinc inde ab extremis æqualiter diſtantium ſemper iidem erunt, adeoque ſi fingatur æquationes iſtas formari ex $yy+yz+1=0$, & æquatione altera cujus coefficientes hinc inde ab extremis æqualiter diſtantes ſint iidem, comparationes terminorum homologorum non erunt plures quam eſt dimidius ludorum numerus, adeoque dimenſiones quantitatis y dimidio ſaltem pauciores erunt quam dimenſiones quantitatis z.

Prob. XXIV. *Poſitis iiſdem ac in* Prob. 20. *habeat* A *nummos* p, B *vero nummos* q: *Quæritur expectatio ipſius* S.

Solutio. Sumatur Binomium $a+b$, & rejectis ſemper terminis in quibus dimenſiones quantitatis a excedunt dimenſiones quantitatis b per q, & terminis in quibus dimenſiones quantitatis b excedunt dimenſiones quantitatis a per p, multiplicentur continuo termini reſidui per $a+b$, & fiant tot multiplicationes quot ſunt unitates in dato ludorum numero unitate diminuto, & habebitur numerator expectationis ipſius S, cujus denominator erit poteſtas binomii $a+b$ deſignata per numerum ludorum.

Exemplum. Sit $p=3$, & $q=2$; numerus ludorum 7.

$$\begin{array}{l}
a+b \\
\underline{a+b} \\
aa \mid + 2ab + bb \\
\underline{\qquad a+b} \\
\qquad 2aab + 3abb \mid + b^3 \\
\underline{\qquad a+b} \\
\qquad 2a^3b \mid + 5aabb + 3ab^3 \\
\underline{\qquad\qquad a+b} \\
\qquad\qquad 5a^3bb + 8aab^3 \mid + 3ab^4 \\
\underline{\qquad\qquad a+b} \\
5a^4bb \mid + 13a^3b^3 + 8aab^4 \\
\underline{\qquad a+b} \\
\qquad 13a^4b^3 + 21a^3b^4 \mid + 8aab^5
\end{array}$$

Ergo erit expectatio ipſius $S = \frac{13a\,b^3 + 21a\,b^4}{\overline{a+b}|^7}$.

Prob. XXV. A & *B colluſores duo, quorum dexteritates ſint in ratione data, hoc pactum ineant, ut non prius ludo deſiſtant quam datus numerus ludorum ſit tranſactus; ſint* R & S *ſpectatores duo, quorum* R *contendat fore ut aliquando ante concluſum certamen vel expirante certamine,* A *victorem ſe præſtiterit pluries quam* B *dato ludorum numero; Quæritur expectatio ipſius* R.

Solutio. Sit n numerus ludorum tranſigendus priuſquam A & B ludo deſiſtant, ſit $n-d$ numerus ludorum de quo R & S contendant, ſit ratio dexteritatum ut a ad b. Elevetur $a+b$ ad poteſtatem n, tunc ſi d ſit numerus impar, ſumantur tot termini iſtius poteſtatis quot ſunt unitates in $\frac{d+1}{2}$; ſumantur etiam tot termini ſequentes quot jam ſumpti fuerunt, ſed mutentur illorum coefficientes, iiſque præfigantur coefficientes terminorum præcedentium ordine retrogrado: Si vero d ſit numerus par, ſumantur tot termini poteſtatis $\overline{a+b}|^n$ quot ſunt unitates in $\frac{d+2}{2}$, ſumantur etiam tot termini ſequentes quot ſunt unitates in $\frac{1}{2}d$, ſed præfigantur illis coefficientes terminorum præcedentium ordine retrogrado, omiſſo ultimo præcedentium, & habebitur numerator expectationis ipſius R, quorum denominator erit $\overline{a+b}|^n$.

Exemp. I. Sit 10 numerus ludorum tranſigendus priuſquam A & B ludo deſiſtant, ſit 3 numerus ludorum quibus aliquando A ſuperaturus eſt ipſum B, ſit ratio dexteritatum ut 1 ad 1: Elevetur $a+b$ ad poteſtatem 10^{am}, videlicet, $a^{10} + 10a\,b + 45a\,bb + 120a\,b^3 + 210a^6b^4 + 252a^5b^5 + 210a\,b^6 + 120a^3b^7 + 45aab^8 + 10ab^9 + b^{10}$.

1°. Eſt $n = 10$; 2°. $n - d = 3$; ergo eſt $d = 7$, & $\frac{d+1}{2} = 4$. Sumantur ergo 4 termini iſtius poteſtatis, videlicet $a^{10} + 10a\,b + 45a\,bb + 120a\,b^3$; ſumantur etiam 4 termini ſequentes, illiſque præfigantur coefficientes terminorum præcedentium ordine retrogrado, & termini ſequentes evadent $120a^6b^4 + 45a\,b^5 + 10a^4b^6 + a\,b^7$. Ergo erit expectatio ipſius $R = \frac{a^{10} + 10a\,b + 45a\,bb + 120a\,b^3 + 120a^6b^4 + 45a\,b^5 + 10a^4b^6 + 1a\,b}{\overline{a+b}|^{10}}$ $= \frac{352}{1024}$.

Exemp. II. Sit $n = 6$, & $n - d = 4$; ergo eſt $d = 2$, & $\frac{d+2}{2} = 2$. Ergo expectatio ipſius R erit $\frac{a^6 + 6a^5b + a^4bb}{\overline{a+b}|^6}$.

N. B. Si d ſit numerus impar, poterunt numerator & denominator expectationis ipſius R dividi per $a+b$.

Prob. XXVI. A & B, *quorum dexteritates ſint in ratione data, videlicet ut* a *ad* b, *hoc pactum ineant, ut non prius ludo deſiſtant quam datus ludorum numerus ſit tranſactus: Adſint ſpectatores duo* R & S, *quorum* R *affirmet,* S *neget, fore ut aliquando ante finitum certamen, vel expirante certamine,* A *ſit ſuperaturus ipſum* B *dato ludorum numero* q; & *fore etiam ut aliquando* B *ſit ſuperaturus ipſum* A *dato ludorum numero* p: *Quæritur expectatio ipſius* R.

Solutio. Inveniatur numerus caſuum quibus *A* ſuperare poſſit ipſum *B* dato ludorum numero *q*, per *Prob.* 25.

Inveniatur numerus caſuum quibus *B* ſuperare poſſit ipſum *A* dato ludorum numero *p*, per idem.

Inveniatur denique numerus caſuum quibus neuter ſuperare poſſit alterum datis ludorum numeris, per *Prob.* 24.

Addantur hi caſus ſimul, & ex eorum aggregato ſubtrahatur $\overline{a+b}|^{n}$, & habebitur numerator expectationis ipſius *R*, cujus denominator erit $\overline{a+b}^{n}$.

Exemplum. Contendat *R* fore ut aliquando *A* ſit ſuperaturus ipſum *B* 2 ludis, & fore etiam ut aliquando *B* ſit ſuperaturus ipſum *A* 3 ludis, & ſit numerus ludorum tranſigendus 7.

Numerus caſuum quibus poſſit *A* ſuperare ipſum *B* 2 ludis, eſt $a^7 + 7a^6b + 21a^5bb + 21a^4b^3 + 7a^3b^4 + aab^5$.

Numerus caſuum quibus poſſit *B* ſuperare ipſum *A* 3 ludis, eſt $14a^4b^3 + 7a^3b^4 + 21aab^5 + 7ab^6 + b^7$.

Numerus caſuum quibus neuter alterum ſuperare poſſit datis ludorum numeris, eſt $13a^4b^3 + 21a^3b^4$.

Summa omnium iſtorum caſuum erit

$$a^7 + 7a^6b + 21a^5bb + 35a^4b^3 + 35a^3b^4 + 22aab^5 + 7ab^6 + b^7.$$

Subtrahatur $\overline{a+b}|^7$ ſeu

$$a^7 + 7a^6b + 21a^5bb + 35a^4b^3 + 35a^3b^4 + 21aab^5 + 7ab^6 + b^7$$

Reſiduum erit $1aab^5$.

Ergo expectatio ipſius *R* erit $\frac{aab^5}{\overline{a+b}|^7}$.

A Solution of the XV. Problem, by M. Nic Bernoulli, n. 341. p. 133.

2. Cum Methodus ſynthetica, qua uſus eſt *D. de Moivre* ad inveniendam cujuſque Colluſoris ſortem, in uſum verti nequeat tunc quando plures quam tres ſunt colluſores, ob vix perſpiciendam legem progreſſionis ſerierum quæ ſe offerunt; oſtendam hic quo modo Analyſis in ejuſmodi Problematibus, ubi depoſitum continuo augetur, adhiberi queat: eumque in finem demonſtrationem dabo analyticam trium Theorematum, quæ inveni, & quidem diu ante viſum D. *Moyvræi* libellum de Menſura Sortis, occaſione triplicis quæſtionis mihi ab amico circa ludum hunc, quem Galli vocant *le Jeu de la Poule*, propoſitæ, pro inveniendis ſcil. probabilitate vincendi, lucro item vel damno cujuſque Colluſoris, & duratione certaminis.

Theorema 1. Si Colluſores aliquot $A, B, C, D, E,$ &c. quorum numerus eſt $n+1$ & dexteritates ſunt æquales, deponant ſinguli 1, & iſtis conditionibus certent. 1°. Ut illorum duo A & B ludum incipiant. 2°. Ut victus locum ſuum tertio C cedat, ita ut ille tertius C jam cum victore contendat, quique ex hoc certamine victor evaſerit cum quarto D ludat, & ita deinceps. 3°. Ut ille depoſitum totum obtineat, qui omnes colluſores ſucceſſive vicerit. Dico probabilitates vincendi duorum quorumlibet colluſorum ſeſe immediate in ordine ludendi ſequentium eſſe in ratione $1+2^n$ ad 2^n, adeoque expectationes luſorum $A\,(B), C, D, E,$ &c. eſſe in progreſſione Geometrica.

Demonſtratio. Ponantur expectationes vincendi ipſius A vel $B=a$, ipſius $C=c$, ipſius $D=d$, ipſius $E=e$, &c. Porro cum accidere poſſit, ut colluſor aliquis prima vice in ludum intrans inveniat adverſarium qui vel nondum, vel ſemel, vel bis, vel ter, &c. jam ſucceſſive victor extitit, vocetur expectatio luſoris illius primo caſu $=z$, ſecundo $=y$, tertio $=x$, quarto $=u$, quinto $=t$, &c. Item cum colluſor aliquis vinci poſſit ab adverſario qui antea jam vel nullum, vel unum, vel duos, vel tres, &c. colluſores ſucceſſive vicit, ita ut exiens e ludo relinquat adverſarium qui vel ſemel, vel bis, vel ter, vel quater &c. victor extitit, vocetur expectatio ſeu probabilitas vincendi ejus qui exit e ludo primo caſu $=h$, ſecundo $=k$, tertio $=l$, quarto $=m$, &c. Hiſce omnibus poſitis habebuntur ſequentes novem ſeries æquationum ſignatæ N°. 1. N°. 2. N°. 3, &c. uſque ad N°. 9. *Tab.* I. Ratio eas inveniendi breviter hæc eſt. Inter æquationes N. 1°. reperitur *ex. gr.* $f=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$. Nam colluſor F certabit vel cum colluſore A, vel B, vel C, vel D, vel E: ut primum vel ſecundum contingat, oportet ut vel A vel B quater ſucceſſive victor exiſtat, cujus eventus probabilitas eſt $\frac{2}{16}$ ſeu $\frac{1}{8}$: Ut tertium contingat oportet ut C ter victor exiſtat, cujus eventus probabilitas eſt etiam $\frac{1}{8}$: Ut quartum contingat oportet ut D bis ſucceſſive vincat, quod probabilitatem habet $\frac{1}{4}$; Ut quintum contingat, oportet ut E ſemel vincat, cujus eventus probabilitas eſt $\frac{1}{2}$; ergo luſoris F probabilitas vincendi eſt $=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$. Sic inter æquationes N°. 2. eſt *ex. gr.* $x=\frac{1}{2}l+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\frac{1}{2^n}\times h+\frac{1}{2^n}\times 1$. Colluſor enim qui certat cum adverſario qui jam bis ſucceſſive victor extitit, vincere poteſt vel omnes colluſores, vel aliquos, vel nullum. Prioris eventus probabilitas eſt $\frac{1}{2^n}$, ſecundi $\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}$, & tertii $\frac{1}{2}$;

ſi primus eventus contingat, probabilitas vincendi evadit certitudo integra ſeu 1; ſi ſecundus, exit e ludo relinquens colluſorem qui ſemel vicit; ſi tertius, exit e ludo relinquens colluſorem qui ter ſucceſſive vicit; adeoque ſors ejus totalis eſt $\frac{1}{2} \times l + \overline{\frac{1}{4} + \frac{1}{8} + \frac{1}{16} \cdots \frac{1}{2^n}} \times b + \frac{1}{2^n} \times 1$. Simili ratiocinio inveniuntur æquationes N°. 3. Colluſor enim qui victus ab adverſario exit e ludo, relinquens *ex. gr.* colluſorem unius tantum ludi victorem, acquirit ſortem vel ipſius *C*, vel ipſius *D*, vel ipſius *E*, vel ipſius *F*, *&c.* prout adverſarius a quo victus eſt vincit vel omnes colluſores præter unum, vel omnes præter duos, vel omnes præter tres, *&c.* unde ſequitur quod $b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d + \frac{1}{2^{n-3}} \times e + \frac{1}{2^{n-4}} \times f +$ *&c.* Æquationes N°. 4. inveniuntur ſubtrahendo æquationes N°. 2. ab invicem: & æquationes N°. 5. ſubtrahendo æquationes N°. 3. ab invicem. Æquationes N°. 6. inveniuntur ſubſtituendo in æquationibus N°. 4. valores inventos per æquationes N°. 5. Æquationes No. 7. inveniuntur quærendo valores ipſarum $z, y, x, u,$ *&c* per æquationes N°. 1. Et his valoribus ſubſtitutis in æquationibus N° 4. habebuntur æquationes N°. 8. quæ comparatæ cum æquationibus N°. 6. dant æquationes N°. 9. ex quibus ſequitur quod $1 + 2^n : 2^n :: a : c :: c : d :: d : e$, *&c.* *Q. E. D.*

Corollarium. Hinc facile inveniuntur probabilitates vincendi ſingulorum Colluſorum, quas habent tum ante ludum inceptum, tum in quolibet ſtatu in quem ludum proſequendo pervenire poſſunt. Si ſint, *ex. gr.* tres colluſores *A*, *B*, *C*, erit $n = 2$, & $1 + 2^n : 2^n :: 5 : 4 :: a : c$: id eſt, probabilitates vincendi ipſorum *A*, *B*, *C*, antequam *A* vicerit *B*, vel *B* vicerit *A*, ſe habent ut numeri 5, 5, 4, adeoque ipſæ probabilitates ſunt $\frac{5}{14}, \frac{5}{14}, \frac{4}{14}$; omnes enim ſimul ſumptæ facere debent 1 ſeu certitudinem integram. Poſtquam *A* vicerit *B*, probabilitates vincendi ipſorum *B*, *C*, *A*, erunt b, y ſeu c, & (quia *A* æqualem habet expectationem ad victoriam, & ad ſortem ipſius *B* obtinendam) $\frac{1+b}{2}$ reſpective, *hoc eſt*, quia per æq. 1. N°. 3. $b = \frac{1}{2^{n-1}} \times c = \frac{1}{2} c$, & $c = \frac{4}{14} = \frac{2}{7}$ ut modo inventum, hæ probabilitates erunt $\frac{1}{7}, \frac{2}{7}, \frac{4}{7}$, ut D. *de Moivre* invenit Coroll. 1. Prob 15. * pag. 242.

Si ſint quatuor colluſores *A*, *B*, *C*, *D*, erit $n = 3$, & $1 + 2^n : 2^n :: 9, 8$, adeoque probabilitates colluſorum ab initio ludi erunt ut $9, 9, 8, \frac{8 \times 8}{9}$, ſive ut 81, 81, 72, 64, *hoc eſt*, ipſæ a, a, c, d, erunt $\frac{81}{298}, \frac{81}{298}, \frac{72}{298}$ & $\frac{64}{298}$. Poſtquam *A* vicerit *B*, probabilitates ipſorum *B*, *D*, *C*, *A*, erunt b, d, c, $\frac{1+3b}{}$

* *ſupr.* p. 207.

$\frac{1+3b}{4}$, eſt autem per æq. 1. N°. 3. $b = \frac{1}{2^{n-1}} \times c + \frac{1}{2^{n-2}} \times d = \frac{1}{4}c + \frac{1}{2}d$, $c = \frac{72}{298} = \frac{36}{149}$, & $d = \frac{64}{298} = \frac{32}{149}$, ut modo inventum: ergo hæ probabilitates erunt $\frac{25}{149}$, $\frac{32}{149}$, $\frac{36}{149}$, $\frac{56}{149}$ reſpective. Poſtquam *A* vicerit *B* & *C*, probabilitates vincendi ipſorum *C*, *B*, *D*, *A*, erunt k, $\frac{c}{2}$, x, $\frac{1+b}{2}$, ſeu (quia per æq. 2. N°. 3. $k = \frac{1}{2^{n-2}} \times d = \frac{1}{2}d$, & per æq. 3. N°. 7. $x = 2d - c$) $\frac{16}{149}$, $\frac{18}{149}$, $\frac{28}{149}$, $\frac{87}{149}$. Et nota quod calculi bonitas confirmetur ex eo, quod ſummæ harum probabilitatum, *hoc eſt*, $\frac{1}{7} + \frac{2}{7} + \frac{4}{7}$ in priori exemplo, & $\frac{25}{149} + \frac{32}{149} + \frac{36}{149} + \frac{56}{149}$, nec non $\frac{16}{149} + \frac{18}{149} + \frac{28}{149} + \frac{87}{149}$ in poſteriori exemplo, ſingulæ ſint $= 1$ ſeu certitudini integræ.

Theorema 2. Poſitis quæ prius & inſuper hac conditione, ut victus ſemper mulctetur ſumma p, quæ depoſito augendo inſerviat; quod depoſitum ſic gradatim auctum illi ſoli cedat, qui omnium ſucceſſive colluſorum victor extiterit; denotatis etiam ut antea per literas minuſculas a, c, d, e, &c. probabilitatibus vincendi ipſorum *A* (vel *B*), *C*, *D*, *E*, &c. reſpective: per eaſdem vero literas majuſculas *A*, *C*, *D*, *E*, &c. ipſorum *A* (vel *B*), *C*, *D*, *E*, &c. expectationibus, *hoc eſt*, portionibus depoſiti quas ſinguli expectant: Dico, fore ſemper $C = \frac{\overline{A+ap} \times 2^n - ncp}{1+2^n}$, $D = \frac{\overline{C+cp} \times 2^n - ndp}{1+2^n}$, $E = \frac{\overline{D+dp} \times 2^n - nep}{1+2^n}$, &c.

Demonſtratio. Denotetur ut prius per literas minuſculas z, y, x, u, t, &c. probabilitas vincendi ludentis cum adverſario, qui jam vel nullum, vel unum, vel duos, &c. colluſores ſucceſſive vicit; per eaſdem vero literas majuſculas *Z*, *Y*, *X*, *U*, *T*, &c. ejus expectatio, quam ſcil. habet diverſis illis caſibus, depoſito exiſtente $n+1$, $n+1+p$, $n+1+2p$, $n+1+3p$, &c. reſpective. Sic etiam per literas minuſculas h, k, l, m, &c. denotetur probabilitas vincendi luſoris victi ab adverſario, qui antea vel nullum, vel unum, vel duos, &c. colluſores ſucceſſive vicerat; quemadmodum per literas majuſculas *H*, *K*, *L*, *M*, &c. ejuſdem expectatio diverſis illis caſibus, depoſito exiſtente $n+1+p$, $n+1+2p$, $n+1+3p$, &c. reſpective. His poſitis, iiſdem quibus antea ratiociniis inveniuntur ſequentes duodecim æquationum ſeries in *Tab.* II. ſignatæ N°. 1. N°. 2. N°. 3.

N°. 3. *&c.* Inter æquationes N°. 1. *ex. gr.* eſt $E = \frac{U}{4} + \frac{X + xp}{4} + \frac{Y + 2yp}{2}$. Luſor enim *E* ludet vel cum luſore *A*, vel luſore *B*, vel *C*, vel *D*. Si ludit cum *A* vel *B*, expectatio ejus erit $= U$, quia ludit cum adverſario qui jam tres adverſarios vicit, depoſito exiſtente $n + 1 + 3p$. Si ludit cum luſore *C*, expectatio ejus erit $= X + xp$, ludit enim cum adverſario qui jam duos colluſores vicit, adeoque ſi depoſitum eſſet $n + 1 + 2p$ ejus expectatio eſſet $= X$: verum quia ludente *E* depoſitum eſt $= n + 1 + 3p$, ob tres colluſores victos & ſumma *p* mulctatos, addenda eſt expectationi *X* portio illa mulctæ unius *p*, quam luſor *E* ſperare poteſt: eſt autem hæc portio (quia probabilitas ejus vincendi eſt x) $= xp$, ejus igitur expectatio totalis tunc erit $= X + xp$. Sic ſi ludit cum luſore *D*, expectatio ejus erit $= Y + 2yp$: additur ad *Y* (quæ eſſet ejus expectatio depoſito exiſtente $n + 1 + p$) portio $2yp$, quæ ipſi debetur de duabus mulctis $2p$, quibus depoſitum $n + 1 + 3p$ majus eſt quam $n + 1 + p$. Simili modo habentur æquationes N°. 2, 3, 4 & 5. Subſtituendo autem primam æquationem N°. 2 *Tab.* I. in æquationibus N°. 4. habentur æquationes N°. 6. Et ſubſtituendo primam æquationem N°. 3. *Tab.* I. in æquationibus N°. 5. habentur æquationes N°. 7. quibus dein in æquationibus N°. 6. ſubſtitutis habentur æquationes N°. 8. Æquationes N°. 9. inveniuntur quærendo valores ipſarum *Z*, *Y*, *X*, *U*, *&c.* per æquationes N°. 1. *Tab.* I. & II. vel N°. 2. *Tab.* II. & N°. 7. *Tab.* I. Et his valoribus ſubſtitutis in æquationibus N°. 4. habentur æquationes N°. 10. Quæ comparatæ cum æquationibus N°. 8. (in quibus pro *z* ſubſtitnatur *a*, per 1. æq. *Tab.* I.) dant æquationes N°. 11. Et hæ æquationes No. 11. comparatæ cum æquationibus N°. 9. *Tab.* I. dant æquationes N°. 12. quæ conſtituunt Theorema, quod demonſtrandum erat.

Corollarium. Hinc quoque facile inveniuntur ſingulorum Colluſorum ſortes ſeu expectationes, ipſorumque adeo lucra vel damna. Sint *ex. gr.* tres colluſores *A*, *B*, *C*: erit $C = \frac{\overline{A + ap} \times 2^n - ncp}{1 + 2^n} =$ (ob $n = 2$) $\frac{4A + 4ap - 2cp}{5} =$ (ob $a = \frac{5}{14}$ & $c = \frac{2}{7}$ per Corollarium Theor. 1.) $\frac{4A + \frac{6}{7}p}{5}$. Unde cum omnium trium expectationes ſimul ſumptæ, *id eſt*, $A + A + C$ æquare debeant id quod ab initio depoſitum fuit, *id eſt* 3, erit $2A + \frac{4A + \frac{6}{7}p}{5} = \frac{14A + \frac{6}{7}p}{5} = 3$, & $14A = 15 - \frac{6}{7}p$, & $A = 15 - \frac{6}{7}p$, & $A = \frac{15}{14} - \frac{3}{49}p =$ expectationi luſoris *A* vel *B*: proinde *C* expectatio luſoris tertii $C = \frac{4A + \frac{6}{7}p}{5} = \frac{6}{7} + \frac{6}{49}p$. A quibus expectationibus ſi ſubtrahatur 1, id quod ab initio ſinguli depoſuerunt, remanebit

bit ibi $\frac{1}{14} - \frac{3}{49}p$, hic $\frac{6}{49}p - \frac{1}{7}$; quemadmodum D. *de Moivre* invenit.

Exempl. 2. Sint colluſores 4, A, B, C, D, erit $C = \frac{\overline{A + ap} \times 2^n - ncp}{1 + 2^n}$ $=$ (ob $n = 3$) $\frac{8A + 8ap - 3cp}{9}$ $=$ (ob $a = \frac{81}{298}$ & $c = \frac{36}{149}$, per Coroll. Theorema 1.) $\frac{8A + \frac{216}{149}p}{9}$; item $D = \frac{\overline{C + cp} \times 2^n - ndp}{1 + 2^n}$ $=$ $\frac{8C + 8cp - 3dp}{9}$ $=$ (ob $d = \frac{32}{149}$ per id. corr.) $\frac{8C + \frac{192}{149}p}{9}$ $=$ $\frac{64A + \frac{3456}{149}p}{81}$: unde habebitur æquatio $2A + C + D = 2A + \frac{8A + \frac{216}{149}p}{9} + \frac{64A + \frac{3456}{149}p}{9} = \frac{298A + \frac{5400}{149}p}{81} = 4$, ſive $149A + \frac{2700}{149}p = 162$, & $A = \frac{162}{149} - \frac{2700}{22201}p$. Hinc $C = \frac{8A + \frac{216}{149}p}{9} = \frac{144}{149} + \frac{1176}{22201}p$, & $D = \frac{64A + \frac{3456}{149}p}{81} = \frac{128}{149} + \frac{4224}{22201}p$. Subtracta autem unitate 1, quam ſinguli ab initio ludi depoſuerunt, remanebit $\frac{13}{149} - \frac{2700}{22201}p$ pro luſore A vel B, $\frac{1176}{22201}p - \frac{5}{149}$ pro C, & $\frac{4224}{22201}p - \frac{21}{149}$ pro D; quæ ſingula indigitabunt lucrum vel damnum, prout pars affirmata præpollet negatæ, vel contra. Simili ratione habebuntur etiam ſortes quas acquirunt in quolibet ſtatu in quem ludum proſequendo pervenire poſſunt.

Theorema 3. Poſitis quæ prius, ſi adſint ſpectatores Q, R, S, T, U, &c. quorum numerus ſit n unitate minor quam numerus colluſorum, quorumque prior Q affirmet certamen finitum iri poſt $n + p$ ludos peractos, R poſt $n + p - 1$, S poſt $n + p - 2$, T poſt $n + p - 3$, U poſt $n + p - 4$, &c. præciſe, non antea; ſintque q, r, s, t, u, &c. ſortes ipſorum Q, R, S, T, U, &c. Dico fore $q = \frac{1}{2}r + \frac{1}{4}s + \frac{1}{8}t + \frac{1}{16}u +$ &c.

Demonſtratio. Vocetur A colluſor ille, qui poſt $n + p$ ludos vincere ſupponitur: hic intrare debet in ludum poſt p ludos peractos, & tum ludet contra adverſarium, qui jam vel unum vel duos, vel tres, &c. colluſores ſucceſſive vicit. Jam cum, ut primus caſus contingat, & ut colluſor A omnes ſuos colluſores præter unum, *id eſt*, $n - 1$ colluſores ſucceſſive vincat, æque probabile ſit quam ut adverſarius ejus vincat $n - 1$ colluſores, *id eſt*, (quia jam unius colluſoris victor fuit) ut certamen finiat poſt $n + p - 1$ ludos peractos, hujuſque eventus probabilitas ſit $= r$: erit probabilitas ut colluſor A unum adhuc colluſorem vincat, *id eſt*, certamen

tamen finiat poſt $n+p$ ludos $= \frac{1}{2} r$. Sic, ut ſecundus caſus exiſtat, & ut *A* omnes colluſores præter duos vincat, æque probabile eſt quam ut certamen finiatur poſt $n+p-2$ ludos, adeoque ut tunc *A* vincat adhuc duos colluſores, *id eſt*, ut certamen finiat poſt $n+p$ ludos, probabilitas erit $= \frac{1}{4} s$. Eodem modo ut, tertio caſu exiſtente, *A* vincat omnes colluſores, probabilitas eſt $= \frac{1}{8} t$; ut quarto $= \frac{1}{16} u$, *&c.* Quare ut indifferenter certamen finiatur poſt $n+p$ ludos, probabilitas eſt $\frac{1}{2} r + \frac{1}{4} s + \frac{1}{8} t + \frac{1}{16} u +$ *&c.* $= q$. *Q. E. D.*

Corollarium 1. Facile hinc invenitur quænam ſit probabilitas ut certamen finiatur intra datum quemvis ludorum numerum. Series enim fractionum incipientium a fractione $\frac{1}{2^{n-1}}$, quarum denominatores creſcant in continua proportione dupla, numerator autem cujuſque fractionis ſit ſumma numeratorum tot fractionum immediate præcedentium quot ſunt unitates in $n-1$, dabit omnes ſucceſſive probabilitates, ut certamen finiatur peractis præciſe n, $n+1$, $n+2$, $n+3$, *&c.* ludis: & per conſequens ſi addantur tot termini hujus ſeriei quot ſunt unitates in $p+1$, ſumma ipſorum exprimet probabilitatem ut certamen finiatur ad minimum ludis $n+p$ peractis. *Ex. gr.* Si ſint colluſores 4, adeoque $n=3$, habebitur hæc ſeries $\frac{1}{4}, \frac{1}{8}, \frac{2}{16}, \frac{3}{32}, \frac{5}{64}, \frac{8}{128}, \frac{13}{256}, \frac{21}{512}$ *&c.* E qua ſi fiat alia $\frac{1}{4}, \frac{3}{8}, \frac{8}{16}, \frac{19}{32}, \frac{43}{64}, \frac{94}{128}, \frac{201}{256}$, *&c.* cujus termini ſint ſummæ terminorum præcedentis ſeriei, denotabunt iidem termini qualis ſit probabilitas ut certamen finiatur ad minimum 3, 4, 5, 6, *&c.* ludis.

Corollarium 2. Poteſt terminus quicunque prioris ſeriei (excepto primo termino,) ut & ſumma omnium terminorum, *id eſt*, terminus quicunque poſterioris ſeriei, per formulam generalem exprimi hoc modo. Si $n+1$ ſit numerus colluſorum, & p ſit numerus terminorum, erit ultimus terminus prioris ſeriei

$$\frac{1}{2^n} - \frac{\overline{p-n+1}}{1 \times 2^{2n}} + \frac{\overline{p-2n} \times \overline{p-2n+3}}{1 \times 2 \times 2^{3n}} - \frac{\overline{p-3n} \times \overline{p-3n+1} \times \overline{p-3n+5}}{1 \times 2 \times 3 \times 2^{4n}} + \frac{\overline{p-4n} \times \overline{p-4n+1} \times \overline{p-4n+2} \times \overline{p-4n+7}}{1 \times 2 \times 3 \times 4 \times 2^{5n}},\ - \text{\&c.}$$

Et ſumma omnium terminorum ſive ultimus terminus poſterioris ſeriei $= \frac{p+1}{1 \times 2^n} -$

$$\frac{\overline{p-n}\times\overline{p-n+3}}{1\times 2\times 2^{2n}}+\frac{\overline{p-2n}\times\overline{p-2n+1}\times\overline{p-2n+5}}{1\times 2\times 3\times 2^{3n}}-$$

$$\frac{\overline{p-3n}\times\overline{p-3n+1}\times\overline{p-3n+2}\times\overline{p-3n+7}}{1\times 2\times 3\times 4\times 2^{4n}}+\&c.$$

Tabula I.

Intrat.		Exit.	
	Sors		Sors
0	z	1	b
1	y	2	k
2	x	3	l
3	u	4	m
4	t		

No. 1.

$$a=z$$

$$c=y$$

$$d=\frac{1}{2}x+\frac{1}{2}y$$

$$e=\frac{1}{4}u+\frac{1}{4}x+\frac{1}{2}y$$

$$f=\frac{1}{8}t+\frac{1}{8}u+\frac{1}{4}x+\frac{1}{2}y$$

No. 2.

$$z=\frac{1}{2}+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1$$

$$y=\frac{1}{2}k+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1.$$

$$x=\frac{1}{2}l+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1$$

$$u=\frac{1}{2}m+\overline{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots\frac{1}{2^n}}\times b+\frac{1}{2^n}\times 1.$$

No. 3.

$$b=\frac{1}{2^{n-1}}\times c+\frac{1}{2^{n-2}}\times d+\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\cdots$$

$$k=\frac{1}{2^{n-2}}\times d+\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\cdots$$

$$l=\frac{1}{2^{n-3}}\times e+\frac{1}{2^{n-4}}\times f+\cdots$$

$$m=\frac{1}{2^{n-4}}\times f+\cdots$$

No. 4. No. 6. No. 8.

$$z-y=\frac{1}{2}b-\frac{1}{2}k\quad=\frac{1}{2^n}\times c\quad=a-c$$

$$y-x=\frac{1}{2}k-\frac{1}{2}l\quad=\frac{1}{2^{n-1}}\times d\quad=2c-2d$$

$$x-u=\frac{1}{2}l-\frac{1}{2}m\quad=\frac{1}{2^{n-2}}\times e\quad=4d-4e$$

Tabula II

Intrat.			*Exit.*				
Depositum		*Sors.*	*Depositum*		*Sors*	*Deposit.*	
$n+1$	0	Z	$n+1+p$	1	H	$n+1+$	$A=Z$
$n+1+p$	1	Y	$n+1+2p$	2	K	$n+1+p$	$C=Y$
$n+1+2p$	2	X	$n+1+3p$	3	L	$n+1+2p$	$D=\frac{1}{2}X+\frac{1}{2}\times\overline{Y+yp}$
$n+1+3p$	3	V	$n+1+4p$	4	M	$n+1+3p$	$E=\frac{1}{4}V+\frac{1}{4}\times\overline{X+xp}+\frac{1}{2}\times\overline{Y+2yp}$
$n+1+4p$	4	T				$n+1+4p$	$F=\frac{1}{8}T+\frac{1}{8}\times\overline{V+up}+\frac{1}{4}\times\overline{X+2xp}+\frac{1}{2}\times\overline{Y+3yp}$

N.° 1

N.° 2

$$Z=\frac{1}{2}\times\overline{H-p}+\frac{1}{4}\times\overline{H-p+hp}+\frac{1}{8}\times\overline{H-p+2hp}+\frac{1}{16}\times\overline{H-p+3hp}+\cdots\cdots\frac{1}{2^n}\times\overline{H-p+nhp-hp}+\frac{1}{2^n}\times\overline{np+n+1}$$

$$Y=\frac{1}{2}\times\overline{K-p}+\frac{1}{4}\times\overline{H-p+2hp}+\frac{1}{8}\times\overline{H-p+3hp}+\frac{1}{16}\times\overline{H-p+4hp}+\cdots\cdots\frac{1}{2^n}\times\overline{H-p+nhp}+\frac{1}{2^n}\times\overline{np+p+n+1}$$

$$X=\frac{1}{2}\times\overline{L-p}+\frac{1}{4}\times\overline{H-p+3hp}+\frac{1}{8}\times\overline{H-p+4hp}+\frac{1}{16}\times\overline{H-p+5hp}+\cdots\cdots\frac{1}{2^n}\times\overline{H-p+nhp+hp}+\frac{1}{2^n}\times\overline{np+2p+n+1}$$

$$V=\frac{1}{2}\times\overline{M-p}+\frac{1}{4}\times\overline{H-p+4hp}+\frac{1}{8}\times\overline{H-p+5hp}+\frac{1}{16}\times\overline{H-p+6hp}+\cdots\cdots\frac{1}{2^n}\times\overline{H-p+nhp+hp}+\frac{1}{2^n}\times\overline{np+3p+n+1}$$

N.° 3

$$H=\frac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\frac{1}{2^{n-2}}\times\overline{D+ndp-2dp}+\frac{1}{2^{n-3}}\times\overline{E+nep-3ep}+\frac{1}{2^{n-4}}\times\overline{F+nfp-4fp}+\cdots\cdots$$

$$K=\frac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\frac{1}{2^{n-3}}\times\overline{E+nep-2ep}+\frac{1}{2^{n-4}}\times\overline{F+nfp-3fp}+\cdots\cdots$$

$$L=\frac{1}{2^{n-3}}\times\overline{E+nep-ep}+\frac{1}{2^{n-4}}\times\overline{F+nfp-2fp}+\cdots\cdots$$

$$M=\frac{1}{2^{n-4}}\times\overline{F+nfp-fp}+\cdots\cdots$$

N.° 4 N.° 6 N.° 8

$$Y-Z=\frac{1}{2}K-\frac{1}{2}H+\frac{1}{4}hp+\frac{1}{8}hp+\frac{1}{16}hp+\cdots\cdots\frac{1}{2^n}\times hp=\frac{1}{2}K-\frac{1}{2}H+zp-\frac{1}{2}hp=-\frac{1}{2^{n-0}}\times C-\frac{ncp}{2^n}+zp$$

$$X-Y=\frac{1}{2}L-\frac{1}{2}K+\frac{1}{4}hp+\frac{1}{8}hp+\frac{1}{16}hp+\cdots\cdots\frac{1}{2^n}\times hp=\frac{1}{2}L-\frac{1}{2}K+zp-\frac{1}{2}hp=-\frac{1}{2^{n-1}}\times D-\frac{ndp}{2^{n-1}}-\frac{cp}{2^n}+zp$$

$$V-X=\frac{1}{2}M-\frac{1}{2}L+\frac{1}{4}hp+\frac{1}{8}hp+\frac{1}{16}hp+\cdots\cdots\frac{1}{2^n}\times hp=\frac{1}{2}M-\frac{1}{2}L+zp-\frac{1}{2}hp=-\frac{1}{2^{n-2}}\times E-\frac{nep}{2^{n-2}}-\frac{dp}{2^{n-1}}-\frac{cp}{2^n}+$$

N.° 5 N.° 7

$$K-H=-\frac{1}{2^{n-1}}\times\overline{C+ncp-cp}+\frac{1}{2^{n-2}}\times dp+\frac{1}{2^{n-3}}\times ep+\frac{1}{2^{n-4}}\times fp+\cdots\cdots=-\frac{1}{2^{n-1}}\times C-\frac{ncp}{2^{n-1}}+hp$$

$$L-K=-\frac{1}{2^{n-2}}\times\overline{D+ndp-dp}+\frac{1}{2^{n-3}}\times ep+\frac{1}{2^{n-4}}\times fp+\cdots\cdots=-\frac{1}{2^{n-2}}\times D-\frac{ndp}{2^{n-2}}-\frac{cp}{2^{n-1}}+hp$$

$$M-L=-\frac{1}{2^{n-3}}\times\overline{E+nep-ep}+\frac{1}{2^{n-4}}\times fp+\cdots\cdots=-\frac{1}{2^{n-3}}\times E-\frac{nep}{2^{n-3}}-\frac{dp}{2^{n-2}}-\frac{cp}{2^{n-1}}+hp$$

N.° 9.

$Z=A$

$Y=C$

$X=2D-C-cp$

$V=4E-2D-C-2dp-2cp$

N.° 10 N.° 8

$$C-A=Y-Z=-\frac{1}{2^n}\times C-\frac{ncp}{2^n}+zp$$

$$2D-2C-cp=X-Y=-\frac{1}{2^{n-1}}\times D-\frac{ndp}{2^{n-1}}-\frac{cp}{2^n}+zp$$

$$4E-4D-2dp-cp=V-X=-\frac{1}{2^{n-2}}\times E-\frac{nep}{2^{n-2}}-\frac{dp}{2^{n-1}}-\frac{cp}{2^n}+zp$$

N.° 11 N.° 12

$$C=\frac{A\times 2^n+ap\times 2^n-ncp}{1+2^n}=\frac{\overline{A+ap}\times 2^n-ncp}{1+2^n}.$$

$$D=\frac{C\times 2^n+cp\times\overline{2^{n-1}-\frac{1}{2}}+ap\times 2^{n-1}-ndp}{1+2^n}=\frac{\overline{C+cp}\times 2^n-ndp}{1+2^n}$$

$$E=\frac{D\times 2^n+dp\times\overline{2^{n-1}-\frac{1}{2}}+cp\times\overline{2^{n-2}-\frac{1}{4}}+ap\times 2^{n-2}-nep}{1+2^n}=\frac{\overline{D+dp}\times 2^n-nep}{1+2^n}$$

I. Senex ſculp

$$\begin{array}{l|l|l}
\text{No. 5.} & \text{No. 7.} & \text{No. 9.}\\
b-k=\frac{1}{2^n-1}\times c & \begin{array}{l} z=a\\ y=c\end{array} & c=a\times\frac{2^n}{1+2^n}\\
k-l=\frac{1}{2^n-2}\times d & \begin{array}{l} x=2d-y=2d-c\\ u=4e-x-2y=4e-2d-e\end{array} & d=c\times\frac{2^n}{1+2^n}\\
l-m=\frac{1}{2^n-3}\times e & & e=d\times\frac{2^n}{1+2^n}
\end{array}$$

Corollarium 3. Poteſt quis, priuſquam ludus inchoetur, in ſe ſuſcipere, ut ſummam $n+1$ de qua colluſores contendunt, & mulctas omnes pendat, ſi ſibi initio in manus datum ſit $n+1+2^n-1\times p$.

Demonſtrationem duorum præcedentium corollariorum curioſis indagandam relinquo.

3. *Deſignationes.* Si *B* & *C* colluſores duo ſimul certent; ad deſignandum *B* victorem eſſe, *C* victum, ſcribatur *BC*; atque viciſſim ad deſignandum *C* victorem eſſe, *B* victum; ſcribatur *CB*: & ſic de cæteris.

***Another general Solution by Combinations and infinite Series's. by Mr.* De Moivre, *ib.* p. 145.**

Ponatur 1° *B* vincere *A*, certamenque concludi tribus ludis

$$\left.\begin{array}{l}\underline{BA}\\ BC\\ BD\end{array}\right\}$$ Sic patet *B* victorem neceſſario evadere.

Ponatur 2° *B* vincere *A*, certamenque concludi quatuor ludis

$$\left.\begin{array}{l}\underline{BA}\\ CB\\ CD\\ CA\end{array}\right\}$$ Sic patet *C* victorem neceſſario evadere.

Ponatur 3° *B* vincere *A*, certamenque concludi quinque ludis

$$\left.\begin{array}{ll}\underline{BA} & \underline{BA}\\ CB* & BC\\ DC & DB\\ DA & DA\\ DB & DC\end{array}\right\}$$ Sic patet *D* victorem neceſſario evadere, idque duplici modo.

Ponatur 4° *B* prima vice vincere *A*, certamenque concludi ſex ludis

$$\left.\begin{array}{lll}\underline{BA} & \underline{BA} & \underline{BA}\\ CB & CB* & BC\\ DC* & CD* & DB\\ AD & AC & AC\\ AB & AB & AD\\ AC & AD & AB\end{array}\right|$$ Sic patet *A* victorem neceſſario evadere, idque triplici modo.

Pona-

Ponatur 5° certamen concludi ſeptem ludis, ponaturque ſemper *B* prima vice vincere ipſum *A*.

$$\left.\begin{array}{cc} B & A \\ \hline C & B \\ D & C \\ A & D^{*} \\ B & A \\ B & C \\ B & D \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B \\ D & C^{*} \\ D & A \\ B & D \\ B & C \\ B & A \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B^{*} \\ C & D \\ A & C \\ B & A \\ B & D \\ B & C \end{array}\quad\begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ A & D^{*} \\ C & A \\ C & B \\ C & D \end{array}\quad\begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ D & A \\ C & D \\ C & B \\ C & A \end{array}\right\}$$

Sic patet *B* vel *C* neceſſario victores evadere, *B* triplici modo, *C* duplici.

Ponatur 6° certamen concludi octo ludis,

$$\begin{array}{cc} B & A \\ \hline C & B \\ D & C \\ A & D \\ B & A^{*} \\ C & B \\ C & D \\ C & A \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B \\ D & C \\ A & D^{*} \\ A & B \\ C & A \\ C & D \\ C & B \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B \\ D & C^{*} \\ D & A \\ B & D \\ C & B \\ C & A \\ C & D \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B \\ C & D \\ A & C \\ B & A^{*} \\ D & B \\ D & C \\ D & A \end{array}\quad\begin{array}{cc} B & A \\ \hline C & B^{*} \\ C & D \\ A & C \\ A & B \\ D & A \\ D & B \\ D & C \end{array}\quad\begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ A & D \\ C & A^{*} \\ B & C \\ B & D \\ B & A \end{array}\quad\begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ A & D^{*} \\ A & C \\ B & A \\ B & D \\ B & C \end{array}\quad\begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ D & A \\ C & D \\ B & C \\ B & A \\ B & D \end{array}$$

Sic patet *C* victorem evadere triplici, *D* duplici, *B* triplici modo, &c.

Nunc ordine ſcribantur literæ quibus victores deſignantur.

$$\begin{array}{r|l} 3, & 1\,B \\ 4, & 1\,C \\ 5, & 2\,D \\ 6, & 3\,A \\ 7, & 3\,B + 2\,C \\ 8, & 3\,C + 2\,D + 3\,B \\ 9, & 3\,D + 2\,A + 3\,C + 3\,D + 2\,A \\ 10, & 3\,A + 2\,B + 3\,D + 3\,A + 2\,B + 3\,A + 2\,C + 3\,D. \end{array}$$

&c.

Perſpecta illarum formatione, patebit 1° literam *B* in ordine aliquo ſemper toties reperiri, quoties *A* in ordine ultimo & penultimo reperitur: 2° *C* in ordine aliquo toties reperiri quoties *B* in ordine ultimo & *D* in penultimo reperiuntur: 3° *D* in ordine aliquo toties reperiri quoties *C* in ultimo & *B* in penultimo: 4° *A* in ordine aliquo ſemper toties reperiri quoties *D* in ordine ultimo & *C* in penultimo reperiuntur.

Sed numerus variationum dato cuilibet ludorum numero competens, duplus eſt numeri variationum omnium dato ludorum numero unitate diminuto competentis: adeoque Probabilitas quam habet Colluſor *B* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *A* ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat idem *A*, ut vinceret dato ludorum numero minus duobus: & ſic de cæteris.

Probabilitas quam habet *C*, ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *D* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *C*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *B*, ut vinceret dato ludorum numero minus duobus.

Probabilitas quam habet *A* ut vincat dato ludorum numero, eſt ſubdupla probabilitatis quam habebat *D*, ut vinceret dato ludorum numero minus uno; atque etiam ſubquadrupla probabilitatis quam habebat *C* ut vinceret dato ludorum numero minus duobus.

Ex jam obſervatis facile eſt componere Tabulam Probabilitatum, quas *B*, *C*, *D*, *A* habent ut victores evadant dato ludorum numero, atque etiam illorum ſortium ſeu expectationum.

Tabula Probabilitatum, &c.

		B	C	D	A
'	3	$\frac{1}{4}\times\overline{4+3p}$			
''	4		$\frac{1}{8}\times\overline{4+4p}$		
'''	5			$\frac{2}{16}\times\overline{4+5p}$	
''''	6				$\frac{3}{32}\times\overline{4+6p}$
v	7	$\frac{3}{64}\times\overline{4+7p}$	$\frac{2}{64}\times\overline{4+7p}$		
v'	8	$\frac{3}{128}\times\overline{4+8p}$	$\frac{3}{128}\times\overline{4+8p}$	$\frac{2}{128}\times\overline{4+8p}$	
v''	9		$\frac{3}{256}\times\overline{4+9p}$	$\frac{6}{256}\times\overline{4+9p}$	$\frac{4}{256}\times\overline{4+9p}$
v'''	10	$\frac{4}{512}\times\overline{4+10p}$	$\frac{2}{512}\times\overline{4+10p}$	$\frac{6}{512}\times\overline{4+10p}$	$\frac{9}{512}\times\overline{4+10p}$
'x	11	$\frac{13}{1024}\times\overline{4+11p}$	$\frac{10}{1024}\times\overline{4+11p}$	$\frac{2}{1024}\times\overline{4+11p}$	$\frac{9}{1024}\times\overline{4+11p}$
x	12	$\frac{18}{2048}\times\overline{4+12p}$	$\frac{19}{2048}\times\overline{4+12p}$	$\frac{14}{2048}\times\overline{4+12p}$	$\frac{4}{2048}\times\overline{4+12p}$
		&c.		&c.	&c.

Jam vero Series iſtæ ſunt convergentes, adeoque ſingulæ ſummari poſſunt per vulgarem Arithmeticam; & obtinebuntur vel ſummæ accuratæ ſi poſſint, vel ſaltem approximatæ, ſi non liceat terminos multos adhibere.

Invenire ſummas Probabilitatum ad infinitum uſque pergentium, quas Col*luſores habent ut victores evadant.*

Sint Probabilitates omnes ipſius B ad infinitum, nempe

$$B' + B'' + B''' + B'''' + B^{v} + B^{v\prime}, \&c. = y$$

Probabilitates ipſius C

$$C' + C'' + C''' + C'''' + C^{v} + C^{v\prime}, \&c. = z$$

Probabilitates ipſius D

$$D' + D'' + D''' + D'''' + D^{v} + D', \&c. = v$$

Probabilitates ipſius A

$$A' + A'' + A''' + A'''' + A^{v} + A^{vi}, \&c. = x$$

Scribantur autem in Scala perpendiculariter deſcendente, ad hunc modum.

$$\begin{aligned} B' &= B' \\ B'' &= B'' \\ B' &= \tfrac{1}{2} A'' + \tfrac{1}{4} A' \\ B'''' &= \tfrac{1}{2} A''' + \tfrac{1}{4} A'' \\ B^{v} &= \tfrac{1}{2} A'''' + \tfrac{1}{4} A''' \qquad \text{Proinde } y = \tfrac{1}{4} + \tfrac{3}{4} x. \\ B^{v\prime} &= \tfrac{1}{2} A^{v} + \tfrac{1}{4} A'''' \end{aligned}$$

Ergo $y = \frac{1}{4} + \frac{1}{2} x + \frac{1}{4} x$.

Demonſtratio. Etenim prima columna perpendicularis $= y$, ex Hypoth.

Eſt vero $A' + A'' + A''' + A'''' + A^{v}$ &c. $= x$, ex hypotheſi;

Ergo $\frac{1}{2} A' + \frac{1}{2} A'' + \frac{1}{2} A''' + \frac{1}{2} A'''' + \frac{1}{2} A^{v}$ &c. $= \frac{1}{2} x$.

Proinde $\frac{1}{2} A'' + \frac{1}{2} A''' + \frac{1}{2} A'''' + \frac{1}{2} A^{v}$ &c. $= \frac{1}{2} x - \frac{1}{2} A'$.

Et $B' + B'' + \frac{1}{2} A'' + \frac{1}{2} A''' + \frac{1}{2} A''''$ &c. $= \frac{1}{2} x - \frac{1}{2} A' + B' + B''$.

Sed $\frac{1}{2} A' = 0$, $B'' = 0$ & $B' = \frac{1}{4}$, ut patet ex Tabula.

Ergo ſecunda columna perpendicularis $= \frac{1}{4} + \frac{1}{2} x$.

Sed tertia columna perpendicularis $= \frac{1}{4} x$.

erit igitur $y = \frac{1}{4} + \frac{3}{4} x$.

Simili modo ſcribantur

$$\begin{aligned} C' &= C' \\ C' &= C' \\ C''' &= \tfrac{1}{2} B'' + \tfrac{1}{4} D' \\ C'''' &= \tfrac{1}{2} B''' + \tfrac{1}{4} D'' \qquad \text{hoc eſt } z = \tfrac{1}{2} y + \tfrac{1}{4} v. \\ C^{v} &= \tfrac{1}{2} B'''' + \tfrac{1}{4} D''' \\ C^{v\prime} &= \tfrac{1}{2} B^{v} + \tfrac{1}{4} D'''' \\ &\&c. \end{aligned}$$

Ergo $z = \frac{1}{8} + \frac{1}{2} y - \frac{1}{8} + \frac{1}{4} v$.

Scribantur etiam

$$\begin{aligned} D' &= D' \\ D'' &= D'' \\ D''' &= \tfrac{1}{2} C' + \tfrac{1}{4} B' \\ D'''' &= \tfrac{1}{2} C'' + \tfrac{1}{4} B'' \\ D^{v} &= \tfrac{1}{2} C''' + \tfrac{1}{4} B''' \\ D^{vi} &= \tfrac{1}{2} C^{v} + \tfrac{1}{4} B'''' \\ &\&c. \end{aligned}$$

& pari Argumento patebit

$$v + \tfrac{1}{2} z + \tfrac{1}{4} y$$

Scribantur denique

$$\begin{aligned} A' &= A' \\ A'' &= A'' \\ A''' &= \tfrac{1}{2} D'' + \tfrac{1}{4} C' \\ A'''' &= \tfrac{1}{2} D''' + \tfrac{1}{4} C'' \\ A^{v} &= \tfrac{1}{2} D'''' + \tfrac{1}{4} C''' \\ A^{vi} &= \tfrac{1}{2} D^{v} + \tfrac{1}{4} C'''' \\ &\&c. \end{aligned}$$

Unde concludetur:

$$x = \tfrac{1}{2} v + \tfrac{1}{4} z$$

Reſolutis autem quatuor iſtis æquationibus, reperietur

$$B' + B'' + B''' + B'''' \ \&c. = y = \frac{56}{149}$$

$$C' + C'' + C''' + C'''' \ \&c. = z = \frac{36}{149}$$

$$D' + D'' + D''' + D'''' \ \&c. = v = \frac{32}{149}$$

$$A' + A'' + A''' + A'''' \ \&c. = x = \frac{25}{149}$$

Valoribus iſtis inventis, ponatur jam $\frac{56}{149} = b$, $\frac{36}{149} = c$, $\frac{32}{149} = d$, $\frac{25}{149} = a$.

Iterum ſit,

$$\begin{aligned} 3B'p + 4B''p + 5B'''p + 6B''''p \ \&c. &= py. \\ 3C'p + 4C''p + 5C'''p + 6C''''p \ \&c. &= pz. \\ 3D'p + 4D''p + 5D'''p + 6D''''p \ \&c. &= pv. \\ 3A'p + 4A''p + 5A'''p + 6A''''p \ \&c. &= px. \end{aligned}$$

$$\begin{aligned} 3B' &= 3B' \\ 4B'' &= 4B'' \\ 5B''' &= \tfrac{5}{2} A'' + \tfrac{5}{4} A' \\ 6B'''' &= \tfrac{6}{2} A''' + \tfrac{6}{4} A'' \\ 7B^{v} &= \tfrac{7}{2} A'''' + \tfrac{7}{4} A''' \\ 8B^{vi} &= \tfrac{8}{2} A^{v} + \tfrac{8}{4} A'''' \\ \text{Ergo } y &= \tfrac{3}{4} + \tfrac{3}{4} x + a. \end{aligned}$$

Etenim prima Columna perpendicularis $= y$, ex Hypotheſi:

$3B' + 4B'' = \frac{3}{4}$: Nam eſt $B' = \frac{1}{4}$, & $B'' = 0$.

$3A' + 4A'' + 5A'''$ &c. $= x$ ex Hypotheſi.

$A' + A'' + A'''$ &c. $= a$, ut repertum eſt.

Eſt igitur $4A' + 5A'' + 6A''' + 7A'''' \text{ \&c.} = x + a$

Et $\frac{4}{2}A' + \frac{5}{2}A'' + \frac{6}{2}A''' + \frac{7}{2}A'''' \text{ \&c.} = \frac{1}{2}x + \frac{1}{2}a.$

Sed A' 0

Ergo ſecunda Columna perpendicularis $= \frac{3}{4} + \frac{1}{2}x + \frac{1}{2}a.$

$$3A' + 4A'' + 5A''' + 6A'''' \text{ \&c.} = x$$
$$2A' + 2A'' + 2A''' + 2A'''' \text{ \&c.} = 2a$$

Eſt igitur $5A' + 6A'' + 7A''' + 8A'''' \text{ \&c.} = x + 2a.$

Et $\frac{5}{4}A' + \frac{6}{4}A'' + \frac{7}{4}A''' + \frac{8}{4}A'''' \text{ \&c.} = \frac{1}{4}x + \frac{1}{2}a.$

Eſt igitur tertia Columna perpendicularis $= \frac{1}{4}x + \frac{1}{2}a.$

Erit igitur $y = \frac{3}{4} + \frac{1}{2}x + \frac{1}{2}a + \frac{1}{4}x + \frac{1}{2}a$

ſive $y = \frac{3}{4} + \frac{3}{4}x + a$, quod erat probandum.

$$3C' = 3C'$$
$$4C'' = 4C''$$
$$5C''' = \frac{5}{2}B'' + \frac{5}{4}D'$$
$$6C'''' = \frac{6}{2}B''' + \frac{6}{4}D''$$
$$7C^{V} = \frac{7}{2}B'''' + \frac{7}{4}D'''$$
$$8C^{VI} = \frac{8}{2}B^{V} + \frac{8}{4}D''''$$

&c.

Ergo $z = \frac{1}{2}y + \frac{1}{2}b + \frac{1}{4}v + \frac{1}{2}d.$

Etenim prima Columna perpendicularis $= z$, ex Hypotheſi.

$$3C' + 4C'' = \tfrac{1}{2}.$$
$$3B' + 4B'' + 5B''' + 6B'''' \text{ \&c.} = y.$$
$$B' + B'' + B''' + B'''' \text{ \&c.} = b.$$

Eſt igitur $4B' + 5B'' + 6B''' + 7B'''' \text{ \&c.} = y + b.$

Sed $4B' = 1.$

Ergo $5B'' + 6B''' + 7B'''' \text{ \&c.} = y + b - 1.$

$$\frac{5}{2}B'' + \frac{6}{2}B''' + \frac{7}{2}B'''' \text{ \&c.} = \frac{1}{2}y + \frac{1}{2}b - \frac{1}{2}.$$

Ergo ſecunda Columna perpendicularis $= \frac{1}{2} + \frac{1}{2}y + \frac{1}{2}b - \frac{1}{2} = \frac{1}{2}y + \frac{1}{2}b.$

Iterum, $3D' + 4D'' + 5D''' + 6D'''' \text{ \&c.} = v$

$$2D' + 2D'' + 2D''' + 2D'''' \text{ \&c.} = 2d$$

Eſt igitur $5D' + 6D'' + 7D''' + 8D'''' \text{ \&c.} = v + 2d.$

Et $\frac{5}{4}D' + \frac{6}{4}D'' + \frac{7}{4}D''' + \frac{8}{4}D'''' \text{ \&c.} = \frac{1}{4}v + \frac{1}{2}d.$

Ergo tertia Columna perpendicularis $= \frac{1}{4}v + \frac{1}{2}d.$

Eſt igitur $z = \frac{1}{2}y + \frac{1}{2}b + \frac{1}{4}v + \frac{1}{2}d$, quod erat probandum.

Eodem prorſus ordine ſcribantur.

$$3D' = 3D'$$
$$4D'' = 4D''$$
$$5D''' = \frac{5}{2}C'' + \frac{5}{4}B'$$
$$6D'''' = \frac{6}{2}C''' + \frac{6}{4}B''$$
$$7D^{V} = \frac{7}{2}C'''' + \frac{7}{4}B'''$$
$$8D^{VI} = \frac{8}{2}C^{V} + \frac{8}{4}B''''$$

&c.

Unde $v = \frac{1}{2}z + \frac{1}{2}c + \frac{1}{4}y + \frac{1}{2}b.$

$$3A' = 3A'$$
$$4A'' = 4A''$$
$$5A''' = \frac{5}{2}D'' + \frac{5}{4}C'$$
$$6A'''' = \frac{6}{2}D''' + \frac{6}{4}C''$$
$$7A^{V} = \frac{7}{2}D'''' + \frac{7}{4}C'''$$
$$8A^{VI} = \frac{8}{2}D^{V} + \frac{8}{2}C''''$$

&c.

Et $x = \frac{1}{2}v + \frac{1}{2}d + \frac{1}{4}z + \frac{1}{2}c.$

Quæ quidem Concluſiones eodem modo demonſtrantur ac ſuperiores. Solutis autem quatuor iſtis æquationibus, elicietur

$$y = \frac{45536}{149|^2},\ z = \frac{38724}{149|^2},\ v = \frac{37600}{149|^2}\ x = \frac{33547}{149|^2} = \frac{33547}{22201}$$

Ergo, ſi velint *B*, *C*, *D*, *A* vendere Spectatori cuidam *R* ſummas quas ſinguli obtinere ſperant, æquum erit ut emptor *R* pendat

$$\text{ipſi } B \quad 4 \times \frac{56}{149} + \frac{45536}{22201}p, \qquad \text{ipſi } C \quad 4 \times \frac{36}{149} + \frac{38724}{22201}p.$$

$$\text{ipſi } D \quad 4 \times \frac{32}{149} + \frac{37600}{22201}p, \qquad \text{ipſi } A \quad 4 \times \frac{25}{149} + \frac{33547}{22201}p.$$

Invenire Probabilitates quas habent B, C, D, A, *ut mulctentur, dato ludorum numero.*

Si Ludi duo tantum ſint, erunt hoc modo,

$$\left.\begin{array}{cc} B & A \\ \hline C & B \end{array} \quad \begin{array}{cc} B & A \\ \hline B & C \end{array}\right\} \text{Unde patet } B \text{ vel } C \text{ neceſſario mulctari.}$$

Si Ludi tres fuerint, hoc modo ſe res habet,

$$\left.\begin{array}{cc} B & A \\ \hline C & B \\ D & C \end{array} \quad \begin{array}{cc} B & A \\ \hline C & B \\ C & D \end{array} \quad \begin{array}{cc} B & A \\ \hline B & C \\ D & B \end{array} \quad \begin{array}{cc} B & A \\ \hline B & C \\ B & D \end{array}\right\} \begin{array}{l} \text{Hinc patet } C, \text{ vel } D \text{ vel } B \\ \text{neceſſario mulctari.} \end{array}$$

Si vero quatuor Ludi fuerint,

$$\left.\begin{array}{cc} B & A \\ \hline C & B \\ D & C \\ A & D \end{array} \quad \begin{array}{cc} B & A \\ \hline C & B \\ D & C \\ D & A \end{array} \quad \begin{array}{cc} B & A \\ \hline C & B \\ C & D \\ A & C \end{array} \quad \begin{array}{cc} B & A \\ \hline C & B \\ C & D \\ C & A \end{array} \quad \begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ A & D \end{array} \quad \begin{array}{cc} B & A \\ \hline B & C \\ D & B \\ D & A \end{array}\right\} \begin{array}{l} \text{Debet igitur } A \\ \text{triplici modo, } D \text{ du-} \\ \text{plici, } C \text{ ſimplici,} \\ \text{mulctari,} \end{array}$$

Et ſic de cæteris. Ex quibus manifeſta eſt Compoſitio Tabulæ ſubjunctæ Probabilitatum quas *B*, *C*, *D*, *A*, habent ut mulctentur, dato ludorum numero.

	Num. Lud.	B	C	D	A
′	2	$\frac{1}{2}$	$\frac{1}{2}$		
″	3	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	
‴	4		$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$
⁗	5	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{3}{16}$
v	6	$\frac{6}{32}$	$\frac{5}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
v′	7	$\frac{6}{64}$ &c.	$\frac{8}{64}$	$\frac{8}{64}$	$\frac{4}{64}$

Sint autem y, z, v, x ſummæ omnium Probabilitatum quas B, C, D, A habent reſpective ut mulctentur.

Scribantur eodem ordine ac in præcedentibus.

$$\begin{array}{llll}
B' = B' & & C' = C' & \\
B'' = B'' & & C'' = C'' & \\
B''' = \frac{1}{2}A'' & + \frac{1}{4}A' & C''' = \frac{1}{2}B'' & + \frac{1}{4}D' \\
B'''' = \frac{1}{2}A''' & + \frac{1}{4}A'' & C'''' = \frac{1}{2}B''' & + \frac{1}{4}D'' \\
B^{V} = \frac{1}{2}A'''' & + \frac{1}{4}A''' & C^{V} = \frac{1}{2}B'''' & + \frac{1}{4}D''' \\
B^{VI} = \frac{1}{2}A^{V} & + \frac{1}{4}A'''' & C^{VI} = \frac{1}{2}B^{V} & + \frac{1}{4}D'''' \\
\&c. & & \&c. &
\end{array}$$

Ergo $y = \frac{3}{4} + \frac{1}{2}x + \frac{1}{4}x.$
$ = \frac{3}{4} + \frac{3}{4}x.$
 Ergo $z = \frac{1}{2} + \frac{1}{2}y + \frac{1}{4}v.$

Scribantur deinde

$$\begin{array}{llll}
D' = D' & & A' = A' & \\
D'' = D'' & & A'' = A'' & \\
D''' = \frac{1}{2}C'' & + \frac{1}{4}B' & A''' = \frac{1}{2}D'' & + \frac{1}{4}C' \\
D'''' = \frac{1}{2}C''' & + \frac{1}{4}B'' & A'''' = \frac{1}{2}D''' & + \frac{1}{4}C'' \\
D^{V} = \frac{1}{2}C'''' & + \frac{1}{4}B''' & A^{V} = \frac{1}{2}D'''' & + \frac{1}{4}C''' \\
D^{VI} = \frac{1}{2}C^{V} & + \frac{1}{4}B'''' & A^{VI} = \frac{1}{2}D^{V} & + \frac{1}{4}C'''' \\
\&c. & & \&c. &
\end{array}$$

Ergo $v = \frac{1}{4} + \frac{1}{2}z + \frac{1}{4}y.$ Ergo $x = \frac{1}{2}v + \frac{1}{4}z.$

Reſolutis autem quatuor iſtis æquationibus, invenietur

$$y = \frac{243}{249} \qquad z = \frac{252}{149} \qquad v = \frac{224}{149} \qquad \&\ x = \frac{175}{149}$$

Ergo ſi velit Spectator aliquis S mulctas omnes ſuſtinere, æquum erit ut ipſi S

$$B \text{ tradat } \frac{243}{149}p \qquad C\,\frac{252}{149}p \qquad D\,\frac{224}{149}p \qquad \&\ A\,\frac{175}{149}p.$$

Sublatis itaque ſummis probabilitatum quas ſinguli Colluſores habent ut mulctentur, è ſummis expectationum quas habent iidem ſi victores abeant, reſtabunt ſortes eorum reſpective: nempe

$$B \text{ recipit ab } R\ \frac{4 \times 56}{149} + \frac{45536}{22201}p$$

$$B \text{ tradit ipſi } S \qquad \frac{243}{1\ 9}p$$

$$\text{Ergo ipſi } B \text{ ſupereſt } \frac{224}{149} + \frac{9329}{22201}p$$

Sed B depoſuerat 1 priuſquam ludus inciperetur.

$$\text{Ergo } B \text{ lucratur } \frac{75}{149} + \frac{9329}{22201}p.$$

C recipit ab *R* $\frac{4\times 36}{149} + \frac{38724}{22201}p$

C tradit ipſi *S* $\frac{252}{149^2}p$

Ergo ipſi *C* ſupereſt $\frac{144}{149} + \frac{1176}{22201}p$

Sed *C* depoſuerat 1.

Ergo *C* lucratur $-\frac{5}{149} + \frac{1176}{22201}p.$

D recipit ab *R* $\frac{4\times 32}{149} + \frac{37600}{22201}p$

D tradit ipſi *S* $\frac{224}{149^2}p$

Ergo ipſi *D* ſupereſt $\frac{128}{149} + \frac{4224}{22201}p$

Sed *D* depoſuerat 1.

Ergo *D* lucratur $-\frac{21}{149} + \frac{4224}{22201}p.$

A recipit ab *R* $\frac{4\times 25}{149} + \frac{33547}{22201}p$

A tradit ipſi *S* $\frac{175}{149^2}p$

Ergo ipſi *A* ſupereſt $\frac{100}{149} + \frac{7472}{22201}p$

Sed *A* depoſuerat $1+p$, nempe 1 priuſquam ludus inchoaretur, & p poſtquam ſemel victus fuerat à *B*:

Ergo *A* lucratur $-\frac{49}{149} - \frac{14729}{22201}p.$

Lucrum ipſius $B = +\frac{75}{149} + \frac{9329}{22201}p$

ipſius $C = -\frac{5}{149} + \frac{1176}{22201}p$

ipſius $D = -\frac{21}{149} + \frac{4224}{22201}p$

ipſius $A = -\frac{49}{149} - \frac{14729}{22201}p$

Summa Lucrorum $= \quad 0 \qquad\qquad 0$

Summa autem lucrorum ipſorum B & $A = \frac{26}{149} - \frac{5400}{22201}p$; ſed poſueramus *B* viciſſe ipſum *A* ſemel, priuſquam Colluſores pacta inirent cum *R* & *S*. Priuſquam vero ludus inchoaretur, *A* poterat æqua ſorte expectare ut vinceret ipſum *B*, adeoque ſumma lucrorum $\frac{26}{149} - \frac{5400}{22201}$ in duas partes æquales eſt dividenda, ita ut utriuſque lucrum cenſendum ſit $\frac{13}{149} - \frac{2700}{22201}p$.

Ponatur $\frac{13}{149} - \frac{2700}{22201}p = 0$, & erit $p = \frac{1937}{2700}$.

Ergo ſi ſit mulcta *p* ad ſummam quam ſinguli deponunt ut 1937 ad 2700, *A* & *B* nihil lucrantur, nihil perdunt. Verum hoc in Caſu *C* lucratur $\frac{1}{225}$, quam *D* perdit.

Coroll. 1. Spectator *R*, priuſquam ludus inchoetur, id ſuſcipere in ſe poterit, ut ſummam 4 de qua Colluſores contendunt, & mulctas omnes pendat, ſi ſibi initio in manus darentur $4 + 7p$.

Coroll. 2. Si dexteritates Colluſorum ſint in ratione data, Sortes Colluſorum eadem ratiocinatione determinabuntur.

Coroll. 3. Si Series aliqua ita ſit conſtituta, ut continuò decreſcat, & terminus quivis ad præcedentes quoſlibet habeat rationes datas, ſive eaſdem ſive diverſas, ſeries iſta accurate ſummabitur. Inſuper ſi termini omnes hujus Seriei multiplicentur per terminos progreſſionis Arithmeticæ, ſinguli per ſingulos, Series nova reſultans accurate ſummabitur.

Coroll. 4. Si ſint Series plures collaterales, ita relatæ ut terminus quilibet cujuſque Seriei ad præcedentes quoſlibet aliarum Serierum habeat rationes datas, ſive eaſdem ſive diverſas, ita ut Series iſtæ collaterales ſe decuſſent data qualibet lege conſtanti, Series iſtæ accurate ſummabuntur. Inſuper ſi termini omnes harum Serierum multiplicentur ordinatim per terminos Progreſſionis Arithmeticæ, ſinguli per ſingulos, Series novæ ex hac multiplicatione reſultantes etiamnum accurate ſummabuntur.

Clavis ad Problema generale.

Si ſint Colluſores quotcunque *v. g.* Sex, *B*, *C*, *D*, *E*, *F*, *A* & Probabilitates quas habent ut victores evadant, ſive ut mulctentur, dato Ludorum numero, denotentur reſpective *B*, *C*, *D*, *E*, *F* & *A*; & Probabilitates dato Ludorum numero his proximo & minori competentes, per $B_{,,}$, $C_{,,}$, $D_{,,}$, $E_{,,}$, $F_{,,}$, $A_{,,}$; & Probabilitates dato Ludorum numero his itidem noviſſimis proximo & minori competentes, per $B_{,,,}$, $C_{,,,}$, $D_{,,,}$, $E_{,,,}$, $F_{,,,}$, $A_{,,,}$ & ſic deinceps, erit ſemper,

$$
\begin{aligned}
B' &= \tfrac{1}{2} A_{,,} + \tfrac{1}{4} A_{,,,} + \tfrac{1}{8} A_{,,,,} + \tfrac{1}{16} A^{v} \\
C_{,} &= \tfrac{1}{2} B_{,,} + \tfrac{1}{4} F_{,,,} + \tfrac{1}{8} E_{,,,,} + \tfrac{1}{16} D^{v} \\
D_{,} &= \tfrac{1}{2} C_{,,} + \tfrac{1}{4} B_{,,,} + \tfrac{1}{8} F_{,,,,} + \tfrac{1}{16} E^{v} \\
E_{,} &= \tfrac{1}{2} D_{,,} + \tfrac{1}{4} C_{,,,} + \tfrac{1}{8} B_{,,,,} + \tfrac{1}{16} F^{v} \\
F_{,} &= \tfrac{1}{2} E_{,,} + \tfrac{1}{4} D_{,,,} + \tfrac{1}{8} C_{,,,,} + \tfrac{1}{16} B^{v} \\
A_{,} &= \tfrac{1}{2} F_{,,} + \tfrac{1}{4} E_{,,,} + \tfrac{1}{8} D_{,,,,} + \tfrac{1}{16} C^{v}
\end{aligned}
$$

Et fiat ſemper retrogreſſus ordinatim ad tot literas minus duobus quot ſunt Colluſores, omittaturque ſemper litera *A*, prima æquatione excepta ubi litera *A* terminos omnes præter primam occupat.

XXXIII. *Papers omitted.*

1. Solutio Problematis à Cl. Viro D. Johan. Bernoullio in Diario Gallico, Feb. 1703. propoſiti; Quam D. G. Cheynæo communicavit Jo Craig. n.289. p.1527.

2. Vindiciæ Matheſeos univerſalis Gregorianæ contra ſecundos Abbatis Galloyſii impetus in Hiſtoria Acad. Scient. *Ann.* 1703. n.308. p.2336.

3. A Letter from Monſ. *l'Abbé Conti*, to the late Mr. *Leibnitz*, concerning the Diſpute about the Invention of the Method of Fluxions, or Differential Method; with Mr. *Leibnitz*'s Anſwer. n.359. p.923.

4. Apologia D. Brook Taylor, J. U. D. & R. S. S. contra V. C. J. Bernoullium, Math. Prof. Baſileæ, de contumeliis quibuſdam ab illo in Actis Lipſienſibus inſertis. n.360. p.955.

XXXIV. *Accounts of Books omitted.*

1. Apollonii Pergæi Conicorum Libri Octo, & Sereni Antiſſenſis de Sectione Cylindri & Coni Libb. duo. Fol. Reg. E Theatro Oxon. 1710. n.354. p.732.

2. Euclidis quæ ſuperſunt omnia, Gr. Lat. ex recenſione Davidis Gregorii, M. D. Aſtr. Prof. Savil. Fol. Oxon. 1703. n.289. p.1558.

3. Methodus Incrementorum. Auctore Brook Taylor, LLD. R. S. Secr. Lond. 1715. wherein he corrects a Miſtake in the 25th Propoſition of the Second Part. n.345. p.339.

4. De Locis ſolidis ſecunda Divinatio Geometrica, in quinque Libros injuria temporis amiſſos, Ariſtæi Senioris Geometræ. Autore Vincentio Viviani, Magni Ducis Etruriæ Mathematico primario, & R. S. S. Fol. 1701. n.291. p.1607.

5. Commercium Epiſtolicum Collinii & aliorum De Analyſi promota; publiſh'd by order of the Royal Society, in relation to the Diſpute between Mr. *Leibnitz* and Mr. *Keill*, about the Right of Invention of the Method of Fluxions, by ſome call'd the Differential Method. n.342. p.173.

CHAP. II.

Opticks.

I. *A univerſal Spherico-Catoptrick Theorem, by Mr.* Ditton. n.295. p.1810. *Plate* 3. *Fig.* 36.

SIt DEF Speculi Spherici concavi portio, cujus Centrum B, ſemidiameter BE vel BD: Sit etiam A punctum radians in axe collocatum, a quo profluat radioſa linea AD, quæ ad punctum D reflectatur in DC. Inveſtiganda jam eſt Foci C, a ſpeculi vertice *E* diſtantia.

Notandum vero, quod punctum *D* ipſi *E* proximum ſupponimus. Radii enim remotiores oculum (quem in axe AE conſtituimus) præterlabuntur, nec ad imaginis viſionem aliquid faciunt. Porro, propter arcum

DE indefinite parvum, anguli DAB, ADB (ut & ipſorum ſumma DBC) ſunt quam minimi, ac idcirco eandem habebunt inter ſe rationem, quam ipſis latera oppoſita: quo ratiocinii principio poſito, ad Theorema Dioptricum pervenit D. *Halleius*, Geometriæ Profeſſor apud Oxonienſes. *Abr. Vol.* I. *p.* 183.

Hiſce præmiſſis, ſit AB $= b$. BD $=$ BE $= r$. BC $= z$. CE ($= r - z$, ſed brevitatis cauſa ponatur) $= f$. Quantitates b & r cognitæ ſunt (dantur enim ſemidiameter ſpeculi, ac puncti lucidi a vertice diſtantia) z vero & f quæſitæ ac incognitæ. Jam in Triangulo DAB, erit $\angle$DAB : $\angle$ADB :: $r : b$. Item in Triangulo DBC, $\angle$BDC $=$ $\angle$ADB, ex natura Reflexionis, & $\angle$DBC $=$ $\angle$DAB $+$ $\angle$ADB, ex *Elem. Eucl.* Ergo cum $\angle$DBC ſit ut $r + b$, & $\angle$BDC ut b; erit etiam $\angle$DBC : $\angle$BDC :: $r + b : b$, & (quod ex principio ſupra memorato conſequitur) DC : BC :: $r + b : b$. Sed quoniam punctum D ipſi E proximum eſt, erit DC ipſi CE æqualis æſtimanda, ergo CE : BC :: $r + b : b$; hoc eſt $f : z :: r + b : b$, & (comparando Antecedentium & Conſequentium ſummas ad Antecedentes) $f + z : f :: r + 2b : r + b$; ſed $f + z = r$, ergo $r : f :: r + 2b : r + b$, ergo $f = \frac{rr \times rb}{r \times rb}$. *Q : E : I.*

Si ponatur $r + b$ (= AE) $= d$, Theorema in formam contractiorem redigetur, & ſic ſtabit $f = \frac{rd}{2d - r}$. Sed utrovis modo, focorum inventioni, quæcunque tandem ſit, vel Speculi forma, vel radiorum conditio, aptum evadet.

Coroll. I. Erit $zd = df - rf$, ſive AE × BC $=$ AB × CE, vel quod idem eſt, linea AE harmonice dividitur in punctis A, B, C, E; nam prædicta Rectangulorum æqualitas, lineæ ſecundum proportionem harmonicam ſectæ, propria eſt. Patet hæc veritas: Eſt enim $f = \frac{dr}{2d - r}$, & $z = r - f = r - \frac{dr}{2d - r}$, unde valores hoſce ſubſtituendo, Æquatio manifeſta fiet. Adeo ut in omni Speculo Spherico, lineæ DA, DB, DC, DE, ſunt Harmonicales; & Punctum radians, Centrum, Focus, Vertex ſunt puncta diviſionem Harmonicam efficientia.

Coroll. II. 1ma Poſito $d > r$; erit ex calculo f, ſive $\frac{rd}{2d - r} > \frac{r}{2}$ ſemper. Hoc eſt, ſi puncti radiantis diſtantia major ſit Semidiametro Speculi, foci diſtantia ſemper major erit quarta parte Diametri.

Item, erit $\frac{rd}{2d - r} < r$ ſemper. Hoc eſt, diſtantia foci ſemper erit minor ſpeculi ſemidiametro.

2d. Si ponatur $d = r$, erit $\frac{rd}{2d - r}$, ſive $f = r$. Hoc eſt, ſi punctum radians in centro ſpeculi conſtituatur, Imago ejus ibi cum eo unietur.

3i. Si ponatur $d = r$, tum ipſius f expreſſio erit vel poſitiva vel negativa vel infinita, prout quantitas $2d$ quantitate r vel major eſt vel minor, vel ei æqualis.

Si $2d > r$, hoc eſt, ſi $d > \frac{r}{2}$, tum punctum radians & focus ad eaſdem partes ſpeculi jacent.

Si $2d < r$, vel $d < \frac{r}{2}$, tum Imago, in axe ultra ſpeculi verticem producto, ſita eſt.

Si $2d = r$, vel $d = \frac{r}{2}$, Imago infinite diſtat, ſive radius reflexus axi parallelus evadit.

Coroll. III. Calculi hujus ope expedite determinari poteſt, quomodo objecti radiantis (ſpeculi reſpectu) motui, ipſius Imaginis motus reſpondet. Sit (ut antea) Imaginis a ſpeculo diſtantia $= \frac{dr}{2d-r}$, quando objecti diſtantia eſt d. Mutetur jam utcunque objecti diſtantia, & ex d, fiat nd, quantitate n Numerum vel integrum vel fractum deſignante: & ſic loco prioris Æquationis, $f = \frac{dr}{2d-r}$, habebimus pro Novo Foco aliam Æquationem, $F = \frac{ndr}{2nd-r}$. Et quidem ſi n Numerum integrum exprimere ſupponatur, ſecunda hæc objecti diſtantia primâ major erit, ſi vero ſit fractus, tum minor erit prima.

Hiſce poſitis, ſi $d > r$, & n ſit integer, erit $F < f$, id eſt, erit $\frac{ndr}{2nd-r} < \frac{dr}{2d-r}$ ſive $2nddr - ndrr < 2nddr - drr$, quod manifeſtum eſt. Hoc eſt, ſi in ſpeculo concavo objecti diſtantia major ſit ſemidiametro, tum recedente objecto a ſpeculo, Imago verſus ſpeculum accedet. Rurſus, deſignet n Numerum fractum, & tunc reperietur $2nddr - ndrr > 2nddr - drr$, ſive $F > f$. Hoc eſt, accedente objecto ad ſpeculum recedet Imago.

Supponatur jam $d < \frac{r}{2}$; ut & alia quæcunque ſit objecti diſtantia nd intelligatur ea ſemper minor eſſe quam $\frac{r}{2}$. Tum erunt $2nddr - ndrr$, & $2nddr - drr$, quantitates negativæ; ſive $ndrr - 2nddr$, & $drr - 2nddr$ quantitates poſitivæ. Et quidem ſi n numero integro æquetur, erit $ndrr - 2nddr > drr - 2nddr$, ſive $F > f$; ſi vero n fractio ſit, tum erit $ndrr - 2nddr < drr - 2nddr$, ſive $F < f$. Hoc eſt, ſi in ſpeculo concavo objecti diſtantia minor ſit ſpeculi Diametri quarta parte, tum recedente objecto a ſpeculo, recedet & Imago; vel accedente objecto verſus ſpeculum, Imago etiam accedet.

Et hæc omnia (quæ calculi veſtigia premendo deduximus) Scholio unico concluſit, & in ſua Catoptrica tradidit D. *Gregorius* apud Oxonienſes Aſtronomiæ Profeſſor.

Coroll. IV. In Æquatione $f=\frac{dr}{2d-r}$, ſi ponatur d infinita, erit $f=\frac{r}{2}$; quæ regula eſt pro Radiis parallelis, ſive pro objecto radiante ad diſtantiam infinitam remoto. Idem ſequetur, poſito b infinito in Æquatione $f=\frac{rr+rb}{r+2b}$.

Coroll. V. In Æquatione $\frac{dr}{2d-r}$, mutato quantitatis r ſigno negativo in poſitivum, erit $f=\frac{dr}{2d+r}$; vel in æquatione $f=\frac{rb+br}{r+2b}$, mutato ſigno poſitivo in negativum, erit tunc $f=\frac{rb-rr}{2b-r}$, quæ regulam exhibet pro ſpeculo verſus objectum radians convexo. Patet hæc mutatio ſigni; nam ſicut in ſpeculo concavo $d=r+b$, ſic in convexo $d=b-r$.

Coroll. VI. In ſpeculo convexo (ſtantibus quæ ad *Cor.* III. annotavimus de Concavo) patebit quod (ſi n ſit numerus integer) $2rndd+ndrr > 2rndd+drr$; & (n fractione exiſtente) quod $2rndd+ndrr < 2rndd+drr$. Hoc eſt, quod recedente objecto a ſpeculo vel verſus idem accedente, Imago ſimiliter recedet vel accedet.

Patet etiam in ſpeculo convexo, objecto ad immenſam uſque diſtantiam retrocedente, Imaginem tamen illius non ultra Diametri partem quartam abire a vertice, ſed ibi, in puncto, centrum inter & verticem medio, ſe ſiſtere. Poſito enim d vel b infinito, erit $f=\frac{dr}{2d}$ vel $\frac{br}{2b}$, id eſt (utrovis modo) $=\frac{r}{2}$.

Hiſce adjungi poteſt & Problematis Catoptrici ſolutio, *Radiantis poſitionem reſpectu ſpeculi dati talem invenire, ut radians ad ipſius Imaginem a ſpeculo factam, datam habeat rationem.* Sit Ratio data $r:q$, & ſymbolo O deſignetur Objectum, I Imago, d diſtantia objecti, & f imaginis a ſpeculo. Jam (quod demonſtravit D. *Greg.*) erit $O:I::d:f$, (hoc eſt Objectum & Imago ſunt diſtantiis ſuis a ſpeculi vertice directe proportionales) & quoniam requiritur ut ſit $O:I::r:q$, debet etiam eſſe $d:f::r:q$, vel (ipſius f expreſſionem ſcribendo) $d:\frac{dr}{2d-r}::r:q$, unde $2ddq-rdq=rdr$, & $2dq=rr+qr$, & $d=\frac{rr+rq}{2q}$. Unde quoniam $dr=\frac{rrr+rrq}{2q}$, & $2d-r=\frac{rr}{q}$, erit etiam f ſive $\frac{dr}{2d-r}=\frac{rrr+rrq}{2q}:\frac{rr}{q}=\frac{qrrr+qqrr}{2qrr}=\frac{r+q}{2}$, quæ eſt ipſius f, ſive imaginis a ſpeculo diſtantia, huic objecti diſtantiæ congrua. Ergo ſi ſtatuatur objectum ad diſtantiam $\frac{rr\times rq}{2q}$, ipſius Imago facta ad diſtantiam $\frac{r+q}{2}$ ei comparata,

ata, eandem habebit rationem, quam $q : r$, ſive erit $O : I :: r : q$. Nam

$$O : I :: d : f :: \frac{rr + rq}{2q} : \frac{r + q}{2} :: r : q. \quad Q : E : I.$$

Objectum Radians & Imaginem hic tanquam lineas conſideravimus. Si enim Superficies ſunt, tum erit $O : I :: d^2 : f^2$, & $d^2 : f^2 :: r : q$, ſic ut ultimo deveniatur ad Æquationem $4dd - 4qdr = r^3 - qrr$, e qua radicis d valor, Methodis vulgaribus facillime inveniri poteſt.

II. *Exp.* 1. Having ſew'd together end-wiſe two Pieces of Ribbon four Inches long each, the one blew, the other red, whoſe common Breadth was $\frac{1}{4}$ of an Inch; I caus'd it to be held in ſuch manner, that the Light which fell from the Clouds thro' the Window was ſo reflected, that the Angle made by the Rays of Light, which came in at the Middle of the Window, with the Plane of the Ribbon produced, was equal to the Angle made by a Line drawn from the Ribbon to my Eye and the ſaid Plain of the Ribbon. My Eye was plac'd as far behind the Ribbon as the Window was before it, the Diſtance from which to me was about 12 Feet. Then looking thro' a Priſm at the Ribbon, it appear'd broken aſunder in the Place where the blew and red Half joyn'd. If the Priſm was held with the refracting Angle downwards (or laid with one of its Planes flat upon the Noſe) the blew Half of the Ribbon appear'd to be carried down lower than the red, as at *B R*, but if the refracting Angle of the Priſm was turn'd upwards (as when the Priſm has one of its Planes laid flat to the Forehead) then the blew Half of the Ribbon was lifted up, as at $\beta\rho$.

Some Experiments of Light and Colours, mentioned by Sir Iſaac Newton in his Opticks, lately repeated by Dr. Deſaguliers. n. 348 p. 433.

Fig. 1.

The Priſm was of white Glaſs, having every Angle of 60 Degrees: but when inſtead of it, one of a greeniſh ſort of Glaſs, ſuch as Object-Glaſſes of Teleſcopes are made of, was uſed, having the refracting Angle which I look'd thro' of about 48 Degrees: the ſame Phænomenon was more diſtinct, this Glaſs having no Veins, but the Red and Blue were nearer to a ſtreight Line: in ſuch manner that if *A* repreſent the Ribbon ſeen through the firſt Priſm, *B* will repreſent the Ribbon ſeen thro' the ſecond Priſm, *Fig.* 2. If the refracting Angle of the laſt Priſm had been as great as that of the firſt, the Light being tranſmitted thro' too great a Body of greeniſh Glaſs, the Phænomenon would not have ſucceeded ſo well.

The blue Ribbon being ſomewhat too pale, and the red a little dull; I repeated the Experiment with a Skeen of blue, and one of red Worſted join'd together in the Middle as the Ribbons were before; and, the Colours of both being very intenſe, the Experiment ſucceeded better with both Priſms. All that were preſent trying the Experiment found it to ſucceed, and that every Circumſtance anſwer'd to the Account given in *Prop.* 1. *Theor.* 1. *Book* 1. of Sir *Iſaac Newton's Optics*, as far as the Directions there given were followed. So that it appear'd that the Blue being carried lower than the Red in the firſt caſe, and lifted higher in the ſecond, was owing to the greater Refraction of the blue Ray : for tho' each Part of the Ribbon or Worſted reflected all manner of Rays, yet the Phænomenon was very apparent; as alſo that the blue Ribbon or Worſted reflected the blue Rays more copiouſly than the red Rays, and that the red Ribbon or Worſted reflected the red Rays more than the blue ones, becauſe the Red of the blue Half ſeen thro' the Priſm was leſs intenſe than that of the red Half, and the Blue or Purple of the red Half ſeen thro' the Priſm was leſs intenſe than that of the blue Half.

N. B. If the Ribbon or Worſted is laid upon any enlightned Body, the Phænomenon will not ſucceed ſo well; the Colours of the Body ſeen thro' the Priſm mixing with thoſe of the Ribbon or Worſted. Even a black Body will not do, if Light falls upon it : but there muſt be a black Cloath behind, in ſuch manner that no Light falling upon it can be reflected ſo as to diſturb the Phænomenon. And if a ſhort-ſighted Perſon looks through the Priſm, a concave Lens between his Eye and the Priſm will render the Phænomenon more diſtinct than it wou'd otherwiſe be.

Exp. II. Some Days after, the Sun ſhining, I made two Holes *H*, *b*, in the Window Shut *S s*, of a darkned Room; thro' which letting the Suns Beams paſs, by means of two Priſms *A*, *B*, (one near each Hole) I open'd the Rays coming from the Sun into the two colour'd *Spectra* α, β, where the following Colours were very diſtinct, *viz.* Red, Orange, Yellow, Green, Blue, Purple and Violet. Now the Reaſon of their being more diſtinct than ordinary, was, that the Priſms which I made uſe of were made of the greeniſh Glaſs mentioned before; which is very free from thoſe Veins by which the Colours are too much thrown into one another, by the beſt white Priſms of the common ſort.

The forementioned colour'd *Spectra* being thrown into the Room, to the Diſtance of about 20 Feet from the Window where the Sun's Light came in, I caus'd a Piece of white Paper π, $\frac{3}{4}$ Inch broad and 5 Inches long, to be held within the refracted Rays, (at the Diſtance of 10 Feet from the Windows,) which produc'd theſe Colours in ſuch manner, that by turning the Priſms round their Axes, I cou'd make the red Ray of the *Spectrum*, made by the one Priſm, fall upon one half of the Paper, and the purple Ray of the *Spectrum* made by the other Priſm fall upon the other Half; for the *Spectra* were both vertical, the Lines which terminated the long Sides of them towards each other juſt touching, as appears in *Fig.* 3. Then at the Diſtance of 9 Foot, looking thro' the Priſm C at the Paper thus colour'd, the red Half appear'd very much ſeparated from the Purple, the one ſeeming lifted up from the other; the Red or the Purple appearing the higheſt, according as the refracting Angle of the Priſm was either held upwards or downwards. The Phænomenon is much more diſtinct this way than any other; for the Paper not only ſeems divided into two, when it is coloured by a red and a purple Ray, but alſo by a Red and Blue, (*Fig.* 4.) by a red and a green Ray (*Fig.* 5.) or indeed by any two Colours that are different how near ſoever

ver their Places in the *Spectra* be to each other. The Halves of the Paper appear, when view'd thro' the Priſm, to be farther from each other, when the Paper is ting'd with ſuch Colours as are farther from each other in the Series of Colours in the *Spectrum*: and neareſt, tho' ſtill divided, when neighbouring Colours fall upon the Paper, as Yellow and Green, or a light and a deep Green. But the Paper appears no way divided, when colour'd with the Red of the two *Spectra*, (*Fig.* 6.) if thoſe Reds are equally intenſe: and ſo of the other Colours.

Exp. III. I held a Lens of about 3 Foot Radius at the Diſtance of Six Feet from the oblong Paper (on which a red and purple Ray falling, made it look half Red and half Purple) and I projected the Image of the ſaid colour'd Paper at the Diſtance of about Six Foot on the other Side of the Lens, on a white Sheet of Paper; where it was obſervable, that when the red Half was diſtinctly painted on the white Paper (which was known by the Edges of the Image being regularly terminated) then the blue Half of the Image was confus'd: but if the white Paper was brought about two Inches nearer to the Lens, the Image of the Blue Half became diſtinct, and that of the red Half confus'd.

I try'd the Experiment with a Paper colour'd half red and half blue, the red with Carmine and the blew with Smalt, making the Candle to enlighten the Paper (the Room being otherwiſe dark) and the Experiment ſucceeded in the ſame manner. The Experiment thus made is the ſame that Sir *Iſaac Newton* gives an Account of, *Book* 1. *Part* 1. *Theor.* 1. *of his Optics*. Only it is to be obſerv'd that when the oblong Paper is colour'd with red and blue from the Priſms, the focal place, where the red part of the Image is diſtinct, is more diſtant from the place where the blue part of the Image is diſtinct, than when the Paper is colour'd with the Painter's Powders, and much more vivid.

The 7*th* Figure ſhews the Projection of the Paper ting'd with the Rays; and *Fig.* 8. the Projection of it when painted: where a black Thread is wrapp'd round the red and the blue part, that the Diſtinctneſs of the Image of the Thread may ſhew when the red or when the blue part of the Image of the Paper is moſt diſtinct.

N. B. When the Candle enlightens the painted Paper, ſet an opaque Body as *B* between the Candle and Lens; leſt the Image of the Candle being alſo projected ſhould diſturb the Experiment.

Exp. IV. Having made an Hole of $\frac{1}{4}$ Inch Diameter in the Window-Shut of the darkned Room, I ſuffer'd a Sun-Beam to come into the Room, which I intercepted with a Priſm at the Diſtance of 5 Inches from the Hole; and after its Refraction in paſſing thro' the Priſm, I receiv'd it upon a Sheet of white Paper, where it was colour'd, making an oblong Image of the Sun or *Spectrum* of about 9 Inches in length and 2 in breadth, which Breadth was nearly equal to the Diameter of the round Image of the Sun received upon a Paper at the ſame Diſtance from the Hole, which here was 18 Foot. Or if the ſun be too high, a Looking-Glaſs being put in the room of the Priſm will throw a white round *Spectrum* upon the Pa-blue Ray: for tho' each Part of the Ribbon or Worſted reflected all manner of Rays, yet the Phænomenon was very apparent; as alſo that the blue Ribbon or Worſted reflected the blue Rays more copiouſly than the red Rays, and that the red Ribbon or Worſted reflected the red Rays more than the blue ones, becauſe the Red of the blue Half ſeen thro' the Priſm was leſs intenſe than that of the red Half, and the Blue or Purple of the red Half ſeen thro' the Priſm was leſs intenſe than that of the blue Half.

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The Colours of the *Spectrum* were theſe; Red, Orange, Yellow, Green, Blue, Purple and Violet, tho' the Violet was ſo faint in this as to be ſcarce perceivable. *See Fig.* 9.

N. B. The Axis of the Priſm in this, and all the other Experiments hereafter mention'd muſt be perpendicular to the Ray that falls on it; and the Plane into which the Ray enters muſt be held in ſuch a Poſition, that the Angle which ſuch a Ray makes with that Plane when it enters, may be equal to the Angle made by the middle Line of thoſe Rays which emerge after Refraction, on the other Side of the refracting Angle of the Priſm, with the Plane out of which they emerge. That is $\sphericalangle BDG = \sphericalangle AEH$.

If the Plane *AC*, on which the Sun-Beam falls, be turned nearer to a perpendicular to the Sun Beam than before, the *Spectrum* will be much longer: if it be more inclin'd to the ſaid Beam, the *Spectrum* will be ſhorter; and in both caſes leſs diſtinct. See the *Spectrum DE* and the *Spectrum de* in *Fig.* 10. and 11. where *H*, *h*, repreſents the Hole in the Window Shut in each Caſe; *AC*, *ac* the Plane of the Priſm on which the Rays enter; *BC*, *bc*, that out of which they emerge; *P*, *p* the perpendicular, and *C*, *c* the refracting Angle.

If the Plane *AC* be ſtill more oblique to *HF*, all the Light will be reflected, and there will be no colour'd Image or *Spectrum* made by Refraction at all. *Fig.* 12.

But if it be held ſo as to be more nearly perpendicular to the Sun Beam than in *Fig.* 10. the whole Beam will indeed enter the Priſm, but meeting with *BC* the lower Surface of the Priſm, or rather the Surface of the Air contiguous to it, ſome of the Light will by the Plane *BC* be reflected to *de*, paſſing almoſt perpendicularly thro' *AB*; and the reſt will emerge thro' *BC*, and by Refraction make the Imperfect Spectrum *DE*. See *Fig.* 10.

If the Sun Beam enter *AC* perpendicularly and in the middle of it, the Light will be all reflected as in *Fig.* 13. ſome of it by the Plane *BC* to *R*, and the reſt by the Plane *AB* to *r*. But if the Beam fall nearer to *A* (ſtill perpendicularly) it will all be reflected by the Plane *AB*; if nearer to *B*, it will be all reflected by the Plane *BC*.

In order therefore to have the colour'd *Spectrum* as it ought to be, care muſt be taken that the emerging coloured Light may make the ſame Angle with the Plane *BC*, as the immerging Light does with the Plane *AC*; that is, the Angle *AEH* muſt be equal to *BDG*; as was ſaid before, *Fig.* 9. which may alſo be ſeen on the enlightned Duſt in the Air. But the beſt way is to turn the Priſm on its Axis, and at the ſame time look at the colour'd *Spectrum*, which will riſe and fall, and become longer or ſhorter, as you turn the Priſm; and between the Aſcent and Deſcent of the Image, it will appear Stationary: there ſtop the Priſm, and the Reflection will be ſuch as is required for all the Experiments hereafter mention'd.

In order to have the Priſm move freely on its Axis, and ſtop any where, I fix'd each End of it into a triangular Collar of Tin, from the End of which came a Wire, which was the Axis of the Priſm produc'd; and ſo I laid it on two wooden Pillars, with a Notch on the Top to receive the Wires, and fix'd it to a ſmall Board juſt broad euough to ſtand faſt, See *Fig.* 14.

Exp. V. I took the Priſm *C D*, and through it look'd at the coloured *Spectrum RP*, which appear'd then round and white as at *S*, juſt as if it had been the Sun's Light received on a Paper from the Hole *H*, and ſeen with the naked Eye. In this Caſe the Priſm *C D* muſt be held *in directum* with *A B*, and the refracting Angles in the two Priſms muſt be equal. This *Spectrum* appearing white but juſt in one Point, is not ſo readily found; but the beſt way is to look through the ſame Priſm *A B* which makes the *Spectrum*, which may eaſily be done if it be pretty long, and then *R P* will be ſeen white and round, and as at *S*, as if coming directly from *H*. *See* Fig. 15.

Exp. VI. I held a broad *Lens L l*, ground to a Radius of 2 $\frac{1}{2}$ Feet, in ſuch manner that the whole colour'd *Spectrum* fell upon it; and after Refraction all the Colours appear'd to converge, if receiv'd on a Paper at *pp*; but when the Paper was held in the Focus at *F* in the Poſition $\pi F \pi$, the *Spectrum* was round and perfectly white by the Union of all the colour'd Rays. If the Paper was held at $\Pi\Pi$, the Colours appear'd to diverge from each other, but then the Red was uppermoſt, which before us'd to be the loweſt; and ſo on in an inverted Order.

I try'd the ſame Experiment with a *Lens* of one Foot Radius, with one of 9 Inches, and with another of 7; and the Succeſs was the ſame. See the 16th Figure, where the *R*, *O*, *Y*, *G*, *B*, *P*, *V*, expreſs the Colours.

N. B. Care muſt be taken that the very end of the Red, and the Extremity of the Violet be taken in by the *Lens*; otherwiſe the *Spectrum* will not be perfectly white at the Glaſs's Focus.

There is no fix'd Diſtance of the Priſm from the Lens, but it ought to be brought ſo near the Priſm that the two Ends of the *Spectrum* may fall nearer the Axis of the Lens than the Edges of the Lens; becauſe there the Refraction is not ſo regular.

Behind the Lens *L*, which made the Colours converge into White at the diſtinct Baſe or Focus *F*, I plac'd the Lens *l*, which made the White be at *f* the diſtinct Baſe of the two Glaſſes combin'd; and the Experiment ſucceeded as before. Fig. 17.

When the Paper was held in the Focus of the Lens, ſo as to receive the white Image of the colour'd *Spectrum* projected by the Lens; if with a Card I intercepted the red Ray, the white appear'd ting'd with Purple, and if I intercepted the Violet or purple Ray, or both, the white appear'd ting'd with red; and if the red was intercepted at the ſame time, the *Spectrum* appear'd to be a mixture of yellow, green and blue. If any ſingle colour was ſuffer'd to fall upon the Lens, the reſt being intercepted, that colour wou'd continue the ſame; only it wou'd be more intenſe in the Focus of the Lens.

Exp. VII. I took a Board (*Fig.* 18.) *q b s* which ſtood reclining on a Prop *t*, having an Hole of a Quarter of an Inch Diameter at *b*, and behind it a Priſm *B* ſupported on two Props, as above-mention'd, ſo as to turn eaſily about its Axis; and having ſet this Board on the Ground with the Priſm behind it at *B*; by turning the Priſm *AC* about its Axis, I firſt made the red Ray of the colour'd *Spectrum* paſs through the Hole *b*, and fall obliquely upon the ſecond Priſm *B*. This Ray after its Refraction in paſſing through the ſecond Priſm, was carried up to the Ceiling of the Room at the place mark'd *R*: then I made the purple Ray fall upon the Board, and paſs through the Hole *b*, as the red had done before; and after Refraction through the Priſm *B* it was carried up to the Ceiling at *P*. And the green Ray being afterwards made to paſs the ſecond Priſm in the ſame manner, went up to *G*: and ſo of all the intermediate Rays, which were by this ſecond Refraction thrown to the intermediate places on the Ceiling between *R* and *P*.

Care is to be taken that the ſecond Priſm be plac'd oblique to the Rays which come through the Hole *b*; leaſt they be reflected, as they wou'd be, if the Board being in the Poſition *Q S*, and the ſecond Priſm in the Poſition *L N M*, the Ray from the firſt Priſm be *ρ b*: for then it will be reflected upwards to *σ* inſtead of being refracted (*Fig.* 19.) Neither muſt the Plane of Immerſion be too oblique, leaſt the Incident Ray be reflected downwards by it, as the Ray *R b* is by the Priſm *B* thrown to *E*, in *Fig.* 20. Several have confeſs'd to me that they at firſt us'd to fail in this Experiment, for want of ſetting the ſecond Priſm in a due Inclination.

Tho' the Colours by the ſecond Refraction on the Ceiling appear'd unchang'd, when ſeen by the naked Eye, yet if view'd through a Priſm, they afforded new Colours (except ſome part of the red, and ſome part of the Violet) which was owing to their not being fully ſeparated; for which reaſon I made the following Experiment, to prove that if the Colours be well ſeparated, they are truly homogeneal and unchangeable.

N. B. When the Priſms are good, and no Clouds are near the Sun, the Extremity of the red or Violet will afford unmix'd Colours in this Experiment; otherwiſe not.

Exp. VIII. Having made a Hole in the Window-Shut 2 Inches wide (*Fig.* 21.) I applied to it a Tin Plate, which ſliding up and down hid all this Hole in the Wood, and only tranſmitted a ſmall Beam through it's own Hole *H*, whoſe Diameter was = $\frac{1}{16}$ Inch. This Beam, by means of the Looking Glaſs *L*, plac'd on the Board of the Window *X W*, I reflected horizontally to the other end of the Room. But to correct the Irregularity of the Reflection of the Looking-Glaſs, I made uſe of the Frame of Paſt-Board *P p*, which had an Hole in it *b* of $\frac{1}{16}$ Inch likewiſe; and placing it at *Pp* I ſuffer'd ſome of the reflected Beams to paſs through it, ſo as to fall upon the Lens *F E* (convex on both Sides, and ground to a Radius of 4 $\frac{1}{2}$ Feet) at the Diſtance of 9 Feet, ſo that the Image of the Hole *b* was projected to *f* on the other ſide of the Glaſs, at the Diſtance of 9 Feet more. Juſt behind the Lens, which by a Screw in the Stand *S* might be rais'd or let down, ſo as always to receive the Beam along its Axis, I plac'd a Priſm *A* (upright on one of its Ends and eaſily moveable about its Axis, by reaſon of its Wire turning freely in an Hole in the ſolid piece of Wood *T*, which ſtood on another Stand behind the Lens) as near as I cou'd to the Lens *E F*, ſo that the Image of *b* inſtead of being round, white, and projected to *f*, was caſt ſidewiſe on a white Paper ſtretch'd on a Frame, and appear'd colour'd, and 30 or 40 times its Breadth, as at *M N*. The Colours in this Caſe were very vivid and well ſeparated, only the Violet had ſome pale Light darting from its End, upon account of ſome Veins in the Priſm *A*, and the Light not coming directly from the Sun, but reflected; which ought not to have been, if the Sun had been low enough to have thrown the Rays a good way into the Room without the help of a Looking Glaſs.

To ſhew that the Colours in this *Spectrum* were ſimple and homogeneal Lights, I made the following Experiments.

Exp. IX. Having made an Hole *b* in the Paper which receiv'd the colour'd *Spectrum*, I ſuffer'd the red Light to paſs; which being refracted by a ſecond Priſm fell upon another Paper at *T*, where it appear'd ſtill red whether ſeen with the naked Eye or Priſms of different refracting Angles. To the Eye which ſaw it through the Priſm *V*, it appear'd indeed lower as at *t*, but red, round and unchang'd. I made the Experiment upon all the Colours, which by this means appear'd to be ſimple and homogeneal. See *Fig.* 22. where the ſame Letters denote the Lens, Priſm and firſt Paper.

Through the ſame Lens and Priſm the *Spectrum* was made to fall on a Book; then through the Priſm *F* it appear'd unchang'd; and the Letters in the Book, which croſs'd the *Spectrum*, were as diſtinct as when ſeen with the naked Eye. See *Fig* 23.

N. B. The Axis of the Priſm *F* ought to be perpendicular to the long Axis of the *Spectrum s m* thrown on the Book, which will appear as at σ μ; and the Priſm in the Poſition repreſented at *F*, with its flat Side towards the Noſe: for that is the moſt convenient Poſition for looking at the *Spectrum* in theſe Experiments.

I ſuffer'd the purpleRay only to paſs through the Hole *b*, and fall upon a Book at *P*, the Letters of which appear'd at π, and were as diſtinct through the Priſm *Q* as when ſeen with the naked Eye: and I had the ſame Succeſs with all the other Rays. See *Fig.* 24.

But if a Sun-Beam as *r* comes through the Hole *H* directly upon the Book at *W*, an Eye looking at it through a Priſm at *X* will ſee this Beam at *Y* oblong and colour'd; and the Letters on which it falls, confus'd. See *Fig.* 24.

N. B. The Lens ought to be very good, without Veins or Blebs, and ground to no leſs a Radius than I mention'd in the Experiment; though a Radius of a Foot or two longer is not amiſs. The Priſm ought to be of the ſame Glaſs as the Object-Glaſſes of Teleſcopes, the white Glaſs, of which Priſms are uſually made, being commonly full of Veins. And the Room in theſe laſt Experiments ought to be very dark.

A few Days after, having got very good Priſms made for the purpoſe of the above-mention'd Glaſs, I made all the Experiments over again with better Succeſs; and had the *Spectrum* very regularly terminated, without any pale Light darting from the ends of it.

Sir Iſaac Newton's *Doctrine of the different Refrangibility of the Rays of Light confirm'd, by the ſame, ib. p.* 448.

2. After the *Experimentum Crucis* made by two Priſms, I ſhou'd not give the following Experiment, but that it is ſo eaſy to be made, that by it thoſe who want the *Apparatus* (or are unwilling to be at the pains) to make the *Experimentum Crucis*, may at any time ſatisfy themſelves of the Truth of the fore-mention'd Doctrine.

Let the Candle *A* be ſet before the Bar of a Chimney Looking-Glaſs, ſuch as is repreſented by *HH* (*Fig.* 25.) which is a piece of Looking-Glaſs Plate conſiſting of four Planes, ſeen in the Section of it $\alpha f d \beta$, *viz.* $d \beta$ which is quick-ſilver'd behind, $f \alpha$ a Plane parallel to it, *fd* one of the Side-planes bezell'd towards $d \beta$, or inclin'd to it in an Angle of about 40 Degrees (tho' from 30 to 40 will do, but the greater the Angle the better, if it does not exceed 45°). $\alpha \beta$ the other Side Plane inclin'd in the ſame Angle to βd.

The Rays of the Candle which come from *A* to γ fall obliquely on the Plane $\alpha \beta$, ſo that inſtead of going on to *a*, they are by Refraction made to incline more towards the Perpendicular *p p*, namely to go on in the Line γc and then are reflected from the Point *c* on the quick-ſilver'd Surface, in the Direction $c \kappa$, ſo as to make the Angle $\kappa c d = \gamma c \beta$. Now as the Rays which would go to κ, if not refracted, emerge obliquely from the Plain $\alpha \beta$, they leave the Direction $c \kappa$, and decline from the perpendicular $\pi \pi$, and, being differently refracted, open into four differently colour'd Rays; *viz.* *b R* a red Ray, *b TO* a Ray made up of Orange and Yellow; *b G B* a Ray made up of Green and Blue or a Sea-Green, and *b P* a purple Ray.

If from the place *E e* you look full upon the point *b*, the *Spectrum* or Image of the Candle at *b* will appear doube, abut not mix'd; that is, there will appear a Sea-green Spot and a red Spot, s it were, one upon another; but not ſo as to produce a mix'd or intermediate Colour. Then if the right Eye or Eye at *E* be ſhut, there will appear only a green Spot to the Eye at *e*; if the Eye at *e* be ſhut, the Eye at *E* will ſee only a red Spot.

If you come nearer to *b*, ſo that the Eyes at *e* 1, *e* 2 receive the moſt and the leaſt refrangible Rays, there will be a double *Spectrum*, *viz.* a red and purple one juſt touching, or upon one another: and the Phænomenon will anſwer as before. *Fig.* 25.

If keeping both Eyes open, you direct their Axes towards *O* a point nearer than the uſual place of the compound *Spectrum S*, (*Fig.* 26.) which point is in a Line from the Noſe *N* to the point *S*; or in other Words, if you look full ar *O*, or at the End of your Finger held in *O*; the red and the blue (or purple Spot) will appear to be divided from each other after the manner repreſented at *p r* (in *Fig.* 27) where the red will appear to be on the Right-hand, and the blue on the Left.

To make plain what is meant by *ſeeing* the *Spectra p* and *r* whilſt we look full at *O*, I beg leave to explain the Diſtinction between *looking* and *ſeeing*; that I may the better ſhew how this Phænomenon proves that the Senſation of different Colours is caus'd by Rays differently refracted.

Definition I. The *Optic Axis* is a Line which going through the Center of the Convexity of all the Coats and Humours of the Eye, falls upon the middle of the *Retina*, as *α a* or *A a Fig.* 28.

Def. II. To *look* at any point, is to turn both Eyes towards it in ſuch manner, that the *Optic Axes* making an Angle at the ſaid point as *a*, the Rays from *a* may have the *Optic Axis* for their Axis, and (by their Convergence upon the *Retina* after Refraction in the *Eye*) may paint the Image of the ſaid point upon the middle of the *Retina* of each Eye, where the Optic Axis in each Eye falls.

Def III. To *ſee* without *looking*, is to direct the Optic Axes to ſome other place than to the point which is then ſeen; and in ſuch a Caſe, the Image of the point ſeen will be projected upon a part of the *Retina* of each Eye, where the Optic Axis does not fall, namely either nearer to the Noſe *N* as in (*Fig.* 26.) at the points of the *Retina* mark'd *n n*; or farther from the Noſe than the middle of the *Retina*, as at *o o* in *Fig.* 29.

Whatever is *ſeen*, by being *look'd* at with both Eyes, always appears ſingle, by reaſon of the Communication between the middle of the *Retina* in one Eye, and the middle of the *Retina* of the other: there being no ſuch Communication between any other part of the *Retina* in one Eye, and the Correſpondent part of the *Retina* in the other, when theſe Correſpondent parts are equally diſtant from the Noſe.

There is indeed a Communication between the Nervous Fibres on the Right-ſide of the Retina *of one Eye, and the nervous Fibres on the Right-ſide of the* Retina *of the other Eye, and ſo of thoſe on the Left: but no ſingle Object can be ſo painted in each Eye, as to have its Image on the Right or Left part of one* Retina *that communicates with the Right or Left part of the other, of the ſame bigneſs and at the ſame time as in the other; becauſe in whatever Poſition the Object is, it muſt be nearer to one Eye than to the other, except it be juſt in a Line from the Noſe betwixt the two Eyes ſtreight forward.*

Hence it is that if there be two Candles ſet before any one, the firſt at the diſtance of one Foot, and the ſecond at the diſtance of two Feet, from the Eyes; he that looks at the ſecond Candle at *B* will ſee it ſingle, but ſee the firſt Candle or the Candle *A* double; one Appearance being in the Line *A D γ*, the other in *o A E*, becauſe it paints it ſelf upon *o o* in the Retina of each Eye, which points are not the middle points, but farther from the Noſe than the middles *m m*.

So if *B* be the firſt Candle, and *C* the ſecond, he that looks at *B* will ſee C double, becauſe it is painted in the *Retina* at the points *n n* nearer the Noſe than *m m*; and ſo will appear to be in the ſame Poſition as *p r*. in *Fig.* 27

If *γ ε* be two Candles ſo diſpoſed, *Fig.* 30. that by the Interpoſition of a perforated Board *F F*, *γ* can paint it ſelf only in the Eye *R*, and in the Eye *L*. Upon making the Optic Axes meet at *B* and to tend towards ρ and γ, ρ and γ will each paint an Image on the middle of the *Retina* of each Eye, by croſſing their Rays at *B*: and thus the two Candles will appear to be but one, or rather to be in one Place, upon the account of the Communication of the middle of each *Retina*. But if inſtead of the Candles, ρ be a piece of red Silk, and γ a piece of green Silk, the ſame Poſition of the Eyes will make an Image at *B*, appearing like a red and green Spot together without a Mixture of the Colours. If ρ be a red hot Iron, and γ a Candle of Sulphur, the Phænomenon will be more diſtinct. If the Optic Axes be turn'd directly towards γ and ρ, as if there was no Board *F F* in the way, there will appear two Holes in the Board; the one having the red hot Iron in it, the other the Candle.

Now if, of the refracted Rays of the Candle in the firſt Caſe (*Fig.* 25.) thoſe which diverge from each other, ſo as to fall into each Eye, cauſe the ſame Senſations reſpectively; as the Rays which come from a red hot Iron and thoſe which come from a blue Candle; it is evident that the Candle in the firſt Caſe affords red-making and blue-making Rays after Refraction, and that thoſe Rays are differently refrangible; the red *b R* (*Fig.* 25.) the leaſt refrangible, as declining leſs from the perpendicular π π; and the Purple as *b P* declining moſt from the ſaid perpendicular.

The ſame will (*cæteris paribus*) be found true in the intermediate Rays; and to be certain that the Experiment is as I have related it, the Planes α *f* and *f d* of the Bar may be covered with Paper.

Of Microſcopes by Mr. Leewenhoeck *n.* 273. *p.* 903.

III. It has often happen'd that in the burſting of Glaſs, or of Sparks flying out of Wood-Coals (my Eye being a little too near) ſmall particles of the Glaſs or Fire came into my Eye, and cauſed it to ſmart, upon which I uſed to arm my Eyes with Spectacles, againſt the like Accidents for the future.

Now I obſerve when I look through one of my Glaſſes by Candle-Light, that near the upper part of my Eye in the *Tunica cornea*, there appears a fine ſmall Flame of a Candle inverſed, no bigger than the common Letters we uſe in writing, and oppoſite to it appear two round clear Lights, ſo very ſmall that the Flame of the Candle is not be perceived therein.

From hence I conclude, that the *Tunica Cornea* of the Eye, by the Wound it receiv'd from thoſe Particles of Glaſs, has loſt ſomething of its Roundneſs, which occaſion'd thoſe Appearances, and that when the wounded part ſtood juſt before the Sight it obſtructed it, &c.

I obſerved alſo, that in ſeveral places of the *Tunica Cornea*, there lay Veins no longer than the Breadth of 2 or 3 Hairs put together, wherein I could perceive the Globules of Blood very diſtinctly; theſe Veſſels were ſo ſmall, that they could contain but one Globule in the Diameter of them, and the Blood had no manner of Motion.

The Veſſels ſeem to me to be broken off from other Blood-Veſſels, and when the Particles of Blood are a little crowded together, or when one of thoſe longiſh Veſſels are ſomewhat bended, it appears to the Sight, as if one ſaw a thick Cloud.

With this Cloud of ſmall Particles, the Eyes are ſurrounded but more one time than another, for ſome are diſperſt, and then others ariſe in their places; when we view theſe Veſſels with their Globules of Blood, thro' one of my Glaſſes againſt a Candle or other ſtrong Light, they ſeem to be in a continual Motion, whereby thoſe Particles that are in the *Tunica Cornea,* be their Motion never ſo ſmall, ſeem to us, as if they were moving in the Air, but by ſtrict Examination we ſhall find, that they are one and the ſame Particles, which ſometime appear Aſcending, and at other times deſcending.

Beſides the aforemention'd Blood Veſſels, we find in the *Tunica Cornea* round Particles that lye ſcatter'd about, which Particles I judge to be Globules of Blood; an ignorant perſon ſeeing theſe Particles in continual Motion, for as I ſaid before, ſometimes they appear aſcending, and ſometimes deſcending, would be apt to ſay, that theſe Particles were not in his Eye, but in the Air before the Glaſs; and perhaps too, that the deſcending Particles were the Influxes of the Star, and thoſe that ſeem'd aſcending the Exhalations of the Earth, or other Bodies.

It has often befallen too, when I lookt againſt a ſtrong Light, through my Microſcopes, that I ſaw an infinite number of exceeding ſmall Particles, that had all a glittering Motion.

I never imagined that theſe Particles were in the Air, as others would, but rather that they were in the Chriſtalline humor of the Eye between the *Tunica Cornea* and the *Criſtalina,* the Motion of which ſmall particles is occaſioned by preſſing the *Tunica Cornea,* when we ſhut our Eye cloſe together.

Of the manner of making Microſcopes, by Dr. Arch. Adams, *n.* 325. *p.* 24.

IV. I had not an opportunity of examining Mr. *Leeuwenhoeck*'s Glaſſes particularly, which is a Favour he allows to none; therefore I am not capable at this diſtance to deſcribe either their Make or Uſe, any further than that to me they appear'd to be Spherules lodg'd between two Plates of Gold or Braſs, in a hole whoſe Diameter might not be bigger than that of a ſmall Pins head, and the Objects I ſaw through them were pretty and diverting; but ſtill their Make and Truth are unknown.

Mr. *Butterfield* is very curious in melting his Glaſs, but I ſuppoſe unſucceſsful in caſting his Spheres; for beſides that a ſufficient quantity of beaten Glaſs cannot ſtick to the moiſtned point of a fine Needle; ſo neither can it run equally, hold the Needle how you will, nor the *Globule* when run ſtick to the Needle, but muſt unavoidably drop; and whereſoever it happens to fall, it muſt in that almoſt liquid State receive Impreſſions ſufficient to ſpoil the Figure of a Sphere.

Mr. *Gray* has ſhown the defect of his Method, which he us'd to recover by grinding and poliſhing his Glaſſes on a Braſs Plane, and ſo reduce 'em to Hemiſpherules; but how far ſhort poliſh'd Glaſſes (I ſpeak of ſmall ones) come of thoſe which are caſt, I leave to any one to judge who has ſeen both. His Water and Quickſilver Microſcopes I never ſaw, ſo can ſay little to them.

After what manner Mr. *Wilſon's* Glaſſes are made I know not, but ſure his greateſt Magnifyers are ill plac'd, they being ſunk to ſo great a diſtance from the Eye, the Object cannot appear to that Advantage it otherwiſe would; if therefore inſtead of a hollow Cap he would contrive a plain Plate of any Metal for the Reception of the Glaſs, then the Eye and the Object might come to their due diſtance; neither ought there to be any Calx or Glaſs between the Object and the Spherule, when we uſe the greateſt Magnifyers, becauſe if the *Focus* of a Sphere be upon the Extremity of its Circumference, any ſmall diſtance from that muſt ſpoil the Truth of the Objects appearance.

I cannot ſay, that the Glaſſes I have made are without fault, but I think they magnify more than any I have yet ſeen; and were they plac'd to the beſt advantage, they would magnify much more than they do: They are made thus.

I take a piece of fine Window Glaſs, and I raſe it with a Diamond into as many lengths as I think needful, not exceeding an eighth of an Inch in breadth; then holding one of theſe lengths between the Fore-Finger and Thumb of each Hand, over a very fine Flame, until the Glaſs begin to ſoften, I draw it out till it be as fine as a Hair and break: Then inuring each of the ends into the pureſt part of the Flame, I have two Spheres preſently, which I can make larger or leſs as I pleaſe; if they ſtay long in the Flame, they'll have ſpots, ſo I draw 'em out preſently after they turn round. As for the Stem, I break it off as near the Ball as I can, and lodging the remainder of this Stem between the Plates, and by drilling the Hole exactly round, all this Protuberance is bury'd between the Plates, and the Microſcope performs to Admiration; inſomuch, that the ſame Thread of very fine Muſlin appeared 3 or 4 times bigger in one of theſe, than it did in the firſt or ſecond of Mr. *Wilſon*'s. I thought I ſaw Animals in fine old Brandy, but they were ſo nimble in their Motion, that I can give no particular Deſcription of them.

A Way for Myopes *to uſe Teleſcopes without Eye-Glaſſes, by Dr.* Deſaguliers, *n.* 361. *p.* 1017.

V. *Lemma* 1. What is requir'd of a Teleſcope is to give large, and diſtinct Viſion; that is, to make the Object (as in *Galilæo*'s Teleſcope) or its Image (as in the Teleſcopes made up of Convex *Lentes*) appear under a great Angle, and to have all the Rays of thoſe Pencils that enter the Eye, meet in a point upon the *Retina* of the Eye, on their reſpective Axes.

The firſt Figure repreſents the Combination of two Convex *Lentes* for the Aſtronomical or inverting Teleſcope; where the above-mentioned Requiſites are obtain'd. *A B* is the Object ſuppos'd at a vaſt diſtance from the Objective *Lens L L*, ſo that Rays coming from the Extremity *A* of the Object, will fall upon the *Lens L L*, in the ſame manner as if they were parallel to their Axis *A X*; and after paſſing the Glaſs unite at *a*, where they project the Image of the point *A*; from whence diverging, they fall on the Eye Glaſs *l l*, and having paſs'd through it, go on parallel to each other, and enter the *Cornea* of a common Eye *E*, which unites thoſe parallel Rays upon its *Retina R R R* at α, where the

Image of a is projected: The ſame may be ſaid of the Rays that come from B, and after their ſeveral Refractions through the two Glaſſes and the Coats and Humours of the Eye, meet upon the *Retina* at β, where they project the diſtinct Image of the Point b. The Rays that come from all the Points of the Object $A B$ being affected after the ſame manner, give a diſtinct Image of thoſe points upon the *Retina*, and therefore the Object doth appear diſtinct.

The Object will alſo appear magnified in the ſame proportion as the Angle $l C l$ = to $b M a$ (under which its Image is ſeen,) is greater than the Angle $A C B$ under which the Object $A B$ would be ſeen by the naked Eye; as is more at large demonſtrated by Dioptrical Writers.

Lemma 2. If parallel Rays fall upon the *Cornea* of a *Myops*, or ſhort-ſighted Perſon, they will unite in the Eye, before they come to the *Retina*, the farther from it the more Convex the Eye is; but if the Rays which fall upon the *Cornea* diverge in proportion to the too great Convexity of the Eye, as from D, ſuch Rays will be ſo refracted by the Coats and Humours of the Eye as to meet in one point upon the *Retina* $R R$, ſee *Fig.* 2 and 3. Where I have in the Scheme neglected the Refraction of the Rays paſſing out of the *Cryſtalline* K into the *Vitreous* Humour V, as I do in the other Caſes.

This *Lemma* is alſo demonſtrated by Dioptrical Writers.

Lemma 3. If two Pencils of Rays (in each whereof all the Rays are parallel to the Axis, as $a C$) fall upon different parts of the *Cornea*, at the greateſt diſtance from one another that can be allow'd for thoſe Rays to enter the Pupil $P P$, their *Axes* will, after entring the *Aqueous* Humour, converge, and meet either in the Vitreous or Cryſtalline Humour, according to the Convexity of the *Cornea* through which they paſs'd, and diverge again before they come to the *Retina*; the Rays of each Pencil converging upon their reſpective Axes, to the place where the ſaid Axes croſs one another, *Fig.* 4.

Demonſtration. The Axes $a C a$, $\alpha C \alpha$, falling obliquely upon the *Cornea* at C C: and entring from *Air* into the *Aqueous* Humour, will be refracted towards the Perpendicular to K: where ſtriking more directly upon the *Cryſtalline*, they will go on to α, a, upon the *Retina* $R R R R$, decuſſating at V within the *Vitreous* Humour. The other Rays r, r; ρ, ρ, after their Refraction in the *Aqueous* Humour, fall more obliquely on the *Cryſtalline*, and therefore are refracted again ſo as to meet at V, where the Axes alſo meet, and thence go on to the *Retina* $R R R R$, *Fig.* 4.

Lemma 4. But if the Axes of the above-mention'd Pencils are Parallel, the Rays that accompany them diverging from a Point ſo near the Eye, that the divergence may be proportionable to the too great Convexity of the Eye; then only the Axes will meet in the Eye before they come to the *Retina* (by *Lemma* 3.) but the other Rays will not unite upon their reſpective Axes, till they come to the *Retina*, (by *Lemma* 2.)

Prop. I ſuppoſe the Eye of the *Myops* ſo Convex that he can ſee no farther than a common Eye, with the Eye-Glaſs of a Teleſcope before it : then the Eye of the *Myops* being in the place of the Eye-Glaſs, will receive the Rays diverging from the ſeveral points of the Image (projected by the Object-Glaſs in its *Focus,*) in ſuch manner, that they will after their ſeveral Refractions meet in reſpective Points on the *Retina*; and the Axes of the Pencils which come from the Extremities of the Object, will, in the Eye, make the Angle *B V A*= to *b c a*, under which the Image *a b* is ſeen, by *Lemma* 4. The *Cornea* and *Aqueous* Humour here ſupply the place of the Eye-Glaſs, and the *Cryſtalline* and *Vitreous* Humours that of a common Eye, See the 5th *Fig.* wherein *R* is the *Retina*, *V* the *Vitreous* Humour, and *K K* the *Cryſtalline* Humour; and the Image *b a* is ſuppos'd to be brought down from the *firſt Fig.* which repreſents the *Aſtronomick Teleſcope*: the too great Convexity of the Eye here being in the place of an Eye-Glaſs.

An Objection may be made to this, *viz.* that *P P* the Pupil of the Eye being ſmall, will take in but a very little Image, or a ſmall part of the Object: But then if the Eye be mov'd ſucceſſively to all the parts of the Space where the Eye-Glaſs was, it can take any part of the Object; and if the Object-Glaſs be large, which may more eaſily be made than a large Eye-Glaſs, and the Tube a Foot wide or wider, as much may ſucceſſively be taken in, as if an Eye-Glaſs might be had of a Foot Diameter. A little practice may make any *Myops* ſo ready, as to keep an Object when once found, though the place where he ſtands be ſhaken. It would not be amiſs to hold a *Lens* in one's Hand (for an Eye-Glaſs) to find the Object at firſt, till Cuſtom has made it eaſy without it: when once the Object is found, it may be eaſily kept.

An Eye more ſhort-ſighted than I have ſuppos'd, will perform the Office of a more Convex Eye-Glaſs, being brought nearer to the diſtinct Baſe of the Object Glaſs; and an Eye leſs Convex, the Office of a leſs Convex Eye-Glaſs: but with this difference, the more Convex the Eye is, the eaſier may the Object be found, and the larger and more lucid will it appear.

I have ſeen *Saturn*'s Ring very plain with an Object Glaſs of little more than 6 Foot *Radius*, without an Eye-Glaſs.

I have alſo found out a new way for the *Presbytæ* to make uſe of an Object-Glaſs, by placing the Eye nearer the *Lens* than its *Focus*, by ſo much as their Eye is flatter than a common Eye, ſo as to make, as it were, the Teleſcope of *Galilæo*: the flat Eye ſerving as a common Eye arm'd with a Concave *Lens*. I have ſo fixt the Teleſcope as to make a *Presbyta* read at a great Diſtance a ſmall Print.

If this Experiment be made at Sea with a very large Tube, big enough to put in the Head, and move it about, and the Object Glaſs large, it may poſſibly enable one to obſerve the Eclipſes of *Jupiter*'s Satellits.

Of the Invention of Teleſcopick Sights, by Mr. Derham, *n.* 352 *p.* 603.

VI. In *Monſieur de la Hire's* firſt Part of his *Tabulæ Aſtron.* publiſh'd in 1687. I find theſe Words, *Paucis abhinc annis D. Picard inſignis Aſtronomus,* *atque in eadem Academia* [*Regia Scientiarum*] *Socius, Dioptrarum crenas ab inſtrumentis ſuſtulit, eorumque loco ſubſtituit Teleſcopia; quæ res Presbytis & Myopibus*, &c. In which it is not indeed expreſly ſaid that Mr. *Picard* was the Inventor of this way, but only that he applied Teleſcopes. But by reaſon it implies that it was Mr. *Picard*'s Invention, I think my ſelf bound to do Mr. *Gaſcoigne* the Juſtice to aſſert his Invention to him.

That Mr. *Gaſcoigne* was the firſt that applied Teleſcopick Sights to Aſtronomical Inſtruments, appears from a Letter of his now in my Hands, to Mr. *Crabtrie, Jan.* 25. 164$\frac{1}{2}$. wherein he ſaith, *If here*, (that is in the Diſtinct-Baſe) *you place the Scale that meaſures—, or if here an Hair be ſet, that it appear perfectly through the Glaſs—, you may uſe it in a Quadrant, for the finding of the Altitude of the leaſt Star viſible by the Perſpective wherein it is. If the Night be ſo dark, that the Hair or Pointers of the Scale be not to be ſeen, I place a Candle in a Lanthorn, ſo as it caſt Light ſufficient into the Glaſs; which I find very helpful when the Moon appeareth not, or it is not otherwiſe light enough.*

In another Letter, dated on *Chriſtmas-Eve*, 1641. he tells Mr. *Crabtrie* how he had applied his Teleſcopick Sights to a Sextant. Saith he, *Mr.* Horrox's *Theory of the Moon I ſhall be ſhortly furniſhed to try. For I am fitting my Sextant for all manner of Obſervations, by two Perſpicills with Threads. And alſo I am conſulting my Workman about the making of Wheels like* β, γ, δ, ε, *of* † *Diagr.* 3, *to uſe two Glaſſes like a Sector. If I once have my Tools in readineſs to my Deſire, I ſhall uſe them every Night. I have fitted my Sextant by the Help of the Cane, two Glaſſes in it, and a Thread, ſo as to be a pleaſant Inſtrument, could Wood and a Country-Joiner or Workman pleaſe me.*

† *This Diagram is wanting in the Letter.*

In another Letter, (the Date of which is worn out, but is, in Mr. *Crabtrie*'s Hand, called his oth Letter to him) he ſaith, *I have given order for an Iron Quadrant of Five Foot, which will give me the* 1000*th part of one Degree, which ſhall be furniſhed like my firſt Scale; only my Workman is ſo* * *throng for my Father, that I fear it will not be finiſhed before the Eclipſe. I have cauſed a very ſtrong Ruler to be exactly made, and intend to fit it with Curſors of Iron, with Glaſſes in them and a Thread, for my Sextant.*

* A Yorkſhire *Phraſe for fully employ'd.*

To theſe I could have added many other Paſſages of the like Nature: but theſe may be ſufficient, to ſhew that Mr *Gaſcoigne*, as early as 1640, made uſe of Teleſcopes on Quadrants and Sextants, as well as in his Invention of the Micrometer.

In Mr. *Crabtrie*'s ſecond Letter, *October* 30. 1640. after a Demonſtration that the Solar Spots are not Planets at a Diſtance from the Sun, but ſomething adhering to, or very near the Sun's Body: He ſaith, *Something I am ſure you were telling me concerning a way of obſerving the Places of the Planets by your Glaſſes. But I have not a little lamented that my Time cut me ſo ſhort, when I was with you, that I could not more fully ruminate and digeſt thoſe ſtrange Inventions which you ſhewed me, and told* *me of. You told me (as I remember) you doubted not in time to be able to make Obſervations to Seconds. I cannot but admire it and yet, by what I ſaw, believe it: but long to have ſome farther Hints of your Conceit for that Purpoſe. One Means, I think, you told me was, by a ſingle Glaſs in a Cane, upon the Index of your Sextant, by which (as I remember) you find the exact Point of the Sun's Rays. But the way how, I have quite forgotten, and much deſire. Your Device for the exact Diviſion of a Quadrant, by dividing 11 Degrees into 10 Parts, I did then underſtand, but do not now fully remember. If it might not be too much Trouble to you, I ſhould intreat you to give me ſuch a Paper-Demonſtration thereof as you ſhewed me, and two or three Lines plainly of the Uſe thereof, how to find thoſe ſmall Parts.*

I cannot conceal how much I am tranſported beyond my ſelf with the Remembrance (of that little I do remember) of thoſe admirable Inventions which you ſhewed me when I was with you. I ſhould not have believed the World could have afforded ſuch exquiſite Rarities, and I know not how to ſtint my longing Deſires, without ſome further Taſte of theſe ſelected Dainties. Mr. *Crabtrie* in his Letter of *Dec.* 28. 1640. hath theſe Words, *My Friend Mr.* Horrox *profeſſeth, that little Touch I gave him of your Inventions, hath raviſhed his Mind quite from it ſelf, and left him in an Extaſie between Admiration and Amazement.*

Thus, among many, I have related ſome of the Paſſages of Mr. *Gaſcoigne*'s and Mr. *Crabtrie*'s Letters relating to *Teleſcopick Sights.* From whence it is very manifeſt, that long before the *French* Gentleman's Claims, our Countryman Mr. *Gaſcoigne* had made uſe of thoſe Sights in his Aſtronomical Inſtruments; particularly in two or more Sorts of *Micrometers* (as I plainly find) and in his *Quadrant* and *Sextant.* He was ſcarce 20 Years of Age when he held theſe Correſpondencies with Mr. *Crabtrie.* And at the Age of 23. he was killed at *Marſton-Moor-Battle,* on *July* 2. 1644. fighting for King *Charles* I.

VII. *A Paper Omitted.*

n. 281. *p.* 1241. The Deſcription and Manner of uſing a late invented Set of ſmall Pocket-Microſcopes, made by Mr. *James Wilſon*; which with great eaſe are apply'd in viewing Opake, Tranſparent and Liquid Objects: as the *Farina* of the Flowers of Plants, *&c.* The Circulation of the Blood in Living Creatures, *&c.* The *Animalcula* in *Semine,* &c.

CHAP. III.

Aſtronomy.

I. *A new way to draw the* Meridian *Line, by Mr.* Stephen Gray, *n.* 268. *p.* 639.

I Have thought of a new contriv'd Inſtrument for drawing the Meridian Line, which is eaſy in its Uſe, and ſufficiently exact. Take the *Gnomon* of an Horizontal Dial for the Latitude of the Place, and to the Hypotenuſa fix two Sights, whoſe Centers may be parallel to the ſame: let the Eye-ſight be a ſmall Hole, but the others Diameter muſt be equal to the Tangent of the double Diſtance of the North Star, from the Pole; the Diſtance of the Sights being made Radius, let the Stile be rivetted to the end of a ſtreight Ruler; then when you would make uſe of it, lay the Ruler on an Horizontal Plane, ſo that the end to which the Stile is fixt may over hang, then look through the Eye-ſight, moving the Inſtrument till the North Star appears to touch the Circumference of the whole in the other Sight, on the ſame Hand with the Girdle of *Caſſiopeia*, or on the oppoſite Side to that whereon the Star in the Great *Bear's* Rump is at that time, then draw a Line by the Edge of the Ruler, and it will be a true Meridian Line.

Of Drawing one by the Pole-Star, by the ſame, n. 270. *p.* 815.

II. Let there be taken a Teleſcope of about 16 Foot, or longer if you pleaſe, in the Plane of its Focus place a Ring of Braſs at right Angles to the Axis of the Glaſs, the Diameter of the inward Circle equal to the double Tangent of the Pole-Star from the Pole, the focal length of the Object Glaſs being made Radius, as was ſaid in the Deſcription of the Meridian Inſtrument; let the Ring be divided into 24 Hours, with their Minutes number'd from the Right-hand towards the Left, as in our common Nocturnals; the Eye Glaſs muſt be equal in its Diameter to the horary Ring: But becauſe this may be too chargeable, this Contrivance will ſerve; the Eye Glaſs muſt lye in a broad Index towards one end, this is to turn on a Center Pin, that lies in the Center of the Glaſs, and conſequently over the Center of the horary Ring, from which it muſt be equal to the diſtance of the focus of the Eye Glaſs, then let the Tube be elevated to the height of the Pole, and directed to the Pole Star, till by turning the Index through the Eye Glaſs, you perceive the Star to touch the Horary Ring on that ſide the Star in the great Bears Rump lies, or on the oppoſite to that in the Hip of *Caſſiopeia*; whereas had not the Glaſs inverted the Object, it ſhould have been the contrary; then bring one of the 12s to be in a perpendicular to the other by a Plum Line, ſo will the Star ſtand at its Horary diſtance from the Meridian; or if the Latitude of the Place be unknown, by the Right Aſcenſion of the Sun and Star the Line of its coming to the Meridian will be eaſily obtained, and then the Hour of the Night found, will as eaſily give the Stars horary diſtance from the Meridian; then elevate the Tube towards the Star, bringing the Meridian, or 12 and 12 into the plain of the perpendicular, turn the Glaſs about till you ſee the Pole Star ſtand at its horary diſtance from the Meridian; ſo will the Inſtrument when fixed, ſhew the horary diſtance throughout the whole Day, or as long as it remains in this Poſition, by the apparent Motion of the Star in the Ring. The beſt time to fix the Inſtrument, will be when this, or any of the other two Stars above-mentioned, are about 6 Hours from the Meridian. The Latitude of the place is now given with the utmoſt Preciſeneſs, for the Axis of the Glaſs lies now in the Axis of the World, and if one of the ſides of the Tube be parallel thereto, as it ought to be at the upper end, hang a Line and Plummet from the point of the Suſpenſion; find another point equal in diſtance to the length of the Line, or a knot towards the lower end, the diſtance from this knot to the former point will be but the Chord of the Latitude, and if from the ſame edge of the Index, another Line and Plummet be hung towards the lower end of the Tube, theſe two Lines, when at reſt, will be in the plain of the Meridian.

This Inſtrument may be made to ſhew the Hour with as much Facility, as a Clock or Sun-Dial, if the Horary Ring be made to move within a larger fixed one, and the outward Circle of the former be divided into the Days of the Month, reſpect being had to the right Aſcenſion of the Sun and Star; then by bringing the 2 oppoſite Points in the fixed Circle to the Perpendicular, which is done at the fixing the Inſtrument, move the Circle till the Day of the Month come to any of theſe, and the Ring is rectify'd for that Day, and if the Air be clear, you will ſee the Star ſtand at the true time of the Day or Night.

If when by the annual increaſe of its Declination, the Pole-Star by moving in a leſſer Circle be brought too far from the edge of the Ring, that the exact Hour and Minute cannot well be diſtinguiſhed; this Inconveniency may be eaſily remedied by making a leſſer Ring, or by extending a fine thread of Silk croſs the Ring, till it cuts the Star, and at the ſame time it gives the Hour, or which will yet make this Inſtrument commodious for other Purpoſes, there may be made an Index to move on the Center of the Hour-wheel, which being brought to cut the Star with the edge that proceeds from the Center, it will at the ſame time cut the Hour, and now we need not be ſollicitous about the exact Diameter of the Ring, provided it do but a little exceed the diſtance of the Pole-Star from the Pole, the focal length of the Glaſs being made Radius.

Mr. *John Flamſteed*, Math. Reg. and F. R. S. has lately diſcovered, that there is a Parallaxis of the Earth's annual Orbit at the Pole-Star of about 40 or 45 Seconds, whereby the Diameter of the Stars parallel is greater in *June* than in *December* by about one Minute, two Seconds, which he has evinced from 7 Years ſucceſſive Obſervations, whereby the Earths Motion is indubitably demonſtrated.

Now if on the edge of this Index there be drawn a Scale of Degrees, Minutes and Seconds, to the Radius of the Glaſs, we ſhall not only have a very accurate Inſtrument for the Hour, but be furniſhed with one whereby we ſhall ſee the truth of the Earth's Motion, confirmed by the acceſs and receſs of our Star, towards and from the Pole, according to the Earth's place in the Ecliptick, as that learned Perſon above-mentioned has diſcovered; and that not only when the Star tranſits the Meridian, but in clear Air at any time of the Day; one ſhall likewiſe obſerve that annual increaſe of the Pole-Stars Declination, cauſed by the Preceſſion of the Equinox. The Pole-Star may be ſeen in the Day time with a Teleſcope of 16 Foot, for with one of this length I ſaw that Star *April* 26. 1701. from 4 in the Morning till 7, and cou'd have ſeen it longer, had not Clouds interpoſed; and again the 1ſt of *May*, when the Sun had been up more than half an Hour, *viz.* at 5 in the Morning, I ſoon found it, and ſaw it afterwards as oft as I pleaſed, till half an Hour after 9 the ſame Morning.

An Inſtrument to obſerve the Sun, &c. *paſs the* Meridian, *by Mr.* Derham, *n* 291. *p.* 1579.

III. This Inſtrument I have made uſe of for ſeveral Years, and would recommend it, upon my own Experience, for a very nice way to find the Meridian of any place, and to ſee the *Tranſits* of the Celeſtial Bodies over it, either Northward or Southward.

Plate 4.

The Inſtrument is thus made of Wood, or rather Iron, or Braſs, to indure the Weather, without ſwelling or contracting, *viz.* Prepare a ſmall flat Iron Bar, as in *Fig.* 8. C. C. at each end of which rivet on two upright Sights, to turn ſtiffly, at the Joynts, *I. I.* Let one of the Sights *c.d.* have a Perforation big enough to ſee the *Pole-Star* through it, the other Sight *a.b.* a very ſmall Perforation, to ſee the *Sun* through. Juſt behind the Joynts fix two upright Arms, *C.D.* and *C. D*, but to bend off, ſo as to be out of the way of the Sights, when you look through them. Theſe Arms ought to be long enough for the Plumb-lines to reach the Polar-Star, on one ſide; and the Sun at his greateſt height, on the other, when you look through either of the Sights. The Plumb-lines therefore are Tangents to their oppoſite Sights, and their lengths may be found by a Table of natural Tangents, and making the diſtance of the two Sights Radius. Thus in the Latitude of *London*, if the Inſtrument be two Feet from Sight to Sight, the Southern Plumb-line hath need to be near four Feet, and the Northern Plumb-line about two Feet ten Inches. On the tops of theſe two Arms place two ſmall croſs pieces *D. E.* and *D. E.* to turn with a Joynt at *D.* which croſs-pieces are to hold the Plumb-lines *E. F.* and *E. F.* and to turn off and on, ſo as to bring the Plumb-lines to the Sights exactly. Place this Inſtrument on a Pedeſtal *H.* to turn round on it ſtiffly at the Pin *G.*

Your Inſtrument being thus prepared, the way to *Set and Uſe it* is thus; Plant it in a convenient place, where the Polar Star may be ſeen by Night, and the Sun by Day. When that Star is on the Meridian, is the time to ſet this Inſtrument, which is thus to be done; *viz.*

Through the Sight with the larger Hole *c. d.* look at the Pole-Star, and turn the whole Inſtrument about, until you ſee the oppoſite Plumb-line nicely to interſect the Pole-Star. Or when you have brought the Plumb-line near the Star, you may more eaſily bring the Plumb-line to interſect, by moving the Sight *c. d.* backward or forward, at the Joynt *I,* inſtead of moving the whole Inſtrument. And that you may more eaſily ſee the Pole-Star through the Sight, let the Plumb-line be a very fine Cats-gut ſtring or Horſe-hair, *&c.* And if it be white, or ſome ſuch light colour, it will be the better ſeen, with the help of a Candle ſhining on it by Night; which is neceſſary.

The Sight *c. d.* and oppoſite Plumb-line being thus ſet in a direct line with the Polar-Star on the Meridian, it is manifeſt, that the Inſtrument lyeth exactly in the Meridian, ſo as to ſee any Star on the Meridian to the North. And that you may ſee the ſame Southerly; the next Day, or when you pleaſe, you may hang up the Plumb-line *E. F.* upon the Southern Arm *C. D.* ſo as that the Plumb-line may exactly interſect the Perforation *c. d.* This may eaſily be done by moving the top Joynt, with the Plumb-line on its croſſpiece backward and forward, till the Plumb-line hangeth to your Mind. If the Sight with the leſſer Perforation *a. b.* be not exactly under the Northern Plumb-line, it muſt be brought to be ſo by turning the Sight, by help of its Joynt at *I.* And then all the Inſtrument is ſet right, ſo as to ſee the Sun, Moon or Stars come on the Meridian towards the South.

But to ſee the Sun tranſit the Meridian, it is neceſſary to guard the Eye, with a coloured Glaſs, or a Glaſs darkened with the ſmoak of a Lamp or Candle. Which for the ſake of thoſe who do not know the way of it, it may be neceſſary to deſcribe.

A Glaſs to look upon the Sun.

Chuſe two pieces of Glaſs cut into the ſame ſize and figure; but take care they do not refract vitiouſly. Which may be known by moving the Glaſs before the Eye. If the Objects you look on, ſeem to dance about, the Glaſſes are falſe and refract; but true if all ſeems ſteady. Smoak one of theſe Glaſſes over the Flame of a Lamp or Candle, until it be obſcured enough to take off the Sun's Rays ſufficiently, but not ſo as to darken it too much. This may be ſeen by looking upon the Sun with it, or upon the Candle. One of the Glaſſes being thus darkened, lodge them both together, and faſten them in a little Caſe fit for the Purpoſe, with the ſmoaked ſide innermoſt, and an edging of Card between, to keep the Glaſſes aſunder, ſo as that the Soot may not be rubb'd off, or diſordered.

'Tis good to have two Glaſſes thus prepared, one for a ſtrong Sun; the other leſs darkened, for the Sun behind a thin Cloud, or Miſt, *&c.*

With one of theſe Glaſſes held behind, or before the Sight *a. b.* you may plainly ſee the Sun paſs.

To imitate the aforeſaid Inſtrument in a Journey, or where-ever you come. Inſtead of an intire Inſtrument, you may prepare only two Sights with Perforations as before. Let theſe Sights be nailed or ſcrewed down, upon the tops of two Stakes at *I. I*, ſo as to turn ſtiffly upon them. The Plumb-lines (one at leaſt) may be hung up at the end of an Houſe or on the Bough of a Tree (if the Wind would not ſhake it) or any where you ſee fit. And the Sights muſt be ſtuck up, ſo as to bring the Pole-Star to interſect, and all be performed, as hath been before directed.

This, although in a manner the ſame with the Inſtrument before, yet is more convenient in ſome reſpects, chiefly becauſe the Plumb-lines may be made longer, and the Sights ſet farther aſunder, than in the Inſtrument before can conveniently be done; which is ſome, although no great advantage for ſeeing the Tranſits. Alſo, theſe Sights may be made ſo light, as to be eaſily carried about; or they may be eaſily made, or imitated in any place where-ever you come.

To know when the Polar-Star cometh on the Meridian. The way is this, Subſtract the *Right Aſcenſion* of the *Sun* from the *Right Aſcenſion* of the *Pole-Star*, the *Remainder* giveth the Degrees, Minutes and Seconds when the Pole Star tranſits the Meridian above the Pole. Divide theſe Degrees by 15, it giveth the Hours, and every Degree under 15 multiplied by 4, giveth the Minntes; and every Minute multiplied by 4, giveth the Seconds, of Apparent time of the Pole Stars Southing. I ſcarce need to ſay that it cometh under the Pole at 12 Hours diſtance, only making ſome ſmall allowance for the Alteration of the Sun's Right Aſcenſion in that 12 Hours time. But you may ſhorten your Labour, by uſing Tables of the Sun's R. Aſc. in *Time*, inſtead of his R. Aſcenſion in *Degrees*, &c. which may be found in Sir *Jon. Moor*'s *Math. Compend.* and in divers other Books.

If the Sun's R. Aſc. exceedeth the Pole-Stars, add 360 Degrees, or 24 Hours, and then ſubſtract. The R. Aſc. of the Pole-Star is determin'd by Mr. *Flamſteed* to be $0^{h}\ 33'.\ 4''$ of *Time*, Anno 1690, and the increaſe of its R. Aſc. in 10 Years $1'\ 16''$. of *Time*. Therefore in 1703, the R. Aſc. of the Pole-Star was $0^{h}\ 35'.\ 22''$. of *Time*. Or you may ſee when the Pole-Star cometh to the Meridian, by hanging up a Plumb-line, and obſerving when the *Thill-Horſe* in *Charles*'s *Wain*, called *Alioth*, comes near the Line, together with the Pole-Star, on one ſide the Pole; or the bright Star of the third Magnitude in *Caſſiopeia*'s *Thigh* on the other ſide.

The foregoing Inſtruments may be ſet by any Star; but the Pole-Star, in our Northern Hemiſphere, is moſt convenient, becauſe it maketh but a ſmall Circle round the Pole, and therefore moveth ſlower, and conſequently is longer in tranſiting the Meridian. And therefore a ſmall Error in Calculation, or a little Expence of Time in ſetting the Inſtrument, may be admitted, as little deſerving regard.

The Uſes of theſe Meridian Inſtruments.

1ſt, You may ſee with all imaginable Exactneſs, when it is Noon even to 1, 2, or at moſt 3 Seconds of Time. For you may ſee when the very Limb of the Sun toucheth the Meridian, and whilſt all his Diſk is paſſing it. So that by much it exceedeth all Sun-Dials: ſo far that if you once uſe this Inſtrument, you will be ready to lay aſide all Sun-Dials; the beſt of which (unleſs we except Mr. *Molineux's*) can never ſhew the Time to one or many Seconds.

But beſides all this, another vaſt Conveniency is, That it will *fit moſt Latitudes.* So that there is no need of having a ſtrict regard to the Elevation of the Pole, nor any danger of Error in making and ſetting, as is in moſt other Inſtruments, but all is with Eaſe and Certainty performed. Therefore,

2dly, Into whatſoever place you come, you may eaſily ſee the Errors of the Sun-Dials there, and which go trueſt, and which falſe.

3dly, As the Sun, ſo alſo the fixt Stars may be ſeen to tranſit the Meridian, whereby the *Hour of the Night* may as exactly be known, as of the Day by the Sun, knowing the R. Aſc. of the Star that tranſits. For (as before for the Pole-Star) ſubſtract the R. Aſc. of the Sun from the R. Aſc. of the Star, the Remainder converted into Time, is the Time of that Star's Culmination or Southing. And if 12 Hours be added or ſubſtracted (making due Allowance for the Alteration of the Sun's R. Aſc. in that time) it ſheweth the exact time of that Star's coming to the Meridian Northward.

4thly, The Hour of the Day and Night being thus to 1, 2 or 3 Seconds diſcoverable by the aforeſaid Inſtruments, I doubt not but that they may be uſeful in finding the exact *Differences of Meridians*, either by the *Eclipſes* of *Jupiter's Satellits*, or the Occultations of the Fixt-Stars by the Moon.

Theſe Meridian Inſtruments may be of Service where-ever long Teleſcopes are us'd, for ſeeing the Appulſes of the Moon to the Fixt-Stars or the Eclipſes of *Jupiter's Satellits*; which is only on Land. Unleſs (which I have thought feaſible) a convenient ſtanding for a Man, and a Teleſcope might be hung penduloufly in a Ship, which (eſpecially in a calm Sea) may be as little ſubject to diſturbance as the Pendula of Watches are, which will retain their Motion at Sea.

5thly, You may with all Exactneſs continue a Meridian Line for 20 Miles, by looking through either Sight, and ſeeing what Objects are interſected by the Plumb-Lines.

6thly, Theſe Inſtruments are prepar'd with little coſt or trouble, and eaſily carried about or imitable in any place, the latter eſpecially, which may be made pocketable, or ſoon gotten.

Of the Maxima, & Minima, *occurring in the Celeſtial Motions. by* Mr. De Moivre, *n.* 360. *p.* 952.

IV. Sit *ABP* Orbis Planetæ Ellipticus, *AP* Axis Tranſverſus, *CB* Semiaxis conjugatus, *S* Sol, *Q* Focus alter Ellipſeos. Per *S* ducatur *SM* ipſi *CB* parallela: & erit punctum *M* in quo *Maxima* cum velocitate creſcit vel decreſcit diſtantia a Sole, & $SM = AC - \frac{SC^2}{AC}$.

Plate 3. *Fig.* 37.

Si vero capiatur *SL* media proportionalis inter Semiaxes *AC*, *CB*, erit punctum *L* in quo *Maxima* fit æquatio Centri, ut vocant; ſive ubi

motus angularis fit æqualis medio Motui: Quod ſi Eccentricitas non major ſit quam in pleriſque Planetis, $BL = \frac{1}{2} BM$ quamproxime: Eſt vero $SL = \sqrt{\sqrt{AC^4 - AC^2 SC^2}}$.

Si quæratur punctum *N*, in quo fit *Maxima* mutatio Velocitatis motus realis in Curva, Problema Solidum eſt. Eſt enim $2NS = 4AC - 2NQ$ ad $3NQ - AC$ ut $AC^2 - CS^2 = CB^2$ ad NQ^2; adeoque ſi ponatur $AC = a$, $CB = c$ & $NQ = y$, habebitur æquatio $y^3 - 2ayy + \frac{3}{2}ccy - \frac{1}{2}acc = 0$. Qua reſoluta erit *y* ſive *N Q* diſtantia puncti quæſiti *N* ab altero Ellipſeos foco. In Orbibus autem parum Eccentricis, quales ſunt Planetarum, ſi fiat $CD = SQ$, & junctæ *A D* æqualis ponatur *A K*, erit reliqua pars Axis $KP = NS$ diſtantiæ puncti *N* a Sole quamproxime. Si vero Orbis fuerit Parabolica, erit *S N* ad *S P* ut 5 ad 4, anguluſque *N S P* erit 53°. 8′ fere, cujus Sinus eſt $\frac{4}{5}$ Radii.

At Punctum *O*, in quo motus apparens ſive angularis acceleratio Planetæ deſcendentis, vel retardatio aſcendentis *Maxima* fit, hoc modo obtinebitur. In *A C* capiatur $CG = \frac{1}{6} AC$, ac fiat angulus *C S F* 30 *gr.* ductæque *S F* æqualis ponatur *C E*, ipſique *G E* ſit *G H* æqualis. Dico, ſi diſtantia *S O* fiat æqualis ipſi *P H*, quod in puncto *O* proveniet *Maxima* mutatio motus angularis Planetæ in Orbe Elliptico *A B O P* gyrantis; eo ſcilicet in Orbis loco ſecundæ differentiæ æquationum centri Planetæ reperientur *Maximæ*. Eſt autem $SO = \frac{7}{6}AC - \sqrt{3\frac{1}{6}AC^2 + \frac{1}{3}SQ^2}$. Quod ſi Orbis Parabolica fuerit, ut in Cometis, fiet *S O* ad *S P* ut 8 ad 7, anguluſque *O S P* fiet 41°. 24′$\frac{1}{2}$, ſive cujus Sinus ſit ad Radium ut $\frac{1}{4}\sqrt{7}$ ad 1.

Denique *Minima* cum Velocitate mutatur directio Tangentis Orbitæ in puncto *R*, ſi fiat *S R* æqualis duabus tertiis Axis majoris *A B*. Quod ſi Eccentricitas *S C* minor fuerit quam $\frac{1}{3}PC$, *Minimum* hoc non locum habet, ſed decreſcit ſemper hæc Velocitas quacum revolvitur Tangens, uſque in ipſum Aphelion; quemadmodum ſe res habet in omnium Planetarum motibus. Neque etiam in orbe Parabolico obtinet, ob Axem ejus in infinitum protenſum *.

* *v. ſupra Chap.* I. *p.* 1.

A new Star in the Swan's *Neck, by Mr.* Kirch, *n.* 343. *p.* 226.

V. Ut mutabilem apparentiam ſtellæ χ in collo Cygni deprehenderem, occaſionem dedit ſtella capiti Cygni vicina, quam *Hevelius*, Aſtronomus ſolertiſſimus, Anno 1670 & 71. obſervavit. Cum enim ſpem conciperem fore ut eadem ſtella nunc iterum ſæpius eſſet apparitura, non ſecus ac ſtella in collo Ceti, quam *Hevelio* poſt primam diſparitionem mox iterum apparuiſſe conſtabat, quærebam eam 1° & 6 to (11° & 16°) Julii 1686. noctibus ſerenis, non autem reperiebam; ſed potius animadvertebam ſtellam illam 5 æ magnitudinis in collo Cygni, a *Bayero* græca litera χ ſignatam, deſiderari. Die vero 9 (19.) Octobris deprehendi eam nudis oculis diſtincte omnino. Et quia facile adducebar ut exiſtimarem eandem nudis noſtris oculis iterum diſparituram, delineavi aliquot ipſam circumſtantes ſtellulas, ope bipedalis magnæque capacitatis Tubi, ut ex harum cum illa comparatione magnitudinem ejus, cum decreſceret, expenderem, uti Fig. *A.* exhibetur.

Plate 3.

Deprehenſum eſt quoque, ſtellam iſtam paulatim decreviſſe, donec eam Tubo 8 pedum non potuerim amplius aſſequi, cum tamen aliam illam, in collo Ceti, quando nudo oculo non amplius patet, per tubum 4 pedum ſemper dignoſcere poſſim.

Ab illo tempore quærebam ſtellam iſtam variis noctibus fruſtra, tandem vero tamen eam reperi 6^{to} (16°) Aug. 1687. iterum ope octopedalis Tubi, at vero exiguam valde. Inde de die in diem creviſſe deprehendebatur, & factum eſt ut die 23 Octob. (Novemb.) iterum prima vice, nudo oculo, ſeſe conſpiciendam præberet, valde licet adhuc dum exiguam. Die 2^{do} (12°) Nov. optime erat conſpicua, etiam poſt 26^{tum} Nov. (6. Dec.) ut hoc ultimo die jam iterum in ſtatu decreſcendi exiſteret. Poſtmodum non niſi per Tubos dignoſci potuit, tandemque adeo exigua evaſit, ut iterum Tubo 8 pedum eam deprehendere non potuerim. Atque ita animadverſum eſt hac vice, ab una diſparitione uſque ad alteram, annum unum, menſem unum, unamque hebdomadam circiter effluxiſſe. Sequentes quoque obſervationes docuerunt hancce ſtellam tempus ſatis conſtans in ſua apparitione ſervare, non tamen quavis vice ad æqualem magnitudinem pervenire. Imo aliquando accidit, ut nudo oculo plane inviſibilis maneat, dum per Tubum eſt conſpicua, & maximam ſuam magnitudinem aſſecuta eſt; prout Anno 1688 in fine, & 1689 in principio anni accidit. E contrario, ſequente Anno 1690, ſtella hæc eo melius videri poterat, & quidem notabiliter major quam ſua vicina, quam *Bayerus* juxta χ collocavit extra collum Cygni, nullaque litera notavit, quam ego ſolius memoriæ juvandæ gratia, Hebraica litera ג notavi. Et poſtquam hujus ſtellæ apparentiam & diſparentiam ſæpius obſervavi, comperi eam valde eſſe regularem, revolutionemque 404½ dierum ſervare.

A Remark by Dr. Ed. Halley, *ib. p.* 228.

Cum Miſcellanea Berolinenſia *ſerius ad nos perlata ſint, non ante annum ultimo elapſum hanc Stellam novam ſecundam* D. Kirchii *monitum conſpeximus; idque juxta Idus Julii,* ſt.vet. *cum multo clarior quam vicina* ג, *ac fere æqualis mediæ in collo Cygni* Bayero *η) apparuit: ſed poſt menſem nudis oculis inconſpicua facta, tandem etiam Teleſcopio evanuit.*

A Hiſtory of new Stars, by Dr. Halley, *n.* 346. *p.* 354.

VI. The firſt new Star in the *Chair* of *Caſſiopeia,* was not ſeen by *Cornelius Gemma* on the Eighth of *November* 1572, who ſays, he that Night conſidered that part of Heaven in a very ſerene Sky, and ſaw it not: but that the next Night, *November* 9°. it appeared with a Splendour ſurpaſſing all the fixt Stars, and ſcarce leſs bright than *Venus.* This was not ſeen by *Tycho Brahe* before the 11th of the ſame Month, but from thence he aſſures us that it gradually decreaſed and died away, ſo as in *March* 1574, after ſixteen Months, to be no longer viſible; and at this Day no Signs of it remain. The Place thereof in the Sphere of Fixt Stars, by the accurate Obſervations of the ſame *Tycho,* was 0^s 9°. 17′. *a* 1^{ma} ✱ $♈^{is}$, with 53°. 45′. North Latitude.

Such another Star was ſeen and obſerved by the Scholars of *Kepler,* to begin to appear on *Sept.* 30°. *ſt. vet. anno* 1604, which was not to be ſeen the Day before: but it broke out at once with a Luſtre ſurpaſſing

that of *Jupiter*; and like the former it died away gradually, and in much about the ſame time diſappeared totally, there remaning no Footſteps thereof in *January* $160\frac{5}{6}$. This was near the Ecliptick, following the Right Leg of *Serpentarius*; and by the Obſervations of *Kepler* and others, was in 7^s 20° 00′ a 1^{ma} ✱. ♈, with North Latitude 1° 56′. Theſe two ſeem to be of a diſtinct *Species* from the reſt, and nothing like them has appear'd ſince.

But between them, *viz.* in the Year 1596, we have the firſt Account of the wonderful Star *in Collo Ceti*, ſeen by *David Fabricius* on the third of *Auguſt*, ſt. vet. as bright as a Star of the third Magnitude, which has been ſince found to appear and diſappear periodically: its Period being preciſely enough 7 Revolutions in Six Years, though it return not always with the ſame Luſtre. Nor is it ever totally extinguiſh'd, but may at all times be ſeen with a Six-Foot-Tube. This was ſingular in its Kind, till that in *Collo Cygni* was diſcovered. It precedes the firſt Star of *Aries* 1°. 40′. with 15°. 57′ South Latitude.

Another New Star was firſt obſerved by *Will. Janſonius* in the Year 1600, *in Pectore* or rather *in eductione Colli Cygni*, which exceeded not the third Magnitude. This having continued ſome Years, became at length ſo ſmall, as to be thought by ſome to diſappear entirely: but in the Years 1657, 58 and 59, it again aroſe to the third Magnitude, tho' ſoon after it decay'd by Degrees to the fifth or ſixth Magnitude, and at this Day is to be ſeen as ſuch in 9 · 18°. 38′. a 1^{ma} ✱. ♈, with 55°. 29′. North Latitude.

A fifth New Star was firſt ſeen by *Hevelius* in the Year 1670, on *July* 15. *ſt. vet.* as a Star of the third Magnitude, but by the Beginning of *October* was ſcarce to be perceived by the naked Eye. In *April* following it was again as bright as before, or rather greater than of the third Magnitude, yet wholly diſappeared about the middle of *Auguſt*. The next Year, in *March* 1672. it was ſeen again, but not exceeding the ſixth Magnitude: ſince when it has been no further viſible, though we have frequently ſought for its Return; its place is 9^{s}· 3°. 17′. *a* 1^{ma} ✱. ♈. and has Lat. North. 47° 28′.

The Sixth and laſt is that we above deſcribed from the *Acta Berolinenſia*, of the Year 1710. diſcovered by Mr. *G. Kirch* in the Year 1686, and its Period determined to be of $404\frac{1}{2}$ Days: and though it rarely exceed the fifth Magnitude, yet is it very regular in its Returns, as we found in the Year 1714. Since then we have watched, as the Abſence of the Moon and the Clearneſs of Weather would permit, to catch the firſt beginning of its Appearance in a Six-Foot-Tube, that bearing a very great Aperture diſcovers moſt minute Stars. And on *June* 15. laſt, it was firſt perceived like one of the very leaſt Teleſcopical Stars: but in the reſt of that Month and *July* it gradually encreaſed, ſo as to become in *Auguſt* viſible to the naked Eye; and ſo it continued all the Month of *September*. After that it again died away by degrees, and on the 8th of *December* at Night was ſcarce diſcernible by the Tube, and as near as could be gueſſed, equal to what it was at its firſt Appearance on *June* 25th: ſo that this Year it has been ſeen in all near Six Months, which is but little leſs than half its Period: And the middle, and conſequently the greateſt Brightneſs, falls about the 10th of *September.*

Of Nebulæ *or lucid Spots among the Fixt Stars by* —— *n.* 347. *p.* 390.

VII. Not leſs wonderful are certain luminous Spots or Patches, which diſcover themſelves only by the Teleſcope, and appear to the naked Eye like ſmall fixt Stars; but in reality are nothing elſe but the Light coming from an extraordinary great Space in the Ether; through which a lucid *Medium* is diffuſed, that ſhines with its own proper Luſtre. This ſeems fully to reconcile that Difficulty which ſome have moved againſt the Deſcription *Moſes* gives of the Creation, alledging that Light could not be created without the Sun. But in the following Inſtances the contrary is manifeſt; for ſome of theſe bright Spots diſcover no ſign of a Star in the middle of them; and the irregular Form of thoſe that have, ſhews them not to proceed from the Illumination of a Central Body. Theſe are, as the aforeſaid New Stars, ſix in Number, all which we will deſcribe in the order of Time, as they were diſcovered; giving their Places in the Sphere of Fixt Stars, to enable the Curious, who are furniſhed with good Teleſcopes, to take the Satisfaction of contemplating them.

The firſt and moſt conſiderable is that in the middle of *Orion*'s Sword, marked with θ by *Bayer* in his *Uranometria*, as a ſingle Star of the third Magnitude; and is ſo accounted by *Ptolemy*, *Tycho Brahe* and *Hevelius*: but is in reality two very contiguous Stars environed with a very large tranſparent bright Spot, through which they appear with ſeveral others. Theſe are curiouſly deſcribed by *Hugenius* in his *Syſtema Saturninum* p. 8. who there calls this Brightneſs, *Portentum cui certe ſimile aliud nuſquam apud reliquas Fixas potuit animadvertere*: affirming that he found it by chance in the Year 1656. The middle of this is at preſent in ♊ 19°. 00, with South Lat. $28^{\circ}\frac{3}{4}$.

About the Year 1661, another of this ſort was diſcovered (if I miſtake not) by *Bullialdus, in Cingulo Andromedæ.* This is neither in *Tycho* nor *Bayer*, having been omitted, as are many others, becauſe of its Smallneſs: But it is inſerted in the Catalogue of *Hevelius*, who has improperly call'd it *Nebuloſa* inſtead of *Nebula*; it has no ſign of a Star in it, but appears like a pale Cloud, and ſeems to emit a radiant Beam into the North Eaſt, as that in *Orion* does into the South Eaſt. It precedes in Right Aſcenſion the Northern in the Girdle, or ν *Bayero*, about a Degree and three Quarters, and has Longitude at this time ♈. 24°. 00′. with Lat. North $33^{\circ}\frac{1}{3}$.

The third is near the Ecliptick between the *Head* and *Bow* of *Sagittary*, not far from the Point of the Winter Solſtice. This it ſeems was found in the Year 1665. by a *German* Gentleman *M. J. Abraham Ihle*, whilſt he attended the Motion of *Saturn* then near his *Aphelion*. This is ſmall but very luminous, and emits a Ray like the former. Its Place at this time is ♑ $4^\circ\frac{1}{2}$ with about half a Degree South Lat.

A fourth was found by Dr. *Edm. Halley* in the Year 1677, when he was making the Catalogue of the Southern Stars. It is in the *Centaur*, that which *Ptolemy* calls ὁ ἐπὶ τῆς τοῦ νώτου ἐκφύσεως, which he names *in dorſo Equino Nebula*, and is *Bayers* ω; it is in appearance between the fourth and fifth Magnitude, and emits but a ſmall Light for its Breadth, and is without a radiant Beam; this never riſes in *England*, but at this time its Place is ♏ $5^\circ\frac{3}{4}$ with $35\frac{1}{6}$ South Lat.

A fifth was diſcovered by Mr. *G. Kirch* in the Year 1681, preceding the Right Foot of *Antinous*: It is of its ſelf but a ſmall obſcure Spot, but has a Star that ſhines through it, which makes it the more luminous. The Longitude of this is at preſent ♑. 9°. *circiter*, with $17'\frac{1}{6}$. North Latitude.

The ſixth and laſt was accidentally hit upon by Dr. *Edm. Halley* in the Conſtellation of *Hercules*, in the Year 1714. It is nearly in a right Line with ζ and η of *Bayer*, ſomewhat nearer to ζ than η: and by comparing its Situation among the Stars, its Place is ſufficiently near in ♏ $26\frac{1}{2}$. with 57°. 00. North. Lat. This is but a little Patch, but it ſhews it ſelf to the naked Eye, when the Sky is ſerene and the Moon abſent.

There are undoubtedly more of theſe which have not yet come to our Knowledge, and ſome perhaps bigger, but though all theſe Spots are in Appearance but little, and moſt of them but of few Minutes in Diameter; yet ſince they are among the Fixt Stars, that is, ſince they have no annual Parallax, they cannot fail to occupy Spaces immenſely great, and perhaps not leſs than our whole Solar Syſtem. In all theſe ſo vaſt Spaces it ſhould ſeem that there is a perpetual uninterrupted Day, which may furniſh Matter of Speculation, as well to the curious Naturaliſt as to the Aſtronomer.

Of the Change of Latitude of ſome of the Fixt Stars, by Dr. Halley, *n.* 355. *p.* 736.

VIII. Having of late had occaſion to examine the quantity of the Preceſſion of the Equinoctial Points, I took the pains to compare the Declinations of the fixt Stars delivered by *Ptolomy*, in the 3d Chapter of the 7th Book of his *Almag.* as obſerved by *Timocharis* and *Ariſtyllus* near 300 Years before *Chriſt*, and by *Hipparchus* about 170 Years after them, that is about 130 before *Chriſt*, with what we now find: and by the reſult of very many Calculations, I concluded that the fixt Stars in 1800 Years were advanced ſomewhat more than 25 degrees in Longitude, or that the Preceſſion is ſomewhat more than 50″ *per Ann.* But that with ſo much uncertainty, by reaſon of the imperfect Obſervations of the Ancients, that I have choſen in my Tables to adhere to the even proportion of five Minutes in ſix Years, which from other Principles we are aſſured is very near the Truth. But while I was upon this Enquiry, I was ſurprized to find the Latitudes of three of the principal Stars in Heaven directly to contradict the ſuppoſed greater *Obliquity* of the *Ecliptick*, which ſeems confirmed by the Latitudes of moſt of the reſt:

they being ſet down in the old Catalogue, as if the Plain of the Earth's Orb had chang'd its Situation, among the fixt Stars, about 20′ ſince the time of *Hipparchus*. Particularly all the Stars in *Gemini* are put down, thoſe to the *Northward* of the Ecliptick, with ſo much leſs Latitude than we find, and thoſe to the *Southward* with ſo much more *Southerly* Latitude. Yet the three Stars *Palilicium* or the *Bulls* Eye, *Sirius* and *Arcturus* do contradict this Rule directly: for by it, *Palilicium* being in the Days of *Hipparchus* in about 10 *gr.* of *Taurus* ought to be about 15 Min. more *Southerly* than at preſent, and *Sirius* being then in about 15 of *Gemini* ought to be 20 Min. more *Southerly* than now; yet *e contra Ptolomy* places the firſt 20 Min. and the other 22 more *Northerly* in Latitude than we now find them. Nor are theſe Errors of Tranſcription, but are proved to be right by the Declinations of them ſet down by *Ptolomy*, as obſerved by *Timocharis*, *Hipparchus* and himſelf, which ſhew that thoſe Latitudes are the ſame as thoſe Authors intended. As to *Arcturus*, he is too near the Equinoctial Colure, to argue from him concerning the change of the Obliquity of the Ecliptick; but *Ptolomy* gives him 33′ more *North* Latitude than he now has; and that greater Latitude is likewiſe confirmed by the Declinations delivered by the aforeſaid Obſervers. So then all theſe three Stars are found to be above half a degree more *Southerly* at this time than the Antients reckoned them. When on the contrary at the ſame time the bright Shoulder of *Orion* has in *Ptolomy* almoſt a degree more *Southerly* Latitude than at preſent. What ſhall we ſay then? It is ſcarce credible that the Antients could be deceived in ſo plain a matter, three Obſervers confirming each other. Again theſe Stars being the moſt conſpicuous in Heaven, are in all probability the neareſt to the Earth, and if they have any particular Motion of their own, it is moſt likely to be perceived in them, which in ſo long a time as 1800 Years may ſhew it ſelf by the Alteration of their places, though it be utterly imperceptible in the ſpace of a ſingle Century of Years. Yet as to *Sirius* it may be obſerved that *Tycho Brahe* makes him 2 Min. more *Northerly* than we now find him, whereas he ought to be above as much more *Southerly* from his Ecliptick, (whoſe Obliquity he makes 2½ greater than we eſteem it at preſent) differing in the whole 4½ Min. One half of this difference may perhaps be excuſed, if Refraction were not allowed in this Caſe by *Tycho*; yet two Minutes, in ſuch a Star as *Sirius*, is ſomewhat too much for him to be miſtaken.

But a more evident proof of this Change is drawn from the Obſervation of the Application of the Moon to *Palilicium* at *Athens Anno Chriſti* 509 *Mart.* 11°. when in the beginning of the Night the Moon was ſeen to follow that Star very near, and ſeem'd to have Eclipſed it.* Now from the undoubted Principles of Aſtronomy, it was impoſſible for this to be true at *Athens*, or near it, unleſs the Latitude of *Palilicium* were muchleſs than we at this time find it.

* v. Bulliald. *Aſtr. Philolaica.* p. 172.

This Argument ſeems not unworthy of the *Royal Society's* Conſideration, to whom I humbly offer the plain Fact as I find it, and would be glad to have their Opinion.

But whether it were really true, that the Obliquity of the Ecliptick was, in the time of *Hipparchus* and *Ptolomy*, really 22 Min. greater than now, may well be queſtion'd; ſince * *Pappus Alexandrinus*, who lived but about 200 Years after *Ptolomy*, makes it the very ſame that we do.

* v. Pappi *Coll. Lib.* VI: *Prop.* 35.

IX. Satis notum eſt Orbitæ *Caſſinianæ* hanc eſſe naturam. Si a datis duobus punctis F & G ad quodvis curvæ punctum H ducantur rectæ FH, GH; rectangulum ſub FH, GH æquale eſt dato ſpatio. Recta FG hinc inde producta donec curvæ occurrat, oſtendit Vertices A & B; & AB eſt Axis principalis; mediumque inter vertices punctum C eſt figuræ Centrum; & DE, per C ad AB normalis, Axis minor; punctaque F & G Foci.

Of the Orbit of Caſſini, *by Dr.* Gregory, *n.* 293. *p.* 1704. *Plate* 3. *Fig.* 38.

In hac figura ſi axis minor excedat diſtantiam focorum, Curva figuram terminans eſt ubique verſus centrum cava, qualis vulgo habetur. Si, manente axe principali, diſtantia focorum minuatur, augebitur axis minor, qui tamen minor manet axe Ellipſis eodem axe principali, iiſdemque focis deſcriptæ; donec tandem, coeuntibus focis, ille evadat æqualis axi majori, & figura abeat in Circulum. Si vero, e contra, diſtantia focorum augeatur; minuetur axis minor, fietque æqualis dictæ diſtantiæ cum hæc eſt ad axem principalem ſicut unitas ad medium proportionalem inter unitatem & ternarium.

Si ulterius augeatur diſtantia focorum, minuetur adhuc axis minor, & Curva non erit amplius ad hujus extrema verſus centrum cava, ſed convexa, ut in *Fig.* 39. donec aucta eouſque diſtantia focorum, ut hæc ſit ad axem majorem ſicut latus quadrati ad ejuſdem diametrum, axis minor fiat nullus, & curva pertingat ad centrum hinc inde.

Fig. 39.

Si diſtantia focorum fuerit major quam pro dicta ratione, axis minor fit impoſſibilis, & figura in duas conjugatas abit, ut in *Fig.* 40. quæ aucta focorum diſtantia minuentur, donec tandem figura in bina puncta conjugata abeat.

Fig. 40.

Creſcente porro diſtantia focorum, rurſus emergunt binæ figuræ conjugatæ, quæ ſimiliter creſcunt atque prius decreverant a prioribus diverſæ in focorum verticumque ordine, augenturque donec infinitæ evadant. Poſteaque Syſtema hoc iiſdem gradibus ad circulum rurſus accedet quibus ab illo receſſit.

Ex his vel primo intuitu ſatis patet Figuram hanc ad conſtituendam Planetæ Orbitam minime idoneam eſſe. Ut enim taceam caſus ubi in duas conjugatas abit, Orbitæque naturam deponit, nimirum ubicunque tanta ſit ejus excentricitas, quantam Cometæ (ſi circa Solem Planetarum inſtar verſantur, quod veriſimillimum eſt) ad curſum ſuum deſcribendum poſtulant: ut hos inquam caſus præteream: ſunt etiam in iis caſibus, ubi in ſe redit orbitamque perficit, quædam ejus excentricitates ita amplæ, ut curva prope D & E (*Fig.* 39.) verſus Solem convexa evadat; adeoque vi a Sole centrifuga Planetæ opus eſſet, ut hanc Orbitæ ſuæ partem percurreret, dum interim in locis propioribus & remotioribus *B* & *A* vis ad Solem centripeta requiritur. Id eſt corpora circumſolaria ea lege moveri poſſe concedendum eſſet, ut in paribus a Sole intervallis hic vis centripeta, illic centrifuga obtineret, quod quam ſit a naturæ legibus alienum facile omnes perſpicient. Et licet nullius e Planetis tanta ſit excentricitas; cum tamen Geometris notum ſit, figuræ, cujus ſpecies omnes ultra certum terminum muneri cuivis naturæ obeundo ineptæ fuerint, ejus ſpecies reliquas citra dictum terminum quaſi eidem muneri idoneas, admitti non poſſe: Neceſſe eſt *Curvam hanc Caſſinianam ex Aſtronomia rejicere*, non ſolum ob rationes *Prop.* VIII. *Lib.* III. *Elem. Aſtr.* adductas, nempe *quod neque obſervatis cœleſtibus congruat propter minoris axis brevitatem, neque rationes Phyſicæ reſpondeant, cum ad illam deſcribendam opus eſſet vi centripeta ad Solem abhorrente ab illa per rerum naturam uſurpata*, ſed etiam propter abſolutam impoſſibilitatem. Impoſſibile namque eſt hujus figuræ ſpeciem quamcunque poſſe a Planeta percurri, ita ut anguli ad focum a Sole diverſum proportionales ſint temporibus; ſic enim area per radium vectorem deſcripta non eſſet tempori proportionalis. Non enim aucto angulo ad focum unum æqualibus incrementis, areæ ad alterum incrementa ſimul facta etiam æqualia ſunt, uti perperam nuper ſentiebam.

In duobus ultimis ſchematibus maxima figuræ latitudo invenitur, ſi centro *C* per focos deſcribatur circulus; ſecabit namque hic Curvam in punctis *L*, *L* quæſitis. Eſtque maxima ordinata *K L* tertia proportionalis rectis *G F* & *F D* in horum primo, vel quarta proportionalis ipſis *G F*, *G A*, & *A F* in utroque.

Superſtite *D E*, ordinata ex foco *F P* æqualis eſt ſemi-axi minori *C D*, quando axis minor eſt ad diſtantiam focorum ut latus quadrati ad diametrum. Si diſtantia focorum fuerit major quam pro hac ratione, *F P* excedet *C D*.

Solution of Kepler's *Problem, by Dr.* John Keill, *n.* 337. *p.* 1.

X. Ob æquabilem Arearum deſcriptionem (a *Keplero* demonſtratam,) quo Planetarum loca in propriis Orbitis ad datum tempus determinentur, neceſſe eſt ut ſolvatur Problema quod ſequitur.

Invenire Poſitionem rectæ, quæ per datæ Ellipſeos focum alterutrum tranſiens, abſcindat Aream motu ſuo deſcriptam, quæ ſit ad Aream totius Ellipſeos in ratione data.

Fig. 41.

Sit nempe Ellipſis *A P B*, cujus focus alteruter *S*. Invenienda eſt poſitio rectæ *S P*, quæ abſcindat Aream trilineam *A S P*, ad quam Area totius Ellipſeos eandem rationem habet, quam tempus Periodicum Planetæ Ellipſim deſcribentis, ad aliud tempus datum; qua inventa dabitur punctum *P* ubi planeta ad tempus illud datum verſatur. Vel ſit *A Q B* ſemicirculus ſupra Ellipſeos axem majorem deſcriptus, ducenda eſt per *S* recta *S Q* abſcindens Aream *A S Q*, ad quam Area totius circuli eſt in eadem ratione: ſi enim ex *Q* demittatur in Axem perpendicularis *Q H*, Ellipſi occurrens in *P*, ducta *S P* dabit Aream Ellipticam quæſitam; & punctum *P* erit locus Planetæ ad datum tempus. Eſt enim ſemiſegmentum Ellipticum APH ad ſemiſegmentum circulare AQH, ut HP ad HQ, hoc eſt, ut area totius Ellipſeos ad aream totius Circuli; ſed eſt triangulum SPH ad triangulum SQH in eadem ratione PH ad QH: Adeoque Area ASP eſt ad Aream totius Ellipſeos, ut area ASQ ad aream totius Circuli. Unde ſi habeatur methodus ſecandi in data ratione Aream circuli, recta ducta per datum punctum S, facile erit hac ipſa ratione ſecare Aream Ellipticam.

Ipſi *Keplero*, qui primus Problema propoſuit, nulla innotuit Methodus directa computandi locum Planetæ ex dato tempore; ſed illi neceſſe fuit, per ſingulos gradus ſemicirculi AQB progrediendo, ex dato arcu AQ, quam vocat Anomaliam Excentri, tam tempus per aream ASQ, quæ Anomaliæ mediæ eſt proportionalis, quam angulum ASP, hoc eſt locum Planetæ, ſeu Anomaliam coæquatam huic tempori reſpondentem, calculo eruere.

Cum itaque difficilis fuit hujus Problematis ſolutio, Aſtronomi ad alias tranſiverunt Hypotheſes, fingendo punctum aliquod circa quod motus foret æquabilis, ſeu tempori proportionalis, & exinde data Anomalia media, coæquatam determinabant. Sed computus hiſce hypotheſibus innixus, obſervationibus non congruere deprehenſus eſt: Itaque Geometræ varias adhibuerunt approximationes, quibus ex data Area ASQ tempori Analoga, angulus ASP, hoc eſt Planetæ locus, quam proxime eliciatur. At horum omnium facillima, & ad Praxim maxime expedita, mihi videtur eſſe illa methodus quam tradit Dominus *Newtonus* in *Principiis*, *pag.* 111 & 112. *Edit.* 1*ma.* quæ fere ſimilis eſt ei, qua ex æquationibus affectis extrahunt Radicem Analyſtæ; & quidem tanto magis eſt æſtimanda, quod non ſolum exhibeat Planetarum loca, quorum orbitæ ad Circuli formam proxime accedunt, ſed eadem fere facilitate inſervit etiam Cometis, qui in orbitis maxime excentricis moveantur. Hanc itaque methodum in gratiam Artificum, qui Tabulas Aſtronomicas ſecundum veras motuum leges, & non ex fictis hypotheſibus condere volunt, hic exponendam duxi.

Sit itaque AQB ſemicirculus ſupra Axem majorem Ellipſeos deſcriptus, cujus centrum C, & focus in quo Sol locatur ſit S. Ducatur CQ, in quam (ſi opus ſit) productam cadat perpendicularis SF. Eſt Area $ASQ =$ ſectori $ACQ +$ Triang. $CSQ = \frac{1}{2} CQ \times AQ + \frac{1}{2} CQ \times SF$; adeoque ob datam $\frac{1}{2} CQ$ erit Area ASQ ſemper proportionalis arcui $AQ +$ recta SF, cum ſcil. motus ſit ab Aphelio verſus Perihelium: at cum a Perihelio ad Aphelion tendit Planeta, ut in Fig. 43. ſit Area $BSQ =$ ſectori $BCQ -$ Triang. CSQ, adeoque erit illa proportionalis arcui $BQ -$ recta SF. Hinc ſi capiatur Arcus AN in Fig. 42: & 44. & BN in Fig. 43. temporibus proportionalis, erit $AQ + SF = AN$, & $BQ - SF = BN$: unde SF erit $= QN$, modo arcus AN vel BN ſint proportionales temporibus quibus deſcribuntur areæ ASQ vel BSQ. Ut vero inveniatur in gradibus eorumque partibus menſura arcus in peripheria AQB, qui ſit æqualis rectæ SF, Fiat ut CQ ad CS, ita arcus graduum 57,29578 (qui æqualis eſt CQ radio) ad arcum quartum, qui æqualis

Fig. 42, 43, 44.

lis erit CS. Sit arcus ille B. Eſt autem CS ad SF ut Radius ad ſinum anguli SCF vel ACQ. Fiat itaque ut Radius ad ſinum anguli ACQ vel arcus AQ, ita arcus B ad alium D; erit arcus ille D æqualis rectæ SF, adeoque ſi, ad datum tempus, Area ASQ eſſet tempori proportionalis, eſſet Arcus $D = NQ$: & capiendo arcum $NP = D$, punctum P caderet in Q. Si vero Area ASQ non exacte tempori reſpondeat, punctum P cadet ſupra vel infra Q, prout Area ASQ major ſit vel minor vera Area quæ tempori reſpondeat. Sit ea ASq & in Cq cadat perpendicularis SH: erit per hactenus demonſtrata $SH = Nq$. At eſt $SF = NP$, unde erit $SH - SF$ vel $SF - SH$, hoc eſt fere $HE = qP \;\; QP - Qq$ vel $= Qq - QP$: Et ſi angulus QCq ſit parvus, erit $CH : CQ :: HE : Qq :: QP - Qq : Qq$, unde $CQ + CH : CQ :: QP : Qq$, cum arcus AQ eſt quadrante minor. At cum is eſt quadrante major, erit $CQ - CH : CQ :: QP : Qq$. Et ſimiliter cum arcus BQ eſt quadrante minor, erit $CQ - CH : CQ :: QP : Qq$.

Si angulus ACQ vel BCQ parvus ſit, *h. e.* ſi Planeta prope Apſides verſetur, erit ut $CA \pm CS : CA :: QP : Qq$.

Fiat ut CS ad CQ ita Radius R ad longitudinem quandam L, erit $CQ = \frac{CS \times L}{R}$. Eſt vero Radius ad coſinum anguli ACQ ut SC ad CF vel CH (ſunt enim CH & CF fere æquales) quare erit $CH = \frac{SC \times \mathit{coſ}.\, ACQ}{R}$, adeoque $QP : Qq :: \frac{CS \times L + CS \times \mathit{coſ}.\, ACQ}{R} : \frac{CS \times L}{R} :: L + \mathit{coſ}.\, ACQ : L$, cum arcus AQ ſit quadrante minor. At ſi AQ ſit quadrante major, erit $QP : Qq :: L - \mathit{coſin}.\, ACQ : L$.

Atque hac ratione ſi capiatur utcunque arcus AQ, qui aliquantiſper minor ſit aut major vero, invenietur exinde arcus Qq huic addendus aut demendus, qui facit ut Area ASq ſit quam proxime tempori proportionalis. Et ſi loco AQ capiatur arcus Aq, & inſtituatur proceſſus priori ſimilis, invenietur alius Aq, qui ſimiliter eundem repetendo proceſſum dabit alium Aq, atque ſic quantumvis proxime ad veritatem accedere licebit.

Invento angulo ACq, facile habebitur angulus ASq, cum in triang. qCS dentur latera Cq & CS & angulus qCS. Dabitur exinde angulus CSq cujus tangens diminuendus eſt in ratione axis minoris Ellipſeos ad majorem, ut tandem habeatur tangens anguli ASP. Vel ſic forte facilius inveſtigatur angulus ASP. Sit F numerus qui exprimit longitudinem CS in partibus qualium CQ eſt 100000: a puncto q ad axem demittatur perpendicularis qr, qui erit ſinus arcus dati Aq, & erit Cr ejuſdem coſinus & $Sr =$ ſummæ vel differentiæ rectarum Cr, CS, hoc eſt $Sr = F \pm$ coſin. ACQ: adeoque in rectangulo triangulo rSq, datis Sr, rq, invenietur angulus rSq. Hinc ſi in unam ſummam addantur ſinus Log. ang. ACq, complementum Arithmeticum Logarithmi Sr, & Logarithmus rationis axis minoris Ellipſeos ad majorem dabitur Tangens anguli ASP.

Tanta autem eſt hujus methodi facilitas ut ea exemplis magis quam ulteriori explicatione indigeat; adeoque licebit eam in motibus Planetæ Martis experiri; in cujus orbita, ſecundum Tabulas *Carolinas*, Excentricitas eſt ad diſtantiam mediam ut 14100 ad 152369, adeoque Logarithmus arcus B, qui æqualis eſt rectæ SC, erit 0,7244451. Erit etiam in hoc exemplo L partium 1080631 qualium Radius eſt 100000: Inveniendus ſit angulus ACQ cum motus medius, ſeu arcus tempori proportionalis ab Aphelio computatus, ſit unius Gradus. Quoniam CS ſit hic fere pars decima ipſius CA, pono Arcum AQ eſſe 0,9 grad, decima ſcil. parte minorem motu medio. Addatur ſinus Logarithmicus arcus AQ ad Log. B, & fit ſumma $8{,}9205471 =$ Log. numeri 0.083281 qui numerus exprimit arcum æqualem rectæ $SF = NP$. Et ſi arcus AQ eſſet recte aſſumptus, foret $AN - NP = AQ$, & $QP = 0$. At hic eſt $QP = 0{,}016719$, a quo ſi auferatur ejus pars undecima, cum AS ſuperat AC undecima circiter ipſius parte, reſtabit $Qq = 0{,}0152$; qui additus ad AQ dat $Aq = 0.9152$, qui ne milleſima gradus parte a vero Aq differt. Sit ſecundo Arcus AN ſeu motus medius = 2 gr. Pono $AQ = 1{,}83$ prioris Aq fere duplum, & ad ejus ſinum Log. addatur Log. B. erit ſumma $9.2286997 =$ Log. numeri 0,16931, unde erit QP 0,00069; a quo ſi ſubducatur ejus pars undecima, fit $Qq = 0{,}00063$, & $Aq = 1{,}83063$, qui ne decies milleſima gradus parte a vero Aq diſcrepat. Eodem modo ſit motus medius ſeu arcus tempori proportionalis grad. 3. Fiat arcus AQ $2{,}745 = 1{,}83 + 0{,}915$, & ad ejus ſinum Log. addendo Log. B, habebitur Log. numeri $0{,}25392 = NP$, & $AN - NP = 2{,}74608$, adeoque $QP = 0{,}00108$, unde Qq fere $= 0{,}001$ & $Aq = 2{,}746$. Sic unica duorum Logarithmorum additione invenietur arcus Aq, qui erit verus ad gradus partes milleſimas.

Si jam non gradatim ſed per ſaltum pergendo, inveniendus ſit angulus ACq, cum motus medius eſt grad. 45. Pono arcum AQ eſſe graduum 40, & ad ſinum ejus Logarithmicum addendo Log. B fit ſumma $0.5325125 =$ Log. numeri 3,4081; qui numerus a 45 ſubductus relinquit $AN - NP = 41{,}5919$, cujus exceſſus ſupra arcum AQ eſt 1,5919. Unde ſi fiat ut $L +$ coſ. ACQ ad L ita 1,5919 ad alium, invenietur arcus Qq eſſe graduum 1,4865, adeoque $Aq = 41{,}4865$, qui non multum ſupra milleſimam gradus partem a vero differt. Verum abſque hac proportione inveniri poteſt Aq, capiendo novum arcum AQ, qui ſit aliquantulum minor quam $AN - NP$, eidem tamen ſere æqualis; ſcil. ſit $AQ = 41.50$, & addendo Log. datum B ad ejus ſinum Log. habebitur alter $NP = 3.5131$, qui ab AN ſubductus dat 41,4869 pro novo Aq: & hic arcus minore labore eruitur, & aliquanto propius ad verum accedit quam prior Aq.

Poſt inventum Aq correſpondentem motui medio 45°, rurſus gradatim pergendo, unica duorum Logarithmorum additione, habebitur Aq, ad omnes motus medii gradus ſubſequentes. Nempe cum motus medius ſit grad. 46, pono $AQ = 42{,}40$. & addendo ejus ſinum Log. ad conſtantem B, fiet $AN - NP = 42{,}4249$; cui arcui ſi novus AQ æqualis ponatur, habebitur Aq, qui ne milleſima gradus parte a vero Aq diſcrepabit. Sic cum motus medius ſit 47°, pono $AQ = 43{,}36 =$

priori $Aq +$ incremento iſtius arcus pro uno gradu motus medii, & addendo ejus ſinum Log. ad Log. B, fit ſumma$=$Log. numeri 3,6402, qui ab AN ſubductus relinquit $AN - NP = 43{,}3598 =$ novo Aq, qui circiter gradus parte decies milleſima a vero Aq diſcrepat.

Si omiſſis gradibus intermediis inveniendus eſſet arcus Aq, cum motus medius ſit gr. 100. Pono AQ grad. 96, & addendo ejus ſinum Log. ad Log. B, fit ſumma$=$Log. numeri 5,273 unde $AN - NP = 94{,}727$. Itaque pono ſecundo $AQ = 94{,}72$ & addendo ejus ſinum ad Log. B, habebitur Log. numeri 5,285, qui ab AN ſubductus dat $AN - NP = 94{,}715 = Aq$ quam proxime. Similiter ſi motus medius ſit grad. 101, pono AQ eſſe 95,71, cujus ſinus Log. ad Log. B additus dat Log. numeri 5,2756; quo numero ab 101 ſublato, reſtabit $AN - NP = 95{,}7244 = Aq$. Atque hac ratione, dato motu medio, ſi gradatim fiat proceſſus, habebitur angulus ad centrum per unicam tantum duorum Logarithmorum additionem; quorum unus, qui conſtans eſt, in charta ſeorſim ſervandus, quo labori ſæpius eundem exſcribendi parcatur.

Tranſeamus jam ad Orbitam alterius ſpeciei, talem nempe ut diſtantia Aphelii ſit ad diſtantiam Perihelii ut 70 ad 1; qualis fere eſt iſtius Cometæ Orbita quem Periodum ſuam annis $75\frac{1}{2}$ complere primus deprehendit Sagaciſſimus Aſtronomus & Geometra D. *Edmundus Halleius*, Geometriæ Profeſſor *Savilianus*. In hac Orbita erit AC vel CQ partium 35,5, & CS 34,5 qualium SB eſt una. Et inveniendus eſt arcus Bq, cum motus medius eſt gradus pars centeſima. Quoniam media diſtantia trigeſies & quinquies circiter ſuperat diſtantiam minimam, pono $BQ = 0{,}35$, cum motus medius eſt 0,01. In hac Orbita invenitur conſtans Log. $B = 1.7457133$. Hic itaque Log. ad ſinum Log. arcus 0,35 additus dat Log. numeri 0,34013, qui ad arcum 0,01 additus erit$=$0,35013. Si hæe ſumma eſſet æqualis 0,35, arcus BQ eſſet recte aſſumptus: ſed differentia eſt 0,00013. Unde quoniam CB eſt ad SB ut 35,5 ad 1, multiplicetur differentia 0,00013 per 35,5 & prodibit $Qq = 0{,}004615$; unde erit arcus $Bq = 0{,}354615$, qui vix per partes tres decies milleſimas a vero diſcrepat.

Sit ſecundo motus medius 0,02, & ponatur BQ eſſe 0,71. Ad ejus ſinum Log. addendo Log. B, fit ſumma$=$Log. numeri 0,68998, unde $BN + NP = 0{,}70998$, adeoque arcus aſſumptus $BQ = 0{,}71$ nimius fuit: & eſt differentia$=$0,00002, quæ ſi per 35,5 multiplicetur, & productus a BQ ſubducatur, reſtabit $Bq = 0{,}7092$, vix gradus parte decies milleſima a vero aberrans.

Sit motus medius 0,03. Ponatur BQ eſſe 1°,06: addendo ejus Log. ſin. ad Log. B, fit ſumma$=$Log. numeri 1,03008, cui ſi addatur $BN = 0{,}03$, fit ſumma 1,06008, qui numerus major eſt quam BQ; quare ſi differentia 0,00008 per 35,5 multiplicetur & ad BQ addatur, erit $Bq = 1{,}06284$. Similiter cum motus medius ſit 0,04, pono $BQ = 1°{,}40$ & invenio $NP = 1{,}3604$; ad quem numerum addendo $BN = 0{,}04$ fit ſumma$=$1,4004 qui ſuperat 1,40 per 0,0004. Multiplicetur hæc differentia per 35,5 & productus 0,01420 erit æqualis Qq, unde $Bq = 1{,}41420$. In hiſce omnibus errores ſunt admodum exigui, & raro milleſimam gradus partem tranſcurrentes.

Inveniendus ſit, jam arcus Bq, cum motus medius ſit æqualis uni gradui. Pono $B\mathcal{Q} = 20°$, & addendo ejus ſinum Log. ad Log. B, habebitur Log. numeri 19,045; cui addendo $BN = 1°$, ſumma 20,045 ſuperat 20 per ,045: Et cum in hoc caſu L—coſin. $B\mathcal{Q}$ eſt ad L ut 1 ad 11.5 fere, multiplico differentiam ,045 per 11,5, & productus ,5175 ad $B\mathcal{Q}$ additus facit 20,5175. Pono igitur ſecundo $B\mathcal{Q} = 20{,}51$, & prodibit, ſimiliter ut in præcedentibus, $NP = 19.5092$; cui addendo BN fit ſumma 20,5092, quæ minor eſt quam $B\mathcal{Q}$: unde ſi differentia 0,0008 multiplicetur per 11,5, & productus 0,0092 ſubſtrahatur a $B\mathcal{Q}$, reſtabit $Bq = 20{,}5008$.

Sit denique motus medius æqualis duobus grad. Pono $B\mathcal{Q}$ grad. 30, & invenitur $NP = 27{,}84$, cui addendo gradus duos, ſumma 29.84 minor eſt quam 30; & ſi multiplicetur differentia 0,16 per 6,3 (nam L—coſ. $B\mathcal{Q}$ eſt ad L ut 1 ad 6,3 fere) fiet $1{,}008 = \mathcal{Q}q$; adeoque hic arcus a $B\mathcal{Q}$ ſubductus dat $Bq = 28{,}982$: Ut vero corrigatur Bq, aſſumo ſecundo $B\mathcal{Q} = 29°$, & ſimili proceſſu invenietur $Bq = 28{,}9672$.

A new way to find the Sun's Parallax, by Dr. Halley, *n.* 348. *p.* 454.

XI. Plurima ſunt maxime quidem paradoxa, omnemque fidem apud vulgus ſuperantia, quæ tamen adhibitis Mathematicarum Scientiarum principiis levi negotio enodantur. Ac ſane nullum Problema magis arduum & difficile videbitur; quam eſt Solis a Terra diſtantiam vero proximam determinare: quod tamen obtentis accuratis quibuſdam obſervationibus, ad electa & præviſa tempora peractis, non multo opere efficietur. Id quod inclytæ huic Societati, quam immortalem fore auguror, in hac diſſertatione ob oculos ponere libet, ut junioribus noſtris Aſtronomis, quibus forſan hæc obſervare ob minorem ætatem obtingere poteſt, viam præmonſtrem, qua immenſam Solis diſtantiam intra quingenteſimam ſui partem rite dimetiri poterint.

Notum eſt Solis a Terra diſtantiam a diverſis Aſtronomiæ authoribus diverſam fingi, prout cuique ex conjectura probabile viſum eſt; a *Ptolemæo* quidem ejuſque aſſeclis, uti & *Copernico* & *Tychone Brahæo*, *Terræ* ſemidiametris mille & ducentis, *Keplero* ter mille quingentis fere. *Ricciolus* diſtantiam *Keplerianam* duplicat, quam tamen *Hevelius* dimidio tantum auget. At vero viſis in *Solis* diſco ope Teleſcopii Planetis *Venere* & *Mercurio* mutuato fulgore nudatis, tandem compertum eſt Planetarum diametros viſibiles multo minores eſſe quam eatenus haberentur; *Veneriſque* Semidiametrum e *Sole* viſam, non niſi quartam minuti primi partem vel quindecim ſecunda ſubtendere; *Mercuriique* ſemidiametrum, ad mediam ipſius a *Sole* diſtantiam, ſub angulo decem tantum ſecundorum conſpici; atque ſub eodem etiam *Saturni* ſemidiametrum e *Sole* videri. *Jovis* autem Planetarum maximi ſemidiametrum non niſi tertiam minuti primi partem apud *Solem* ſubtendere. Unde, ſervata analogia, nonnullis e modernis Aſtronomis viſum eſt, *Terræ* quoque ſemidiametrum e *Sole* conſpectam, medio loco inter *Jovis* majorem & *Saturni* & *Mercurii* minorem angulum ſubtendere, *Veneriſque* æqualem, nempe quindecim ſecundorum: adeoque *Solem* a *Terra* quatuordecim fere millibus ſemidiametrorum *Terræ* diſtare. Iiſdem autem Authoribus, aliud argumentum paulo ampliavit hanc diſtantiam: quoniam enim *Lunæ* diameter paulo major eſt quarta parte diametri *Terræ*, ſi Parallaxis *Solis* ponatur quindecim minutorum ſecundorum, fieret *Lunæ* corpus corpore *Mercurii* majus, Planeta ſcil. ſecundarius primario major; quod concinnitati Syſtematis mundani contrariari videretur. E contra vero *Venerem* inferiorem & Satellitio deſtitutam, majorem eſſe *Terra* noſtra ſuperiori & tam inſignem comitem nacta, vix concedere videtur eadem concinnitas. Ut itaque medio loco incedamus, ponatur *Terræ* ſemidiameter e *Sole* viſa, ſeu quod idem eſt, *Solis* Parallaxis horizontalis, duodecim ſecundorum cum ſemiſſe: unde *Luna* minor erit *Mercurio* & *Terra Venere* major; ac proveniet *Solis* a *Terra* diſtantia ſedecies mille cum quingentis *Terræ* ſemidiametris proxime. Huic autem diſtantiæ in præſentiarum aſſenſum præbeo, uſq; dum Experimento quod proponimus quanta ſit certius conſtet. Nec moror authoritatem quantumvis gravem eorum qui *Solem* ultra hos terminos in immenſum evehunt, freti obſervationibus vibrantis Penduli, determinandis his angulorum minutiis, uti videtur, haud ſatis fidis: ſaltem hac methodo tentanti Parallaxis aliquando nulla, aliquando etiam negativa occurret; hoc eſt diſtantia vel infinita fiet, vel infinito major: quod abſurdum. Et, ut verum fatear, minuta ſecunda vel etiam dena ſecunda inſtrumentis quantumvis affabre factis certo diſtinguere vix homini datum eſt; atque adeo minime mirandum, ſi tantorum Artificum multos & ingenioſos conatus hactenus eluſerit rei ipſius maxima ſubtilitas.

Dum autem ante 40 fere annos, in Inſula *Sanctæ Helenæ*, ſyderum polum Auſtralem ambientium obſervationibus operam darem; contigit mihi *Mercurium* ſub *Solis* diſco tranſeuntem omni adhibita diligentia obſervare: quodque mihi præter ſpem feliciter ſucceſſit, momentum quo *Mercurius* ingrediens *Solis* limbum interius contingere viſus eſt, pariterque momentum quo egrediens limbum Solis ſtrinxit, facto angulo contactus interioris, Tubo optimo viginti quatuor pedum accuratiſſime obtinui. Unde pro comperto habui intervallum quo *Mercurius* totus intra *Solis* diſcum tum temporis apparuit, etiam abſque errore unius minuti ſecundi temporis: Nam filum luminis Solaris, inter limbum planetæ obſcurum & *Solis* lucidum interceptum, quantumvis tenue in oculos incurrere viſum eſt; & in ictu oculi, denticulus in limbo *Solis* a *Mercurio* ingrediente factus evaneſcere, uti ab egrediente factus quaſi momento incipere. Hoc autem perſpecto ſtatim intellexi *Solis* Parallaxin ex hujuſmodi obſervationibus rite concludi poſſe, ſi modo *Mercurius Terris* vicinior majorem haberet parallaxin a *Sole*; etenim hæc parallaxium differentia tantilla eſt, ut ſemper minor ſit ipſa Solari quam quærimus; proinde *Mercurius*, licet frequenter intra *Solem* videndus, huic noſtro negotio vix ſatis aptus habebitur.

Reſtat itaque *Veneris* tranſitus per *Solis* diſcum, cujus parallaxis quadruplo fere major Solari, maxime ſenſibiles efficiet differentias, inter ſpatia temporis quibus *Venus Solem* perambulare videbitur, in diverſis *Terræ* noſtræ regionibus. Ex his autem differentiis debito modo obſervatis, dico determinari poſſe *Solis* parallaxin etiam intra ſcrupuli ſecundi exiguam partem. Neque alia inſtrumenta poſtulamus præter *Teleſcopia*

leſcopia & *Horologia* vulgaria ſed bona: & in Obſervatoribus non niſi fides & diligentia, cum modica rerum Aſtronomicarum peritia deſiderantur. Non enim opus eſt ut Latitudo Loci ſcrupuloſe inquiratur, nec ut Horæ ipſæ reſpectu meridiani accurate determinentur: ſufficit, Horologiis ad Cæli revolutiones probe correctis, ſi numerentur tempora a totali Ingreſſu *Veneris* infra diſcum *Solis*, ad principium Egreſſus ex eodem; cum ſcilicet primum incipiat Globus *Veneris* opacus limbum *Solis* lucidum attingere; quæ quidem momenta, propria experientia novi, ad ipſum ſecundum temporis minutum obſervari poſſe.

Ob leges autem motuum admodum arctas, rariſſime intra *Solis* orbem conſpicitur *Venus*, ac per plus quam centum & viginti annorum decurſum, ne ſemel quidem ibidem videbitur; nempe ab anno 1639. (cum præclaro Juveni *Horroxio* noſtro, eique primo & ſoli a rerum conditu, jucundiſſimum hoc ſpectaculum obtigit,) uſque in annum 1761, quo juxta Theorias quas hactenus cœlo conformes experimur, Stella *Veneris* iterum ſubtercurret *Solem*, *Maii* 26. mane; * ita ut *Londini*, hora fere ſexta matutina in medio diſci Solaris expectanda ſit, nec niſi quatuor minutis centro *Solis* Auſtralior. Duratio autem hujus tranſitus erit octo fere Horarum, nempe a ſecunda uſque in decimam fere matutinam. Atque adeo ingreſſus minime *Anglis* conſpicuus erit: cum autem Sol tum temporis occupaturus ſit 16. *Geminorum* gradum, viginti tres ferme gradus in Boream declinans; per totam quaſi Zonam frigidam Septentrionalem inocciduus conſpicietur: ac proinde qui littus *Norwegiæ* incolunt ultra Urbem *Nidroſiam*, quam *Drontem* vocant, uſque ad Promontorium ejus *Boreale*, *Venerem Solis* diſcum ſubingredientem obſervare poterunt; ac fortaſſe *Scotis* Borealioribus & Inſulæ *Hetlandiæ*, olim *Thylen* dictæ, incolis, in oriente Sole ingreſſus ille conſpici poterit. Quo tempore vero *Venus Solis* centro proxima erit, *Sol* verticalis erit ſupra littora Borealia ſinus *Gangetici*, vel potius regni *Peguani*; ac proinde in Regionibus circumvicinis, cum *Sol* in ingreſſu *Veneris* quatuor fere horis diſtabit ad ortum, & in egreſſu totidem fere ad occaſum, accelerabitur motus apparens *Veneris* intra *Solem* duplo fere parallaxeos horizontalis *Veneris* a *Sole*; quia *Venus* tunc ab ortu in occaſum fertur retrograde, interea dum oculus ad *Terræ* ſuperficiem poſitus in contrarias partes ab occaſu in ortum gyratur.

Poſita autem parallaxi *Solis*, uti diximus, duodecim ſecundorum cum ſemiſſe, erit parallaxis *Veneris* 43^{um} ſecundorum; & ſublata parallaxi *Solis*, reſtabit ſaltem ſemiminutum pro parallaxi Horizontali *Veneris* a *Sole*, ac proinde dodrante ſaltem minuti promovebitur *Veneris* motus a parallaxi illa, interea dum *Solis* diſcum percurrit, in iis ſcilicet Poli altitudinibus quæ Tropico vicinæ ſunt; atque adhuc amplius in vicinia Æquatoris. *Venus* autem tum temporis ſatis accurate quatuor minuta prima ſingulis horis intra *Solem* conficiet; ac propterea dodranti minuti undecim ſaltem temporis minuta prima competunt, quibus duratio *Eclipſeos* hujus *Venereæ* ob parallaxin contrahetur. Atque ex hac contractione ſola liceret de parallaxi quam quærimus tuto pronunciare, ſi modo

* *Vide Phil. Tranſact.* No. 193. *Abr.* Vol. I. *p.* 436.

modo darentur *Solis* diameter *Veneriſque* Latitudo in minimis accuratæ; quas tamen ad computum poſtulare, in re tam ſubtili, haud integrum eſt.

Procuranda eſt igitur alia obſervatio, ſi fieri poſſit, in locis illis ubi medium *Solis* occupat *Venus* in ipſo Medinoctio; nempe ſub Meridiano priori oppoſito, *i. e.* ſex quaſi horis vel 90 gradibus *Londino* occidentaliore, & ubi *Venus* paulo ante occaſum *Solem* ſubintrat, paulo poſt ortum exit; id quod fiet in dicto Meridiano, ſub altitudine Poli Borei quinquaginta ſex circiter graduum: hoc eſt, in eo Sinu qui *Hudſoni* dicitur, ad Portum ejus cui nomen *Nelſoni* inditum. In locis enim huic circumvicinis parallaxis *Veneris* durationem tranſitus protrahet, & ſex ſaltem temporis minutis longiorem efficiet; quia dum *Sol* ab occaſu in ortum ſub Polo tendere videtur, ea loca in diſco *Terræ* motu contrario in occaſum ferri videbuntur, hoc eſt motu cum motu proprio *Veneris* conſpirante; proinde tardius moveri videbitur *Venus* intra *Solem*, ac cum diuturniore mora diſcum ejus pertranſire.

Si itaque in utroque loco hic tranſitus ab Artificibus idoneis contigerit debite obſervari, manifeſtum eſt totis ſeptendecim minutis longiorem futuram eſſe moram in portu *Nelſoni* obſervabile, quam quæ apud *Indos* orientales expectanda eſt: nec multum refert an ad Fortalitium *Sancti Georgii* vulgo *Maderas* dictum, vel ad *Bencoulam* in litore occiduo Inſulæ *Sumatræ* prope æquatorem capiatur obſervatio, ſi *Anglis* tum temporis hæc ſtudia curæ fuerint. Si vero *Gallis* his rebus invigilare placuerit, non incommode apud *Poudechery* ſe ſiſtet Obſervator in litore *Sinus Gangetici* occidentali, ſub altitudine Poli duodecim fere graduum. *Batavis* autem celeberrimum *Bataviæ* ſuæ Emporium Obſervatorium huic negotio ſatis aptum miniſtrat, ſi modo illis etiam animus fuerit hac in parte cœlorum ſcientiam promovere. Ac ſane vellem diverſis in locis ejuſdem Phænomeni obſervationes a pluribus inſtitui, tum ad majorem adſtruendam ex conſenſu fidem, tum ne Nubium interventu fruſtraretur ſingularis Spectator, eo ſpectaculo quod neſcio an denuo viſuri ſunt hujus & ſubſequentis ſeculi Mortales; & a quo pendet Problematis nobiliſſimi & aliunde inacceſſi ſolutio certa & adæquata. Curioſis igitur ſyderum ſcrutatoribus, quibus, nobis vita functis, hæc obſervanda reſervantur, iterum iterumque commendamus ut, moniti hujus noſtri memores, obſervationi peragendæ ſtrenue totiſque viribus incumbant; iiſque fauſta omnia exoptamus & vovemus, præprimis ne nubili cœli importuna obſcuritate exoptatiſſimo ſpectaculo priventur; utque tandem Orbium cœleſtium magnitudines intrà arctiores limites coercitæ in eorum gloriam famamque ſempiternam cedant.

Diximus autem hac ratione Solis Parallaxin intra quingenteſimam ſui partem inveſtigari poſſe, id quod nonnullis mirum ſine dubio videbitur. Veruntamen ſi in utroque e locis nuper deſignatis accurata habeatur obſervatio; jam monſtravimus, totis ſeptendecim minutis differre inter ſe durationes *Eclipſeωn* harum *Venerearum*, ex Hypotheſi ſcilicet quod Solis parallaxis fuerit duodecim cum dimidio minutorum ſecundorum. Quod ſi major vel minor reperiatur ex obſervatione hæc differentia,

tia, in eadem fere ratione major vel minor erit Solis parallaxis. Cumque 17 minuta prima temporis competant duodecim ſecundis cum dimidio parallaxeos Solaris; pro unoquoque parallaxeos minuto ſecundo, orietur differentia pluſquam 80 ſecundorum minutorum temporis; adeoque ſi habeatur differentia hæc intra bina ſecunda vera & comprobata, intra quadrageſimam partem unius ſecundi minuti conſtabit quanta ſit Solis Parallaxis; ac proinde diſtantia ejus determinabitur intra quingenteſimam ſui partem, ſaltem ſi parallaxis non minor reperiatur ea quam ſuppoſuimus: quadragies enim duodecim cum dimidio fiunt quingenti.

Hactenus Aſtronomice doctis ſatis ſuperque rem indicaſſe mihi videor, quos etiam monitos velim, me in hoc argumento, Latitudinis Planetæ rationem non habuiſſe, tum ad vitandas calculi intricatioris moleſtias, concluſionem etiam minus evidentem reddituras; tum ob motum Nodorum *Veneris* nondum compertum, nec niſi ex hujuſmodi corporalibus Planetæ cum *Sole* Conjunctionibus rite determinandum. Non enim concluſum eſt *Venerem* quatuor minuta infra *Solis* centrum tranſituram, niſi ex Hypotheſi quod Planum Orbitæ *Veneris*, in Sphæra ſtellarum fixarum immobile, Nodos ſuos iiſdem in locis habiturum ſit, ubi anno 1639 inventi ſunt. Quod ſi tramite Auſtraliori tranſeat anno 1761, liquido patebit Nodos regredi; ſi vero Borealiori, progredi inter Fixas; idque in ratione 5 ½ *min.* in centum annis *Julianis*, pro unoquoque minuto, quo via *Veneris* tum temporis plus vel minus diſtabit a *Solis* centro quam dictis quatuor minutis. Differentia autem inter durationes harum Eclipſium paulo minor fiet ſeptendecim minutis, ob Latitudinem *Veneris* Auſtralem; major vero futura, ſi procedentibus Nodis, ad Boream centri *Solem* tranſierit.

In eorum autem gratiam, qui cum obſervandis ſyderibus oblectentur, nondum tamen integram Parallaxium doctrinam hauſerint, libet Schemate ſimulque Calculo paulo accuratiore, rem plenius exponere.

Ponamus igitur, anno 1761, *Maii* 25°. 17^{h}. 55'. *Londini*, *Solem* occupaturum ♊ 15°. 37'. ac proinde ad centrum ejus Eclipticam tendere in Boream angulo 6°. 10'. *Veneris* autem viſibilem intra *Solis* diſcum Viam tum temporis deſcendere in Auſtrum, facto angulo cum Ecliptica 8°. 28': proinde via *Veneris* tendet parum in Auſtrum reſpectu æquatoris, interſecans declinationis parallelas angulo 2°. 18'. Ponamus etiam *Venerem* ad dictum tempus *Solis* centro proximam fore, ac ab eodem quatuor minutis diſtare ad Auſtrum; ſinguliſque horis etiam quatuor minuta prima intra *Solem* motu retrogrado deſcribere. Erit autem *Solis* Semidiameter 15'. 51''. proxime, *Veneris* vero 0'. 37''½. Ac ſupponamus, experimenti gratia, differentiam parallaxium Horizontalium *Veneris* & *Solis*, quam quærimus, 0'. 31''. eſſe, qualis ex ſuppoſita *Solis* Parallaxi 0'. 12''½ elicitur. Deſcribatur itaque centro C circellus *AEBD*, cujus ſemidiameter ſit 0'. 31''. diſcum Terræ repræſentans, & in eo Ellipſes parallelorum 22 & 56 *grad.* Latitudinis Borealis, modo jam ad conſtruendas Eclipſes Solares ab Aſtronomis uſitato, ut *D a b E, c d e*: ſit autem *B C A* Meridianus in quo *Sol*; ad quem inclinetur recta *F H G* Viam *Veneris* deſignans angulo 2°. 18'. quæque diſtet a centro C 240 partibus qualium *B C* eſt 31; & de *C* cadat recta *C H* ipſi *F G* perpendicularis. Ac poſito planeta in *H* ad 17^{h}. 55′, vel 5^{h}. 55′ mane, dividatur recta *F H G* in ſpatia Horaria III. IV, IV. V, V. VI, *&c.* ipſi *C H*, hoc eſt quatuor minutis æqualia. Fiat etiam recta *K L*, æqualis differentiæ apparentium Semidiametrorum *Solis* & *Veneris* ſive 15′. 13″½. Et Circulus radio *K L*, centro vero quolibet puncto intra circellum Diſci *Terræ* deſcriptus, occurret rectæ *F G* in puncto denotante quota hora *Londini* numerabitur, cum in eo *Terræ* ſuperficiei loco, qui ſumpto in diſco puncto ſubjacet, *Venus* angulo contactus interioris *Solis* limbum continget. Ac ſi centro *C* radio *K L* deſcriptus circulus occurrat ipſi *F G* in punctis *F* & *G* erunt recte *F H*, *H G*=14′. 41″ id quod percurrere videbitur Venus tribus horis cum 40 Min. Cadet igitur *F* in II^{h}. 15′. *Londini*; *G* vero in IX^{h}. 35′ mane. Unde manifeſtum eſt quod, ſi Terræ magnitudo, ob immenſam diſtantiam, quaſi in punctum evaneſceret: vel ſi motu diurno deſtituta *Solem* haberet eidem puncto *C* ſemper verticalem, Eclipſeos hujus Mora integra per ſeptem horas cum triente duraret. Verum Terra interea motu motui *Veneris* contrario gyrata per 110 *grad.* Longitudinis ſuæ, ac proinde contracta dictæ moræ duratione, puta 12 Min. proveniet ea 7^{h}. 8′. proxime, ſive 107 *grad.*

Plate 5.

Jam in ipſo Meridiano *Venus Solis* centro proxima erit ad Oſtium orientale fluminis *Gangis*, ubi poli altitudo eſt 22 *grad.* circiter. Locus igitur ille utrinque æqualiter diſtabit a Sole, in momentis introitus & exitus planetæ, nempe 53½ *grad.* ut ſunt puncta *a*, *b*, in parallelo majore *D a b E*. Erit autem Diameter *A B* ad diſtantiam *a b* ut quadratum Radii ad contentum ſub Sinubus 53½ & 68 *grad.* hoc eſt, ut 1′. 02″ ad 0′. 46″. 13‴; ac calculo rite inſtituto (quem ne Lectori tædio ſit, omittere præſtat) invenio quod circulus centro *a* & radio *K L* deſcriptus occurret rectæ *F H*, in puncto *M*, ad II^{h}. 20′. 40″; centro vero *b* deſcriptus occurret ipſi *H G* in *N*, ad IX^{h}. 29′ 22″; horis ſcilicet *Londini* numeratis; proinde tota *Venus* intra *Solem* conſpicietur ad *Gangis* ripas, per 7^{h}. 8′. 42″. Recte igitur poſuimus durationem fore 7^{h}. 8′; cum pars minuti hic nullius ſit momenti.

Aptato autem calculo ad *Portum Nelſoni*, invenio, quod Sole jamjam occaſuro, diſcum ejus ſubitura ſit Venus; ſtatim vero ab ortu ejus exitura ab eodem; Loco illo interea per Hemiſphærium a Sole averſum de *c* ad *d* tranſlato, motu motui *Veneris* conſpirante. Mora igitur *Veneris* intra *Solem* diuturnior fiet ob Parallaxin, puta quatuor minutis; ut ſit omnino 7^{h}. 24′. ſive 111 *grad.* æquatoris. Cumque Latitudo Loci ſit 56 *gr.* erit ut Quadratum Radii ad contentum ſub Sinubus 55½ & 34 *grad.* ita *A B*= 1′, 02″ ad *c d*=28″. 33‴. Ac calculo rite peracto conſtabit, circulum centro *c* radio *K L* deſcriptum rectæ *F H* occurſurum in *O*, ad II^{h}. 12′ 45″, centro vero *d* deſcriptum ipſi *H G* in *P*, ad IX^{h}. 36′. 37″. Quocirca duratio Moræ ad *Nelſoni* portum erit 7^{h}. 23′. 52″; major ſcilicet quam ad oſtia *Gangis* totis 15′. 10″ temporis, Quod ſi *Venus* abſque Latitudine tranſierit, fiet dicta differentia 18′. 40″; Si vero quatuor minutis

Solis

Part I. p.248.

Limbus Solis Or.
Figura Veneris Solis Discum intrantis.
Limbus Solis Occ.
Figura Veneris excuntis.
c B d
e
E D
b C a
A
O F M
III
Via
IV
visibilis Veneris
V
infra
VI
Solem
VII
Maij 26. 1761
VIII
Mane
IX
N G P
K
L
differentia Semidiam. Solis et Veneris. $\overset{h}{3}.\ \overset{\prime}{48}.\ \frac{3}{8}$

Solis centro fuerit Borealior, ad $21'. 40''$ augebitur eadem differentia, multo major futura aucta Planetæ Latitudine Borea.

Londini autem, ex prædictis Hypotheſibus, conſequitur *Venerem* jam tum infra *Solem* ingreſſam orituram; & ad $9^h. 37'$ mane, in Egreſſu *Solis* limbum interius contacturam; ac denique non niſi hora $9^h. 56'$, orbem ejus integrum relicturam eſſe.

Iiſdem etiam Hypotheſibus conſtat *Venerem* extremum *Solis* limbum Boreum quaſi centro ſuo ſtringere debere, Anno 1769. *Maii* 23°. $11^h. 00'$, ita ut, ob Parallaxin, in Borealibus *Norwegiæ* partibus, tota intra *Solem* inocciduum apparere poterit: dum in litoribus *Peruviæ* & *Chili*, vix exiguo ſui ſegmento cadentis *Solis* diſco quaſi inequitare videbitur; uti in Inſulis *Moluccis* earumque vicinia, oriente *Sole*. Quod ſi Nodi *Veneris* retrocedere reperiantur (ut ob nuperas quaſdam obſervationes ſuſpicio eſt) tum toto corpore intra orbem *Solis* ubique conſpicua, maxima harum Eclipſeωn differentia argumentum Parallaxeos *Solaris* præbebit adhuc multo luculentius.

Quomodo autem ex obſervatis alicubi apud *Indos* Orientales, anno 1761. Ingreſſu & Egreſſu *Veneris*, & cum Exitu ejus apud Nos obſervabili collatis, eadem Parallaxis derivari poterit; aptando ſcilicet angulos Trianguli ſpecie dati in trium Circulorum æqualium circumferentias, alia occaſione docebitur.

***To uſe the Micrometer in Solar Obſervations, by Mr.* Derham, *n.* 288. *p.* 1505.**

XII. My Micrometer is not as uſually to be put into a Tube, but is to meaſure the Species of the Sun on Paper (of any *Radius*) or to meaſure any part of it. By this means, I can eaſily, and very exactly, with the help of a fine Thread, take the Declination of a Spot at any time of the Day; and by my half Seconds Watch and a fine croſs Hair, I can meaſure the diſtance of the Spots from the Sun's Eaſtern or Weſtern Limb.

This croſs fine Hair ſhould be ſet, not (as uſually) at the exact focal diſtance from the Eye-Glaſs, but a little out of that diſtance, nearer the Object Glaſs, becauſe the Shadow of the Star will thereby be much narrower, and appear more ſtrong croſs the Species of the Sun receiv'd on the Paper, which I obſerve, becauſe it may be of good uſe in taking the Sun's Altitude, meaſuring his Diameter, *&c.*

***Spots in the Sun, by Mr.* St. Gray, *n.* 288. *p.* 1502.**

XIII. *June* 15. 1703. between 4 and 5 *P. M.* I ſaw a Spot in the Sun, by placing a white Paper ſo far behind the Teleſcope of 6 Feet, as to give the Image of the Sun 9 Inches Diameter; the Spot was in the lower Right-hand Quadrant of the Sun's Disk, its form almoſt round, inclining to an Ellipſis, it was diſtant from the Limb of the Sun about 6 or 7 Minutes, its Diameter I judg'd to be about 10 or 12 Seconds; a little before the Sun ſet, I ſaw the Spot with a 16 Foot Teleſcope, and could perceive it environ'd with a Miſtineſs. The 16th I ſaw the Spot again about 2 in the Afternoon, and found it advanc'd nearer the Weſtern Limb of the Sun. The 17th was cloudy. The 18th *P. M.* a little before 5, I ſaw the Spot with the 16 Foot Glaſs through thin Clouds, and found it very near the Limb of the Sun, little more then ½ a Minute, 'twas much contracted in breadth, ſo as to be 4 or 5 times longer than broad. The 19th I look'd for it, but could not ſee it, ſo I conclude it was either gone off the Disk of the Sun, or if it adhered to the Limb, the great Tremulation of the Atmoſphere hinder'd me from ſeeing it.

Plate 3.

June the 26, 1703. in the Evening I look'd to ſee whether there were generated any new Spots in the Sun, but found none; but on the 27th about half an Hour after 8 in the Morning, by receiving the Sun's Image on white Paper from the 6 Foot Glaſs, I ſaw a Spot near the Vertical of the Sun towards the lower Limb; betwixt 9 and 10 I elevated the 16 Foot Tube, the Clouds now being of a convenient thickneſs to let me ſee the Sun without Prejudice to my Eyes, and found that this Spot was of a triangular form, and that it was accompanied with 2 other leſſer ones, as is expreſt in Fig. 45. the ſides of the great Spot were curvilinear, this with 2 leſſer ones made an Equicrural Triangle; at 4 in the Afternoon the Triangular Spot had a ſmall Fragment ſeparated from it, and it ſelf was now become Elliptical, the Spot *b. d.* was much augmented, but the Spot *e.* diminiſhed, and become longiſh, as in Fig. 46. At half an Hour after 5 the Fragment from the great Spot was it ſelf divided into 2, and the Spot *c.* was ſo narrow, as ſcarce to be ſeen, as at Fig. 47. At 6 a Clock, and 30 Minutes there was a ſmall Fragment ſeparated from the lower end of the great Spot, as at Fig. 48. At 7 a Clock the Spot *b.* was much encreas'd, but *c.* was vaniſh'd; the Obſervations made this Afternoon with the 16 Foot Glaſs, were when the Air was clear, and ſo to ſecure my Eye, the Eye Glaſs was ſmoak'd with a Wax-Candle.

The 28th about 7 in the Morning, I ſaw that the great Spot was much augmented, but the leſſer ones that Yeſterday attended it, were vaniſh'd, and that there were two new ones generated at about 1½ Minutes diſtance from the great one below, and towards the Left hand of it the great one was a parallelogram, with a very black diagonal croſſing it, ſee Fig. 49. At 10 a Clock there was another diagonal croſſing the former, and the two leſſer Spots which before were longiſh, had now taken a round form, the Spot *c.* being much larger than the other at *b.* Fig. 50.

Solar Spots by Mr. Derham, *ib. p.* 1504.

XIV. The two Circles, Fig. 51, 52, repreſent the Sun's Disk, and N. the Northern part thereof, S. the Southern, E. the Eaſtern, and W. the Weſtern part. The place of the Spots, and the manner of their Appearance every Day, is repreſented with the Day of the Month on the Sun's Disk. But although the Figures of the Spots are done pretty exactly, yet their places on the Sun are not ſo; for being unprovided with convenient Inſtruments for the purpoſe, I could not exactly ſet off their Delineations, nor their diſtances from the Sun's Limb, but was forc'd to repreſent them only as well as I could, by taking the Species of the Sun upon Paper through a Teleſcope, and ſo marking out their places.

Fig. 1
blue
red
Fig. 2
A
B
Fig. 3
red
Purple
Fig. 4
red
blue
Fig. 5
red
green
Fig. 6
red
red
Fig. 7
Fig. 8
Painted red
purple
Fig. 9
Fig. 10
Fig. 11
Fig. 12
Fig. 13
Fig. 14
Fig. 15
Fig. 16
Fig. 17
red purple
Purple from white
Fig. 18
Fig. 19
Fig. 20
Fig. 21
Fig. 22
Fig. 23
Fig. 24
Fig. 25
Fig. 26
Fig. 27
Fig. 28
Fig. 29
Fig. 30

Part I. P. [250.]

I. Senex sculp.t

The Spots ſeemed ſtrong enough to have laſted another, or more Revolutions, but none have been viſible ſince the ſixth of this Month, on which Day I think I had a glimpſe of one on the Sun's Weſtern Limb, about 7 of the Clock in the Morning.

The Appearances of the Spots being in the Figures above ſet with every Day of the Month, I need ſay but little, only take notice of a few things that the Figures do not ſo well expreſs.

Plate 3.

The Spot in Fig. 51. was as repreſented, *viz.* firſt round and ſtrong, afterwards long, and with a *Nucleus.* The very ſame Spot (I doubt not) I ſaw again on the Sun's Eaſtern ſide on *July* 5, but very faint, ſmall and long (as in Fig. 52.) ſo as to be but juſt diſcernible. On *July* 6 it quite diſappeared, both through my Tubes, and on Paper, which is better.

The Spots in Fig. 2. had theſe remarkable Appearances and Variations. On *June* 28. viewing the Sun towards Evening, I eſpy'd a large ſtrong dark Spot, with two or more glaring *Nubeculæ* behind it, ſomewhat like the Repreſentation in the Figure. Theſe the next Day were become four ſtrong dark Spots, the foremoſt with a Tail to it, conjoyning the little Spot next it, as in the Figure. On *June* 30. I ſaw Spots, but it being a cloudy Morning, and I abſent from my Tubes in the Afternoon, the Repreſentation of them in the Figure is not exactly as they were. *July* 4. between two long Spots appeared ſomething like a round *Nubecula,* as in the Figure. The reſt as in the Figures.

The ſingle Spot in *June* may be ſeen to have paſſed above half over the Disk before I had notice of it. And that or ſome others were, I hear, ſeen in *May* ; but I ſaw them not.

—— by *Captain* Stanyan. *n.* 294. *p.* 1756.

XV. On *Saturday, May* 15. 1703. As I was obſerving the Setting of the Sun, in order to examine my Clocks, there appeared two Suns, the Mock Sun ſeemed above the real one, which was then only five degrees above the Horizon. Whereupon I took a good 7 Foot Teleſcope, with a ſmall Aperture, and ſoon diſcovered a Solar Spot near the Sun's Center, which I deſigned to obſerve more exactly the Day following, but it proved Cloudy. *May* the 16th, *Sunday* no Sun-ſhine.

Monday the 17th, at Six a Clock in the Morning I took the ſame Teleſcope, armed with a clouded Eye-glaſs, and immediately perceived that the Spot was advanced conſiderably towards the Sun's Weſtern Limb ; it ſeem'd of a ſtrong conſiſtence, very Compact, reſembling a Face, and was diſtant by Noon from the Anteriour Limb of the Sun's Disk 61 Seconds of Time. See Fig. 54.

Tueſday the 18th, at Noon I found the Spot diſtant from the preceeding Limb 46 Seconds of Time. Fig. 54.

Wedneſday the 19th. At Noon I obſerved the Solar Spot to be moved within 33 Seconds of Time of his Weſtern Limb. Fig. 54.

Thurſday the 20th, at Noon the Spot was arrived within 21 Seconds of Time of the preceding Limb, and moving nearly in a ſtraight

Line: interſecting the Parallel of Declination paſſing through the Sun's Center. Fig. 54.

Friday the 21ſt, we had no Sun-ſhine.

Saturday the 22d, at ſeven a Clock in the Morning I obſervedthe Solar Spot was advanced very near the Limb of the Snn's Disk. Fig. 54.

Sunday May 23d, at 6 in the Morning I ſaw the Spot, which by that time was gotten to the very edge of the Sun's Disk, reſembling a barley Corn, lean and ſlender, and of a duskiſh Colour, wanting only its own ſhorteſt Diameter of the Sun's Limb. At 8 a Clock I obſerved it again: Alſo at 10, and at 12. At 2 I perceived it was ſlid into the very Circumference, and hardly Viſible, had I not had an Eye upon it all the Day long. At Four I examined the Sun's Body with my Eighteen Foot Glaſs, which is a good one, but could not perceive the leaſt Glimpſe of it; ſo that about Three in the Afternoon it totally diſappeared. Fig. 54.

On *Tueſday, June* 3. 1703. about ſix in the Evening, I obſerved with my Eighteen Foot Glaſs four Spots in the Sun's Disk, environ'd with a Miſtineſs, thicker on the Right-Hand than on the Left, ſituated in the upper Left-Hand Quadrant, about the 12 part of the Sun's Diameter diſtant from his neareſt Limb. From the Cloud about them proceeded both ways five long curve Rays, of a yellower Colour than the Sun's Body. Theſe Spots I could never ſee more, though I watch'd them for ſeveral Days together. Fig. 55.

On *Monday, June* the 7th. at three a Clock in the Afternoon I diſcovered the ſame Spot (to my thinking) that I ſaw go off the Sun's Disk on *May* the 23. re-entring the Sun's Face juſt at the time and place that I expected it.

At four of the Clock, the Sun being extreamly clear, I mounted my Eighteen Foot Teleſcope, through which the Spot appeared diſtinct, but ſlender like a Spider, with an Elliptical Speckly Miſt about it, and 5 or 6 light-coloured Streaks. It ſeem'd to me to be as it were divided near the Top, as in the Figure. Fig. 56.

Tue day, June the 8th, at ſix this Morning the Spot was very viſible, and I ſaw it trace again its former Path, coming in exactly where I expected; it kept its ſhape, but thoſe Lemon coloured Streaks diſappeared, though it ſelf and the Miſt about it grew bolder and broader viſibly, as it re-entred the Sun's Disk.

Wedneſday, June the 9th, at five of the Clock this Evening, I obſerved the Spot with the 18 Foot Glaſs, but could not perceive it had altered its Shape, but advanc'd gradually over the Sun's Disk, as it had formerly done.

Thurſday, June the 10th, at Noon, the Sun ſhining very bright, I had an Opportunity of being aſſured it was the ſame Spot; I plainly ſaw it move over its former Path, and was then diſtant from its neareſt Limb 29 Seconds of Time. At five in the Evening I obſerved its Shape (with my 18 Foot Tube) to be altered, appearing bigger and blacker than ever, as in the Scheme, Fig. 57.

Friday, June the 11th was an ill Day for Obſervations. But I had a Sight on't with the 18 Foot Glaſs; it continued black and bold, as before.

Saturday, June the 12th, at 7 a Clock in the Morning, the Sun's Body being very clear, I ſaw the Spot through the 18 Foot Glaſs, retaining its former Shape.

Sunday, June the 13th, by this Day Noon the Spot was arrived at the ſame Point of the Sun's Disk that I found it in on *Monday* at Noon, *May* the 17th; which makes me inclinable to believe it was the very ſame Spot.

Monday, June the 14th, according to Rules received Yeſterday from Mr. *Flamſteed*, I meaſured the diſtance of the Spot from the next Limb of the Sun's Disk, which I found to be 45 Seconds of Time from the anteriour edge of the Sun's Body: And upon *Tueſday, May* the 18th, it was obſerved to be in the very ſame place of its Path, within a ſingle Second of Time. At 4 I obſerved it with my 18 Foot Glaſs, and perceived that it had altered its Shape, appearing as at Fig. 58. I received it on the Scheme, and it was diſtant from the preceding Limb 612 ſuch parts as the Sun's Semidiameter is 900.

Tueſday, June the 15th, at Noon the Solar Spot was diſtant 32 Seconds of Time from the leading Limb of the Sun's Disk, and covered the very place where the ſame Spot had been obſerved on *Wedneſday* the 19th of *May*.

Wedneſday, June the 16th, No Sun-ſhine.

Thurſday, June the 17th, No Sun-ſhine.

Friday, June the 18th, At Noon I obſerved the Solar Spot waxing very ſlender, but, notwithſtanding that, it was black and bold to appearance, the Miſtineſs about it on the Right-Hand perceivable, and that on the Left grown ſlender, in proportion with the Spot it ſelf, and found it diſtant 5 Seconds of Time.

Saturday, June the 19th, At 5 this Morning, it being clear Weather, I ſaw the Spot diſtinctly with my 7 Foot Tube: At 9 a Clock I mounted my 18 Foot Glaſs, obſerving once in half an Hour all the Morning: At 12 I perceived that all the Cloud or Miſty Matter that uſed to ſurround the Spot was inviſible, and the Spot it ſelf reduced to little or no Breadth, in Compariſon to what it had been towards the Sun's Center, and ſo cloſe to the Limb of the Disk, that I could only perceive a ſmall Streak of the Sun's Light between it and the Limb of the Sun's Body; at two a Clock I could juſt perceive it, but grown extreamly ſlender.

The firſt Revolution I ſaw the Spot half in the Circumfererence of the Sun's Limb at two a Clock on *Sunday, May* the 23d: And the ſecond Revolution I juſt perceived it with the eighteen Foot Glaſs, at half an Hour after two a Clock, on *Saturday* the 19th Day of *June*.

On *Sunday, June* the 27th, about ſix a Clock in the Evening, I obſerved ſeveral Spots in the Sun's Disk, but had not the Conveniency to uſe my longeſt Teleſcope, becauſe of ſome Trees that were in my way to Weſtward, ſo that I made no Obſervation till the *Tueſday* following.

Tueſday, *June* the 29th, about ſeven in the Morning I counted ſixteen remarkable Spots in the Sun's Body, and near his Center they appeared *Fig.* 59. as in the Figure, through the eighteen Foot Glaſs; then I took my ſeven Foot Teleſcope and Frame, and obſerved that the foremoſt Center of ſix that looked on the Paper as one Spot, was diſtant from the Sun's Anteriour Limb eighty one Seconds of Time, and the laſt Cluſter 87.

This Day the foremoſt Spot was diſtant from the following Limb, according to the Path of the Spot, juſt fifty five Seconds of Time. The Sun's Diameter was always 136 Seconds in the Tranſit, and the Spot was 126: So that the Spots Path was ten Seconds ſhorter than the Sun's Diameter.

Wedneſday, *June* the 30th, At Eight a Clock this Morning, obſerving the Solar Spots with my eighteen Foot Teleſcope, I perceived very plainly that they had wonderfully increaſed in Number, and ſtrangely changed their places. The Cluſter of ſeven Spots ſeemed to me to move *Fig.* 60. gradually, as the ſingle Solar Spot did in *May*, but the Cluſter 4 went too faſt forward, the twelve Spots without a Miſt about them ſtraggled all manner of ways, and the nine Spots and the five black little ones went backward, and unbent it ſelf at the ſame time as it were into a ſtreight Line. I am apt to believe it went backward, as that the other went too faſt, or faſter than ordinary forward, for in twenty four Hours the foremoſt Cluſter advanced twenty one Seconds of Time, which is more by ſix Seconds than ever the ſingle Spot moved in that Time, even when neareſt the Sun's Center; and the diſtance in time between the firſt and the laſt Cluſter this Day, was greater by three Seconds than the Day before.

The foremoſt Cluſter of four Spots was diſtant from the advancing Limb of the Sun ſixty Seconds of Time.

At half an Hour paſt four, the advancing Cluſter paſs'd the Interſection in fifty five Seconds of Time, after the Sun's foremoſt Limb had paſſed Conformable to the Spots Path; and the laſt Spot paſſed in ſixty three Seconds of Time, the laſt Limb paſſing the Interſection, according to the Path of the Spot, in a hundred and twenty ſix Seconds of Time, the Sun's largeſt Diameter paſſing in a hundred and thirty ſix Seconds, the Spots by this time appeared ſtrangely black, and of very odd Shapes, as in the upper part of the Circle.

Thurſday, *July* the Firſt, At eight a Clock in the Morning I obſerved the Solar Spots with my eighteen Foot Teleſcope, the Weather *Fig.* 61. being good, and ſaw that they rang'd themſelves in reſpect of one another, as is repreſented in the upper part of the Scheme: The leading and largeſt Spot being diſtant from the anteriour Limb forty four Seconds of Time, the laſt Cluſter lying a little awry, paſs'd in fifty three Seconds: After the Anteriour Limb ſo done, the following Limb

also

Fig: I.
R
α
β
R
E
R
l
M
l
b
a
L
X
L
A
B

Fig: II.

Fig: III.

Fig: IV.
V
p
k
e
k
e
p
r
a
r
ρ
α
ρ

Fig: V.
B
k
b
R
V
c
k
p
a
A

alſo paſs'd the Interſection, according to the Path of the Spot, in 125 Seconds of Time.

Friday, } No Sun-ſhine.
Saturday, }

Sunday, July the 4th, This Morning at Eight a Clock the Leading Spot was diſtant from the advancing Limb ten Seconds of Time, the Spots and Cluſters retaining nearly the ſame Shape, but beginning to Contract themſelves, the foremoſt methought look'd ſtrong enough to make another Revolution, and paſs'd in 127 Seconds.

Monday, July the 5th, At ſeven a Clock I found the Spots had quite alter'd their Shape, appearing dull and ſlender, as in the lower part of the Scheme, and diſtant about four Seconds, being all included in a Cloud.

Tueſday, July the 6th, At ten a Clock the Sun's Disk, view'd with my ighteen Foot Teleſcope, was found clear of all Spots.

On the ſeventeenth Day of *July*, about four a Clock in the Afternoon I obſerved ſome Spots in the Sun's Body, reſembling thoſe I ſaw on *Thurſday* the third of *June*, only with this difference, that theſe appeared to me as if they had been heated red hot; they ſeemed to be in he ſame part of the Sun's Disk. I obſerv'd them above an Hour together that Day, but could never afterwards ſet Eye on them, nor diſover whether they were coming in, or going off his viſible Disk. I ontinued to obſerve the Sun, as often as was poſſible, with my eighteen oot Glaſs, till the end of the Month, but without farther Succeſs.

A Table of all the Spots *and* Fæculæ *on the* Sun, *at* Upminſter, *ſince* July 1703.

1703.	*March* 10	*Nov.* 21	*June* 24		*Faint* 6	*March* 12	*No Spot*	
Octob. 9	11	*Dec.* * 2	26	1706.	* 8	14	*nor* *	28
10	13		27	*Feb.* 7	28	18	*Octob.*	31
11		1705.	28	*Mar.* * 7	29	21	*Nov.*	1
Nov. 19	*April* 11	*Jan.* 1	30	27	30	*Jun.* * 29		4
22	12	2	*July* * 1	30	*Dec.* 1	*Extinct* 30		8
	13	3	* 3	*April* 2	2	*July* 1		10
1704.	*May* 1	5	24	*Vaniſhed* 3	3	*Languid* 2	N.B. *This*	15
Jan. 16	* 11	* 25			4	*Scarce*	Nov. 15.	16
17	*June* 23. *none*	*Feb.* * 19	*Sept.* 30	*June* 7	5	*viſible* } 4	*another*	17
18	*Some* 24	*March* 14	*Octob.* 2	8	* 22	*More*	*Spot*	18
19	*Vaniſhed* 25	16	3	*July* 24	* 31	*viſible* } 5	*aroſe on*	19
21	*July* 18		5	25		*Extinct* 6	*the Ea-*	20
22	20	*April* 1	6	*Sept.* 4	1707.	*Appears* 8	*ſtern ſide*	22
23	21	*May* 5	7	5	*Feb.* 14	* 10	*of the*	
	* 23	6		6	15	*Aug.* * 31	*Disk,*	
Jan. 30.	*Sept.* * 10	7	25	7	16	*Sept.* * 12	*whilſt*	
		8	26	8	18	16	*this*	
Feb. 23	*nothing* 14	* 10	30	10	24	*Spot* & 17	*was on*	
25	18	* 22	31	*Faint* 11		19	*the*	
Mar. 7	* 21	* 23	*Nov.* 2	*Extinct* 12	*March* 6	21	*Weſtern.*	
8	*Nov.* 17	*June* 22	4	*Oct.* * 29	9	27		
9	18	23	* 21	*Nov.* 5	11			

XVI. *Spots in the Sun, by Mr.* Derham, *n.*330. *p* 270.

In this Table the *Faculæ* are noted with an Aſteriſk; and the Duration of every Appearance of the ſame Spots or *Faculæ*, or the time they diſappeared, with a Line.

As to the Figure of the Spots. They are well known to change frequently; and therefore I think it of little uſe to give their Figures every time I obſerv'd them. But it is ſomewhat remarkable, that the Spots generally appear longiſh near the extream Parts of the Disk. If they are never ſo round near the middle of the Disk, they become longer and longer towards the Extremes, till (at going off) they ſeem to be nearly a ſtraight Line, nearly parallel to the Sun's Limb. Which is a manifeſt Argument, that the Sun is a Globe, and that theſe Spots are on, or very near its Surface.

Another thing remarkable is, The Mutability of the Shape of the Spots. I have more than once manifeſtly perceived them to change in the very time I have been looking upon them. Thus *Nov.* 19. 1703. I ſaw three or more Spots not far off the middle of the Disk; and whilſt I was looking upon them, they ſeemed to vary, both as to their Shape and Strength; ſometimes ſeeming longer, ſometimes ſhorter; ſometimes ſpiſs, ſometimes languid. And this they ſeemed to do, not only through my 16 Feet Tube, (which I thought at firſt was from the different Diſpoſition of my Eye) but alſo when I received the Sun's Image through a ſix Feet Teleſcope, on a white Paper, in a darkened Room. Theſe mutable Spots the Weather hindred me from ſeeing again till *November* the 22d, following; and then they were become only like a thin Smoak, or *Nebula.*

So again *April* 11. 1704; there were divers Spots with *Umbræ* about them. Theſe *Umbræ* or *Nebulæ* I could plainly perceive, whilſt I was looking on them, to be ſometimes very faint and thin, and ſometimes much darker and thicker. Theſe *Maculæ* and *Umbræ* I obſerved ſuddenly brake out in the Sun: For, on *April* 9. the Disk was free. But this *April* 11, laſt mentioned, I perceived them advanced near a quarter part on the Disk: And conſequently they brake out in the Sun within 48 Hours before. On *April* 13. the *Spots* were become *Umbræ,* in the Morning; and at 4 of the Clock in the Afternoon, there were no Remains of either *Maculæ* or *Umbræ.*

From this ſhort Continuance of theſe Spots on the Sun, it is more than probable, they were in a perpetual Flux and Change; and that thoſe Mutations which I perceived in them, whilſt I was looking on them, were real, not imaginary.

Alſo it may be farther remarked, (which I have frequently obſerved, and which as I remember *Scheiner* obſerved long ago.) That thoſe *Spots* and *Umbræ* which ſuddenly ariſe, do as ſuddenly decay, and are ſoon extinct. And ſuch Spots, I have farther obſerved, do ſeldom turn to *Faculæ,* as they commonly do when longer on the Sun, as I ſhall obſerve by and by.

Again, *May* 5. 1705. I could perceive two Spurs or Branches (running from a Spot) to change, and be ſometimes darker, ſometimes thinner.

So *March* 30. 1706. I obſerv'd ſuch another Variation. This Day, or but little before, *Spots* with *Faculæ* aroſe in the Sun, which remained not above three Days on him. One of theſe Spots I could manifeſtly perceive to be ſometimes quite extinct, and then again immediately to appear: And the *Faculæ* alſo, in half an Hours time, had plainly alter'd their Shapes.

October 29. the ſame Year, I could plainly perceive the *Maculæ* and *Faculæ* both to change: And whilſt I was carefully viewing them, I ſaw a Spot ariſe in one of the brighteſt *Faculæ,* and again nearly diſappear; and then again appear ſtrong and ſpiſs. I ſhould have been glad to have ſeen how they appeared next Day; but the Weather was ſtormy, cloudy, and wet for ſeveral Days after.

Another thing I have obſerved is, That the *Maculæ* do generally, if not always, become *Nebulæ* or *Umbræ* before they quite vaniſh; and after that, very frequently turn to *Faculæ,* or bright golden Spots, more illuſtrious and fulgid than the other Parts of that glorious Globe. If the Spots are of ſhort Duration, *Faculæ* ſeldom enſue; Or if they do, they are commonly the Remains of ſome Spots that had before been on the Sun, and vaniſh'd perhaps on the ſide oppoſite to us. But Spots that long continue, if they vaniſh before that part of the Sun revolveth out of our ſight, do very often become *Faculæ*. Of which the Table affordeth ſeveral Inſtances, particularly *July* 3. 1705.

From theſe preceeding Particulars, and their Congruity to what we perceive in our own Globe, I cannot forbear to gather, *That the Spots on the Sun are cauſed by the Eruption of ſome new Vulcano therein*; which at firſt, pouring out a prodigious quantity of Smoak, and other opacous Matter, cauſeth the Spots: And as that fuliginous Matter decayeth and ſpendeth it ſelf, and the Vulcano at laſt becomes more torrid and flaming, ſo the *Spots* decay and grow to *Umbræ*, and at laſt to *Faculæ*; which *Faculæ* I take to be no other than more flaming brighter Parts than any other Parts of the Sun. Theſe *Faculæ* I have obſerved never continue long on the Sun: And the reaſon I conceive is, becauſe the *Vulcano*, after its Smoak is over, doth not long emit its Flames; by reaſon the fiery *Pabulum* is then near ſpent, when once it begins to flame: After which the torrid *Vulcano* ſoon returneth to the natural Temperature of the Sun, ſo nearly at leaſt as to eſcape our ſight, at ſo vaſt a diſtance as the Sun is from us.

Another thing that may be accounted for, and indeed doth in ſome meaſure confirm alſo what I have ſaid, is the *Nuclei*, or darker part of the Spots; generally in moſt Spots, and towards the middle of them. Now it is very uſual in Culinary Fires in this our Globe, when they emit Smoak, that the middle is the darkeſt part. If for Inſtance, we were from aloft in the Air, to ſee a thick Smoak come tumbling out of a *Chimney*, or the Mouth of a *Vulcano* juſt kindled, we ſhould find the middle part, juſt over the Mouth of the *Chimney*, or *Vulcano*, to be the moſt ſpiſs and dark, and towards the Extremes clearer and thinner. And ſo I take it to be in the Eruptions of the Sun; that the *Nucleus* is juſt over the Mouth of the ignivomous Cavern, and that the miſty parts of the Spot are the thinner parts of the Smoak, ſwimming about in that Fluid, or *Atmoſphere*, which I ſuppoſe doth ſurround the Sun, as well as our Globe, and the Moon manifeſtly; yea, and in all probability, every Planet of this our Solar Syſtem.

From what hath been ſaid, we may give a reaſon why there are ſometimes Spots frequently on the Sun, and ſometimes none in many Years. One thing I believe there is in this, That there may be Spots, but not always ſeen. But there are doubtleſs great Intervals ſometimes when the Sun is free; as between the Years 1660 and 1671, 1676 and 1684. In which time Spots could hardly eſcape the ſight of ſo many curious Obſervers of the Sun, as were then perpetually looking at him with their Teleſcopes in *England*, *France*, *Germany*, *Italy*, and all the World over; whatever might be before, from *Scheiner*'s Time. The reaſon I ſay, of this long diſappearance of the Spots, I take to be from the want of extraordinary Eruptions in that fiery Globe. The ſulphureous or other Matter, or *Pabulum* of thoſe Eruptions, is ſpent or diſſipated, and that Globe continues in its natural ordinary burning State, till there happens to be a freſh Collection of ſmoaking, Diſploſive, and extraordinary Matter, that cauſeth a new Eruption. Which Eruptions generally happen between what we may call the Sun's Tropicks, or in his torrid Zone: For I never obſerved any Spots to be near the Sun's Poles. And if I miſremember not, the Spots in *Scheiner's* Cuts are all about the middle Zone of the Diſk. The greateſt Evagation I ever obſerved of them was *March* 8. $170\frac{3}{4}$. On which Day, beſides the dark Spots in the uſual Zone, I perceived ſome faint Spots, ſcarce viſible, much nearer the Southern Pole than I ever had ſeen them. But this was, no doubt, in ſome meaſure owing to the Poſition of the Earth in reſpect of the Sun, as well as to the ſoutherly Place of the Spots on him: For about the Equinoxes, the Spots ſeem to march pretty far towards the Poles of the Sun, as may be ſeen by the annexed Schemes.

Fig. 62. 63.

Having thus obſerv'd what part of the Sun the Spots commonly poſſeſs, I ſhall next take notice of their *Stages* and *Path* over the Sun. That the Sun moveth round his own *Axis*, is manifeſt, beyond doubt, from the Motion of the Spots. And that the Spots ſeem to traverſe the Sun, ſometimes in ſtraight Lines, ſometimes in curve Lines, curved this way, and that way, is as manifeſt alſo, and well known to the Curious, and is ſet forth in the annexed two Figures: Which Figures ſhew the Stages of the Spots every Day that I obſerved them, and the Lines they deſcribe in ſeveral Months of the Year. The daily Stages in both Figures are exact; or if they ſeem otherwiſe, it is by reaſon the Obſervations were made at different times of the Day; as one in the Morning, the other ſome following Day in the Evening, or Afternoon. But the Declinations of the Spots, or their diſtances from the Sun's Northern or Southern Limb, are leſs exact in Fig. 63. than in Fig. 62. in which latter they are very near the Truth.

And the Cauſes of the Defects in the 63d Scheme I ſhall mention, to prevent the ſame Errors in others I my ſelf ran into.

1. The Diminution of the Sun's vertical Diameter by the Refractions was the principal cauſe of my Errors. This, although I was ſufficiently aware of, yet I did not think had been ſo conſiderable, for want of experimenting, or well conſidering the Matter: For I have ſometimes found the perpendicular, or vertical Diameter of the Sun diminiſhed, from 32′ 21″ on the Meridian, to 36′ 3″ at the Horizon, in one and the ſame Day.

2. For the ſame reaſon I was not aware of the time being ſo long before the Sun goes round, as I found it.

3. Another Error was meaſuring the Sun's Image on the Scene of white Paper, with the Shade of the Micrometer; and not by looking through the Tube, and ſo claſping the Limb of the Diſk with the parallel edges of the Micrometer. The former, although practiſed by ſome eminent Aſtronomers, is a far more eaſy and indulgent, than accurate way.

Since my foregoing Account was drawn up, I have ſeen other Spots on the Sun, whoſe times are expreſſed in this following Table.

1707.		1709.	
Decemb.	4	*Jan.*	15
	10		21
	* 29		22
	* 30	*Auguſt*	13
1708.			* 17
July	31	*Octob.*	8
Auguſt	1	*Novemb.*	1
	5		2
	6		4
	22		5
	23		6
	24	1710.	
	28	*Jan.*	22
Septemb.	1	*April*	* 6
Novemb.	5	*Octob.*	14
Dec.	14		* 18
	26		

From the Spots in this Table I had frequent Occaſions to be aſſured of my Opinion, in the foregoing Paper. Particularly in viewing the Spots of *Auguſt* 1. 1708. (repreſented Fig. 64.) where ſome were large and dark, others leſs and thinner, and all encompaſs'd with *Nebulæ*: In viewing theſe, I ſay, I obſerv'd great Alterations at the very time I was looking on them. Sometimes the *Nuclei* were very dark and black, ſometimes leſs ſo; and the ſame thing I obſerved alſo in the *Nebulæ* encompaſſing them. One of the leſſer Spots *b.* in Fig. 64. which the Day before was ſufficiently viſible and ſtrong, was this Day, now thick and ſtrong, and anon languid and leſs viſible. And from the two Spots *a.* and *d.* I could plainly ſee a Smoak iſſuing out to *c.* and *f.* ſometimes viſible for 5 or 6 Minutes, and then diſappearing for a quarter of an Hour, or more; and then again ſmoaking out, and again diſappearing, as before. All which Particulars, I ſaw over and over again repeated, for a good while together, till I was weary of the Obſervation.

Theſe Spots I was hindered from viewing until *Auguſt* 5. following: And then I found the Spot *b.* quite extinct, (as I expected,) as alſo ſome of the other Spots; together with the *Nebulæ* grown leſs. But the great Spot *a.* continued dark and ſtrong, only ſometimes fainter, and then again ſtronger; and ſometimes like a half, or horned Moon; ſometimes roundiſh, or rather of an oval Figure; of which latter Figure they commonly are, when they are near the Sun's Limb; which this Spot was not far off at this time.

Theſe Particulars are Confirmations of what I ſaid, *That the Solar Spots are no other than a Smoak riſing out of the Body of the Sun.* Of which Opinion I have been almoſt ever ſince I firſt obſerv'd them, and find that I am not ſingular in this Opinion, as I ſhall ſhew from a part of a Letter from Mr. *Crabtrie* to Mr. *Gaſcoigne*, the Inventor of the Micrometer.

'I writ alſo to Mr. *Townley* at that time my Opinion in brief of the 'Sun's Spots, (which you conceive to be Stars,) and it ſeems he, or 'Mr. *Kay*, writ to the ſame purpoſe to you, deſiring your Opinion: 'Which you freely deliver; for which I cannot but commend you, and 'eſpecially for preferring Reaſon before any Man's Authority. Yet 'give me leave *(pace tua Amice deſideratiſſime)* to ſpeak my mind like-

‘ wiſe freely concerning theſe Appearances. I have often obſerved theſe
‘ Spots; yet from all my Obſervations cannot find one Argument to
‘ prove them other than fading Bodies. But that they are no Stars,
‘ but unconſtant (in regard of their Generation) and irregular Excreſcences
‘ ariſing out of, or proceeding from the Sun's Body, many things
‘ ſeem to me to make it more than probable’.

‘ For *Firſt*, For their Form; they are ſeldom round, but of irregular
‘ ſhapes, and, as I have often ſeen, one ſide, or end of the Spot more
‘ thin than the reſt, like to a certain miſty Darkneſs, and by degrees
‘ thicker, groſſer and darker, nearer to the main body of the Spot; juſt
‘ as the Smoak of ſome pitchy Fire, which is in one part very groſs,
‘ and in another more rare and thin, turning at laſt into meer Air: Or
‘ like a Cloud, Fog or Miſt, more thick, dark, and groſs in the midſt;
‘ and more thin, fluid, penetrable, and tranſparent towards the ſides;
‘ which I ſuppoſe is not compatible with any of the Stars.’

‘ *Secondly*, For their Colour: The lightneſs thereof differenceth them
‘ from Stars or Planets; they being never of ſuch abſolute Darkneſs as
‘ I obſerved *Venus* the 24th of *November* laſt: Tho' I have ſeen Spots
‘ ſometimes little leſs than ſhe, yet always of a far paler and whiter
‘ Colour, looking (at leaſt in ſome parts) like ſome thin diſſipated
‘ ſubſtance.’

‘ *Thirdly*, For the manner of their Appearance. I have ſeen many
‘ Spots, which in the middle of the Sun appear of a round body, but
‘ coming towards the ſide of the Sun, appear long. Which (if you
‘ rightly conſider it) is a demonſtrative Argument that they are not
‘ Globes, as all the Planets and Stars are: (for Globes always appear of
‘ one form (round) in every Poſition) but Exhalations, or ſuch like
‘ fluid Subſtances, extended to a broad flat form like our Clouds, which
‘ being over our Heads, and ſo in their full breadth, appear large and
‘ broad; but driven with the Wind, till they turn one edge upon us,
‘ ſeem of a long ſhape. So theſe Solar Clouds, being turned about the
‘ Sun, may in the middle ſhew their full breadth to us, and about both
‘ edges of the Sun, turn their edges to us; which anſwereth to the appearance’.

‘ *Fourthly*, For their Continuance. Some of theſe Spots, ariſing at
‘ the Eaſt-ſide of the Sun, vaniſh before they come to the midſt of the
‘ Sun. Others appear firſt in the middle of the Sun, and vaniſh before
‘ they come to the weſtern Limb; and for the moſt part they vaniſh before
‘ they have made a full Revolution about the Sun. Which argues
‘ them to be but thin, vaniſhing, fading Subſtances, not like the permanent
‘ Bodies of the Stars’.

‘ But to take off theſe Reaſons, you anſwer, That you conceive theſe
‘ Spots to be Stars moving regularly in their own Orbs, which are many,
‘ tho' none of greater extent than about $\frac{1}{10}$ of the ☉ Semidiameter from
‘ its Circumference; and that the ſwifter Movers in the lower Orbs,
‘ overtaking the ſlower in the higher Orbs, cauſe an appearance. You
‘ ſeem

‘ ſeem therefore to think, that they being ſo thin Bodies, the Sun's Rays paſs through them, and ſo one cannot be ſeen alone, till more being together, one heaped behind another, they ſtop the light of the Sun's Rays, and ſo cauſe an appearance. This I conceive is your Meaning: or elſe (as you ſeem to inſinuate afterwards) that the higher reflects the Sun's Rays ſtrongly enough upon the lower (when they come within the Angle of Reflection) to make the interjacent Planet indiſcernable'. But to theſe I anſwer,

1. ‘ If it be by their coming within the Angle of Reflection, that the light of the Sun reflected from the outer Planet upon the inner, doth make it (as you ſpeak) indiſcernable, then that Light ſo reflected, is reflected either upon all places, as the Moons and Planets Light; or but upon one, as is the Reflection of a plain Looking-Glaſs. If the firſt, there would never be many ſeen (ſeldom above one or two) becauſe the outermoſt would continually make the inner indiſcernable. But *Gaſſendus* affirms, There are ſeen ſometimes forty at once in the Sun's Body. If the 2d, there would always be many ſeen, becauſe reflected Light would but occupy a little room, and that but for a ſmall time, till the ſwifter were paſt the place of Reflection: Whereas many Days there are none at all ſeen in the Sun's Hemiſphere: And in both theſe Caſes, the outermoſt Planet of all would always in the ſpace of 27 Days, be ſeen in the ſame place, being never obſcured, none of the inferior being able to reflect Light upon it. Add hereunto, if any kind of Reflection ſhould make them to appear bright like the Sun, and ſo not diſtinguiſhable from the Light of the Sun, what ſhould hinder, but we ſhould ſee them alſo bright Bodies by the ſide of the Sun, when they are paſſing either by the Weſt, or Eaſt-ſide of the Sun's Body? The Light being then reflected upon them by the inferior Planets as well as at other times, and that alſo upon much of that ſide of them which we ſhould behold'.

‘ But if you wave this Conceit, as inſufficient, and fly to your former, That the ſwifter Movers in the lower Orbs, overtaking the ſlower in the higher Orbs, cauſe an appearance. To this I anſwer, 1. The thing you ſuppoſe ſeems to me neither neceſſary nor probable, nor do I conceive why they ſhould not be ſeen, being themſelves alone, as well as conjoined, ſeeing all other Stars and Planets are ſo. 2. If it be becauſe they are of a thin, tranſparent Subſtance, till many, being one behind another, make them to ſeem groſſer; Then they are not of the nature of other Planets, as is proved in ☿ and ♀, who of themſelves appear dark Bodies, when they come between us and the Sun; nay, they muſt be more thin than our Clouds, which will eaſily be ſeen between us and the Sun, and hides it from us. 3. If it be becauſe they are ſo little, that the Imperfection of our Glaſſes cannot diſcover one alone, there muſt be, without doubt, many Millions of them; which how they can be included within the compaſs of $\frac{1}{10}$ of the ⊙ Semidiameter, we ſhall conſider anon. I have ſeen one of an ordinary Darkneſs, (yea darker than many greater) yet not above 5″ Diameter. If this conſiſt of two or many, of themſelves inviſible, how many were in thoſe which *Gaſſendus* ſaw of $1'\frac{1}{2}$ Diameter? 4. The Figure of theſe great ones (being neceſſarily compoſed of Stars of ſuch different Orbs and Motions) would quickly vary, by reaſon of the diverſity of their Motions; like as we ſee in a Flock of ſmall Birds. But 5thly, you ſay the furtheſt of theſe Orbs is not above $\frac{1}{10}$ of the Sun's Semidiameter from its Circumference. But there would not, in that ſmall ſpace, be room enough for ſo many Orbs of Planets, as have been ſeen at once. Which I prove thus. 1. *Gaſſendus* affirms there are ſometimes ſome of about the $\frac{1}{10}$ part of the ☉ Semidiameter; which is the whole ſpace allowed by you for them all. And I my ſelf have ſeen of $\frac{1}{15}$ of the ☉ Semidiameter: And yet you muſt confeſs theſe great ones could only be the Conjunctions of ſome, not all. 2. There are many times ſeen in the ☉ Superficies, a great number of Spots, whoſe Diameters added together, would do more than twice fill the ſpace you ſpeak of. I my ſelf have ſeen it, and ſo I believe have you. *Gaſſendus* affirms, There are ſometimes 40 ſeen at once: If this was by Conjunction of Planets, in every Appearance, there was at leaſt 80 Bodies at once on this ſide the ☉; it may be as many on the other ſide, beſides thoſe unſeen (by your Reflection or otherwiſe) which doubtleſs muſt be far more than ſeen. For it is a moſt rare, and I think unheard of thing to ſee but 3 (which is leſs than the half) of our Planets, conjoin'd in viſible ☌ at once: So that without queſtion, if they be Planets, they are many hundreds; which muſt have ſo many ſeveral Orbs, and which certainly cannot be done in ſo narrow a compaſs, as the $\frac{1}{10}$ of the ☉ Semidiameter. And that they cannot have any larger (I ſuppoſe not ſo large an) extent from the ☉ Superficies, may be proved by their Motion, through the viſible Hemiſphere of the Sun's ſpherical Body, by comparing the Swiftneſs of their Motion towards the middle and ſides together. 6. If one of theſe (imagined) Planets be ſwifter than another, as they muſt needs be, then the ☌ of 2 or 3 ſwifter ones would make a Spot of ſpeedier motion than the ☌ of 2 ſlower ones: But the motion of all about the ☉ Center, is always equal; yea, and the Spots retain the ſame Poſition one to another, (conſidering the Sun's Sphericity, and the Angle of their appearance to us) juſt like the Fixed-Stars. So affirms *Gaſſendus, Moveri omnes eodem & uniformi motu, adeo ut, cum plures fuerint, nulla antevertat aliam, ſed eundem tenorem in diſco ☉ perinde ſervent inter ſe, ac ſervant Fixa in firmamento*'.

‘As for that other annual Motion of the Spots, you ſpeak of, from Weſt to Eaſt, upon their Axis inclined above 8 Degrees to the Ecliptick; I ſuppoſe it is not any real Motion of the Orbs of thoſe ſolar Planets or Spots, but only a viſible Motion ſo appearing, cauſed (in *Kepler*'s Syſtem) by the Sun's rolling upon its own Center in the midſt of all the Orbs, not exactly in the way of the *Temporary* Eclip-

‘ tick, but in the *Via regia* (as *Kepler* calls it) inclined certain Degrees ‘ to the *Temporary*; thereby turning about with him, the ſame way, ‘ his adventitious, or excrementitious Parts, the Spots, by his *Magne*‘*tical* or *Sympathetical Rayes.* And hence may be demonſtrated the ap‘pearance of that annual Motion in the Sun's Spots you ſpeak of. See ‘ *Galilæus, Syſt. Coſm. p.* 339, *& ſeq.* So alſo in *Ptolemy*'s and *Tycho*'s ‘ Syſtem, the ſame appearance may be demonſtrated, ſuppoſing the ⊕ ‘ fixed in the middle of the Univerſe, and the ☉ rolling round upon the ‘ ſame Poles of that *Via regia* (or way of the Spots) and keeping his ‘ Axis in Paralleliſm continually towards one and the ſame part of the ‘ Univerſe. This may be certainly demonſtrated, altho' *Galilæus* there ‘ affirms the contrary. Other *Hypotheſes* of that Motion may be feigned, ‘ as by the annual Converſion of the Poles of the *Via regia* about the ‘ Poles of the Ecliptick in the Sun's Body: But none I conceive ſo com‘pendious, as the one of the former. For my part, I incline to the ‘ firſt: yet if when we ſee you, you ſhew us any more likely Theory, ‘ for my part I ſhall be ready to conſent to you in any thing with ‘ reaſon'.

‘ Thus you have, what for the preſent I conceive of theſe *Maculæ* ‘ *Solares. Fromundus* mentions one *Jo. Tarde Gallus,* who thinks them ‘ to be Secondary Planets; who hath written a Book of that Subject, ‘ and calls them *Aſtra Borbonia:* But I could never yet ſee it. What ‘ you, or he, or others may alledge for that Opinion, I know not. In ‘ the mean time it were too much Levity in me, againſt my Judgment, ‘ to acknowledge them Stars; unleſs I ſee at leaſt ſome poſſibility how ‘ they may be ſo, or ſome probability why they ſhould not rather be ‘ Spots. Which when you, or they do produce from better grounded ‘ Reaſons, Optical Experiments, or Demonſtrations, I ſhall willingly re‘cant my Opinion'.

Plate 3.

Fig. 62. *Shews the Stages and Lines deſcribed by the Spots upon the Sun in* September *and* November 1706. *and in* February *and* March $170\frac{6}{7}$. *and in* September *and* November 1707. Fig. 63. *Shews the Stages and Lines deſcribed by the Spots upon the Sun in* January $170\frac{3}{4}$. *and in* May, June, *and* October 1705.

Eclipſe of the Sun at Cambridge, 4 *Miles from* Boſton *in* NewEngland, June 12 1694, *by* Mr. Tho. Brattle, *n.* 292. *p.* 1630.

XVII. Obſervations made of the Sun's Altitude before the Eclipſe began, in order to rectify the Watch.

	By the Watch				Comp. Altit.		Time by Calcul.			Differ.	
	h	'	"								
at	8	26	37	Mane	49	31	-8	16	40	9	57
		31	27		48	26		21	40	9	47
		38	26		47	20		28	32	9	54

The Eclipſe was firſt perceived at 9h. 25'. by the Watch, at which time the Sun had ſcarcely been eclips'd 1 Minute, ſo that

	By the Watch.		True Time.		
At	9h	24'	9h	14'	It began
	9	32	9	22	about 1 digit eclipſed
	9	48	9	38	full 3 digits
	9	57½	9	48	about 4
	10	06	9	56	near 5
	10	15	10	05	full 6
	10	33	10	23	about 8
	10	43	10	33	full 9
	10	47	10	37	full 9½
	10	53	10	43	full 10
	10	59	10	49	about 10½
	11	03	10	53	better than 10½.
	11	06	10	56	much the ſame.
	11	09	10	59	rather decreaſing.
	11	10½	11	00½	ſenſibly decreaſed near ¼ of a digit
	11	14½	11	04½	neareſt to 10 digits
	11	25	11	15	full 9 digits, *i. e.* full 3 digits reſtored, or the Shadow rather within 9 digits.
	11	29	11	19	8½ compleat
	11	34½	11	24½	full 8 digits
	11	44	11	34	full 7
	11	48	11	38	full 6½ digits
	11	52½	11	42½	juſt 6
	0	02½ P.M.	11	52½	juſt 5
	0	13	0	03 P. M.	full 4
	0	26	0	16	full 2½
	0	32	0	22	better than 2
	0	41	0	31	better than 1
	0	48	0	38	ended.

Obſervations made after the Eclipſe was done, of the Sun's Altitude, in order to rectify the Watch.

	Time by the Watch.				Comp. Altit.		True Time.			Differ.	
At	3h	31'	30"		45	52	3	21	36	9	54
		36	15		46	23		26	16	9	59
		38	10	P. M.	46	45		28	16	9	54
		46	50		48	19		36	48	10	02
		48	10		48	30		38	20	9	50

Hence it appears, that the Watch went about 10 Minutes too faſt during the whole Eclipſe, as we have all the way allowed.

So that the Eclipſe

Began at	9^h. 14′	Mane
Ended-- ———	--0 38	P. M.
Laſting in all	3 24	

In the Calculation the Latitude of *Boſton* was allow'd to be 42° 25′.

2. *Another at the ſame place,* Nov. 27. 1703. *by the ſame, ib.* p. 1634.

At 10^h	00^m	The Sun was not touch'd.
	06	The Moon entred on the S. S. W. Point as near as I could judge.
	15	The Eclipſe was conſiderably advanced.
	20	Seem'd to be about half a Digit eclips'd, rather more than leſs, and the Section to be a ſmall Matter more Weſterly.
10	25	Much the ſame, and near the ſame Point.
	30	Seem'd to be leſs.
	$33\frac{1}{2}$	The middle of the Section nearer the S. W. and the Diameter of the Section leſs every way.
	$37\frac{1}{2}$	Much leſs and nearer the Weſt.
	$44\frac{1}{2}$	It ended, and was juſt over, going off near the S. W. ſo that all the while it was within a Point or two of the place where it firſt came on, or between the S. S. W. and S. W.

I judg'd when it was at the height, that the Chord of the eclips'd part, was neareſt equal to the ſide of an inſcrib'd Decagon, or ſubtended about $\frac{1}{10}$ of the Periphery of the Sun's Disk.

Another May 1. 1706. *at* Greenwich, *by Mr.* Flamſtead, *n.* 306. *p.* 2237.

XVIII. 1. The Morning was cloudy and moiſt, till about Eight a Clock, when the Clouds began to break, and we had ſometimes a ſight of the Sun through the Spaces betwixt them. A ſeven Foot Teleſcope was fitted up with a Scene to receive the Species of the Sun caſt through it, and on which it was about ſeven Inches Diameter, divided into Digits by ſix concentrick Circles. But Clouds coming, the Sun frequently rendred this way of obſerving inconvenient, and therefore laying aſide the *Apparatus* of the Scene, I viewed him through the ſame Teleſcope with ſmoaked Glaſſes, to ſave my Eyes, and noted:

1706 *May* 1ſt *Mane.*	Time corr. by the Pend. Clock.	
	8^h. 21′ 30″	A very ſmall part of the Sun's Diameter was eclips'd.
	28 00	The Chord of the Arch of the Sun's Periphery eclipſed was 14′. 40″. then followed frequent Clouds through the Spaces betwixt; then ſome Zenith Diſtances of the Sun were taken for correcting

			recting the Clock, and afterwards near the middle of the Eclipſe.
9h.	21′	46″	The parts of the Diameter remaining clear —5 00
	26	20	—— —— —— —4 30
10	31	50	Frequent large Clouds again till the Sun appeared through the breaks, and we ſaw the Eclipſe was not ended. Clouds again till
10	33	50	When the Sun ſhone out again we ſaw his Limb entire, and the Eclipſe certainly over.

2. Mr. *Stephen Gray* had prepared a Scene placed behind his 7 Foot Glaſs, ſo that the Species of the Sun projected on it was ſeven Inches over; but having the ſame ſort of Weather we had at *Greenwich*, he ſaw not the beginning, by reaſon of Clouds, but other Phaſes with the end he noted as follows. *At* Canterbury, *ib. p.* 2238.

Correct Time by the Pend. Clock.		
8h.	53′	Digits 5 $\frac{1}{2}$ darkned
9	08	———7
	31	———10 or more
	36	—— The Sun ſhining for a ſhort time, the Eclipſe ſeem'd to decreaſe.
	55	———7 $\frac{1}{2}$ a little clearer.
	57	———6$\frac{3}{4}$
10	02	———6.
	4	———5$\frac{3}{4}$
	14	———4
	16	———3$\frac{3}{4}$
	20	———2$\frac{1}{2}$
	30	———1.
	31	———0$\frac{3}{4}$
10	36$\frac{1}{2}$	The end accurately with a Tube of 16 Foot.

3. Mr. *Abr. Sharp* caſt the Species of the Sun on a Scene-plate, behind his ſeven Foot Glaſs, ſo as it appeared ſeven Inches over. By reaſon of cloudy Weather, he ſaw neither the beginning nor end; But other Phaſes near the middle, as follows. *At* Horton, Yorkſhire, *ib. p.* 2239.

Times correct by the Pend. Clock.				
8h.	35′	00″	digits dark 3	By Ocular Eſtimation.
9	01	00	—— ——7	
	4	54	—— —— 8 $\frac{3}{10}$	Eclipſed on the Scene
	6	33	—— ——8 $\frac{1}{2}$	
	7	53	—— ——8 $\frac{6}{10}$	
	12	50	—— ——9	
	16	08	—— ——9 $\frac{4}{10}$	

18h. 48′ ——— 9½ exactly, the Sun ſhining out clear.
20 45 ——— 9½ the Sun ſtill ſhining clearly. Greateſt Obſcurity.
21 48 ——— 9½ ſtill clear.
28 46 ——— 9
44 45 ——— 7
54 42 ——— 5½
10 06 10 ——— 3½
19 55 ——— 1 preciſely.
24 00 The Sun ſeen through the Clouds, the Eclipſe not ended.
30 00 The Sun ſeen again perfectly round and entire.

At Bern, ib. p. 2240.

4. Captain *Stannyan*, who was there with his Kinſman, Her Majeſty's Envoy, writes the ſame Day to me, 'That the Sun was totally darkned 'there for 4½ Minutes of Time; that a fixt Star and a Planet appeared 'very bright; and that *his getting out of the Eclipſe was preceded by a 'Blood red ſtreak of Light, from its left Limb; which continued not longer than 6 or 7 Seconds of Time*; then part of the Sun's Diſk appear'd, 'all of a ſudden, bright as *Venus* was ever ſeen in the Night; nay, 'brighter; and in that very inſtant gave a Light and Shadow to things, 'as ſtrong as Moon-light uſes to do'.

The Captain is the firſt Man I ever heard of that took notice of a red ſtreak of light preceding the Emerſion of the Sun's Body from a total Eclipſe. And I take notice of it to you, becauſe it infers that *the Moon has an Atmoſphere*; and its ſhort continuance of only 6 or 7 Seconds of Time, tells us that *its height is not more than the 5 or 6 hundredth part of her diameter*.

At Geneva, by Mr. Facio Duiller, ib. p. 2241,

5. The total Eclipſe of the Sun, which happened on the 12th of *May*, 1706. N. S. did preſent to the Inhabitants of *Geneva* a magnificent and ſurprizing Sight. The more learned did obſerve that Eclipſe with much Satiſfaction: But it did ſtrike many of the Common People with a great deal of Terror. A little after the Sun's riſing, the Sky did ſeem clear; though the Air was thick already with ſome Vapours. Many little Clouds did afterwards ariſe here and there, and the Vapours did much encreaſe. For want of a Pendulum Clock, in a convenient place, the Moment of the total Immerſion, the Moment of the firſt Emerſion, and that of the end of the Eclipſe, could not be accurately obſerved. Tho' the Sky was ſomewhat overcaſt, the heat of the Sun was already felt, when the Eclipſe did begin: But a very ſenſible Coldneſs took place, as the Moon did, by degrees, cover a greater and greater part of the Sun, and the light decreaſe. The Eclipſe was obſerved only with ſome Glaſſes, either darkned with Smoak, or but little tranſparent; and by receiving the Sun's Image, through a ſix Foot Teleſcope, which repreſented the Objects inverted, upon a white Paper, placed at ſome diſtance from the Eye-Glaſs. When the Sun was near being totally dark, the bright Creſcent which did remain, was ſeen to diminiſh more and

more, upon the Paper, where its Image was received. And when that Creſcent was reduced to a very narrow Breadth, and to a very little Length, it was ſeen of a ſudden to diſappear: And in that Moment the whole Sun was eclipſed. At the ſame inſtant of Time, the Darkneſs, which was already very conſiderable, did become much greater. The Clouds did change of a ſudden their Colour, and became red, and then of a pale Violet. There was ſeen, during the whole Time of the total Immerſion, a whiteneſs, which did ſeem to break out from behind the Moon, and to encompaſs it on all ſides equally. The ſame whiteneſs was but little determined, in its outward ſide, and was not broad the twelfth part of the Diameter of the Moon. This Planet did appear very black, and her Disk very well defined, within the whiteneſs, which encompaſſed it about, and whoſe Colour was the ſame with that of a white Crown or *Halo*, of about four or five degrees in Diameter, which accompanied it, and had the Moon for its Center. The Star of *Venus* was ſeen, at the ſame time, at ſome diſtance, without that Crown, between the Eaſt and N. E in Reference to the Sun. The Planets of *Saturn* and *Mercury* were ſeen alſo by many, Eaſtward from the Sun's place. And if the Sky had been clear, many more Stars might have been ſeen, and with them the Planets of *Jupiter* and *Mars*; that towards the Eaſt, and this toward the Weſt: And ſo the ſeven Planets might have been ſeen, almoſt all at once. Accordingly ſome Gentlewomen, being in the Country, did tell, as is ſaid, more than ſixteen Stars. And many People, which were on the neighbouring Mountains, did ſee the Sky ſtarry, in ſome places, where it was not overcaſt, as during the Night, in the time of the full Moon. The total Immerſion did begin about three Quarters paſt Nine. The Duration of the total Darkneſs was preciſely three Minutes, or 180 Seconds, to the Moment that the firſt Ray of the Sun did begin to appear again, with much Brightneſs. And this time was obſerved with a ſimple Pendulum; which was afterwards compared with a Pendulum Clock, ſhewing the Seconds, and regulated upon the mean Motion of the Sun. The Council was met during the time of the Eclipſe; but they did riſe from their Seats, a little before the total Obſcuration; becauſe one could neither read nor write. They perceived as they came down the Stair-caſe of the Town-Houſe, ſome Swallows amazed, looking for a reſting place; and many Bats flying out. In other places the Hens and Pigeons would make haſte towards their Houſes.

A little time after the Sun had began to appear again, the whiteneſs and the Crown, which did encompaſs the Moon, did entirely vaniſh. The Sun did then ſhew it ſelf more and more; appearing at firſt as a little Creſcent, which did ſtill increaſe; and whoſe Concave ſide did ſeem terminated, as by an Arch, deſcribed with the Compaſs. A little before the total Obſcuration, the Country on the Weſt ſide did already ſeem overcaſt with Darkneſs; and afrer the total Obſcuration, the Darkneſs was ſeen to leave us more and more, and to fly Eaſtward. According to Mr. Profeſſor *Gautier*'s Obſervations, from the firſt Emerſion of the Sun, to the end of the Eclipſe, there was 1h 9′ 30″. As to the accurate times, they are uncertain, the Pendulum Clock having been ſet only by a ſmall Sun Dial. I ſend you alſo the following Account, which the ſame Gentleman did communicate to me.

‘ Obſervations on the Eclipſe of the Sun, of the 12th of *May*, 1706,
‘ made at *Marſeilles*, in the Obſervatory of the *Jeſuits* of St. *Croix*;
‘ by Monſieur *Chazelles*, Ingineer of the Galleys, and by Father *Laval*,
‘ *Jeſuit*, Royal Profeſſor of Hydrography.
‘ The Eclipſe did begin at — — — — — — 8h. 28′ 40″
‘ It did reach the Sun's Center at — — — — 9 6 11
‘ It was total at — — — — — — — 9 34 15
‘ The Sun did begin to appear again at — — — 9 37 9
‘ The Eclipſe did come again to the Center at — — 10 12 23
‘ It did entirely end at — — — — — 10 47 50
‘ Three Stars were diſtinctly ſeen; and during three Minutes it was
‘ not poſſible to read. And there did remain one bright Digit, all about
‘ the Globe of the Moon’.

The Mannor Houſe of *Duillier* is in the Latitude of 46° 24′. in Longitude it is 4° 13′ 45″ to the Eaſtward of the *Royal Obſervatory* at *Paris*. And St. *Peter*'s Church at *Geneva*, is in Latitude 0° 12′ to the Southward, and in Longitude 0° 5′ 2″ to the Weſtward of *Duillier*. But of this another time.

According to theſe Obſervations, the Altitude of the Moon's Atmoſphere cannot well be ſuppoſed leſs than of 130 Miles, in perpendicular height: of which Miles 60 go to one Degree of the Earth. Neither could that Atmoſphere be diſcovered, before the time of this Eclipſe, by any Refraction of the Stars: Probably becauſe of this Refraction's ſmallneſs; and for want of proper Obſervations. And though it was very plain that the Atmoſphere of the Moon muſt needs ſhew it ſelf, in the time of a total Eclipſe of the Sun; yet I do not know that any body did think of this, till in the laſt Month of *May*, many Perſons did actually ſee it.

Some particular Obſervations, which are intended to be made publick, do evince that our Atmoſphere is ſometimes viſible, all along, from the Surface of the Earth to the perpendicular height of one Semidiameter of the Terreſtrial Globe. And the continued appearance of a Crown, of only four or five degrees Diameter, about the Sun, during the whole time of the total Obſcuration, does ſhew that the Æthereal Matter, in which that Crown was produced, muſt be at a very great height above the Surface of the Earth. But if that Crown was to be ſeen, ſo far as the Weather did permit, in all the Places where the Eclipſe was total, it muſt be concluded, that the Cauſe of it was not in our Air, but in ſome Vapours incompaſſing the Sun: and probably, in thoſe very Vapours, which produce that pointed Light, that has been obſerved lying in a manner along the Ecliptick, and that has the Sun for Center. Now either of theſe Concluſions, *viz.* concerning the great height of the parts of our Atmoſphere, capable of producing that Crown, or elſe concerning a Meteor obſerved, not in our Air, but in the Vapours that incompaſs the Sun, is very ſingular, and deſerves a great deal of Attention. If ever ſuch another appearance ſhould be ſeen, in the time of a total Eclipſe, it would be proper to obſerve accurately the leaſt Diameter of the Crown, from inſide to inſide: And to take notice whether, during the whole time of the total Immerſion, the inward Circle be every where continued, and of an uniform Figure. The leſs the ſaid Diameter, and the greater the Exceſs of the Moon's apparent Diameter above that of the Sun; as alſo the greater the apparent Altitude of the Sun is above the Horizon; the higher the Cauſe which produces the Crown muſt be above the Surface of the Earth. And the Poſition, upon the Moon's Disk, in reference to the Zenith, of the Points of Contact, where the Sun diſappears, or begins to ſhew it ſelf again, is here alſo of ſome Conſideration.

At Zurich, *by Dr.* Scheuchzer, *ib. p.* 2246.

6. Habuimus die 12. *Maii* Eclipſin Solis totalem ſimul & annularem; totalem, quoniam Sol integer a Luna fuit obtectus; annularem autem non proprie ita dictam, ſed per Refractionem, quandoquidem circa Lunam fulgor apparuit rutilans, a radiis per Atmoſphæram Lunæ refractis ortus.

Initium Eclipſeos fuit mane hora	8. 54′.
Medium hora	9. 58′.
Finis hora	11. 12′.
Mora mediæ & plenæ obſcurationis	4′.

XIX. *Another at* Upminſter, Sept. 3. 1708. *by Mr.* Derham, *n.* 320. *p.* 312.

The correct apparent Time.	
	The Beginning we could not ſee for Clouds.
6^{h}. 44′ 15″	The Sun peep'd out of the Clouds, and I judg'd by my Eye, that about one Tenth of a Digit was eclips'd.
	Then Clouds nearly all the time of the Eclipſe.
8 31 15	A little Obſcuration appear'd through the Teleſcope.
8 32 45	A very little Obſcuration through the Teleſcope.
	Then Clouds. And at
8 35 45	We could diſcern no Remains of the Eclipſe through the Teleſcope.

From theſe Obſervations I imagine the end was much about 8^{h}. 33′ in the Morning.

Another April 22. 1715. *by Dr.*Halley. *n.* 343. *p.* 245.

XX. Though it be certain from the Principles of Aſtronomy, that there happens neceſſarily a Central Eclipſe of the Sun in ſome part or other of the Terraqueous Globe, about twenty eight times in each Period of eighteen Years; and that of theſe no leſs than eight do paſs over the Parallel of *London*, three of which eight are total with continuance: yet from the great variety of the Elements whereof the *Calculus* of Eclipſes conſiſts, it has ſo happened that ſince the 20th of *March*, *Anno Chriſti* 1140, I cannot find that there has been ſuch a thing as a total Eclipſe of the Sun ſeen at *London*, though in the mean time the ſhade of the Moon has often paſt over other Parts of *Great Britain*.

I received the Orders of the Society to provide for the Obſervation of this Eclipſe to be made at their Houſe in *Crane-Court*, and accordingly I procured a *Quadrant* of near 30 Inches *Radius*, exceedingly well fixt with Teleſcope Sights, and moved with Screws ſo as to follow the Sun with great Nicety; as alſo a very good *Pendulum* Clock well adjuſted to the mean Time, and ſeveral Teleſcopes to accommodate the more Obſervers.

In order to examine both Clock and Quadrant, I, on the 20th of *April*, obſerved the diſtance of the upper Limb of the Sun from the *Zenith* 36°. 16′, and the next Day 35°. 58′; by which it appeared that the diſtances from the *Zenith* taken by this Quadrant ought to be encreaſed by about one Minute: and that Allowance being made, by ſeveral Obſervations taken before and after Noon on the ſaid 21ſt Day, the Clock was found to anſwer the apparent Time or Hour of the Sun with ſufficient Exactneſs, as not going above 10″ too faſt. The next Day *April* 22°, juſt before the Eclipſe began, we took three Diſtances of the Sun from the *Zenith*, *viz.* at 7^{h}. 42′. 52″. A. M. the correct diſtance of the Sun's Center *a vertice* was 62°. 1′. 40″. at 7^{h}. 45′. 48″. it was 61°. 34′. 40″. And again at 7^{h}. 48′. 55″. it was 61°. 6′. 40″: which with the given Declination of the Sun and Latitude of the Place ſhew the true Times reſpectively to have been 7^{h}. 42′. 38″, 7^{h}. 45′. 35″. and 7^{h}. 48′. 39″: all concurring that the Clock was only 14 Seconds too faſt, and had gained ſcarce any thing ſenſible in a Day's time: ſo that it might be entirely depended upon during the continuance of the Eclipſe.

Having computed that the Eclipſe would begin at 8^{h}. 7′, I attended ſoon after eight with a very good Teleſcope of about ſix Foot, without ſtirring my Eye from that part of the Sun whereat the Eclipſe was to begin: and at 8^{h}. 6′. 20″. by the Clock, I began to perceive a ſmall Depreſſion made in the Sun's Weſtern Limb, which immediately became more conſpicuous; ſo that I concluded the juſt Beginning not to have been above 5″ ſooner; that is, exactly at 8^{h}. 6′. 00″ correct Time.

From this time the Eclipſe advanced, and by nine of the Clock was about ten Digits, when the Face and Colour of the Sky began to change from perfect ſerene azure blew, to a more dusky livid Colour, having an Eye of Purple intermixt, and grew darker and darker till the total Immerſion of the Sun, which happened at 9^{h}. 9′. 17″. by the Clock, or 9^{h}. 9′ 3″. true time. This Moment was determinable with great nicety, the Sun's Light being extinguiſhed at once; and yet more ſo was that of the Emerſion, for the Sun came out in an Inſtant with ſo much Luſtre that it ſurprized the Beholders, and in a Moment reſtored the Day, *viz.* at 9^{h}. 12′. 26″. true time, after he had been totally obſcured for 3′. 23″ of Time. And as near as I could eſtimate the Points on the Moon's Limb; where the laſt Particle of the Sun vaniſhed was about the middle of the *South Eaſt* Quadrant of her Limb, or about 45 Degrees from her *Nadir* to the Left-hand: And the firſt Emerſion was about ten Degrees below the Horizontal Line through the Moon's Center on the Weſt ſide; and at 14 Minutes paſt nine, correct Time, I judg'd the Horns of the Eclipſe to have been exactly perpendicular, and by Conſequence, the Centers of the Sun and Moon to be in equal Altitude.

It was univerſally remarked, that when the laſt part of the Sun remained on his Eaſt ſide, it grew very faint, and was eaſily ſupportable to the naked Eye, even through the Teleſcope, for above a Minute of Time before the total Darkneſs; whereas on the contrary, my Eye could not endure the Splendor of the emerging Beams in the Teleſcope from the firſt Moment. To this perhaps two Cauſes concurred; the one, that the Pupil of the Eye did neceſſarily dilate it ſelf during the Darkneſs, which before had been much contracted by looking on the Sun. The other, that the Eaſtern parts of the Moon, having been heated with a Day near as long as thirty of ours, could not fail of having that part of its Atmoſphere replete with Vapours, raiſed by the ſo long continued Action of the Sun; and by conſequence it was more denſe near the Moon's Surface, and more capable of obſtructing the Luſtre of the Sun's Beams. Whereas at the ſame time the Weſtern Edge of the Moon had ſuffered as long a Night, during which there might fall in Dews all the Vapours that were raiſed in the preceeding long Day; and for that reaſon, that part of its Atmoſphere might be ſeen much more pure and tranſparent. But from whatever Cauſe it proceeded, the thing it ſelf was very manifeſt and noted by every one.

About two Minutes before the total Immerſion, the remaining part of the Sun was reduced to a very fine Horn, whoſe Extremities ſeemed to loſe their Acuteneſs, and to become round like Stars. And for the ſpace of about a quarter of a Minute, a ſmall Piece of the Southern Horn of the Eclipſe ſeemed to be cut off from the reſt by a good interval, and appeared like an oblong Star rounded at both Ends in this Form. which appearance could proceed from no other Cauſe but the Inequalities of the Moon's Surface, there being ſome elevated parts thereof near the Moon's Southern Pole, by whoſe Interpoſition part of that exceedingly fine Filament of Light was intercepted.

A few Seconds before the Sun was all hid, there diſcovered it ſelf round the Moon a luminous Ring, about a Digit, or perhaps a tenth Part of the Moon's Diameter in Breadth. It was of a pale whiteneſs, or rather Pearl colour, ſeeming to me a little tinged with the Colours of the *Iris*, and to be concentrick with the Moon, whence I concluded it the Moon's Atmoſphere. But the great height thereof far exceeding that of our Earth's Atmoſphere; and the Obſervations of ſome, who found the Breadth of the Ring to encreaſe on the Weſt ſide of the Moon, as the

Emerſion approached; together with the contrary Sentiments of thoſe whoſe Judgment I ſhall always revere, makes me leſs confident, eſpecially in a Matter whereto I own I gave not all the Attention requiſite.

Whatever it was, this Ring appeared much brighter and whiter near the Body of the Moon than at a diſtance from it; and its outward Circumference, which was ill defined, ſeemed terminated only by the extream rarity of the Matter it was compoſed of; and in all Reſpects reſembled the Appearance of an enlightned Atmoſphere viewed from far; but whether it belonged to the Sun or Moon I ſhall not at preſent undertake to decide.

During the whole time of the total Eclipſe I kept my Teleſcope conſtantly fixt on the Moon, in order to obſerve what might occur in this uncommon appearance: and I found that there were perpetual Flaſhes or Coruſcations of Light, which ſeemed for a Moment to dart out from behind the Moon, now here, now there, on all ſides; but more eſpecially on the weſtern ſide before the Emerſion; and about two or three Seconds before it, on the ſame weſtern ſide where the Sun was juſt coming out, a long and very narrow Streak of a dusky but ſtrong red Light ſeemed to colour the dark Edge of the Moon; though nothing like it had been ſeen immediately after the Immerſion. But this inſtantly vaniſhed upon the firſt appearance of the Sun, as did alſo the aforeſaid luminous Ring.

As to the Degree of Darkneſs, it was ſuch that one might have expected to have ſeen many more Stars than I find were ſeen at *London*: The three Planets, *Jupiter*, *Mercury* and *Venus* were all that were ſeen by the Gentlemen of the Society from the top of their Houſe, where they had a free Horizon: and I do not hear that any one in Town ſaw more than *Capella* and *Aldebaran* of the fixed Stars. Nor was the Light of the Ring round the Moon capable of effacing the Luſtre of the Stars, for it was vaſtly inferiour to that of the Full Moon, and ſo weak that I did not obſerve that it caſt a Shade. But the under Parts of the Hemiſphere, eſpecially in the *South Eaſt*, under the Sun, had a crepuſcular Brightneſs: and all round us, ſo much of the Segment of our Atmoſphere as was above the Horizon, and was without the Cone of the Moon's Shadow, was more or leſs enlightened by the Sun's Beams: and its Reflection gave a diffuſed Light which made the Air ſeem hazy, and hindred the Appearance of the Stars. And that this was the real Cauſe thereof, appears by the Darkneſs being more perfect in thoſe Places near which the Center of the ſhade paſt, where many more Stars were ſeen, and in ſome not leſs than twenty; though the Light of the Ring was to all alike.

During the Time whilſt the Sun recovered his Light, ſeveral Altitudes were taken to examine the Regularity of the Clock's Motion; and though the Sun now roſe much ſlower than at the beginning, yet they all conſpired within a very few Seconds that the Clock went ſtill one Quarter of a Minute too faſt. And the end of the Eclipſe approaching, I attended the Moment thereof with all the Care I could, and concluded the compleat Separation of the Sun and Moon at 10h. 20′. 15″. by the Clock, or exactly 10h. 20′. correct time. The Lord Chief Juſtice *Parker*, at the Moment of the Emerſion from total Darkneſs, obſerv'd the diſtance of the Planet *Jupiter* from the *Zenith* 48°. 29′. by which the Time thereof is verified.

Monſieur *le Chevalier de Louville* having ſeen the Beginning, applied himſelf to take Digits with his Micrometer, and to obſerve the Occultations of three Spots at that time ſeen in the Sun; and he was pleaſed to communicate the following Notes, *viz.*

At 8h.	28′	20″	Four Digits were Eclipſed.
8	32	57	The firſt and bigger Spot touched the Moon.
8	33	18	The ſame was wholly hid.
8	34	08	The firſt of the two leſſer Spots was hid.
8	34	58	The Second of them was hid.
9	36	01	Emerſion of the greater Spot.
9	38	26	Emerſion of the firſt leſſer Spot.
9	40	25	Emerſion of the ſecond leſſer Spot.
10	20	04	The end of the Eclipſe.

And he determined the time of the total Darkneſs 3′. 22″, or one Second leſs than by my Account.

The Heavens were all the while very propitious to us, and there was very little or no Wind, and not ſo much as one Cloud interrupted our View from the Beginning to the end; but no ſooner was the Eclipſe over, but a great Body of Clouds hid the Sun for many Hours after.

The Reverend Mr. *James Pound*, Rector of *Wanſted* in *Eſſex*, and R.S.S. gives the following Account of the principal Phænomena obſerved there; having rectifyed his Clock by ſeveral Altitudes of the Sun taken both before and after, *viz.*

At 8h.	6′.	37″	The Eclipſe firſt perceived.
9	9	28	The total Immerſion.
9	12	48	The Emerſion.
10	20	32	The juſt end of the Eclipſe.
0	3.	20	The Continuance of total Darkneſs.

The near Agreement of this Obſervation with our own (the difference being only what is due to the difference of our Meridians) makes us the leſs ſolicitous for what was noted at the *Royal Obſervatory* at *Greenwich*, from whence we can only learn that the Duration of total Darkneſs was 3′. 11″.

Mr. *William Derham* Rector of *Upminſter* in *Eſſex*, and R. S.S. made the following Obſervations there,

At 8h.	7′.	41″	The Eclipſe began.
8	33	46	The Moon touched the greater Spot.
8	34	36	She touched the middle Spot.
8	35	41	She touched the third Spot.

At 9^{h}.	10′	58″	The total Darkneſs began on a ſudden, and *Aldebaran* appeared.
9	14	6	The Emerſion or end of total Darkneſs.
0	3	8	Continuance of total Darkneſs.
9	42	41	The third and laſt Spot diſcovered.
10	21	45	The End of the Eclipſe, by a 13½ Foot Glaſs.

And a little before the Beginning of the Eclipſe, he found the greater and preceeding Spot to be more Northerly than the Sun's Center 373½ ſuch parts as the Sun's Diameter was 1647, and that it followed his weſtern Limb 0′. 43″ of Time: by which *data* the Situation of that Spot is well determined.

My worthy Collegue Dr. *John Keill* by reaſon of Clouds ſaw nothing diſtinctly at *Oxford* but the end, which he obſerved at 10^{h}. 15′. 10″. As to the total Darkneſs, he could only eſtimate it by the ſudden Change of the Light of the Sky; and reckoned its Continuance but 3′. 30″; which was certainly too little, the Center of the ſhadow having without doubt paſt very near *Oxford*. And Mr. *Cotes* at *Cambridge* had the misfortnne to be oppreſt by too much Company, ſo that he obſerved only the Occultations of the three Spots, *viz.* of the firſt and greateſt at 8^{h}. 34′. 11″. of the ſecond at 8^{h}. 35′. 15″, and of the laſt at 8^{h}. 36′. 55″. He noted alſo the end of total Darkneſs at 9^{h}. 14′. 37″, and the exact end of the Eclipſe at 10^{h}. 21′. 57″.

We have received ſeveral Accounts from ſome Places which lay near the Track of the Center of the Shade, and which might have been very proper to determine the greateſt Continuance of the darkneſs; as from *Plymouth*, *Exeter*, *Weymouth*, *Daventry*, *Northampton* and *Lynn regis*, all agreeing that the whole Sun was obſcured at thoſe Places full four Minutes, and at ſome of them rather more. But theſe Obſervers give us no Account how they meaſured this Time, and therefore it may well be ſuppoſed they took it in a round Number, and perhaps from pocket Minute-Watches. What I think may beſt be relied on for this Purpoſe, are two correſponding Obſervations made, the one at *Barton* near *Kettering* in *Northamptonſhire*, where by the Obſervation of *John Bridges*, Eſq; Treaſurer of His Majeſty's Revenue of Exciſe, and R. S. S. with a good Pendulum-Clock and all due Care, the whole Sun was hid no more than 3′ 53″. The other was by Mr. *John Whiteſide*, A. M. Keeper of the *Aſhmolean Muſeum* at *Oxford*, and a skilful Mathematician, who obſerved after the ſame manner, at *King's-Walden* in *Hertfordſhire* near *Hitchin*, that the total Eclipſe continued but 3′. 52″. Hence it follows that the Center of the ſhade paſt near the middle between theſe two Places, which are but 30 Geographical Miles aſunder, and ſituate near at right Angles to the Way of the ſhade, and therefore that the total Obſcurity, where longeſt, could laſt but about 3′. 57″, or perhaps a Second or two more at *Lynn*, and leſs at *Plymouth*: the Velocity of the Progreſs of the ſhade gradually decreaſing, and its Diameter encreaſing as it paſt on to the Eaſtwards. And this Situation of the middle Line is confirmed by an Obſervation made at the Seat of the Right Honourable the Lord *Foley* at *Witley* eight Miles beyond *Worceſter*, by his Order, whereby it appears that the total Darkneſs laſted there $3'\ 15''$. Hence it follows that *Witley* was about three or four Miles farther from the Center of the Shade on the Northſide than *London* on the South; and *Witley* being by *Ogilby*'s Menſurations, 118 meaſured Miles from *London*, it is plain that the Center paſt over *Iſlip*, which is, by the ſame Admeaſurement, 57 ſuch Miles on that Road, and about five Miles almoſt due North from *Oxford*; ſo that the Center of the Shade left *Oxford* but very little upon the Right-hand. This Situation agrees perfectly well with the former between *Barton* and *King*'s-*Walden*, and as far as the Geography of our Country may be relied on, I conclude the Center to have entred upon *England* about *Plymouth*, and to have paſt over *Exeter*, the *Devizes*, *Iſlip*, *Buckingham* and *Huntington*, leaving *Oxford* and *Bedford* on the Right, and *Lynn* on the Left, and to have quitted the Coaſt of *Norfolk* about *Wells* and *Blakeney*.

As for the Limits of the Shade, both on the North and South ſide, we have by Enquiry gotten them with all the Exactneſs the thing is capable of, and we ſhould have been glad the *French Aſtronomers* had done the like for the total Eclipſe that paſt over *Languedoc*, *Provence* and *Dauphiny* on the 1ſt of *May* 1706. But as this is the firſt Eclipſe of this kind that has been obſerved with the Attention the Dignity of the Phænomenon requires, we hope thoſe which may happen for the future to traverſe *Europe*, may not paſs by ſo little regarded as hitherto.

As to the Southern Limit or Term where the Eclipſe ceaſed to be total on the South ſide of the Sun, we have received an Account of an Obſervation made at *Nortoncourt* about ten Miles on this ſide *Canterbury*, by Dr. *John Harris*, aſſiſted by Mr. *Stephen Gray*; by which we learn that the Eclipſe began there at $8^h.\ 8'.\ 55''$. and ended at $10^h.\ 24'.\ 47''$; and that the total Darkneſs continued but about one Minute or rather leſs, the middle thereof being at $9^h.\ 13'.\ 52''$. From this Duration it will follow that *Norton-court* was but about 3 or 4 Miles within the Shade. And that it was really ſo is confirmed by the Relation of the Inhabitants of *Bocton*, about Midway between *Norton-court* and *Canterbury*, who aſſured Mr. *Gray*, as he was returning home that ſame Day, that the Eclipſe was not total there, but as one of them expreſt it, before the Sun had quite loſt his Light on the Eaſt-ſide he recovered it on the Weſt: and that there was a ſmall Light left on the lower part of the Sun that appeared like a Star. And from *Cranbrook* in *Kent*, we are inform'd, by the Relation of *William Tempeſt*, Eſq; that he obſerved there the Sun to be extinguiſhed but for a Moment, and inſtantly to emerge again: So that the Limit paſt exactly over this Town, which is about 38 Geographical Miles from *London*, and very near the right Angle where the Perpendicular from *London* falls on the Line of the Limit, being $3'.\ 00$ of Time to the Eaſtwards of *London* in the Latitude of $51°.\ 6'$, as near as I can gather.

How it paſt over *Suſſex* we have not ſo authentick Relations, but have learnt it was total at *Wadhurſt* beyond *Tunbridge-Wells*, as alſo for ſome ſhort time at *Lewis*; but it was not ſo at *Brightling*, which Place being ſituated on an Eminence that has a commanding Proſpect, all the Country to the Northward was ſeen in Darkneſs, whilſt they there had ſome Benefit of a ſmall Remainder of the Sun.

From theſe Obſervations we may conclude that this Limit came upon the Coaſt of *England*, about the middle, between *Newhaven* and *Brighthelmſton* in *Suſſex*, and paſſing by *Cranbrook* and *Bocton*, left *Canterbury* about four Miles on the Right-hand, and quitted the Coaſt of *Kent*, not far from *Hern* toward the antient *Regulbium*, now called *Reculver*. So that it ſeems ſcarce one third part of *Kent*, and not ſo much of *Suſſex*, out of all the South Coaſt of *Great Britain*, eſcaped being involved in this Darkneſs.

The Northern Limit, having paſt over a much greater Space, has had more Obſervers, and is not leſs curiouſly determined than the other. We find by the account given by Mr. *Roger Proſſer*, Rector of *Haverford-Weſt*, that the Eclipſe was total there a Minute and half, whence it follows that *Haverford* was but about 6 Miles within the Limit; and therefore that it entred on *Pembrokeſhire* about the middle of St. *Bride*'s *Bay*, leaving St. *David*'s and *Cardigan* on the Left-hand: and having traverſed thoſe two Counties and *Montgomeryſhire*, it entred on *Shropſhire*, leaving the Town of *Shrewsbury* 1'. 40". in the ſhadow, as was obſerved there by Dr. *Hollings*: whereby it appears that *Shrewsbury* was about 8 Miles within the Limit. Thence it proceeded by the Eaſt-ſide of *Cheſhire*, leaving *Whitchurch* and *Nantwich* a very little without; and paſſing by *Congleton* went over the *Peak* of *Darbyſhire* into *Yorkſhire*, and croſt the great Northern Road between *Pontefract* and *Doncaſter*, ſomewhat nearer the former than the latter. For by the Obſervations of *Theophilus Shelton*, Eſq; at *Darrington*, about two Miles on this ſide *Pontefract*, (in Lat. 53°. 40' and Long. Weſt from *London* 4'. 40". of time, as may be concluded from *Norwood*'s Meaſure of a Degree) the Sun at 9^{h}. 11'. was reduced almoſt to a Point, which both in Colour and Size reſembled the Planet *Mars*; but whilſt he watched for the total Eclipſe, that Point grew bigger, and the darkneſs diminiſhed; whence he argued the Limit to have been very little more Southerly; and ſince he has been informed that it was juſt total in *Barnſdale*, three Miles South from thence. And that it was ſo at *Badſworth* about the ſame diſtance from *Darrington*, we are told by a Letter of Mr. *Daubuz*, that he has a certain Account from that Place, that the luminous Ring round the Moon was ſeen there, which was no where viſible but while the Eclipſe was Total. From theſe *Data* we may ſecurely determine the Remainder of this Track, and that the edge of the ſhadow having paſt over the reſt of *Yorkſhire*, went off to Sea about *Flamborough-head*.

So that of the forty Counties into which *England* is ſubdivided, only the five moſt Northerly have not had the Sun wholly hid from them;

and ſix others have eſcaped but in part, *viz. Shropſhire, Cheſhire* and *Yorkſhire*, and the extream part of *Darbyſhire* on the *North*, and *Kent* and *Suſſex* on the *South*; all the reſt of the Kingdom having more or leſs ſuffered an Interval of total darkneſs.

I ſhall not at preſent conſider this Eclipſe as univerſal, but only as it related to *England*; and it ſhall ſuffice to ſay, that the ſhadow came out of the *Atlantick* Ocean, having paſt over the Iſlands *Azores*; and that the Southern Limit thereof reach'd the Iſle of *Uſhant*, and the North-weſt Coaſts of *Britanny* between *Breſt* and *Morlax*; and dividing our Iſlands of *Guernſey* and *Jerſey*, juſt touched upon the Promontory of *Normandy* called *Cape de Hague*. And that after it had quitted *England* and traverſed the *German Ocean*, it fell on *Jutland* on the Southſide, and *Norway* on the *North*; and thence proceeded to the Eaſtwards over *Sweden*, *Finland*, &c.

It remains now to conſider the Figure, Poſition, Direction, Velocity and Magnitude of the ſhadow as it paſt over us. And firſt as to the Figure, 'tis obvious that the ſhadow of the Moon being a Cone, and the Earth's Surface ſufficiently Spherical, the apparent ſhadow on the Earth will be the common Interſection of a Cone and Sphere, which is a Figure hitherto little conſidered by Geometers; and not being *in Plano* is not to be exactly deſcribed but in the Spherical or Conical Surface. How to find the Points of this Curve in all Caſes is taught by *P. Courſier*, in a very ſcarce Latin Book, printed at *Dijon* in *Burgundy*, and publiſhed at *Paris* in the Year 1663: nor do I hear of any other Author that has handled the ſame Subject ſince, though capable and worthy of further Improvement. By what he there delivers, *Prop.* 11. 12. Lib. I. it will be eaſily underſtood, that the Convexity of ſo ſmall a part of the Earth's Surface as the ſhadow commonly occupies, can produce but an inconſiderable Effect; ſo that without ſenſible Error we may take it for a Plain, and the Section for a true *Apollonian Ellipſis*, whoſe tranſverſe *Axis*, by reaſon of the ſmallneſs of the Angle of the Cone, will be to its Conjugate nearly as *Radius* to the *Sine* of the Sun's Altitude at its Center, eſpecially if he be conſiderably elevated. But when he is near the Horizon, it will be neceſſary to have regard to the true Figure, by reaſon of the great length to which the tranſverſe Axe is extended, and particularly when the ſhade is entring upon or leaving the Earth's Disk. Of theſe perhaps a fuller Account may be given upon a further occaſion.

As to the Poſition of the *Axis* of the ſhadow, it is manifeſt that it muſt always lie in the Plane of a great Circle of the Earth paſſing through the *Axis* of the Cone of the Shade: and therefore it will be only requiſite to obtain the Azimuth and Altitude of the Sun at the Place where the Center of the ſhade at any time is found, to determine the Situation of the Axe and *Species* of the Ellipſe required. Thus the middle of the Eclipſe at *London*, having been obſerved at 9h. 10'. 45", by the given Latitude and Declination we find his Azimuth about 59°. 00'.

and Altitude 40°. 46′. that is juſt 40 Degrees high at the Center of the Shadow. Wherefore the tranſverſe Axe of the Ellipſe was to its Conjugate very near as *Rad.* to the *Sine* of 40°, or as 1000 to 643 *proxime*; and did make an Angle of 59°, or very little more, with the Meridian paſſing at that time through the Center of the Shade.

Next the Direction and the Velocity of the Motion wherewith the Center of the ſhade paſt over *England* comes to be conſidered, wherein the Reader is to be told that the ſhadow paſſes in a very compound Curve, which as the former, is not *in plano*, and only deſcribable on the Surface of the Sphere: nor is its Motion equable, but compounded of very many Elements producing a great Variety. By what Method its Points, and its Tangents in thoſe Points, are to be obtained, I reſerve to the next Opportunity; this Account being deſigned for the Curious in general: only I muſt acquaint them, that for ſo ſmall a part of the Curve as went over *England*, it may be eſteemed a right Line, with more Exactneſs than we uſually find in moſt of our Geographical Charts. And the like may be ſaid for the Velocity, which though in our preſent Inſtance it was continually decreaſing, may for ſo ſhort a time be ſuppoſed to have been the ſame without ſenſible Error.

By a careful Calculation I have determined the Velocity of the Motion, at the time of the middle of the Eclipſe at *London*, to have been 29 Geographical Miles in a Minute of Time *quam proxime*: and that its way made an Angle of 52°. 45′ with the Meridian towards the Eaſtwards of the North; wherefore the ſaid way made an Angle with the Axis of the Ellipſis of 68°. 15′. And the greateſt Duration of total Darkneſs having been 3′, 57″, (as was before ſhewn) it will follow, that that Diameter of the Elliptick Figure according to which the ſhade paſt, was no leſs than 114½ Geogr. Miles. And from the Elements of the Conicks 'tis eaſy to be proved, that ſuppoſing the Figure of the ſhade a true Ellipſe, whoſe Axes are as *Radius* to the *Sine* of 40 Degrees, the greater Axis would be 171 Geographical Miles, and the leſſer 110; and the neareſt diſtance between the Limits ſuppoſed Parallel 164 ſuch Miles.

And this Length of the *Axis* of the Shade, derived purely from the Continuance of total Darkneſs, is fully confirmed by the obſerved diſtance of the Parallel Limits; the one paſſing by *Badſworth* in *Yorkſhire*, the other by *Cranbrook* in *Kent*. For by the two Latitudes 53°. 37′ and 51°. 6′, with the difference of Longitude 7′ and 40″ of Time, or 1°. 55′, the diſtance of theſe two Places is given 166½ Geogr. Miles; with the mean Angle of Poſition 25 Degrees from the North Weſtwards; wherefore this Arch makes an Angle with the Track of the ſhade of 77°¾: and hence the neareſt diſtance of the Parallels becomes 163 ſuch Miles, which by the other way was found 164.

If therefore we conclude the *Axis* of the ſhadow, when the Sun was juſt 40 Degrees high, to have extended over 2°. 50′ of a great Circle, we may ſecurely determine the Difference of the Sun and Moon's Diameters at this time. For the difference of the Horizontal Parallaxes of the Sun and Moon being found to be 60′. 38″. (as ſhall be hereafter ſhewn, but is not required with extream exactneſs for this Purpoſe) the difference of the Parallaxes in Altitude at both ends of the Axis, will be found to be 1′. 56″; and by ſo much did the Diameter of the Moon when forty Degrees high exceed that of the Sun: Hence the horizontal diameter of the Moon in this Anomaly is found 33′. 27″, which may ſerve for a Rule in all other Caſes.

Laſtly, I have added the following *Synopſis* of ſuch Obſervations as have hitherto come to my Hands; acknowledging the Favour of all thoſe, who have been willing to promote our Endeavours to perfect the doctrine of Eclipſes.

Place	Obſerver	Beginn.	Immerſ.	Emerſ.	Tot.	End.
		h. ′ ″	h. ′ ″	h. ′ ″	′ ″	h. ′ ″
Barton	M. *Bridges*				3.53	
Bell-bar	M. *Jones*	8. 6. 25	9. 9. 45	9. 13.27	3.42	
Broadway } *Carmarth.*			8. 47.00	8. 49.30	2.30	
Cambridge	M. *Cotes*			9. 14.37		10. 21.57
Canterbury	M. *Gray*	8. 10. 00				10. 24.30
Cheſter	M. *Ward*	7. 57. 40				10. 6. 35
Crew	M. *Wright*		9. 2. 8		2.00	10. 9. 00
Dublin	L. *Arch.Biſh.*	7.42. 11				9. 49.40
Dublin	M. *Hawkins*	7.41. 30.				9. 48.45
Exon	L. *Biſhop*		8. 55. 0	8. 59. 0	4.00	10. 0. 00
Exon	M. *Hudſon*	7. 47.30			3.30	10. 0. 30
Greenwich	M. *Flamſteed*				3.11	
King's Wald.	M. *Whitſide*				3.52	
Llanidan } *Angleſey*	M. *Rowland*	7.52. 30				
London	R. *Society*	8. 6.00	9. 9. 3	9. 12.26	3.23	10. 20.00
Northampt.	M. *Hawkins*		9. 5. 22	9. 9. 24	4. 2	10. 15.35
Norton-court	D. *Harris*	8. 8. 55	9. 13.23	9. 14.22	0.59	10. 24.47
Oxon	D. *Keill*				3.30	10. 15.10
Paris	R. *Academy*	8. 11.00				10. 28.00
Plymouth	M. *Heines*	7. 41.00	8. 45. 30	8. 50 00	4.30	9. 54.30
Portcheſter	C. *Candler*		9. 2. 25	9. 6. 15	3.50	
Salop	D. *Hollings*	7. 58. 0			1.40	10. 6. 00
Upminſter	M. *Derham*	8. 7. 41	9. 10.58	9. 14. 6	3. 8	10. 21.45
Wanſted	M. *Pound*	8. 6. 37	9. 9. 28	9. 12 48	3.20	10. 20.32
Weymouth	M. *Hobbs*		8. 54.00	8. 58. 00	4.00	
Witley	M. *Baxter*	7. 59. 0			3.15	10. 13.00

n. 345. *p.* 314. Mr. *John Edens*, being on his Voyage to *Teneriff*, obſerved the Eclipſe at Sea, in Latitude, by Obſervation 34°. 20′, and Longitude 0^{h}. 54′. Weſt from *London*, as he concluded by their Diſtance and Poſition from the Iſland *Forte ventura*, which they ſoon after fell in with. He writes that it began at vi^{h}. 49′. and ended at viii^{h}. 47′. this latter very exactly, though not quite ſo nice as to the beginning.

Had this Obſerver ſignified what difference of Meridians there was found between the Place of Obſervation, and the Weſt end of *Forte ventura*, we might without ſenſible Error, have concluded the true Longitude, not only of that Iſland, but alſo of the Pike of *Teneriff*, where Our *Geographers* and the *Dutch* have fixed their firſt Meridian. But this Gentleman being both able and deſirous to render the Publick this ſort of Service, we hope from him ſuch further Obſervations as may put the matter paſt Diſpute. He adds, that the greateſt Darkneſs was about $\frac{3}{4}$ of the Sun's Diameter, or nine Digits on the Northſide.

From *Germany* we have received the following Accounts.

At *Nurenburg*, the beginning and greateſt Obſcurity could not be ſeen for Clouds, but the end happen'd at xi^{h}. 10′.$\frac{1}{5}$.

At *Hamburg*, the beginning was obſerved at viii^{h}. 57′. The greateſt Obſcurity at x^{h}. 5′. 30″, when xi$\frac{1}{2}$ *digg.* were darkned. The end could not be ſeen for Clouds.

At *Kiel* in *Holſtein*, The beginning ix^{h}. 14′. The greateſt Obſcurity x^{h}. 19′. 20″, and the Quantity then eclipſed xi. *digg.* 20′. The end was at xi^{h}. 29′.

At *Berlin*, the beginning could not be ſeen for Clouds, but the greateſt Obſcurity was at 22 *min.* paſt ten, when xi *digg.* were eclipſed. The juſt end was at xi^{h}. 34′.

At *Franckfort* on the *Mein*, The Eclipſe began at viii^{h}. 50′. The greateſt Darkneſs at x^{h}. 11′, *but perhaps ſhould be* x^{h}. 1 *min.* the Digits being x. and 34 *min.* The end was obſerved at 10 *min.* paſt Eleven.

By whom theſe Obſervations were made, and with what Inſtruments, we are not as yet informed; but hope they may be exact enough to confirm the *Longitudes* of thoſe ſeveral places, which are at preſent reaſonably well known.

Since theſe there is lately come to Hand a *Dutch* Print entituled *Nouvelles Literaires*, publiſh'd at the *Hague*, wherein p. 404, 405. there is an Account of the Obſervation of this Eclipſe at *Upſal* in *Sweden*, made by Mr. *Jo. Waller*, Profeſſor of Mathematicks in that Univerſity, the Times being verified by three Clocks perfectly agreeing with one another, and with the Sun; but more eſpecially by a Quadrant of 5 Foot Radius for taking the Sun's Altitude. By this Inſtrument he has determined the height of the Pole at *Upſall* 59°. 51′. 54″. And by the ſame, a little before the beginning of the Eclipſe he found the height of the Sun 39°. 36′. 42″. his Clocks then ſhewing the Hour ix^{h}. 47′. 50″, which proves that they were very near the true Time. At x^{h}. 58′. 15″. the Altitude of the Sun being 44°. 17′ 29″, was the beginning of the total Darkneſs, and at xi^{h}. 2′. 24″. was the end thereof, *alto ſole* 44°. 29′. 13″. ſo that here the Duration of the total Eclipſe was 4′. 9″, and the middle thereof but one third of a Minute after Eleven. And laſtly the end is ſaid to have happen'd about 4 Minutes before Noon, the Sun being 45°. 42′. 6″. high: But in this is a manifeſt Miſtake, for it makes the Time of Emerſion, or from the middle to the end, but 55′ 20″; whereas being ſo near the Meridian, 'tis certain that this Emerſion was the greater part of the Duration of the whole Eclipſe, and conſequently more than an Hour. Perhaps the Times might be deduced from the Altitudes only, and then the Miſtake might be in ſuppoſing the end ſo much before Noon as it was really after it. However, to prevent all Doubts, we have compared this Obſervation with what we obſerved of this Eclipſe at *London*, and find that in the Latitude of 59° 50′, the Place where the middle of total Darkneſs was at xi^{h}. 0′. 20″, was near 19 Degrees more Eaſterly than *London* (that is exactly in the Meridian of *Dantzick*) and that the Eclipſe began there at ix^{h}. 52′ $\frac{1}{2}$, and ended at xii^{h}. 10′. Wherefore the Duration could not be 2^{h}. 7′. 50″, as the *Editor* of the ſaid *Nouvelles* has publiſh'd; not conſidering that the Beginning could not be ſeen for Clouds, as in the very next Words he aſſures us.

As to the Darkneſs, it was ſuch that they could ſcarce diſtinguiſh one another: and beſides *Jupiter*, *Mercury* and *Venus*; of the Fixt Stars *Caſſiopea*, *Capella*, *Oculus Tauri* and *Orion*, (Sirius *not being yet riſen*) were viſible.

XXI. *Feb.* 19. 1717-8. Noribergæ Sol ortus eſt aliquantulum deficiens in limbo ſuperiore, qui quidem defectus accrevit ad 3 plene Dig. Deſiit Eclipſis 8^{h}. 8′. 48″. circa 60 *gr.* a Vertice Solis ad ſiniſtras.

One at Nuremburg, *by Mr.* Wurtzelbau.

2. Berolini Sol ſtatim ab ortu cœpit deficere, Hora 6. 49′. vel 49′. $\frac{1}{2}$. Circa medium Eclipſis nempe 7^{h}. 35′. erant partes lucidæ in Sole reſiduæ 24′. 40″. unde digiti obſcurati 2 dig. 50′. Finis autem in 8^{h}. 28′. 10″.

n. 357. *p.* 822.
At Berlin, ib.

XXII. The Moon roſe eclipſed, and the Horizon was ſo overcaſt, that I deſpair'd of having any Obſervation; but at $\frac{1}{2}$ an Hour paſt 6 ſhe came from under the Cloud, and at 6^{h} 25′. I had juſt a ſight of her, and judge her eclips'd about 5 digits, at

An Eclipſe of the Moon near Boſton. *by* Mr. Brattle, Feb. 11. 1700. *n.* 292. *p.* 1633.

6^{h}. 29′. The Section equidiſtant from M. Ætna and Horminius.
32. Palus Maræotis begins to be ſeen.
34$\frac{1}{2}$ Palus Maræotis and Mons Apollonius $\frac{1}{2}$ out.
37$\frac{1}{2}$. Palus Maræotis quite free, and Palus Maræotis and Palus Mæotis in the perpendicular.
42$\frac{3}{4}$. The ſhadow near an Inch from Palus Maræotis, Mons Horminius and Mons Hercules.
46$\frac{1}{4}$. Palus Maræotis in the Nadir, and that part of Palus Mæotis to my Right-hand in the Prime Vertical.
57. The upper part of the Section is now, and has been for a long time in Inſula Major in Mare Caſpio (and the Section now

perpendicular) and the lower part wheeling about from Palus Maræotis.

7^{h}. 20. Mount Sinai firſt appears at 22′. wholly free.
25 ½. Palus Maræotis and Mons Horminius near perpendicular.
43. The Eclipſe over in the Teleſcope, and at 49′ to the naked Eye.

My Clock was ſet by my Ring-Dial about 9 a Clock in the Morning, as exactly as I could judge, and the Obſervation was made with my 4½ Foot Teleſcope, with all four Glaſſes in it.

XIII. 1. *Another* Dec. 12. 1703. *near* Boſton *at* Cambridge, *by the ſame* n. 292. p. 1635

11^{h}. 45′. That part of the Moon's Disk near Alabaſtrinus, looks ſomewhat duskiſh, and the Eclipſe beginning to enter between Palus Maræotis and M. Porphyritis.
11 53. The true Shadow was well entred.
58. M. Porphyritis juſt cover'd.
12 03 ½. Near 3 Digits darkned.
7 ½. Mount Ætna begins.
9 ½. Quite covered.
14 ½. Lacus Niger major and M. Sinai almoſt equidiſtant from the Section of the Shadow; Lacus Niger Major being ſomewhat the nearer of the two.
18 ¼. Lacus Niger Major begins. 19 ½ quite covered.
21 ½. Mount Sinai begins.
21 ¾. Quite covered, and the Moon about 6 Digits eclipſed.
12 24 ½. Besbicus begins.
26. Quite covered.
28 ¾. Byſantium begins.
29 ½ Covered, and Mount Horminius begins.
32. Apollonia begins.
33. Covered.
37. The Shadow equidiſtant from M. Corax and Mount Paropamiſus, or ſomewhat nearer to M. Corax.
39 ½. Between 9 and 10 Digits eclipſed.
43. M. Corax begins.
44 ¾. Palus Mæotis begins, and at 45 ¼ the inner of M. Paropamiſus begins.
50. Palus Mæotis quite covered.
51 ½. The Moon not quite eclipſed.
52. Nor yet.
53. Nor yet.
54. Scarce.
54 ½. Quite Immerg'd, and the Mora begins.
14 39. Preciſely, ſhe Emerg'd between Palus Maræotis and Mons Porphyritis.
42. Palus Maræotis begins.
43. Quite clear.
47. M. Porphyritis quite clear.

55. About

14 55. About 3 digits reſtor'd.
59. Mount Ætna begins.
15 02. That and Lacus Niger Major at the ſame time clear.
8½ Mount Sinai about half free.
9½ Quite free, and about 6 digits reſtored.
15. Beſbicus free.
19¼ Byſantium free.
29½. About 9 digits ſeem'd to be reſtored.
30¼. Mons Herculis free.
32¾. Palus Mæotis begins.
38½. Quite free.
41½. Inſula Major in Mare Caſpio free, and in the middle of the Section.
42½. Not yet wholly clear.
45. Fully over in the Teleſcope, though a kind of a Smoak remain'd ſome little after to the naked Eye.

In order to the adjuſting of the time, I ſet my Clock with the greateſt exactneſs I could the Morning preceding, both from my Ring-Dial and the riſing of the Sun, which I very narrowly watch'd and obſerved, and found it to agree with the Sun's ſetting the following Evening; ſo that it went all the time the Eclipſe was, very ſteadily and regularly; but for the greater Certainty and Satisfaction, I took the Altitudes of the following Stars with the Braſs Quadrant with Teleſcope Sights out of my Chamber Window, the lowneſs whereof would not permit me to take them, when they were at all higher elevated.

* In dextro humero Orionis.

By the Watch.			Comp. Alt.					Differ.		
	6h	15'	78	18	6h	13'	40"	1'	20"	So that my
	6	21½	77	03	6	20	28	1	02	Clock went
		26¼	76	11	6	25	08	1	07	by theſe
Procyon										Obſervati-
	8	9¼	77	20	8	08	04	1	11	ons near-
		14½	76	20	8	13	32	0	58	eſt 1' too
		21	75	13	8	19	36	1	24	faſt.
Re-	10	8½	77	46	10	07	18	1	12	
gulus		17¼	76	11	10	15	58	1	17	

I had the good fortune to make ſome few Obſervations of the laſt of *December* the 11th, 1704. as follows.

27 *Some Obſervations hereon, by* Mr. Hodgſon, *ib. p.* 1637.

The Heavens being cloudy moſt part of the Night, it was 35' after 4 in the Morning following, before I could perceive that the Moon was eclipſed, and then as near as I could judge, ſhe had been ſo about three or four Minutes at moſt, from whence we may conclude it began at *London* about 31 or 32 Minutes after four the ſame Morning.

Mr. *Brattle* found, that at 44 Minutes after 11 at Night, part of the Moon's Disk look'd ſomewhat duſkiſh, and that at 52 Minutes, the Shadow was well entred, *ſo that from hence, as well as from a Compariſon of the Ingreſs and Egreſs of the principal Spots, it probably began there about* 49 *Minutes after* 11, *whence it follows, that* Cambridge *in* New England *lies* $4^h\ 4'\ 2\frac{1}{2}$, *or* 70° 37′ *to the Weſtward of the Meridian of* London.

I happen'd to ſee the Moon the ſame Morning at 35 Minutes after 5, when ſhe wanted at moſt but 3 Minutes of being totally eclipſed; ſo that at *London* ſhe immerg'd at 38 Minutes paſt 5.

Mr. *Brattle* ſaw her immerge exactly at 54 Minutes after 12, whence it follows, that the difference of the Meridians found by comparing theſe Obſervations, is $4^h\ 43'\frac{1}{2}$, or 70° 52′ agreeing very well with the former; ſo that by taking a Mean between them, the difference of Longitude of the 2 Places is $4^h\ 43'$, or 70° 45′.

XXIV. 1. *Another at* Zurich: Apr. 17. 1707. *N.S. by Dr.* Scheuchzer, *n.*310.*p.*2394.

H.	Min.	S.	
12	9 ad 18		Penumbra ex parte Maræotidis.
	18	40	Umbra vera intra diſcum.
	20	15	Palus Maræotis in Umbra.
	23		Maris Eoi principium.
	25	20	Mons Alabaſtrinus. Medium Maris Eoi.
	27	40	Principium ſinus Sirbonii.
	29		Medium ſinus Sirbonii, & Mare Ægyptiacum.
	29	20	Principium inſulæ Cercinnæ.
	29	30	Lacus Meridionalis.
	30	30	Medium Cercinnæ.
	31	40	Finis Cercinnæ.
	33		Extrema protenſio Montis Sopher.
	34		Inſulæ inter Siciliam & Cercinnam.
	35		Principium Mauritaniæ, & ſinus Hyperborei.
	36	40	Medius ſinus Hyperboreus.
	37	20	Mare Pamphilium.
	37	30	Creta.
	37	40	Principium Ætnæ.
	38	40	Medium Ætnæ. Melos. Carpathos.
	39	30	Finis Ætnæ.
	41		Rhodus.
	43	40	Initium Sinai.
	45		Medium Maris Adriatici, & Sinai.
	45	20	Sinai Finis.
	46	30	Medium Adriatici.
	48	40	Principium Propont. & Maris Hyperborei.
	51	30	Medium Propontidis, & finis Adriatici.
	52	40	Principium Lacus nigri majoris.

H.	Min.	S.	
12	53	10	Lacus Thraſumenus.
	53	30	Medium Lacus nigri minoris.
	54	30	Principium Inſulæ Besbyci.
	55	10	Principium Ponti Euxini in ſinu Salmydeſſo.
	56		Finis Propontidis.
	57	30	Ponti Euxini inferioris principium.
	58		Byzantii principium.
	59	10	——finis.
	59	40	Promontorium Acheruſium.
1	1		Boryſthenis principium. Apollonia.
	1	30	Medium Ponti Euxini.
	3	20	Sinus Athenienſis Medius.
	4	40	Palus Byces.
	5	40	Promontorium Heracleum.
	7	40	Cochilis media. Finis ſinus extremi Ponti.
	8	30	Lacus Corocondæ Medium.
	9	20	Promont. Hercul. & Maris Caſpii initium.
	11	40	Initium Paludis Amadocæ.
	16	20	——Medium.
	18	10	——Finis.
	20		Paludes amaræ, & Lacus minor.
	20	40	Lacus major.
	22	10	Tenuiſſimus margo lucidus.
	23	20	Lunæ corpus totum in umbra.
	24	40	Diſcus Lunæ integer fere, excepto Mari Mediter. diluto quodam fulgore ſplendet, ut Maria diſtingui poſſint per Tubum.
	40		Alia macula præter paludem Mæotidem per tubum diſtingui nequit.
	45		Lunæ diſcus medius magis magiſque obſcuratur, ambitu manente lucidiore.
2	12		Diſcus Lunæ rutilo colore nudo oculo refulgebat, nec per Teleſcopium macula ulla diſtingui potuit.
	15		Totus Diſcus obſcurior magis, & magis, Peripheria manente lucidiuſcula.
	28		Splendidior Diſcus e regione Paludis Maræotidis, umbraque denſiſſima verſus Paludem Mæotidem.
	33		Senſim lucidior redditur integer Diſcus, obſcuritate majore tegente Paludem Mæotidem ejuſque loca vicina.
	51	40	Redeunt ſenſim veſtigia Marium.
	56	30	Pontus Euxinus, & Mare Caſpium, in media obſcuratione manent veluti nebula craſſa perfuſa.
3	5		Diſtingui poteſt Mare Eoum, & vicina, ut ut Luna nondum ex umbra emerſerit.
	9	40	Emerſionis initium verum.

H.	Min.	S.	
3	11	30	Incipit emergere Palus Maræotis.
	13		Evaſit:
	15	40	Incipit Mare Eoum.
	21	30	Evadit Sinus Sorbonius, & Mare Egyptium.
	26		Evaſit Caſſiotis Regio, & aliquot minuta ante Cercinna inſula.
	27	40	Evaſit Athos Mons, & Maltha.
	31		Emerſit Mauritania.
	38		——— Corſica, & Sicilia.
	44		——— Mare Adriaticum.
	45	20	——— Media Propontis.
	49		——— Besbycus.
	52		——— Byzantium.
	57	30	——— Promontorium Acheruſium.
4	5	20	Emerſit Pontus Euxinus, & medium Caſpium.
	6		Incipit Mæotis Palus.
	9		Emerſit Caſpium, & Media Mæotis.
	11	20	Emerſit Mæotis.
	13	40	Penumbra.
	14	20	Luna tota integra.
			In Emerſione videbatur mihi umbra diſtinctior, quam in immerſione.
12	18	40	Eclipſeos initium in umbra vera.
1	23	20	Obſcuratio maxima.
3	9	40	Emerſionis initium.
1	46	30	Duratio totalis obſcurationis.
4	14	20	Emerſionis finis.
3	55	50	Tota Duratio.
1	5	40	Ab initio ad Immerſionem Lunæ totalem.
1	5	40	Ab emerſione totalis Eclipſeos ad finem.

2. *At* Boſton, *by Mr.* Brattle, *n.*312.*p.*2471.

Tempus ex Alt.correct.			*Apr.* 5. *O. S.* Immerſiones.
6h.	52'	"	Penumbra valde notabilis.
6	58	20	Palus Maræotis tegitur.
7	8	15	M. Porphyrites incipit.
7	9	20	Tegitur.
7	16		M. Ætna incipit.
7	17	15	Penitus tectus.
7	21	40	M. Sinaiincipit.
7	22	40	Plane tectus.
7	24		Inſula Corſica tegitur.
7	31	40	Lacus niger major tegitur.
7	33		Inſula Besbicus.
7	36	30	Bizantium.
7	37	20	M. Horminius.
7	40	30	M. Apollonii.

		Tempus ex Alt. correct.		
M. Hercules		7h	44'	30"
M. Corax		7	51	30
Palus Mæotis incipit		7	52	45
Inſul. Maj. in M. Caſpio incipit		7	54	45
Tegitur		7	56	
Palus Mæotis omnino tegitur		7	57	30
Luna plena Immerſa		8	1	15
Comp. Alt. Arcturi	53° 34'	8	28	
	51 30½	8	39	15
Comp. Alt. ſtellæ quæ ſequitur lucidum in corona ſeptentrionali				
Lat. 44° 33'	60° 2'	9	0	30
	56 57	9	17	15
Emerſiones.				
Luna emergere plane incipit		9	46	30
M. Ætna tota illuſtrata		10	9	30
M. Sinai tota apparet		10	10	15
Inſula Besbicus		10	25	
Bizantium		10	28	30
M. Apollonii		10	33	
M. Hercules		10	36	30
Palus Mæotis incipit		10	44	
Inſula Major in M. Caſpio reſtauratur		10	47	
Palus Mæotis tota retecta		10	49	
Luna plene illuminatur		10	54	

XXV. *Sept.* 18. 1708. I obſerv'd for half an Hour or more a thin Shade to poſſeſs that part of the Disk where the Eclipſe began, which remain'd a good while after the Eclipſe was over.

Another at Upminſter, by Mr. Derham, n. 320, *p.* 312.

The correct Appar. Time.			
7h	56'	30"	A thin Penumbra.
7	57	40	A darker Penumbra.
7	59	00	Yet darker, which may paſs for the beginning of the Eclipſe.
8	00	00	The Eclipſe no doubt begun.
9	01	00	The Lucid parts of the Moon, not long before the middle of the Eclipſe, were 925 parts of my Micrometer.
9	16	40	Diameter of the Moon 1634 parts of the Micrometer.
10	23	11	The end of the Eclipſe draws nigh.
10	25	00	A little Obſcuration.
10	26	00	Leſs.
10	28	15	A very little, except the Duskiſhneſs before mention'd.

The Moon's Eclipſe at Streatham, *compar'd with the Calculation, by Mr.* Creſner, *n.* 325. *p.* 16.

XXVI. In the Lunar Eclipſe *Feb.* 2. 1709-10. finding the time of the end to agree with the Calculation (with a very inconſiderable Difference) according to Sir *Iſaac Newton*'s Theory; and there being no Examples of any Calculation according to that Theory, nor of the Theory's Agreement with the Obſervation; I thought proper to offer this one. I have added the Calculation from Mr. *Flamſteed*'s Tables, according to *Horrox*'s Theory, as I find them publiſh'd in Mr. *Whiſton*'s Aſtronomical Lectures, with the *Radix*'s of the Mean Motions, corrected according to their firſt Author's later Obſervations, which are the ſame with thoſe aſſum'd in Sir *Iſaac Newton*'s Theory.

The Obſervation was made at *Streatham*, about ſix Miles near direct South of *London*, with a very good eight Foot Teleſcope. To correct the Clock, (for want of an Inſtrument,) I carried with me next Day two Watches, that were before adjuſted to the Clock, and compar'd them with Mr. *Flamſteed*'s at the Royal Obſervatory, having firſt noted its Error by an Obſervation of the Sun's Tranſit of the Meridian, his Aſſiſtant communicated to me: Upon my return, I found my Watches ſtill to agree together, and to my Clock, which proved them to have gone true, and gave me the exact Error of my Clock, and the true time at Obſervation.

17$\frac{09}{10}$. *Feb.*	D.	H.	M.	Sec.
The Mean Time of the Mean Oppoſition	2	4	9	42
The Mean Time of the true Oppoſition	2	10	54	48
At which the true Place of the Sun is	10	24	55	50
And its Æquation to be added.				
The Place of the Moon at this Time, from Sir Iſaac Newton's *Theory.*				
Mean Motion of the Moon	4	26	57	37
Annual Æquation Subtr.			8	34
The Correct Mean Motion	4	26	49	03
Mean Motion of Apog.	11	18	13	54
Annual Æquation of Apog. Ad.			14	31
Correct Mean Motion of Apog.	11	18	28	25
Second Æq. from the diſt. of Ap. from Sun Ad.			2	57
Place of the Moon the 2d time Æquat.	4	26	52	00
Mean Motion of Node	11	01	34	25
Æquation of Node Subt.			06	54
Correct mean Motion of Node	11	01	27	31
The 3d Æquat. of the Moon from Nodes Aſpect with the Sun Subt.				10
Place of the Moon the 3d time Æquated	4	26	51	50
Second Æquation of Apog. Subtr.		7	45	41
True Place of Apog.	11	10	42	44
Mean Anomaly	05	16	09	06
Æquation of Center Sub.		1	53	31
Moon's Place the 4th time Æquated	4	24	58	19
The Variation. Ad.				11

Moon's

Moons Place the 5th time Æquated	4	24	58	30
The 6th Æquation from the diſtance of the Luminaries and Apog. Ad.			1	20
Moons Place 6th time Æquated	4	24	59	50
The 7th Æquation. Ad.				34
True place of the Moon in its Orbit	4	25	00	24
True Place of the Sun	10	24	55	50
Moon beyond the Oppoſition			4	34
Which divided by the Horary Motion of Moon from Sun gives			7	42
	D.	H.		
The Mean Time therefore of Oppoſition. *Feb.*	2	10	47	06
And the true Time	2	10	32	20

The Place of the Moon at the ſame time from the Tables in Mr. Whiſton's *Aſtronomy, according to* Horrox's *Theory.*

Mean Motion of the Moon	4	26	57	37
Phyſical Parts Sub.			8	21
Correct Mean Motion	4	26	49	16
Mean Motion of Apog.	11	18	13	54
Æquation of Apog. Sub.		7	25	00
Mean Anomaly	5	16	00	22
Æquation of the Center Sub.		1	53	53
Place of the Moon in its Orbit	4	21	55	23
Diſtance from the Oppoſition				27
That is in time to be added				45½
	D.	H.		
The Mean time therefore of true Oppoſition is exactly	2	10	55	33
The Apparent time	2	10	40	41
Place of Moon in Ecliptick	4	24	57	27
Reduction between the true Oppoſition and middle of Eclipſe Ad.			2	47
	D.	H.		
Middle of Eclipſe	2	10	43	34
Continuance of Eclipſe		2	55	06
Digits Eclips'd		9	55	
Beginning of Eclipſe	2	9	16	01
End of Eclipſe		12	11	07
End of Eclipſe by the Moon's Place from Sir *Iſaac Newton*'s Theory.		12	02	00
End by Obſervation		12	01	30
End by Calculation from *Horrox*'s Theory		12	11	08

XXVII. *Eclipſe of the Moon at* Upminſter, Jan. 12. 1711-12. *by Mr.* Derham, *n.* 336. *p.* 512.

6h.	15".	A darkiſhneſs on the N E. ſide of the Moon.
6	36	A thick Penumbra on the Moon.
6	37	The Penumbra ſo denſe, that it may be taken for the beginning of the Eclipſe.
6	39	The Eclipſe undoubtedly is begun.

6^h $41'$ The Shadow ſo dark, that it nearly hid the Moon's NE Limb.
7 21 Moon's Diameter, by the Micrometer 1612 equal parts, equal to 31′. 25″.
7 25 The Shadows diſtance from the oppoſite luminous Limb of the Moon, repreſented by the Line, *l. n.* was 1025 parts of the Micrometer, equal to 20′.
8 31 End of the Eclipſe is very near.
8 32 End of the Eclipſe.
8 32 45″ Eclipſe is undoubtedly ended.
8 36 A Penumbra left.

Having diſorder'd my Micrometer at the beginning, I could not take with any Exactneſs the Inclination of the Cuſps, and ſome other Matters; neither can I warrant the Micrometrical Meaſures of the Moon's Diameter, and her eclips'd parts otherwiſe than ſomewhat near the Truth. I have made a Type of the Eclipſe as well as I could by gueſs. *Fig. 1. Plate 4.*

m. i o. r. repreſents the two Claſpers of the Micrometer parallel to the Equator.

N. the Northern, *S.* the Southern part of the Moon's Disk running between the Claſpers of the Micrometer.

L. n. The enlighten'd part of the Moon, being 1025 Micrometrical parts, or 20′.

XXVIII. *A Lunar Eclipſe and Occultation of a Star, by Dr.* Fr. Blanchini, *n.* 340, *p.* 88. Nov. 21. 1713.

H. post Merid.			
12^h	$53'$	$34''$	Stella *Bayero* τ Tauri proxime appellit ad limbum Lunæ, obſervata per Teleſcopium duodecim palmorum.
12	54	34	Eadem jam occultata eſt ab ea parte Lunaris Limbi, quæ media ferme eſt inter maculas Ariſtarchi & Galilei. Parallelus diurnus a centro Lunæ deſcriptus apparet Auſtralior quam ſtella τ partibus Micrometri $7\frac{1}{2}$, qualium Lunæ diameter ſubtendit 37. Stellæ igitur τ declinatio Borealior eſt declinatione apparente Lunaris centri minutis circuli maximi $5'\frac{7}{8}$ circiter.
14	00	14	Sirius attingit Meridianum: unde verificata ſunt tempora.
14	32	57	Stella τ, quæ aliquot minuta exceſſerat e limbo Lunæ in revolutione diurna præcedit limbum occidentalem Lunæ ſecundis horariis 0′. 33″, eademque præcedit centrum Lunæ ſecundis 103″ ſive 1′. 43″.
14	42	50	Eadem præcedit limbum Lunæ ſecundis 48″, & centrum 1′. 58″.
14	50	37	Differentia Aſcenſionis rectæ ſtellæ & limbi eſt 1′. 03″, centri vero Lunæ & ejuſdem ſtellæ 2′. 13″.
15	0	00	In limbo Lunæ Penumbra, quæ antea erat dilutior, ſenſim fit denſior.
15	2	20	Penumbra fit evidentior, ſed nondum apparet Umbra vera.
15	4	20	Initium incidentiæ Lunæ in Umbram veram, ea in parte limbi quæ proxima eſt maculæ *Schickardi.*
15	5	21	Umbra vera jam obtegit partem unam, qualium Lunæ diameter in Micrometro obtinet 37.

15	7	20	Jam partes duæ obteguntur, qualium Lunæ diameter eſt 37.
15	16	20	Jam obteguntur Lunaris diametri $\frac{5}{35}$.
15	31	20	Latent Lunaris diametri $\frac{2}{37}$.
16	12	00	Jam latent in diametro partes $\frac{15}{37}$.
16	17	20	Partes latentes 15, ut antea.
16	50	20	Jam partes latentes $\frac{12}{37}$.
16	54	35	Incipit emergere prior limbus Tychonis.
16	56	9	Jam totus Tycho emergit.
17	13	30	Latent Lunaris diametri partes 5 è 37.
17	27	45	Umbra vera excedit e limbo Lunæ, in loco deſignato per diametrum dictam inter Ariſtarchum & Platonem ſitu medio.

Hæc occultatio pluris æſtimanda, quod occultatio ſtellæ τ acciderit tam vicina Oppoſito Solis, ut inde locus Solis inter fixas rite examinari poterit.

XXIX.
A Lunar Eclipſe at Wanſtead, *by Mr.* Pound, *n.* 347. *p.* 402.

Obſerv.	Apparent Time. h	'	"	*October* 30. 1715.	'	"
1	15	09	00	The Eclipſe had been for ſometime begun.		
2		17	00	The Moon's Diameter meaſured by a Micrometer.	34	04
3		22	25	The Chord connecting the Horns	30	28
4		35	45	The enlightened part of the Diameter, continued to the Chord between the Horns	19	58
5		43	24	The enlighten'd part of the Diameter	13	52
6		49	50	The ſame repeated	12	02
7		52	43	The ſame repeated	11	44
8		56	51	The inlightened part of the Diameter continued to the Chord between the Horns	15	22
9		59	27	The inlighten'd part of the Diameter	10	35
10	16.	04	04	The ſame repeated	9	43
11		18	34	The ſame repeated	9	07
12		23	45	The Chord between the Horns	32	35
13		26	30	The ſame repeated	33	07
14	16	31	16	The ſame again At which time alſo the Shade paſſed thro' the middle of *Schickardus*.	33	19
15		37	15	The Chord between the Horns, agreeing with the ☽'s Diameter	33	57
16		40	45	The inlightened part of the Diameter	11	56
17		43	40	The ſame produced to the Chord between the Horns	16	13
18		46	55	The ſame repeated	17	28
19		47	57	The inlightned part of the Diameter	13	28
20		52	57	The ſame	15	20

	h	'	"		'	"
21		55	27	The Edge of the Shadow paſſed through the middle of *Gaſſendus*		
22		56	12	The inlightned part produced to the Chord between the Horns	19	58
23	17	02	45	The Chord between the Horns	32	12
24		8	20	The ſame repeated	30	28
25		10	39		29	56
26		13	00		28	31
27		15	29	The ſame again	27	33
28		17	37		26	35
29		19	35		25	36
30		21	47	The ſame again	24	38
31		23	24		23	39
32		24	54		22	40
33		26	27	The ſame again	21	41
34		27	57		20	42
35		29	08		19	43
36		30	20	The ſame again	18	44
37		31	07		17	45
38		32	04		16	46
39		32	50	The ſame again	15	47
40		34	12		13	48
41	17	35	20	The ſame again repeated	11	42

At 17h. 39'. the Eclipſe was thought to be ended; and was viſibly ſo at 17h. 41': But by comparing the laſt Obſervations of the Chords between the Horns, it follows that the true end of the Eclipſe was at 17h. 38'. 20". at 17h. 43'. the Moon's Diameter was 33'. 40".

The middle cannot be ſuppoſed to be very accurately determined by theſe Obſervations, which were not ſufficiently diſtant from the time of the greateſt Obſcuration. However by comparing ſeveral of them together, the middle will be obtained, *viz.*

	h	'	"
By Obſ. 3. compared with Obſ. 24. at	16h.	15'.	21"
By Obſ. 4. compared with Obſ. 22. at	16	15	58
By Obſ. 5. compared with 19. and 20. at	16	16	00
By Obſ. 6. and 7. compared with 16. at	16	15	48

By reaſon of Clouds I could not ſee the Beginning of the Eclipſe, nor make ſuch Obſervations of the Moon's immerging into the Shadow as I did of her emerging out of it.

By Obſervation 11. compared with Obſervation 15. the Digits eclips'd were $8\frac{3}{4}$.

The Angles were meaſured by a *Micrometer* in a 15 Foot Teleſcope, I have not conſidered how far they are conſiſtent with one another; they being ſet down here exactly as they were firſt taken.

This Eclipſe is the more conſiderable, as happening very near the Moon's *Perigee*, and therefore uſeful to verify her *Anomaly*; as alſo to limit the greateſt Diameter of the Shadow of the Earth, and conſequently ly the Parallax of the Moon. This may very properly be compared with that of the 19th of *October*, 1697, whoſe middle was at 7h. 41′ *P. M.* at *London*, and Quantity the ſame as now.

The Times by the Clock were 17′. 45″. ſooner than the apparent time, as was found by the following Obſervations of *Cor Leonis* and *Arcturus*, which through the Clouds were but juſt diſcernible.

Apparent Zenith Diſtance		Time by the Clock			Apparent Time by Calculat.			The Difference.			
of Cor ♌											
°	′	h.	′	″	h	′	″	′	″	Mean Diff.	
70	16½	13	32	43	13	50	35	17	52	′	″
69	38		36	50		54	44	17	54		
69	09		40	06		57	51	17	45	17	50
68	40		43	09	14	00	59	17	50		
68	08		46	37		04	26	17	59		
of *Arctu.*											
65	19	17	37	40	17	55	24	17	44		
65	06		39	12		56	48	17	36		
64	41		41	49		59	29	17	40	17	40
63	47		47	40	18	05	17	17	37		
					Clock too ſlow					17	45

The Latitude of *Wanſted* is 51°. 34.′ Its Longitude is 8″. in time Eaſtward from the *Obſervatory* at *Greenwich*.

The Account given of this Eclipſe by Mr. Derham, *who obſerv'd it at* Upminſter, *is agreeable to this, as far as Clouds would permit him to obſerve.*

XxX. *Another at* Wanſted, *by Mr.* Pound, *n.* 357. *p.* 845.

Obſerv.	*Tempus Apparens* h.	′	″	*Eclipſis* Lunæ *obſervata apud* Wanſted, 29. *Auguſti*, 1718.	′	″
1	6	53	38	Chorda inter Cuſpides Micrometro menſurata	22	37
2		55	8	Eadem repetita	21	14
3		56	31	Repetita	19	51
4		57	49	Iterum	18	28
5		59	38	Denuo	15	00
6	7	2	41	Immerſio Totalis in Umbram		
7	8	36	13	Stella clara in Catalogis omiſſa occultata eſt a *Luna*, infra *Paludem Mareotida Hevelii*	10	2
8	8	48	18	*Luna* cœpit emergere ex Umbra		

9 51 13

Obſ.	T. appar.			*Eclipſis* Lunæ, *Aug.* 29. 1718.	′	″
9	51^{h}	′	13″	Terminus Umbræ per med. *Maræotidis*; ſimul Chorda inter Cuſpides	15	0
10		53	7	Chorda inter Cuſpides	18	28
11		54	16	Eadem repetita	19	51
12		54	59	Iterum	21	14
13	8	56	18	Denuo	22	37
14	9	0	48	*Porphyrites* emerſit ex umbra.		
15		8	3	Mons *Sinai* incepit emergere.		
16		9	17	Umbra per medium *Sinæ*.		
17		10	6	Jam totus *Sinai* extra Umbram.		
18		11	20	Umbra per medium *Ætnæ*.		
19		17	23	Per medium *Corſicæ*.		
20		20	0	Per medium *Lacus Nigri majoris*.		
21		27	54	Per medium *Besbici*.		
22		28	45	Emerſit Stella prædicta.		
23		32	34	*Byzantium* & *Horminius* ſimul emergunt.		
24		33	58	Stella eandem habuit Declinationem cum Cuſpide Auſt. Eclipſeos.		
25		43	28	Chorda inter Cuſpides 18′ 28″.		
26		47	2	Eadem repetita 15 00		
27	9	53	0	Deſiiſſe videbatur Defectus.		

10ʰ. 30′, Capta eſt Lunæ diameter 29′. 45″. Collatis autem inter ſe Obſervationibus, ubi Chordæ partis deficientis æquales deprehenſæ ſunt, provenit Eclipſeos medium.

	Medium.		
Ex Obſerv. prima & decima tertia	7ʰ.	54′	58″
Ex ſecunda & duodecima	7	55	3
Ex tertia & undecima	7	55	24
Ex quarta & decima	7	55	28
Ex quinta & nona	7	55	25
Ex ſexta & octava	7	55	29
Quorum omnium Medium fit	7	55	18

2. *At* London *by*----*ib.p.*857.

Luna per fumum Urbis & Vapores ægre viſa.	6ʰ.	38′.	0″.
Chorda inter Cuſpides utcunque, 21′ 27″	6	54	13
Immerſio Totalis in umbram.	7	2	0
Stella fixa ſatis clara diſtabat in limbo Lunæ orientali 19′ 21″.	7	42	15
Eadem fixa occultata eſt, 10 circiter minutis centro Lunæ Auſtralior.	8	35	18
vel, ut quibuſdam viſum eſt, uno minuto tardius Luna cœpit emergere.	8	45	50
Palus *Mareotis* primo margine emerſit.	8 8	49 50	38 14

9	7	39	Primus margo *Sinæ* emerſit.
9	9	8	Mons *Sinai* totus extra umbram.
9	10	35	Umbra per medium *Ætnæ*.
9	12	0	Totus mons *Ætna* extra umbram.
9	18	51	Umbra per medium *Lacus Nigri* majoris.
9	27	35	Inſula *Beſbicus* tota emerſit.
9	42	21	Chorda inter Cuſpides 19′. 9″.
9	51	25	Finis Eclipſeos ut quibuſdam viſum eſt.
9	52	45	Finis ex præcedente diſtantia Cuſpidum concluſa.
9	56	45	Lunæ diameter 29′ 45″, iterumque 29′ 48″.

Erat autem Umbra admodum diluta, unde orta eſt difficultas in dijudicandis Emerſionis & Finis momentis: Atque Maculæ etiam obſcuriores clare conſpectæ ſunt, pluribus minutis antequam Umbræ marginem attingerent. Stella vero quæ durante Eclipſi occultata eſt, locum tunc habuit ♓ 17°. 16′½ cum Lat. Auſt. 1° 6′ 30″ proxime.

Obſervationes hujus Eclipſeos a Rev. Dom. *Derham*, apud *Upminſter* in agro *Eſſexienſi* habitas; a Dom. *Wright*, apud *Crew* in agro *Ceſtrienſi*; & a Dom. *Hawkins* apud *Wakefield* in *Eboracenſi*, cum præmiſſis ubique fere conſentiunt, ſi adhibeantur meridianorum differentiæ: poſito ſcil. quod *Upminſter* ſit 1⅓ min. *Londino* orientalius, *Crew* vero 10 min. & *Wakefield* 5 min. occidentaliora.

End of a Lunar Eclipſe May 5. 1718. *near the* Cape *of* Good Hope, *to determine its Longitude, by Dr.* Halley, *n.* 361. *p.* 992.

XXXI. I had accidentally a Journal of an Officer of the Ship *Emperor* put into my Hands, who in his return from *India*, on the fifth of *March* 1718. obſerv'd the end of a Lunar Eclipſe, when the viſible altitude of the Moon's Center was 13°. 25′. he being then in the Latitude of 34°. 23′. *South*, and as they found afterwards, juſt 180 Leagues to the Eaſtwards of Cape *Bonne Eſperance*. By Calculation I find that in that Latitude the Moon had that height at 7^h. 17′½ *P.M.* and by comparing this Eclipſe with that we obſerved with great exactneſs on *Feb.* 11°. 1682. (which agrees perfectly well with our Numbers) I conclude the middle of this to have been at *London* at 3^h. 48′. *P. M.* to which adding 1^h. 46′. for the Semiduration (this being very certain from the obſerv'd Continuance of the Eclipſe of 1682.) the end will be found to have been at *London* at 5^h. 34′. The Ship was therefore in a Meridian 26° to the Eaſtwards of *London*: But ſhe was at that time 180 Leagues to the Eaſtwards of the *Cape*, which diſtance in that Latitude gives eleven Degrees of Longitude; this therefore being deducted from the Longitude of the Ship, leaves juſt fifteen *grad.* or one Hour, for the difference of Meridians between *London* and the *Cape*. So that by this account the *Cape* is yet nearer our Meridian than I had * formerly made it, and near ſix Degrees nearer than M. *De la Hire* places it in his *Tables*.

This Eclipſe was attended with all the Circumſtances requiſite to make the Concluſion as certain as the nature of the thing will admit of: For the Moon was nearly *in Perigæo*, and the Eclipſe almoſt central; for which Reaſons ſhe emerged out of the Shadow as ſwiftly as poſſible:

* Ph. Tr. *n.* 185. *Abr. Vol.* I. *p.*567.

The Sea was very ſmooth, there having been little Wind for above 30 Hours before; and the Moon was not too high to be well obſerved with a Foreſtaff. Nor were they long at Sea before they made the Land, for in leſs than five Days, on the tenth of *March* at Noon, they had paſt Cape *d' Agulhas* the moſt Southerly Promontory of *Africa*, which then bore from them *North Eaſt*, about ſeven Leagues diſtant.

It may not be amiſs to inſert an Obſervation or two I procured to be made at the *Cape*, by Mr. *Alexander Brown* a *Scotch* Gentleman, who went to reſide in *India* on our Companies account. He carried with him a very good *Braſs Quadrant* of above two Foot Radius, and at the *Dutch* Settlement at *Table Bay*, having rectify'd his *Pendulum-Clock* by correſpondent Altitudes, on the fourth of *Auguſt* 1694, at 5^{h}. 59′ *Mane*, the diſtance of the bright Limb of the Moon from the right Shoulder of *Orion* was obſerved to be 25°. 3′. And the next Morning, *Aug.* 5. at 5^{h}. 21′. 12″, the ſame Limb was diſtant from *Procyon* 25°. 57′, and at 5^{h}. 36′. 48″. from the *Lucida Arietis* 58°. 29′.

For want of accurate Obſervations made thoſe Mornings at ſome place in *Europe*, whoſe Longitude from them is well known, I had recourſe to the Period of the Lunar Motions, which is performed in eighteen Years and ten or eleven Days, after which the Errors of our Lunar Computations return very nearly the ſame; and I found among my own old Obſervations, one that tallyed well with that of the fourth of *Auguſt*, *Viz.* Anno 1676. *July* 23°. 13^{h}. 11′. 35″. at *Oxford*, I obſerv'd the Moon to apply to the Star *in medio Collo Tauri*, by *Bayer* markt *A.* The Star at that time was diſtant from the Southern and neareſt Cuſp of the Moon by the Micrometer 20′. 32″. and at 13^{h}. 17′. 15″. when it ſeem'd to immerge upon the bright Limb of the Moon, it was diſtant from the Northern Cuſp 23′. 20″; but this leſs certain by reaſon of the hazey Air. The Star at that time was in ♉ 28°. 56′. with 1°. 13′. 20″. *North* Lat. whereby I found that our Lunar Tables, founded on Sir *Iſaac Newton*'s correct Theory of her Motion, gave her place at that time only two Minutes too ſlow; which Error being allowed on the 4th of *Auguſt* 1694. the reſult was, that 5^{h}. 59′. at Cape *Bonne Eſperance* was at *London* 4^{h}. 53′. whence the difference of Longitude 16½ degrees, ſufficiently near what we had before determin'd.

Rectification of the Motion of the Circum-Saturnials, *with Obſervations of them, by Mr.* Pound, *n.* 355. *p.* 768.

XXXII. About thirty Years ago, Mr. *Chr. Huygens* made the *Royal Society* a Preſent of the Glaſſes of a Teleſcope of 125 Foot length, with the *Apparatus* for uſing them without a Tube; But thoſe here that firſt tried to make uſe of this Glaſs, finding for want of Practice, ſome Difficulties in the Management thereof, were the occaſion of its being laid aſide for ſome time. Afterwards it was deſigned for making perpendicular Obſervations of the fixt Stars paſſing by our Zenith, to try if the Parallax of the *Earths* annual Orb might not be made ſenſible in ſo great a Radius, according to what Dr. *Hook* had long ſince propoſed: but in this we miſcarried alſo, for want of a place of ſufficient height and firmneſs, whereon to fix the Object Glaſs; ſo that it lay by neglected for many Years.

In the mean time we could not but remark a great reſerve in the *French* Aſtronomers, in relation to the Satellites, of which they have given us in their Yearly *Memoirs* no Obſervations till very lately, nor have they ſeemed willing to ſhew them in their Glaſſes to ſuch as requeſted it: ſo that it might poſſibly occaſion in ſome Perſons a Suſpicion of the reality of this Diſcovery: And Mr. *Derham*, having borrowed of the Society their long Glaſs, could not thereby aſſure himſelf that the ſmall Stars he ſometimes found about *Saturn*, were really his Satellites, their Situation not agreeing with their places derived from the Tables of their Motions exhibited in * N°. 187. of *Phil. Tranſact.* beſides that he wanted a ſufficient height to raiſe the Object Glaſs, ſo as to view *Saturn* to advantage, above the Vapours of the Horizon. But in the *Memoirs* for 1714, publiſhed but about a Year ſince, M. *Caſſini*, the worthy Succeſſor of his great Father, has given us ſome Obſervations which clear up the Point, and by ſhewing the Errors of thoſe firſt Tables, has enabled us to be aſſured, that we have ſeen the whole *Satellitium* of *Saturn* ourſelves.

Anno 1714. *Maii* 6. *St. N.* about Mid-night, *Saturn* being then Stationary in ♍ 4°. 27′, the fifth and outermoſt Satellite was in its ſuperiour Conjunction with the Planet, and at the ſame time, the Earth was nearly in the Plain of this Satellit's Orbit, ſo that it appeared to paſs very near the Center of *Saturn*: From hence, and from ſome other preceeding Obſervations, Mr. *Caſſini* concludes that the Nodes of this Satellit's Orb are in four degrees of ♍ and ♓, and that its Inclination to the Ecliptick is not much more than half that of the other Satellites. Hence it ſhould follow that the Ellipſes it deſcribes by its apparent Motion about *Saturn*, when in ♊ and ♐ are much flatter and nearer to his Body, than thoſe of the other four, which he allows to move in the plain of the *Ring*, and to have their Nodes in 21 *gr.* of ♍ and ♓, with an Inclination to the Ecliptick of 31 degrees. To confirm this Diſcovery, he produces another Obſervation of his Fathers, near thirty Years before, *viz.* that *Anno* 1685. *Martii* 31. *St. N.* about Noon, the ſame Satellite was obſerved in ſuperiour Conjunction with *Saturn*, with leſs than one Diameter of the Ring North Latitude, *Saturn* being then in ♍ 11°. 18′. So that the Satellite wanted but 7°. 21′. of compleating 134 Revolutions, in the Interval of time between them. From theſe *Data* it was eaſy to ſettle the Theory of this Satellit.

As to the *Fourth* or the *Hugenian* Satellite; in the *Memoirs* for 1715, but juſt now come to Hand, we find a very curious Obſervation of it, and the firſt of its kind, *viz.* that *Mart.* 25°. *S. N.* about 11^{h}. *P.M.* this fourth Satellite, then in *Apogeo*, did immerge behind the Body of *Saturn*. With this Emendation, the place of this Satellit may for the future be computed with a ſufficient Exactneſs.

The Third Satellite, by an original Miſtake in the Letters in N°. 187 is all wrong; its daily Motion being there printed 2^{s}. 18°. 41′. 50″. in-

* *Abr. Vol. I. p.* 376.

ſtead of 2^{s}. 19°. 41′. 50″; as may be perceived by the Period thereof being determined, in the aforeſaid *Memoirs* of 1714, to be 4*d*. 12^{h}. 25′. 12″. that is, that it makes 400 Revolutions in 1807 Days. This Satellite was obſerved by Mr. *Caſſini*, *April* 4to. *St. N.* 10^{h}. *P. M.* to have newly paſt its inferior Conjunction with *Saturn*, and a perpendicular from it fell on the Extremity of the weſtern *Anſe*, ſo that at about 5^{h}. *P. M.* it was with the Center of the Planet then in ♍ 5°. 23′. and conſequently in ♓ 5°. 23′. but *ineunte anno Gregoriano* 1686, the *Epoche* thereof was ♍ 9°. 39′. So that from the Noon of the laſt of *December* 1685, to *April* 4°. 6^{h} 18′. *anno* 1714, that is, in 10320 Days 6^{h}. 18′. there have been made $2284\frac{1}{2}$ Revolutions of this Satellite to the Equinoctial; from which *Data*, the Tables of its Motion are readily derivable.

The Radix of the penintime or ſecond Satellite, according to the aforeſaid Letter, *ineunte anno Greg.* 1686. was in ♍ 9°. 10′. But by the Obſervations of Mr. *Caſſini* made the Nights before and after, this Satellite was in its ſuperior Conjunction *anno* 1714. *April* 4*d*. 21′$\frac{1}{2}$. *St. N.* that is, in ♍ 5°. 21′, where *Saturn* then was: So that *April* 4*d*. 22^{h}. 12′, an entire Number of Revolutions were performed ſince the *Epoche* of 1686, that is, in 10320 Days 22^{h}. 12: which number can be no other than 3771, according to the Period thereof given in this *Memoire*, *viz.* 2 *d*. 17^{h}. 41′. 22″.

Laſtly, the innermoſt or firſt Satellite, at the ſame time, *viz.* 1714, *April* 4°. 21^{h}. 30′. *St. N.* was in its inferiour Conjunction *proxime*, and conſequently in ♓ 5° 21′. But the *Epoche* thereof for 1686, is ♑ 24°. 50′. which place the Satellite had paſt 40 *gr*. 31′. at the time of the Obſervation. This Arch it moves in 5^{h}. 6′: wherefore from the time of the *Epoche* to *April* 4 *d*. 16^{h}. 24′, 1714, or in 10320 Days 16^{h}. 24′. the Satellite has performed 5467 Revolutions, its Period being determined to be 1 Day, 21 Hours, 18′. 27″, in this *Memoire*.

Having by the help of theſe late Obſervations corrected the Motions of the Satellites which it was not poſſible for their firſt Diſcoverer to ſettle truly, in the ſhort Interval before 1687; and having fixed their *Epoches* for the preſent Year, we were enabled to know where to expect them with more Certainty, and to diſtinguiſh them one from another, and from the ſmall fixt Stars appearing with them. And the Reverend Mr. *James Pound*, (whoſe indefatigable Induſtry is no way inferiour to his incomparable Skill in Aſtronomical Matters) having, by means of his Steeple of *Wanſted*, provided a *Gnomon* high enough for the purpoſe, and having fitted a very commodious *Apparatus* for uſing the Societies aforeſaid long Teleſcope, ſoon diſcovered by it all theſe five Satellites; and lately communicated to them the following very curious Obſervations.

1718. *April* 21 *d*. 10^{h}. 40′. The third and fourth Satellits of *Saturn* were in *Apogæo*, a little paſt their Conjunction with *Saturn*: A perpendicular from the fourth to the Tranſverſe *Axis* of the Ring (or Line of the

Plate 4.

the *Anſæ*) fell a little without the Eaſtern *Anſa*; and a Line through the fourth and third touched the Eaſtern Limb of *Saturn*, *Fig.* 2.

The firſt was Northward of the Line of the *Anſæ* (and therefore in the *Apogæon* Semicircle alſo) diſtant from the ſaid Line about as far as the end of the Conjugate Axis of the Ring was from the Center of ♄, *viz.* nearly $\frac{3}{4}$ of *Saturn's* Semidiameter; and it was about a Semidiameter of the Ring from the Weſtern *Anſa*.

The ſecond was a very little Southward of the Line of the *Anſæ* (and therefore in the *Perigæon* Semicircle) above a Semidiameter of the Ring (or about the Semidiameter of the Ring + the Semidiameter of ♄) from the Weſtern *Anſa*. And the third, firſt and ſecond were in a ſtraight Line.

At 10^{h}. 50′. A perpendicular from the 3d to the Line of the *Anſæ* fell almoſt on the middle of the bright part of the Eaſtern *Anſa*, but ſomewhat nearer the Center than the ſaid middle.

April 22*d.* 11^{h}. 5′. The four innermoſt Satellits were all Eaſtward of ♄. The 2d and 4th in the *Apogæon*, and the 1ſt and 3d in the *Perigæon* Semicircle. A Line through the 2d and 4th touched the *South Eaſt* Limb of ♄. A Line paſſing through the 3d and the end of the Conjugate *Axis* of the Ring, was parallel to the Line of the *Anſæ*.

At 11^{h}. 10′. A perpendicular from the firſt to the Line of the *Anſæ* fell on the Eaſtern Extremity of the Ring. *Fig.* 3.

Theſe Diſtances and Directions were taken only by Eſtimation, and not by any actual Meaſurement.

The fifth (or outermoſt) Satellite being at this time near its greateſt Elongation Eaſtward, among ſeveral very ſmall Teleſcopick Stars, he could not determine its Poſition. But by obſerving the Motion of this ſome other Nights before, he was now fully ſatisfied, from the Motions rectified as above, that there are five Satellits of *Saturn*, as Mr. *Caſſini* had long ſince aſſerted.

In the bright part of each *Anſa* was a darkiſh Ellipſe, nearer to the out-ſide than the inſide of the Ring, as if it was compoſed of two Rings near to one another.

On the Body of ♄, beſide the Ring on the Southſide, there appeared on the Northſide a Zone not ſo far from the Center as the Ring, and not much unlike the ſmalleſt of *Jupiter's* Belts. Theſe Appearances were firſt taken notice of by Mr. *Caſſini*, as may be ſeen in *Phil. Tranſ.* N°. 128. p. 690 *. *v. Fig.* 4.

It is not to be expected that theſe Satellites, exceedingly minute in themſelves, and ſo faintly illuminated, ſhould appear when the Air is but ordinarily ſerene, they requiring not only the *Medium* to be *ſummo modo* defecate and limpid, but withal in perfect Darkneſs. For which reaſons it may well be underſtood why the Gentlemen of the *Pariſian* Obſervatory may have ſometimes made a Difficulty to undertake to ſhew them upon Demand.

Circa

* *Abr. Vol.* I. *p.* 367.

Tables of the Motions of Saturn's Satellits n. 356. *p.* 776.

XXXIII. Circa finem Anni 1686. D. *Jo. Dom. Caſſini*, Reg. Soc. Sodalis, & in Aſtronomicis nemini ſecundus, cum Societate noſtra inventa ſua de motibus quinque Satellitum *Saturni* communicavit, Epochaſque ſingulorum ad annum ineuntem 1686. eorumque motus diurnos in Epiſtola * N° 187. harum *Tranſact.* edita exhibuit: E quibus datis motuum Tabulas concinnavimus, dictæque Epiſtolæ ſubjunctas una edidimus. Cum vero deinde per triginta fere annos nullas omnino Obſervationes eorum tradiderint, qui ſoli poterant, Aſtronomi *Galli*; cumque aliunde, ob intervallum temporis nimis breve, non niſi laxe periodos Satellitum, præſertim interiorum, definire potuerit præclariſſimus Inventor; non prius dictarum Tabularum defectus corrigere datum eſt, quam in nuperis Actis Academiæ Regiæ *Pariſienſis* Phyſicis & Mathematicis, obſervata ea, quæ ſupra protulimus, prodiere.

* *Abr. Vol.* I. *p.* 376.

Horum vero ope facta, aliquali Motuum caſtigatione, tum demum Teleſcopio *Hugeniano* omne Saturni Satellitium ipſi agnovimus; adhibitiſque accuratis Rev. D. *Jac. Poundi* obſervationibus, Tabulas ſubſequentes cælo ſatis conſonas obtinuimus. Addendo ſc. motui annuo *Interioris*, 2*gr*. 9′; *Penintimi* vero 3°. 25′. retentis Epochis D. *Caſſini* ad Annum 1686. Augendo etiam motum annuum *Extimi* 9 min. ſublatis vero 16 grad. ab *Epocha*, quæ in Epiſtola dicta N°. 187. perperam ſcribitur ♓ 16°. 19′. pro ♈ 0°. 16′. *Hugenianum* 6′. annuatim tardiorem invenimus. *Tertii* autem Tabellas, ob motum diurnum falſo in Epiſtola illa traditum, de integro recudere neceſſe habuimus, retenta ſaltem *Epocha*.

Tabula

Tabula Mediorum Motuum Intimi Satellitis Saturni à Caſſino *detecti Anno* 1686.

Annis Chriſti ineunt.	Epochæ			Annis	Mot. Med.			Diebus	Mot. Med.			H M	Mot. Med. Sex. ° '			Min.	Motus Medi.	
	s	°	'		s	°	'		s	°	'		°	'	"		°	'
1681	♒	8	48	1	4	4	43	1	6	10	42	1	0	7	57	31	4	6
1686	♈	13	4	2	8	9	25	2	0	21	24	2	0	15	53	32	4	14
1701	♍	6	34	3	0	14	8	3	7	2	6	3	0	23	50	33	4	22
1714	♎	9	57	4	10	29	33	4	1	12	47	4	0	31	47	34	4	30
1715	♒	14	39	5	3	4	16	5	7	23	29	5	0	39	44	35	4	38
1716	♊	19	22	6	7	8	59	6	2	4	11	6	0	47	40	36	4	46
1717	♉	4	47	7	11	13	42	7	8	14	53	7	0	55	37	37	4	54
1718	♍	9	30	8	9	29	6	8	2	25	35	8	1	3	34	38	5	12
1719	♑	14	13	9	2	3	49	9	9	6	17	9	1	11	31	39	5	10
1720	♉	18	55	10	6	8	32	10	3	16	59	10	1	19	28	40	5	18
Menſ. Anni Com.	Mot. Med.			11	10	13	15	11	9	27	41	11	1	27	24	41	5	26
				12	8	28	40	12	4	8	22	12	1	35	21	42	5	34
	s	°	'	13	1	3	23	13	10	19	4	13	1	43	18	43	5	42
Jan.	0	0	0	14	5	8	5	14	4	29	46	14	1	51	15	44	5	50
Febr.	5	1	38	15	9	12	48	15	11	10	28	15	1	59	11	45	5	58
Mar.	3	1	10	16	7	28	13	16	5	21	10	16	2	7	8	46	6	5
April	8	2	48	17	0	2	56	17	0	1	52	17	2	15	5	47	6	13
Maii	6	23	44	18	4	7	39	18	6	12	34	18	2	23	1	48	6	21
Junii	11	25	22	19	8	12	21	19	0	23	16	19	2	30	58	49	6	29
Julii	10	16	19	20	6	27	46	20	7	3	57	20	2	38	55	50	6	37
Aug.	3	17	57	40	1	25	32	21	1	14	39	21	2	46	52	51	6	45
Sept.	8	19	35	60	8	23	19	22	7	25	21	22	2	54	49	52	6	53
Octob.	7	10	31	80	3	21	5	23	2	6	3	23	3	2	45	53	7	1
Nov.	0	12	9	100	10	18	51	24	8	16	45	24	3	10	42	54	7	9
Dec.	11	3	5	120	5	15	37	25	2	27	27	25	3	18	39	55	7	17
								26	9	8	9	26	3	26	35	56	7	25
								27	3	18	50	27	3	34	32	57	7	33
								28	9	29	32	28	3	42	28	58	7	41
								29	4	10	14	29	3	50	25	59	7	49
								30	10	20	56	30	3	58	22	60	7	57

In Anno Biſſextili poſt Februarium *adde unius diei motum.*

Tabulæ

Tabula Mediorum Motuum Satellitis Saturni *Penintimi, à* Caſſino *detecti Anno* 1686.

Annis Jul. ineunt	*Epochæ*			*Annis.*	*Med. Mot.*			*Diebus.*	*Med. Mot.*			*H M*	*Mot. Med. Sex.* ° ′			*Min.*	*Motus Medi.*	
	s.	°	′		s	°	′		s	°	′		°	′	″		°	′
1681	♓	3	23	1	4	10	2	1	4	11	32	1	0	5	29	31	2	50
1686	♉	5	25	2	8	20	4	2	8	23	4	2	0	10	58	32	2	56
1701	♓	22	3	3	1	0	6	3	1	4	36	3	0	16	26	33	3	1
1714	♑	7	5	4	9	21	40	4	5	16	8	4	0	21	55	34	3	7
1715	♉	17	7	5	2	1	42	5	9	27	40	5	0	27	24	35	3	12
1716	♍	27	9	6	6	11	44	6	2	9	12	6	0	32	53	36	3	17
1717	♊	18	43	7	10	21	46	7	6	20	44	7	0	38	22	37	3	23
1718	♎	28	45	8	7	13	20	8	11	2	16	8	0	43	51	38	3	28
1719	♓	8	47	9	11	23	22	9	3	13	49	9	0	49	19	39	3	34
1720	♋	18	49	10	4	3	24	10	7	25	21	10	0	54	48	40	3	40
Menſ.	*Med. Mot.*			11	8	13	26	11	0	6	53	11	1	0	17	41	3	45
Anni				12	5	5	0	12	4	18	25	12	1	5	46	42	3	51
Com.	s	°	′	13	9	15	2	13	8	29	57	13	1	11	15	43	3	56
Jan.	0	0	0	14	1	25	4	14	1	11	29	14	1	16	44	44	4	1
Feb.	3	27	34	15	6	5	6	15	5	23	1	15	1	22	13	45	4	7
Mar.	6	20	32	16	2	26	40	16	10	4	33	16	1	27	42	46	4	12
April	10	18	6	17	7	6	42	17	2	16	5	17	1	33	11	47	4	17
Maii	10	4	7	18	11	16	42	18	6	27	37	18	1	38	39	48	4	23
Junii	2	1	41	19	3	26	46	19	11	9	9	19	1	44	8	49	4	2
Julii	1	17	43	20	0	18	20	20	3	20	41	20	1	49	37	50	4	34
Aug	5	15	17	40	1	6	40	21	8	2	13	21	1	55	6	51	4	39
Sept	9	12	51	60	1	25	0	22	0	13	45	22	2	0	35	52	4	45
Octob.	8	28	53	80	2	13	20	23	4	25	17	23	2	6	4	53	4	5
Nov.	0	26	27	100	3	1	40	24	9	6	49	24	2	11	32	54	4	56
Dec.	0	12	28	120	3	20	0	25	1	18	22	25	2	17	1	55	5	1
								26	5	29	54	26	2	22	30	56	5	7
In Anno Biſſextili poſt Februarium adde unius diei motum.								27	10	11	25	27	2	27	59	57	5	12
								28	2	22	58	28	2	33	27	58	5	18
								29	7	4	30	29	2	38	56	59	5	23
								30	11	16	2	30	2	44	25	60	5	29

Tabula Mediorum Motuum Satellitis Saturni *Medii, à* Caſſino *detecti Anno* 1671.

Annis Jul. ineunt	*Epochæ* s.	°	'	*Annis.*	*Med. Mot.* s	°	'	*Diebus.*	*Med. Mot.* s	°	'	*H M*	*Mot. Med. Sex.* °	'	"	*Min.*	*Motus Medi.* °	'
1681	♍ 12	16		1	9	17	2	1	2	19	41	1	0	3	19	31	1	43
1686	♏ 27	6		2	7	4	3	2	5	9	23	2	0	6	38	32	1	46
1701	♎ 1	17		3	4	21	5	3	7	29	4	3	0	9	58	33	1	49
1714	♎ 11	43		4	4	27	48	4	10	18	46	4	0	13	17	34	1	53
1715	♋ 28	45		5	2	14	50	5	1	8	27	5	0	16	36	35	1	56
1716	♉ 15	47		6	0	1	52	6	3	28	9	6	0	19	55	36	2	0
1717	♉ 22	30		7	9	18	53	7	6	17	50	7	0	23	15	37	2	3
1718	♓ 9	32		8	9	25	36	8	9	7	31	8	0	26	34	38	2	6
1719	♐ 26	34		9	7	12	38	9	11	27	13	9	0	29	53	39	2	10
1720	♎ 13	35		10	4	29	40	10	2	16	54	10	0	33	12	40	2	13
Menſ. Anni Com.	*Med. Mot.*			11	2	16	42	11	5	6	36	11	0	36	31	41	2	16
				12	2	23	25	12	7	26	17	12	0	39	51	42	2	19
	s	°	'	13	0	10	26	13	0	15	59	13	0	43	10	43	2	23
Jan.	0	0	0	14	9	27	28	14	1	5	40	14	0	46	29	44	2	26
Feb.	10	10	24	15	7	14	30	15	3	25	21	15	0	49	48	45	2	29
Mar.	0	21	44	16	7	21	13	16	6	15	3	16	0	56	8	46	2	33
April	11	2	9	17	5	8	15	17	9	4	44	17	0	56	27	47	2	36
Maii	6	22	51	18	2	25	16	18	11	24	26	18	0	59	46	48	2	39
Junii	5	3	16	19	0	12	18	19	2	14	7	19	1	3	5	49	2	43
Julii	0	23	58	20	0	19	1	20	5	3	49	20	1	6	24	50	2	46
Aug	11	4	23	40	1	8	2	21	7	23	30	21	1	9	44	51	2	49
Sept.	9	14	47	60	1	27	4	22	10	13	11	22	1	13	3	52	2	53
Octob.	5	5	39	80	2	16	5	23	1	2	53	23	1	16	22	53	2	56
Nov.	3	15	54	100	3	5	6	24	3	22	34	24	1	19	41	54	2	59
Dec.	11	6	37	120	3	24	7	25	6	12	16	25	1	23	11	55	3	3
In Anno Biſſextili poſt Februarium adde unius diei motum.								26	9	1	57	26	1	26	20	56	3	6
								27	11	21	39	27	1	29	39	57	3	9
								28	2	11	20	28	1	32	58	58	3	13
								29	5	1	1	29	1	36	17	59	3	16
								30	7	20	43	30	1	39	36	60	3	19

Tabula Mediorum Motuum Penextimi Satellitis Saturni *ab* Hugenio *inventi Anno* 1655.

Annis Jul. ineunt.	*Epochæ.* s	°	′	*Annis*	*Med. Mot.* s	°	′	*Diebus*	*Med. Mot.* s	°	′	*H M*	*Mo. Me.* ° ′	′ ″	*Min.*	*Motus Medi.* °	′
1641	♑	2	48	1	10	20	35	1	0	22	35	1	0	56	31	29	10
1661	♓	13	23	2	9	11	10	2	1	15	9	2	1	53	32	30	6
1681	♈	27	58	3	8	1	45	3	2	7	44	3	2	49	33	31	3
1686	♏	3	28	4	7	14	55	4	3	0	18	4	3	46	34	31	59
1701	♊	12	33	5	6	5	30	5	3	22	53	5	4	42	35	32	55
1714	♓	17	53	6	4	26	5	6	4	15	28	6	5	39	36	33	52
1715	♒	8	28	7	3	16	40	7	5	8	2	7	6	35	37	34	48
1716	♐	29	3	8	2	29	50	8	6	0	37	8	7	31	38	35	45
1717	♐	12	13	9	1	20	25	9	6	23	12	9	8	28	39	36	41
1718	♏	2	48	10	0	11	0	10	7	15	46	10	9	24	40	37	38
1719	♍	23	23	11	11	1	35	11	8	8	21	11	10	21	41	38	34
1720	♌	13	58	12	10	14	45	12	9	0	55	12	11	17	42	39	31
1721	♋	27	8	13	9	5	20	13	9	23	30	13	12	14	43	40	27
Menſ.	*Mot.*	*Med.*		14	7	25	55	14	10	16	5	14	13	10	44	41	24
An. Co.	s	°	′	15	6	16	30	15	11	8	39	15	14	7	45	42	20
Jan.	0	0	0	16	5	29	40	16	0	1	14	16	15	3	46	43	17
Febr.	11	9	54	17	4	20	15	17	0	23	48	17	16	0	47	44	13
Mar.	8	12	2	18	3	10	50	18	1	16	23	18	16	56	48	45	10
April	7	21	55	19	2	1	25	19	2	8	58	19	17	52	49	46	6
Maii	6	9	14	20	1	14	35	20	3	1	32	20	18	49	50	47	3
Junii	5	19	7	40	2	29	10	21	3	24	7	21	19	45	51	47	59
Julii	4	6	26	60	4	13	45	22	4	16	42	22	20	42	52	48	56
Aug.	3	16	18	80	5	28	20	23	5	9	16	23	21	38	53	49	52
Sept.	2	26	12	100	7	12	55	24	6	1	51	24	22	35	54	50	49
Octob.	1	13	30	120	8	27	30	25	6	24	25	25	23	31	55	51	45
Nov.	0	23	24	140	10	12	5	26	7	17	0	26	24	27	56	52	42
Dec.	11	10	42	160	11	26	40	27	8	9	35	27	25	24	57	53	38
								28	9	2	9	28	26	20	58	54	35
In Anno Biſſextili poſt Februarium *adde unius diei motum.*								29	9	24	44	29	27	17	59	55	31
								30	10	17	18	30	28	13	60	56	27

Tabula Mediorum Motuum Satellitis Saturni *Extimi, a* Caſſino *detecti Anno* 1671.

Annis Julia. ineunt	*Epochæ*			*Annis.*	*Med. Motus*			*Diebus.*	*Med. Motus*			*H M*	*Mo. Me.*		*M*	*Med. Mot.*	
	s	°	′		s	°	′		s	°	″		° ′	′ ″		′	″
1681	♈	8	40	1	7	6	32	1	0	4	32	1	0	11	31	5	51
1686	♈	15	50	2	2	13	3	2	0	9	5	2	0	23	32	6	3
1701	♉	11	53	3	9	19	35	3	0	13	37	3	0	34	33	6	14
1714	♓	20	20	4	5	0	39	4	0	18	9	4	0	45	34	6	25
1715	♎	26	52	5	0	7	10	5	0	22	42	5	0	57	35	6	37
1716	♊	3	23	6	7	13	42	6	0	27	14	6	1	8	36	6	48
1717	♑	14	27	7	2	20	13	7	1	1	46	7	1	19	37	7	0
1718	♌	20	58	8	10	1	17	8	1	6	18	8	1	31	38	7	11
1719	♓	27	30	9	5	7	49	9	1	10	51	9	1	42	39	7	22
1720	♏	4	2	10	0	14	20	10	1	15	23	10	1	53	40	7	34
Menſ.	*Med. Mot.*			11	7	20	52	11	1	19	55	11	2	5	41	7	45
Anni				12	3	1	56	12	1	24	28	12	2	15	42	7	56
Com.	s	°	′	13	10	8	27	13	1	29	0	13	2	27	43	8	8
Jan.	0	0	0	14	5	14	59	14	2	3	32	14	2	39	44	8	19
Febr.	4	20	41	15	0	21	30	15	2	8	5	15	2	50	45	8	30
Mar.	8	27	46	16	8	2	34	16	2	12	37	16	3	1	46	8	42
Apr.	1	18	27	17	3	9	6	17	2	17	9	17	3	13	47	8	53
Maii	6	4	37	18	10	15	37	18	2	21	42	18	3	24	48	9	4
Jun.	10	25	18	19	5	22	9	19	2	26	14	19	3	35	49	9	16
Julii	3	11	27	20	1	3	13	20	3	0	46	20	3	47	50	9	27
Aug.	8	2	9	40	2	6	26	21	3	5	18	21	3	58	51	9	38
Sept.	0	22	50	60	3	9	38	22	3	9	51	22	4	9	52	9	50
Oct.	5	8	59	80	4	12	51	23	3	14	23	23	4	21	53	10	1
Nov.	9	29	41	100	5	16	4	24	3	18	55	24	4	32	54	10	12
Dec.	2	15	50	120	6	19	17	25	3	23	28	25	4	43	55	10	24
								26	3	28	0	26	4	55	56	10	35
In Anno Biſſextili poſt Februarium								27	4	2	32	27	5	6	57	10	46
adde unius Diei motum.								28	4	7	5	28	5	17	58	10	58
								29	4	11	37	29	5	29	59	11	9
								30	4	16	9	30	5	40	60	11	21

Motibus mediis Satellitum ad hunc modum conſtitutis, proveniunt Revolutiones eorum jam veris proximæ, ſcilicet

Primi ſive Intimi	1*d.*	21h.	18′.	26″½
Secundi Penintimi	2.	17	41.	10½
Tertii ſive *Medii*	4.	12	25.	10
Quarti Hugeniani	15.	22	41.	28
Quinti ſive *Extimi*	79.	7	46.	00

Poſito autem, juxta regulam Naturæ (ſaltem in hoc noſtro Syſtemate) univerſalem, quæque tam in Jovialium ac Lunæ motibus, quam Planetarum Primariorum circa Solem, obtinet, Vires centrum Saturni petentes eſſe in duplicata ratione diſtantiarum reciproce, ac proinde Cubos diſtantiarum a centro eſſe ut quadrata Temporum periodicorum; ex data diſtantia & periodo *Hugeniani*, fiunt reliquorum diſtantiæ ut ſequitur.

	Semidiam. Annuli,	*Semidiam. Globi* ♄
Diſt. Primi	1.9289	4,3400
Secundi	2 4708	5,5593
Tertii	3.4508	7,7643
Quarti	8.0000	18,0000
Quinti	23.3146	52,4578

Quæ quidem diſtantiæ cum D. *Caſſini* obſervatis ſatis quadrant. Quatuor autem interiores Satellites juxta planum Annuli Saturni orbitas ſuas deſcribunt proxime; in plano, *ſc.* Æquatoris noſtri plano quoad ſenſum parallelo, quicquid in contrarium proferant nonnulli. Quintum vero orbem ſitu paulo diverſum a cæteris deſcribere, nuper deprehendit D. *Jac. Caſſinus* prioris filius, ut videre eſt in Actis Academiæ Scientiarum *Pariſienſis* Anni 1714.

Occultation of a Fixt Star by Jupiter, n. 351. p. 546.

XXXIV. D. *Martinus Folkes Londini*, præſentibus aliis nonnullis e Societate Regia, *Jan.* 11. 1717. 8h. P. M. vidit *Jovis* centrum una diametro corporis ejus Fixam ſequi, quæ dicto centro Borealior erat quaſi dodrante ſemidiametri Jovis. Poſtea Nubes Jovem occuparunt, ſed habita ratione motus Jovis, paulo poſt medium Noctis ſtellam Jovi conjunctam fuiſſe, & a Borea diſci ejus parte occultatam, concluſit.

Reverendus Dominus *J. Theoph. Deſaguliens*, R. S. S. & D. *Stephanus Grey*, *Weſtmonaſterii*, viderunt Fixam, Hora Sexta veſpertina, integra Jovis diametro diſtare a limbo ejus, Corum verſus. Unde & ex ſequentium dierum Obſervationibus, circa medium noctis incidiſſe conjunctionem evincitur.

Reverendus quoque D. *J. Pound*, apud *Wanſted*, infraſcriptas nactus eſt obſervationes, quas utique accuratiſſimas, Tubo ſcil. prælongo & Micrometro captas, huc tranſcribere non pigebit.

Itaque *Januarii* Quinto 5h. 6′. T. æq. Jovis centrum diſtabat a dicta Fixa 31′. 49″. quam 5h. 38′. ſequebatur 34′. 12″. Aſcenſionis rectæ: ſimulque limbus *Jovis* Auſtrinus eandem habuit Declinationem cum ſtella.

Die autem Nono ſequente $6^{h}. 6'$. Jovis centrum diſtabat a ſtella $10' 49''$; & poſt octo minuta erat differentia Aſcenſionum rectarum $11'. 32''$: & tum centrum Planetæ, tantillo, ita ut vix perciperetur, erat Stella Auſtralius.

Die Undecimo $5^{h}. 30'$ T. æq. erat diſtantia centrorum $1'. 24''$. ſimulq; viſa eſt ſtella quaſi quadrante diametri Jovis Borealior centro ejus. Diameter autem minima *Jovis* inventa eſt $0' 43''$. Deinde Nubes.

Die vero Duodecimo $5^{h}. 17'$. erat diſtantia centrorum $3'. 7''$; ac $5^{h}. 50'$. Jupiter ſtellam præcedebat $3'. 30''$. Aſcen. Rect. Eodemque tempore limbus Jovis Boreus eandem habuit Declinationem quam Fixa accurate.

Collatis autem his Obſervationibus manifeſtum eſt Fixam hanc *Jovi* conjunctam, *Januarii* undecimo 13^{h}. circiter, non niſi $17''$. vel $18''$. centro ejus Borealiorem fuiſſe, ac proinde occultatam.

Fixa hæc, etiamſi nulli Catalogo hactenus aſcripta, Locum tunc habuit ♊ 22°. $13'$. cum Lat. Auſt. 0°. $13'\frac{1}{2}$; Comitemque habet 17 min. eam Præcedentem & 7 min. Borealiorem, ſive in ♊ 21°. $56'$. cum Lat. Auſt. 0°. $6'\frac{1}{2}$, cui *Jupiter* conjungi viſus eſt *Jan.* 16. $6^{h}. 30'$. veſperi.

Sic ſpatio minus bimeſtri *Jupiter* corporaliter eclipſavit duas Fixas, cujus rei ne ſingulare quidem exemplum ab invento Teleſcopio extat: proinde hæc obſervata inter pretioſiſſima Uraniæ κειμήλια, in uſum Poſterorum, merito reponenda ſunt.

Noſtra autem ſtellula anno 1634. *Feb. 6. Jovi* Stationario conjuncta, tribus ejus diametris Auſtralior erat, obſervante *Gaſſendo*: unde conſtabit, calculo rite inſtituto, *Joves* Nodos quoad ſenſum immobiles hæſiſſe, per 83 annos ultimo elapſos, idque ad 2^{s} 8°. $35'$. a 1^{a} * ♈.

Tranſit of Jupiter's 4^{th} *Satellite over the Disk of the Planet, by Mr.* Pound, *n.* 359. *p.* 900.

XXXV. *Feb.* 16. 1719. At $6^{h}\frac{1}{4}$ through a ſhort Tube, we ſaw all the 4 Satellites, the 3 outermoſt on the Eaſt ſide of *Jupiter*, and the innermoſt near the Weſtern Limb approaching to an Eclipſe. The Fourth at that time was about half a Semidiameter of *Jupiter* from the Eaſtern Limb. Then it proved cloudy till about 8^{h}, at which time (through the long Glaſs) we could ſee only the ſecond and third Satellites, the firſt being behind *Jupiter* in the Shadow, and the fourth entred upon the Diſque. We ſaw at this time a dark Spot, a little Northward of the great Northern Zone, and near the Eaſtern Limb, where the Satellite was to enter on the Diſque; which Spot we took for the Shade of the Satellite. The Clouds then again intercepted our View, till $8^{h}. 53'$. *Æq.* T. at which time the firſt Satellite was lately emerged out of the Shadow, and the Spot advanced ſo far, that we perceived it would arrive at the middle of *Jupiter*, near two Hours ſooner than the Shade ought to have done by our Computation; but not imagining that this dark Spot could be any thing elſe but the Shade, we concluded there had been ſome Error in the Calculation, which we thought to re-examine afterwards. On this Preſumption we left off obſerving till $9^{h}. 35'$. at which time we were ſurprized to ſee a Notch in the Limb of *Jupiter*,

near the place where the former Spot entred. This laſt Appearance agreeing well with the time that the Shade of the Satellite ought to have entred the Diſque, ſoon made us alter our former Opinion, and conjecture that this, and not the other Spot was the ſaid Shade. At $9^{h}.\ 39'\frac{1}{2}$ *Æq. T.* the Notch vaniſhing, a round black Spot appeared within the Limb, but in contact with it. At $9^{h}.\ 45'$. we judged the firſt Spot, and at $11^{h}.\ 45'$. the ſecond, to be in the middle of *Jupiter*.

At $11^{h}.\ 50'$. the firſt Spot touched the Limb, being within the Diſque, ſoon after which the Limb in that place ſeem'd a little protuberant. At $12^{h}.\ 5'$. appeared the fourth Satellite juſt come out of the Diſque, and touching the Limb in the place where the Protuberancy was. At $12^{h}.\ 7'$ we could perceive the Satellite ſeparated from the Limb. At $13^{h}.\ 56'$ the ſecond black Spot, ſtill within the Diſque, juſt touched the Weſtern Limb; ſoon after which there appeared a Notch in this part of the Limb, as it did on the other at the coming on of this Spot. At $14^{h}.\ 6'$. the Spot was all gone off, and the Limb appeared clear and entire. The firſt Spot, when in the middle of *Jupiter*, was almoſt as black as the ſecond when near the Limb, but ſomewhat leſs, and a little more *Northerly*.

At the time that the firſt Spot was in the middle of the Diſque, the three innermoſt Satellites appeared to the *Eaſt* of *Jupiter*; the firſt (as aforeſaid) having lately emerged out of the Shadow; the ſecond being almoſt at its greateſt diſtance; and the third having paſſed the Axis of the Shade about twelve Hours before, and appearing at this time about three Diameters of *Jupiter* from his Limb. The times that theſe Spots arrived at the middle of the Diſque are agreeable to the times found by Calculation, in which the fourth Satellite and its Shade ought to have appeared there. From all which 'tis very plain, that the firſt of theſe Spots was the fourth Satellite it ſelf, and the ſecond its Shadow.

We have ſeen the firſt and ſecond Satellites appearing not as dark Spots, but as bright ones (ſomewhat different from the light of *Jupiter*) for ſome little time after they entred his Diſque, but as they approached nearer the middle we loſt ſight of them. And we have frequently obſerved that the ſame Satellites appear brighter at ſome times than at others; and that when one of them hath ſhined with its utmoſt Splendour, the Light of another hath been conſiderably diminiſhed. From whence 'tis very probable at leaſt, not only that the Satellites revolve upon their proper Axes, but alſo that ſome parts of their Surfaces do very faintly (if at all) reflect the Solar Rays to us *.

Emerſion of the innermoſt Circumjovial at Rome, *n.* 340. *p.* 89.

XXXV. *Sept.* $\frac{11}{22}$ 1713. Romæ, $8^{h}.\ 38'.\ 20''$. *P.M.* Intimus Jovis Satelles incipit emergere, e regione ſpatii inter utramque Jovis faſciam protenſi. Obſervatio peragebatur Teleſcopio D. *Andreæ Chiarelli* longitudinis 40 palm. Romanorum. Deinde $8^{h}.\ 44'$. Tertius Satelles ita apparebat Quarto conjunctus, ut ambo viderentur unicus Satelles, Diſtabant a centro Jovis

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* *v. Mem. Ac. Roy. for* 1707. *and* 1714.

diametris Jovialibus circiter 5 ¼. Hora vero 9ʰ. 4′, jam disjuncti videbantur. Quartus ſitu inverſo apparuit paulo depreſſior Tertio, & paulo elongatior a Jove: quare erat Tertio Borealior.

Sept. $\frac{18}{29}$ 10ʰ. 36′. 23″, Primus ſeu intimus Satelles incipit emergere ab Umbra, Tubo 25 Palm. Domini *Campani*.

November $\frac{12}{23}$ 7ʰ 32′ 22″, Primus Satelles incipit emergere, conſpectus per Tubum Domini *Chiarelli* palm. 40. Deinde eadem nocte 7ʰ, 46′. Primus & Secundus proximi ſunt, & 7ʰ. 53′ iidem ita ſunt vicini ut vix punctulo diſtinguantur.

Decemb. 9. *N.S.* vel *Novemb.* 28. *V.S.* 5ʰ. 45′. 45″, Primus Satelles incipit emergere ab umbra Jovis.

Decemb. 21. *V. S.* 5ʰ. 50′. 22, iterum viſus eſt primus Satelles incipiens emergere ex umbra.

Remarks, ib. p.90.

Ex his obſervationibus accurato calculo ſubjectis, manifeſtum eſt æquationem ſecundam, quam a motu Luminis progreſſivo ortam ſupponimus, neceſſario locum habere. Nam poſt 57 ſatellitis intimi revolutiones, quibus Jupiter a Terra pluſquam Radio Orbis magni receſſit, novem fere minutis tardius conſpecta eſt Eclipſis ultima, quam debuit juxta tenorem Obſervationis primæ: quod quidem Hypotheſibus D. Caſſini *conſonum eſt.*

Ex iiſdem etiam confirmatur (quod nos quoque antea annotavimus, nempe) *quod motus Intimi Satellitis Jovis paulo celerior ſit quam in Tabulis elaboratiſſimis D. Caſſini, ante viginti annos cum publico communicatis, & calculi facilitate plurimum ſe commendantibus. Errorculus autem iſte vix excedere videtur duo temporis minuta in ſingulis Jovis revolutionibus, ſive duodecim annis; quibus cœlum anticipat* Caſſini *calculum. Hac vero adhibita correctione, ſatis accuratus habebitur conſenſus.*

Tables for computing of the Eclipſes of the firſt Satellite of Jupiter, *by Addition only, n.361. p.1021.*

XXXVI. In *Numb.* * 214. of theſe *Tranſactions*, we exhibited an Epitome of Mr. *Caſſini*'s curious Tables then newly publiſhed, for computing the Eclipſes of the firſt Satellite of *Jupiter*, without the help of any other Numbers. The eaſe of this *Calculus* gave great Satisfaction to thoſe that delight in Teleſcopick Obſervations; and has been of good uſe to encourage Aſtronomers to aſcertain the Geographical Longitudes of many places, by help of theſe Eclipſes, whoſe frequency ſeems to afford us the propereſt means for that purpoſe.

But it being now twenty ſix Years ſince thoſe Tables were publiſhed, length of time has diſcovered that this *Satellit*'s Motion is a ſmall matter ſwifter than Mr. *Caſſini* had ſuppoſed it; and the Reverend Mr. *Pound* being provided with all the Qualifications requiſite for ſuch a Work, has of late apply'd himſelf to rectify by frequent Obſervation what he found amiſs in the aforeſaid *Calculus*; and withal has put it into another Form yet much more eaſy and compendious, by bringing what M. *Caſſini* had given us in odd Numbers, to the Milleſimals of a Circle, both as to *Numb.* I. which he calls *Numb. A.* being the mean Anomalie of *Jupiter* in ſuch parts; as alſo to *Numb.* II. or our *Numb. B*, which is the diſtance of the mean place of *Jupiter*,

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* *Abridg. Vol.* I. *p.*409.

from the true place of the *Sun*, and which with the Addition of the Equation of *Numb. B.* gives the true Angle of *Commutation* in the ſame Milleſimals of a Circle. And having deducted from the *Epoches* the greateſt Equations both of *Numb. A.* and *B.* he reſtores them by adding as much to the Equations themſelves, by which means they all become Affirmative, ſo that the whole Computation is performed by Addition only.

The Reader is ſuppos'd to be acquainted with the Method of M. *Caſſini's Calculus*, which is at large explain'd in the aforeſaid *Tranſact. Numb.* 214. For which reaſon this ſhort Deſcription may ſuffice at preſent.

EPOCHÆ

Epochæ Conjunctionum Primi Satellitis cum Jove.

An. Jul. Curr.	Conjunct.				Num. A.	Num. B.	An. Jul. Curr.	Conjunct.				Num. A.	Num. B.
	D.	H.	′	″				D.	H.	′	″		
1719	1	6	11	13	872	396	1749	0	11	9	34	400	866
1720	0	20	22	40	956	310	1750	0	1	21	1	485	780
21	1	5	2	44	40	229	51	1	10	1	5	569	698
22	0	19	14	11	125	143	52	1	0	12	33	653	612
23	0	9	25	38	209	57	53	1	8	52	37	738	531
1724	1	18	5	42	293	971	1754	0	23	4	4	822	445
1725	0	8	17	10	377	889	1755	0	13	15	32	906	359
26	1	16	57	13	462	808	56	0	3	27	0	990	273
27	1	7	8	41	546	722	57	0	12	7	3	75	191
28	0	21	20	8	630	636	58	0	2	18	30	159	110
1729	1	6	0	12	715	554	1759	1	10	58	34	243	24
1730	0	20	11	39	799	468	1760	1	1	10	1	328	938
31	0	10	23	7	883	382	61	1	9	50	35	412	856
32	0	0	34	34	967	296	62	1	0	1	2	496	770
33	0	9	14	38	52	215	63	0	14	13	0	580	684
1734	1	17	54	41	136	133	1764	0	4	24	27	665	598
1735	1	8	6	9	220	47	1765	0	13	4	31	749	517
36	0	22	17	36	305	961	66	0	3	15	58	833	431
37	1	6	57	40	389	880	67	1	11	56	2	918	349
38	0	21	9	7	473	794	68	1	2	7	29	2	263
1739	0	11	20	35	557	708	1769	1	10	47	33	86	182
1740	0	1	32	2	642	622	1770	1	0	59	0	171	96
41	0	10	12	6	726	540	71	0	15	10	28	255	10
42	0	0	23	33	810	454	72	0	5	21	56	339	924
43	1	9	3	37	895	373	73	0	14	2	0	423	842
1744	0	23	15	4	979	287	1774	0	4	13	27	508	761
1745	1	7	55	8	63	205	1775	1	12	53	31	592	675
46	0	22	6	35	148	119	76		3	4	58	676	589
47	0	12	18	3	232	33	77	1	11	45	1	761	507
48	0	2	29	30	316	947	78	1	1	56	28	845	421
1749	0	11	9	34	400	866	1779	0	16	7	56	929	335

Revolutiones Primi Satellitis Jovis in menſibus:

Januarii.				N.	Nu.
D.	h.	′	″	A.	B.
1	18	28	36	0	5
3	12	57	12	1	9
5	7	25	48	1	14
7	1	54	24	2	18
8	20	23	0	2	23
10	14	51	36	2	27
12	9	20	12	3	32
14	3	48	48	3	37
15	22	17	24	4	41
17	16	46	0	4	46
19	11	14	36	4	51
21	5	43	12	5	55
23	0	11	47	5	60
24	18	40	23	6	64
26	13	8	59	6	69
28	7	37	35	7	73
30	2	6	11	7	78
31	20	34	47	7	82
Februarii					
0	20	34	47	7	82
2	15	3	23	8	87
4	9	31	59	8	92
6	4	0	35	9	96
7	22	29	11	9	101
9	16	57	47	9	105
11	11	26	23	10	110
13	5	54	59	10	114
Februarii.				N.	Nu.
D.	h.	′	″	A.	B.
15	0	23	35	11	118
16	18	52	11	11	123
18	13	20	47	11	128
20	7	49	23	12	132
22	2	17	59	12	137
23	20	46	35	13	141
25	15	15	11	13	146
27	9	43	47	13	150
Martii.					
1	4	12	23	14	155
2	22	40	59	14	159
4	17	9	35	15	164
6	11	38	10	15	168
8	6	6	46	16	173
10	0	35	22	16	177
11	19	3	58	16	182
13	13	32	34	17	186
15	8	1	10	17	190
17	2	29	46	18	195
18	20	58	22	18	199
20	15	26	58	18	204
22	9	55	34	19	208
24	4	24	10	19	213
25	22	52	46	20	217
27	17	21	22	20	221
29	11	49	58	20	225
31	6	18	34	21	230

Aprilis

Revolutiones Primi Satellitis Jovis in menſibus.

Aprilis.				N.	Nu.
D.	h.	′	″	A.	B.
0	6	8	34	21	230
2	0	47	10	21	235
3	19	15	46	22	239
5	13	44	22	22	244
7	8	12	58	22	248
9	2	41	34	23	252
10	21	10	10	23	257
12	15	38	46	24	261
14	10	7	22	24	265
16	4	35	58	25	270
17	23	4	33	25	274
19	17	33		25	279
21	12	1	45	26	283
23	6	30	21	26	287
25	0	58	57	27	292
26	19	27	33	27	296
28	13	56	9	27	300
30	8	24	45	28	304
Maii					
0	8	24	45	28	304
2	2	53	21	28	309
3	21	21	57	29	313
5	15	50	33	29	317
7	10	19	9	29	322
9	4	47	45	30	326
10	23	16	21	30	330
12	17	44	57	31	335
14	12	13	33	31	339

Maii.				N.	Nu.
D.	h.	′	″	A.	B.
16	6	42	9	31	343
18	1	10	45	32	348
19	19	39	21	32	352
21	14	7	57	33	356
23	8	36	33	33	361
25	3	5	9	33	365
26	21	33	45	34	369
28	16	2	21	34	373
30	10	30	57	35	378
Junii					
0	10	30	57	35	378
1	4	59	32	35	382
2	23	28	8	36	386
4	17	56	44	36	391
6	12	25	20	36	395
8	6	53	56	37	399
10	1	22	32	37	403
11	19	51	8	38	408
13	14	19	44	38	412
15	8	48	20	38	416
17	3	16	56	39	420
18	21	45	32	39	425
20	16	14	8	40	429
22	10	42	44	40	433
24	5	11	20	40	438
25	23	39	56	41	442
27	18	8	32	41	446
29	12	37	8	42	450

Revolutiones Primi Satellitis Jovis in menſibus.

Julii. D.	h.	′	″	N. A.	Nu. B.
1	7	5	44	42	455
3	1	34	20	42	459
4	20	2	56	43	463
6	14	31	32	43	468
8	9	0	8	44	472
10	3	28	44	44	476
11	21	57	20	45	480
13	16	25	55	45	485
15	10	54	31	45	489
17	5	23	7	46	493
18	23	51	43	46	498
20	18	20	19	47	502
22	12	48	55	47	506
24	7	17	31	47	510
26	1	46	7	48	515
27	20	14	43	48	519
29	14	43	19	49	523
31	9	11	55	49	528
Auguſti.					
0	9	11	55	49	528
2	3	40	31	49	532
3	22	9	7	50	536
5	16	37	43	50	541
7	11	6	19	51	545
9	5	34	55	51	549
11	0	3	31	51	554
12	18	32	7	52	558
14	13	0	43	52	562

Auguſti. D.	h.	′	″	N. A.	Nu. B.
16	7	29	19	53	567
18	1	57	55	53	571
19	20	26	31	54	575
21	14	55	7	54	580
23	9	23	43	54	584
25	3	52	18	55	588
26	22	20	54	55	593
28	16	49	30	56	597
30	11	18	6	56	602
Septembris.					
1	5	46	42	56	666
3	0	15	18	57	610
4	18	43	54	57	615
6	13	12	30	58	619
8	7	41	6	58	624
10	2	9	42	58	628
11	20	38	18	59	632
13	15	6	54	59	637
15	9	35	30	60	641
17	4	4	6	60	646
18	22	32	42	60	650
20	17	1	18	61	655
22	11	29	54	61	659
24	5	58	30	62	663
26	0	27	6	62	668
27	18	55	42	62	672
29	13	24	18	63	677

Revolutiones Primi Satellitis Jovis in menſibus.

D.	h.	′	″	N. A.	Nu. B.
Octobris.					
1	7	52	54	63	681
3	2	21	30	64	686
4	20	50	6	64	690
6	15	18	41	65	695
8	9	47	17	65	699
10	4	15	53	65	704
11	22	44	29	66	708
13	17	13	5	66	713
15	11	41	41	67	717
17	6	10	17	67	721
19	0	38	53	67	726
20	19	7	29	68	730
22	13	36	5	68	735
24	8	4	41	69	739
26	2	33	17	69	744
27	21	1	53	69	749
29	15	30	29	70	753
31	9	59	5	70	758
Novembris.					
0	9	59	5	70	758
2	4	27	41	71	762
3	22	56	17	71	767
5	17	24	53	71	772
7	11	53	29	72	776
9	6	22	5	72	781
11	0	50	41	73	785
12	19	19	17	73	790
14	13	47	53	74	794
16	8	16	29	74	799
18	2	45	5	74	804
19	21	13	40	75	808
21	15	42	16	75	813
23	10	10	52	76	817
25	4	39	28	76	822
26	23	8	4	76	827
28	17	36	40	77	831
30	12	5	16	77	836
Decembris.					
0	12	5	16	77	836
2	6	33	52	78	840
4	1	2	28	78	845
5	19	31	4	78	849
7	13	59	40	79	854
9	8	28	16	79	859
11	2	56	52	80	863
13	21	25	28	80	868
14	15	54	4	80	873
16	10	22	40	81	877
18	4	51	16	81	882
19	23	19	5	82	886
21	17	48	28	82	891
23	12	17	4	82	897
25	6	45	40	83	900
27	1	14	16	83	905
28	19	42	52	84	909
30	14	11	28	84	194

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num. A	Æquat. Conjun. Adde.		Æq Nu. B	Num. A	Æquat. Conjun. Adde.		Æq Nu. B	Num. A	Æquat. Conjun. Adde.		Æq Nu. B	Num. A.	Æquat. Conjun. Adde.		Æq Nu. B
	′	″			′	″			′	″			′	″	
0	39	8	15	128	12	7	26	256	0	1	31	384	11	52	26
4	38	12	16	132	11	27	26	260	0	0	31	388	12	37	26
8	37	16	16	136	10	47	26	264	0	1	31	392	13	23	25
12	36	21	16	140	10	9	27	268	0	3	31	396	14	11	25
16	35	26	17	144	9	31	27	272	0	7	31	400	14	59	25
20	34	30	17	148	8	45	27	276	0	12	31	04	15	48	24
24	33	35	17	152	8	19	27	280	0	19	31	408	16	38	24
28	32	40	18	156	7	44	28	284	0	28	30	412	17	30	24
32	31	45	18	160	7	10	28	288	0	38	30	416	18	22	23
36	30	50	19	164	6	38	28	292	0	50	30	420	19	15	23
40	29	56	19	168	6	7	28	296	1	3	30	424	20	9	23
44	29	3	19	172	5	37	28	300	1	17	30	428	21	4	22
48	28	10	20	176	5	8	29	304	1	33	30	432	22	59	22
52	27	16	20	180	4	41	29	308	1	50	30	436	22	55	22
56	26	23	20	184	4	15	29	312	2	8	30	440	23	53	21
60	25	30	21	188	3	49	29	316	2	28	30	444	24	51	21
64	24	38	21	192	3	24	29	320	2	51	30	448	25	49	21
68	23	47	21	196	3	1	29	324	3	15	29	452	26	48	20
72	22	56	22	200	2	40	30	328	3	40	29	456	27	48	20
76	22	5	22	204	2	20	30	332	4	6	29	460	28	48	19
80	21	15	22	208	2	1	30	336	4	34	29	464	29	49	19
84	20	26	23	212	1	42	30	340	5	3	29	468	30	50	19
88	19	37	23	216	1	25	30	344	5	34	29	472	31	51	18
92	18	48	23	220	1	10	30	348	6	5	28	476	32	53	18
96	18	0	24	224	0	58	30	352	6	38	28	480	33	55	17
100	17	14	24	228	0	47	30	356	7	13	28	484	34	57	17
104	16	28	24	232	0	36	30	360	7	50	28	488	35	59	17
108	15	42	24	236	0	26	30	364	8	27	27	492	37	1	16
112	14	57	25	240	0	18	30	368	9	6	27	496	38	5	16
116	14	13	25	244	0	12	31	372	9	46	27	500	39	8	15
120	13	30	25	248	0	7	31	376	10	27	27	504	40	11	15
124	12	48	26	252	0	4	31	380	11	9	26	508	41	15	14
128	12	7	26	256	0	1	31	384	11	52	26	512	42	17	14

Primæ

Primæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Num. A.	Æquat. Conjunct. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq. Nu. B.	Num. A	Æquat. Conjun. Adde. ′	″	Æq Nu. B.	Num. A.	Æquat. Conjun. Adde. ′	″	Æq Nu. B
512	42	17	14	640	70	26	3	768	77	40	0	896	61	48	6
516	43	19	14	644	71	3	3	772	77	29	0	900	61	2	7
520	44	21	13	648	71	38	3	776	77	18	0	904	60	15	7
524	45	23	13	652	72	11	2	780	77	6	0	908	59	28	7
528	46	25	13	656	72	42	2	784	76	51	1	912	58	39	8
532	47	26	12	660	73	13	2	788	76	34	1	916	57	50	8
536	48	27	12	664	73	42	2	792	76	15	1	920	57	1	8
540	49	28	11	668	74	10	2	796	75	56	1	924	56	11	9
544	50	28	11	672	74	36	1	800	75	36	1	928	55	20	9
548	51	28	11	676	75	1	1	804	75	15	1	932	54	29	9
552	52	27	10	680	75	25	1	808	74	52	1	936	53	38	10
556	53	25	10	684	75	48	1	812	74	27	1	940	52	46	10
560	54	23	9	688	76	8	1	816	74	1	2	944	51	53	10
564	55	21	9	692	76	26	1	820	73	35	2	948	51	0	11
568	56	17	9	696	76	43	0	824	73	8	2	952	50	6	11
572	57	12	8	700	76	59	0	828	72	39	2	956	49	13	11
576	58	7	8	704	77	13	0	832	72	9	2	960	48	20	12
580	59	1	8	708	77	26	0	836	71	38	3	964	47	26	12
584	59	54	7	712	77	38	0	840	71	6	3	968	46	31	12
588	60	46	7	716	77	48	0	844	70	32	3	972	45	36	13
592	61	38	6	720	77	57	0	848	69	57	3	976	44	41	13
596	62	28	6	724	78	4	0	852	69	21	3	980	43	46	13
600	63	17	6	728	78	9	0	856	68	45	4	984	42	50	14
604	64	5	5	732	78	13	6	860	68	7	4	988	41	55	14
608	64	53	5	736	78	15	0	864	67	29	4	992	41	0	14
612	65	39	5	740	78	16	0	868	66	49	4	996	40	4	15
616	66	24	5	744	78	15	0	872	66	9	5	1000	39	8	15
620	67	7	4	748	78	12	0	876	65	28	5	1004	38	12	16
624	67	49	4	752	78	9	0	880	64	46	5	1008	37	16	16
628	68	30	4	756	78	4	0	884	64	3	5	1012	36	21	16
632	69	10	4	760	77	58	0	888	63	19	6	1016	35	26	17
636	69	49	3	764	77	50	0	892	62	34	6	1020	34	30	17
640	70	26	3	768	77	40	0	896	61	48	6	1024	33	35	17

Secundæ Æquationes Conjunctionum Primi Satellitis cum Jove.

Addenda.

Num. B. Æqu	0	100	200	300	400	500	600	700	800	900
	Æqu.	Æqu.	Æq.	Æq.	Æq.	Æq.	Æq.	Æq.	Æqu.	Æqu.
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	14 0	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52
4	14 0	12 46	9 36	5 20	1 30	0 0	1 44	5 40	9 54	12 58
8	13 59	12 40	9 26	5 9	1 23	0 1	1 52	5 51	10 3	13 2
12	13 59	12 35	9 17	4 59	1 16	0 2	1 59	6 1	10 12	13 7
16	13 58	12 29	9 7	4 48	1 9	0 3	2 7	6 11	10 21	13 11
20	13 57	12 23	8 58	4 38	1 3	0 4	2 15	6 22	10 31	13 16
24	13 56	12 17	8 48	4 28	0 57	0 5	2 24	6 33	10 40	13 20
28	13 54	12 11	8 38	4 18	0 52	0 7	2 32	6 44	10 49	13 25
32	13 53	12 4	8 28	4 8	0 46	0 10	2 41	6 55	10 57	13 29
36	13 51	11 56	8 17	3 58	0 40	0 13	2 50	7 5	11 5	13 33
40	13 49	11 49	8 7	3 58	0 35	0 16	2 59	7 16	11 11	13 36
44	13 47	11 42	7 57	3 38	0 31	0 19	3 9	7 26	11 20	13 38
48	13 44	11 34	7 47	3 29	0 27	0 23	3 19	7 36	11 27	13 41
52	13 41	11 27	7 36	3 19	0 23	0 27	3 29	7 47	11 34	13 44
56	13 38	11 20	7 26	3 9	0 19	0 31	3 38	7 57	11 42	13 47
60	13 36	11 13	7 16	2 59	0 16	0 35	3 48	8 7	11 49	13 49
64	13 33	11 5	7 5	2 50	0 13	0 40	3 58	8 17	11 56	13 51
68	13 29	10 57	6 55	2 41	0 10	0 46	4 8	8 28	12 4	13 53
72	13 25	10 49	6 44	2 32	0 7	0 52	4 18	8 38	12 11	13 54
76	13 20	10 40	6 33	2 24	0 5	0 57	4 28	8 48	12 17	13 56
80	13 16	10 31	6 22	2 15	0 4	1 3	4 38	8 58	12 23	13 57
84	13 11	10 21	6 11	2 7	0 3	1 9	4 48	9 7	12 29	13 58
88	13 7	10 12	6 1	1 59	0 2	1 16	4 59	9 17	12 35	13 59
92	13 2	10 3	5 51	1 52	0 1	1 23	5 9	9 26	12 40	13 59
96	12 58	9 54	5 40	1 44	0 0	1 30	5 20	9 36	12 46	14 0
100	12 52	9 45	5 30	1 37	0 0	1 37	5 30	9 45	12 52	14 0

Te[illegible]æ Æquationes Addendæ.				*Semidurationes Eclipſium Primi Satellitis Jovis.*															
Nu. A.	*Æquationes*		*Num.* A.	*Nu.* A.	*Semidurationes.*			*Nu.* A.	*Semidurationes.*			*Nu.* A.	*Semidurationes*			*Nu.* A.	*Semidurationes*		
	′	″			H.	′	″		H	′	″		H.	′	″		H.	′	″
0	3	30	1000	0	1	5	9	250	1	7	0	500	1	5	9	750	1	7	46
20	3	29	980	10	1	4	56	260	1	7	15	510	1	4	53	760	1	7	57
40	3	28	960	20	1	4	44	270	1	7	31	520	1	4	39	770	1	8	7
60	3	25	940	30	1	4	33	280	1	7	45	530	1	4	26	780	1	8	15
80	3	19	920	40	1	4	23	290	1	7	57	540	1	4	15	790	1	8	22
100	3	12	900	50	1	4	13	300	1	8	7	550	1	4	7	800	1	8	26
120	3	4	880	60	1	4	7	310	1	8	15	560	1	4	3	810	1	8	28
140	2	56	860	70	1	4	4	320	1	8	22	570	1	4	1	820	1	8	30
160	2	46	840	80	1	4	2	330	1	8	27	580	1	4	0	830	1	8	28
180	2	34	820	90	1	4	0	340	1	8	28	590	1	4	3	840	1	8	26
200	2	22	800	100	1	4	2	350	1	8	29	600	1	4	7	850	1	8	22
220	2	10	780	110	1	4	3	360	1	8	27	610	1	4	13	860	1	8	16
240	1	57	760	120	1	4	6	370	1	8	24	620	1	4	23	870	1	8	8
260	1	44	740	130	1	4	12	380	1	8	17	630	1	4	35	880	1	8	0
280	1	30	720	140	1	4	21	390	1	8	9	640	1	4	49	890	1	7	50
300	1	17	700	150	1	4	31	400	1	7	58	650	1	5	4	900	1	7	37
320	1	5	680	160	1	4	42	410	1	7	46	660	1	5	19	910	1	7	22
340	0	53	660	170	1	4	55	420	1	7	31	670	1	5	36	920	1	7	8
360	0	41	640	180	1	5	9	430	1	7	14	680	1	5	54	930	1	6	55
380	0	31	620	190	1	5	23	440	1	6	58	690	1	6	10	940	1	6	40
400	0	22	600	200	1	5	39	450	1	6	40	700	1	6	28	950	1	6	23
420	0	14	580	210	1	5	55	460	1	6	20	710	1	6	46	960	1	6	8
440	0	8	560	220	1	6	11	470	1	6	2	720	1	7	2	970	1	5	54
460	0	4	540	230	1	6	26	480	1	5	45	730	1	7	17	980	1	5	37
480	0	2	520	240	1	6	43	490	1	5	26	740	1	7	33	990	1	5	22
500	0	0	500	250	1	7	0	500	1	5	9	750	1	7	46	1000	1	5	9

The uſe of the foregoing Tables.

The Eclipſes of the firſt Satellite of *Jupiter*, as has been already ſaid, afford the beſt means of determining the Longitude of places on the Land, where Teleſcopes of a convenient length may be uſed; thirteen of theſe Eclipſes happening every 23 Days; but it is requiſite that the Obſerver know near the matter when theſe Opportunities offer themſelves, leaſt on the one Hand he let them ſlip, or elſe grow weary by a too long Attendance on them.

Thoſe therefore who are curious to obſerve them, may readily compute the times of the Immerſions or Emerſions of this Satellite, and that with great Exactneſs, by the following very ſhort Precepts, which admit of no Exception or Caution, *viz.*

Out of the firſt Table take the *Epoche* for the Year, with its correſponding *Numb. A.* and *Numb. B*; and to them add, out of the Tables of Months, the Day, Hour, Minute and Second, neareſt leſs than the time of the Eclipſe you ſeek for, together with its *Numb. A* and *B*: the Sum of the Times is the mean time of the middle of the Eclipſe. 2. With *Numb. A.* thus collected, take out the firſt Æquation of the Conjunctions; as alſo the Æquation of *Numb. B.* always to be added to *Numb. B.* before found. 3. With *Numb. B.* ſo equated, take out the ſecond Æquation of the Conjunctions; and in the laſt Table, the third Æquation, as alſo the Semi-duration of the Eclipſe anſwering to *Numb. A.* 4. To the mean time of the middle of the Eclipſe, add all thoſe three Æquations; the Sum ſhall be the true equated time of the middle of the Eclipſe ſought. 5. If *Numb. B.* equated be leſs than 500, ſubſtract the Semiduration, and you will have the time of the Immerſion, or if it be more than 500, adding the ſame, it will give the time of the Emerſion.

But Note, the times thus found are equal time, ſtill to be reduced to the Apparent: and that in the *Biſſextile* Year, after *February*, one Day is to be deducted from the Day of the Month.

The leſs ſkilful may perhaps be pleas'd with an Example or two, which may ſerve them to imitate. Let it be required to find the time of the Immerſion of this Satellite into *Jupiter*'s Shadow, *November* the 9th 1719. in the Morning. The Work ſtands thus,

	D.	h.	'	"	Nu. A.	Nu. B.	
1719.	1.	6.	11.	13	872	396	
Novemb.	7.	11.	53.	29	72	776	
Conj. Med.	8.	18.	4.	42	944	172	
Æquat. I.			51.	53		10	Æq. B.
Æquat. II.			10.	26			
Æquat. III.			3.	26		182	B. Æquat.
	8.	19.	10.	27			
		1.	6.	33	*Semidur. Sub.*		
Novemb.	8.	18.	3.	54			

So that by this *Calculus*, on the ninth of *Novemb.* at 4 Minutes after 6 in the Morning, equal Time, may be ſeen the *Immerſion* of this Satellite into *Jupiter*'s Shadow.

Another Example ſhall be of the *Emerſion* on the fifth of *April* 1720, *viz.*

	D.	H.	′	″		Nu.A.	Nu. B.	
1720.	0.	20.	22.	40		956	310	
April	4.	13.	44.	22	Biſs.	22	244	
Conj. Med.	5.	10.	07.	02		978	554	
Æquat. I.			44.	13			13	Æq. B.
Æquat II.			0.	45			567	B. Æquat.
Æquat. III.			3.	29				
		1.	5.	40	*Semidur. Add.*			
April	5.	12.	01.	09				

Hence it appears that at one Minute after Midnight following the fifth of *April*, equal Time, will happen the Emerſion required. Nor do we doubt but that the Event will very nearly anſwer.

We have learnt, by the Experience of many Years Obſervation, that the ſecond inequality of this Satellite proceeds from the progreſſive Propagation of Light, and is common to all the reſt of the Satellites; Light being found to proceed in about ſeven Minutes of time as far as from the Sun to the Earth, whether with an equable Motion or otherwiſe is ſtill a Queſtion. For this reaſon we have added a *third Æquation*, whereby to account for the greater diſtance of *Jupiter* from the *Earth* in *Aphelio* than in *Perihelio*, as the *ſecond Æquation* anſwers to the greater diſtance of the Planet when near the Conjunction of the Sun, than when near his Oppoſition.

Tranſit of Mars, *near a Fixt Star, n.* 351. *p.* 548.

XXXVIII. *Februarii* Quinto Mane, vel Quarto 16^h. 1717. *Mars* viſus eſt adeo vicinus Boreæ Frontis Scorpii ut ea nudis oculis non conſpiceretur; ſed per Teleſcopium inventa eſt ſupra & ad ortum, adeoque *Mars* nondum ei conjunctus. Hora 16^h. 10′. T. app. *Mars* erat in recta cum Borea frontis & Teleſcopica quæ eam ſequitur ad Boream, ad diſtantiam octo circiter minutorum 16^h. 35′. *Mars* intermedius erat in recta cum Borea & Media Frontis; & poſt horæ quadrantem, cum Auſtrina Frontis, ita ut 16^h. 54′. T. app. æſtimabatur Conjunctio ipſa quoad Longitudinem, quo tempore *Mars* ſat accurate duobus tantum minutis auſtralior erat ſtella. Obſervavit etiam D. *Pound* Conjunctionem reſpectu Aſcenſionis Rectæ 17^h. 25′. T. app. cum diſtantia centrorum 2′. 07″. Jucundum autem erat ſpectaculum, *Martem* videre ſtellam pedetentim aggredientem, motumque ſuum, lentiſſimum licet, manifeſte prodentem.

Conferatur cum hac Obſervatio *Horroxii* noſtri anno 1638. *Februarii* Septimo mane, quam vide in Epiſtolis ejus pag. 304. Tunc enim *Mars* ad eandem ſtellam appulſus, etiam multo propius ad eam acceſſit, ſed ante ortum ejus præterierat Conjunctio.

His adde *Saturni* obſervationem *Januarii* 25to. $12^h. 25'$. T. æq. a D. *Pound* habitam. Cum Planeta diſtabat a ſtella 58va. *Virginis Catal. Brit.* $13'. 16''$. verſus Auſtrum, eamque ſequebatur $2'. 30''$. Aſc. Rect. Stella in ♎ $19. 21'. 52''$. cum lat. Bor. $2^{\circ}. 47'. 25''$.

The Cauſe of the Appearance of Venus, *for many Days together in the Day time, by* Dr. Halley, *n.* 349. *p.* 466.

XXXIX. The Appearance of *Venus* in the Day-time, in the Summer, 1716. generally taken notice of about *London* and elſewhere, was by ſome reckoned to be prodigious. This put me upon the Enquiry, how it came to paſs that at that time the *Planet* ſhould be ſo plainly ſeen by Day, whereas ſhe rarely ſhews her ſelf ſo, unleſs to thoſe who know exactly where to look for her. To reſolve this, the following Problem aroſe, *viz.* To find the Situation of the *Planet* in reſpect of the *Earth*, when the *Area* of the illuminated part of her Disk is a *Maximum.*

To inveſtigate this *Maximum,* I found it requiſite to aſſume the following *Lemmata.* I. That the viſible *Areas* of the Disk of the ſame *Planet*, at differing Diſtances, are always reciprocally as the Squares of thoſe Diſtances; which is evident from the firſt Principles of *Opticks.* II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the illuminated part thereof, as the Diameter of a Circle to the Verſed-Sine of the exteriour Angle at the *Planet*, in the Triangle at whoſe Angles are the *Sun*, *Earth*, and *Planet.* III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the exceſs of the Square of the Sum of the Sides above the Square of the Baſe, as the Diameter is to the Verſed Sine of the Complement of the contained Angle to a Semicircle, which I call the exterior Angle: This is a new *Theorem* of good uſe in *Trigonometry*, and eaſily proved from the 12th and 13th of the II. *Elem. Euclid.*

This premiſed, putting m for the Diſtance of the *Sun* and *Earth*, and n for that of the *Sun* and *Venus*, and x for the diſtance of the *Earth* and *Venus*, or the third ſide of the Triangle which we ſeek; by the third *Lemma*, $4nx$, will be to the exceſs of the Square of $n+x$, above the Square of m, as the *Area* of the whole Disk of *Venus* to the *Area* of the part illuminated; and by the firſt *Lemma*, the *Area*'s of her whole Disk are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn+2nx+xx-mm}{4nx^3}$ will in all Caſes be proportional to the *Area* of the illuminated part.

Now that this ſhould be a *Maximum*, it is required that the Flux-n thereof be equal to 0, or that the Negative parts thereof be l to the Affirmative, that is, that $\overline{2n\dot{x}+2x\dot{x}}\times 4nx^3 = 12nx^2\dot{x}$ $\overline{+2nx+xx-mm}$; and dividing all by $4nx^2x$, the Equation becomes

comes $2nx + 2xx = 3nn + 6nx + 3xx - 3mm$. Conſequently $3nn + 4nx + xx = 3mm$, and therefore $x = \sqrt{3mm + nn} - 2n$.

From hence a ready and not inelegant Geometrical Conſtruction (if I may be allowed to ſay ſo) becomes obvious; for with the Center *S* and Radius *S T=m*, deſcribe the Semicircle *T D A*; and with the ſame Center and Radins *S E=n*, the Semicircle *E V B*; which two Semicircles ſhall repreſent the Orbs of the *Earth* and *Venus*. Make the Chord *A D* equal to the Radius *S T*, and from *D* towards *A*, lay off *D F=S E*; draw *T F*, and thereon place *F G=B E=2n*, and with the Center *T*, and *Radius T G* deſcribe the Arch *G V*, cutting the Semicircle *B V E* in *V*; and draw the Lines *S V*, *T V*. I ſay the Triangle *S T V* is Similar to that at whoſe Angles are the *Sun*, *Earth* and *Venus*, at the time when the *Area* of the enlightned part of that Planet's Disk, as ſeen from the *Earth*, is greateſt. How this Geometrical Effection follows from the Equation is too evident to need Repetition.

Fig. 5. *Plate* 4.

In conſequence of this Solution, I find this *Maximum* always to happen, when the Planet is about forty Degrees diſtant from the *Sun*; and the times thereof, about the middle between her greateſt Elongations on both ſides from him, and her retrograde Conjunctions with him; when little more than a quarter of her viſible Disk is luminous, and reſembling the Moon of about five days old; and notwithſtanding that her Diameter is at that time but 50 Seconds, yet ſhe ſhines with ſo ſtrong a Beam, as to ſurpaſs the united Light of all the Fixt Stars that appear with her, and caſts a very ſtrong Shade on the Horizontal plain whereon they all ſhine: an irrefragable Argument to prove that the Disks of the fixt Stars are unconceivably ſmall, and next to nothing; ſince ſhining with a native Light, ſo many of them do not equal the reflex Light of one quarter of a Disk of leſs than a Minute diameter.

In this Situation *Venus* was found in *July* laſt, on the tenth day, about which time, when the *Sun* grew low, ſhe was very plainly ſeen in the day-time, for many days together; as ſhe might have been in the Mornings, about the latter end of *September*. But this ariſing from the Cauſes we have now ſhewn, is nothing uncommon; for every eighth Year it returns again, ſo that the Planet may be ſeen on the ſame day of the Month and Hour, very nearly in the ſame place; as all acquainted with the Heavenly Motions muſt know.

Laſtly, it may not be amiſs to note that the Equation $x = \sqrt{3mm + nn} - 2n$ has a Limit; for if n be equal to $\frac{1}{4}m$. the point *V* will fall on *B*; and the whole Disk of a Planet at that diſtance from the Sun would be the *Maximum*, *viz.* when in its ſuperior Conjunction with the *Sun*. And the like if n were leſs than $\frac{1}{4}m$; the Arch *G V* in ſuch Caſe not interſecting the Semicircle *B E*.

XL. Veteres *Ægyptii & Chaldæi*, ſiqua fides *Diodoro Siculo*, longa obſervationum ſerie inſtructi, Cometarum ἐπιτολὰς prænuntiare valuerunt. Cum autem iiſdem artibus etiam Terræ motus ac tempeſtates prævidiſſe dican-

The Aſtronomy of Comets, by Dr. Halley, *n.* 297. *p.* 1882.

dicantur, extra dubium eſt Aſtrologiæ potius calculo fatidico, quam Aſtronomicis motuum Theoriis eorum de his rebus ſcientiam referendam eſſe. Ac vix alia a Græcis utriuſque populi victoribus reperta eſt apud eos doctrina; adeo ut eam, quam nunc eouſque proveximus Aſtronomiam, Græcis ipſis, præſertim magno *Hipparcho*, uti inventoribus, acceptam debeamus. Apud hos vero *Ariſtotelis* ſententia, qui Cometas nihil aliud eſſe voluit quam Vapores ſublunares vel etiam Meteora aerea, tantum effecit, ut hæc Aſtronomicæ ſcientiæ pars longe ſubtiliſſima omnino neglecta manſerit, cum nemini operæ pretium viſum fuerit, vagas & incertas fluitantium in æthere vaporum ſemitas adnotare ſcriptiſque mandare; unde factum ut ab illis nihil certi de motu Cometarum ad nos tranſmiſſum reperiatur.

Seneca autem Philoſophus, perpenſis duorum inſignium ſui temporis Cometarum Phænomenis, non dubitavit iis loca inter corpora cœleſtia aſſignare, Sydera eſſe cum mundo duratura exiſtimans, quanquam Motus eorum legibus nondum compertis regi fateatur. Tandemque Vaticinio non irrito promittit aliquando futura ſecula, quibus hæc tam occulta *dies extraheret ac longioris ævi diligentia*: quibuſque admirationi foret hæc *Veteres* neſcire potuiſſe; poſtquam *Demonſtraverit aliquis* Naturæ Interpres *in quibus Cœli partibus Cometæ errent, quanti, qualeſque ſint.* Ab hac autem *Senecæ* ſententia in diverſas partes abiit pene omnis Aſtronomorum Cohors; ac ipſe *Seneca*, neque Phænomena Motus quibus opinionem hanc tueretur, neque tempora adſcribere dignatus eſt, quæ poſteris ad hæc definienda uſui forent. Ac evolutis plurimis Cometarum Hiſtoriis, nihil omnino invenio quod huic negotio inſervire poſſit, ante annum a *Chriſto* nato 1337. quo *Nicephorus Gregoras* Hiſtoricus & Aſtronomus *Conſtantinopolitanus* nobis Cometæ ſemitam inter fixas ſatis accurate deſcripſit: tempora autem nimis laxe conſignavit, ita ut non niſi quod abhinc quadringentis pene Annis apparuerit, lubricus & incertus hic Cometa Catalogo quem damus inſeri mereatur. Dein Cometa anni 1472 omnium velociſſimus ac terris proximus *Regiomontanum* habuit obſervatorem. Hic magnitudine ac Coma terribilis, unius diei ſpatio 40 gradus ſub circulo Cœli maximo emenſus eſt, ac omnium primus eſt de quo obſervata idonea ad nos pervenere. Quotquot autem Cometas conſiderarunt, uſque ad tempora *Tychonis Brahe* magni illius Aſtronomiæ reſtauratoris, eos ſublunares eſſe autumarunt, adeoque parvi penderunt, utpote pro Vaporibus habitos.

Anno autem 1577, (*Tychone* jam ſtudio aſtrorum ſerio incumbente, comparatiſque Machinis ingentibus pro dimetiendis cœli arcubus, majori cum cura & certitudine quam Veteribus ſperare fas erat) Emerſit Cometa ſatis conſpicuus, cui obſervando ſtrenue ſeſe accinxit *Tycho*: multiſque & fidis experimentis deprehendit, nulli quæ ſentiretur Parallaxi diurnæ obnoxium fuiſſe, adeoque non tantum non fuiſſe Vaporem aereum, ſed & etiam multo ſuperiorem extitiſſe Luna: immo nihil obſtabat quin inter ipſos Planetas collocaretur; fruſtra interim contra obſtrepentibus Scholaſticorum nonnullis.

Tychonis vero eximiam in obſervando induſtriam excepit *Kepleri* ſagaciſſimum & pene divinum ingenium. Hic *Tychonis* laboribus fretus Syſtema Mundi verum & Phyſicum adinvenit, ac ſcientiam Aſtronomicam in immenſum auxit; Monſtrato ſc. Planetas omnes in Planis per Solis centrum tranſeuntibus revolvi, Curvaſque Ellipticas deſcribere, ea lege, ut Areæ Sectorum Ellipticorum, ad centrum Solis in Ellipſeos foco conſtituti, temporibus, quibus deſcribantur arcus, ſemper proportionales ſint. Invenit etiam Diſtantias Planetarum a Sole eſſe in ſeſquialtera ratione temporum periodicorum, ſive Cubos diſtantiarum eſſe ut Quadrata Temporum. Tanto autem Artifici affulſere duo Cometæ, quorum alter maxime illuſtris. Ex horum obſervatis concluſit *Keplerus* non uno parallaxis annuæ indicio, Cometas inter Orbes planetarum liberrime quaquaverſum ferri, motu quidem non multum a rectilineo diverſo, ſed quem nondum definire licuit. Ac *Hevelius*, *Tychonis* æmulus, *Kepleri* veſtigiis inſiſtens, eandem Hypotheſim Motus rectilinei amplexus eſt, ipſe plurium Cometarum Obſervator perquam ſubtilis. Cœlo tamen Calculum ſuum non penitus conſentire queſtus eſt, Viamque Cometicam verſus Solem incurvari ſuboluit.

Tandem de ſummo Cœlo lapſus eſt prodigioſus ille Cometa Anni 1680. quaſi Caſu perpendiculari Solem petens, & exinde pari velocitate aſſurgens: Hic per quatuor Menſes continuos viſus, inſigni ac peculiari Curvitate Orbitæ ad inveſtigationem Motus Theoriæ præ cæteris idoneus erat: Inſtructis autem jampridem Regiis Obſervatoriis, *Pariſienſi* & *Grenovicenſi*, ac Aſtronomorum Clariſſimorum curæ commiſſis, accidit ut hujus Cometæ Motus apparens, quantum forſan mortalibus fas eſt, accuratiſſime a *Caſſino* & *Flamſtedio* obſervaretur.

Non multo poſt, dum Geometrarum Princeps illuſtriſſimus *Newtonus* operam dabat *Principiis Philoſophiæ Mathematicis*; non ſolum Inventa *Kepleri* in Syſtemate Planetario neceſſario locum habere demonſtravit, verum etiam Cometarum Phænomena omnia ex iiſdem Principiis evidenter conſequi. Id quod exemplo prædicti Cometæ anni 1680 abunde illuſtravit, modumque docuit Geometrice conſtruendi Orbitas Cometarum; Problemaque arduum ac tanto Oedipo dignum ſumma cum omnium admiratione reſolvit. Cometam autem hunc in orbe parabolico Solem circumiſſe probat, ita ut Areæ ad Centrum Solis æſtimatæ Temporibus proportionales fuerint.

Tanti viri veſtigia inſecutus eandem methodum calculo Arithmetico accommodare aggreſſus ſum, nec irrito Conamine. Undique enim conquiſitis Cometarum Obſervationibus, Tabellam hanc, immenſi pene Calculi fructum, obtinui, exiguum quidem ſed non ingratum Aſtronomis munus. Hi etenim numeri vim habent omnia quæ de motu Cometarum hactenus obſervata ſunt accuratiſſime repræſentandi, ope ſolius Tabulæ Generalis inſequentis, cui adornandæ nullis ſane peperci laboribus, ut perfecta prodiret, utpote Poſteritati conſecrata, ac cum Scientia Aſtronomica duratura.

Cometarum Omnium hactenus rite Obſervatorum, Motuum in Orbe Parabolico Elementa Aſtronomica.

Cometæ Anni.	Nodus Aſcend.	Inclin. Orbitæ.	Perihelion in Orbe.	Perihelion in Ecliptica.	Latitudo Perihelii.	Diſtantia Perihelia a Sole.	Log. diſt Periheliæ à Sole.	Temp. æquat. Perihelii Londini.
	gr. ′ ″	gr. ′ ″	gr. ′ ″	gr. ′ ″	gr. ′ ″			die. h. ′
1337	♊ 24. 21. 0	32. 11. 0	♉ 7. 59. 0	♉ 12. 45. 15	22. 40. 30 B	40666	9. 609236	*Junii* 2. 6. 25
1472	♑ 11. 46. 20	5. 20. 0	♉ 15. 33. 30	♉ 15. 40. 20	4. 25. 50 A	54273	9. 734584	*Feb.* 28. 22. 23
1531	♉ 19. 25. 0	17. 56. 0	♒ 1. 39. 0	♒ 0. 48. 15	17. 3. 05 B	56700	9. 753583	*Aug.* 24. 21. 18½
1532	♊ 20. 27. 0	32. 36. 0	♋ 21. 7. 0	♋ 16. 59. 40	15. 57. 00 B	50910	9. 706803	*Oct.* 19. 22. 12
1556	♍ 25. 42. 0	32. 6. 30	♑ 8. 50. 0	♑ 11. 6. 00	31. 10. 20 B	46390	9. 666424	*Apr.* 21. 20. 3
1577	♈ 25. 52. 0	74. 32. 45	♌ 9. 22. 0	♍ 7. 53. 00	69. 35. 20 A	18342	9. 263447	*Octo.* 26. 18. 45
1580	♈ 18. 57. 20	64. 40. 0	♋ 19. 5. 50	♋ 19. 17. 10	64. 40. 0 B	59628	9. 775450	*Nov.* 28. 15. 00
1585	♉ 7. 42. 30	6. 4. 0	♈ 8. 51. 0	♈ 8. 59. 10	2. 55. 25 A	109358	0. 038850	*Sept.* 27. 19. 20
1590	♍ 15. 30. 40	29. 40. 40	♏ 6. 54. 30	♏ 2. 55. 50	22. 45. 50 A	57661	9. 760882	*Jan.* 29. 3. 45
1596	♒ 12. 12. 30	55. 12. 0	♏ 18. 16. 0	♏ 22. 44. 35	54. 44. 30 B	51293	9. 710058	*Julii* 31. 19. 55
1607	♉ 20. 21. 0	17. 2. 0	♒ 2. 16. 0	♒ 1. 29. 40	16. 10. 5 B	58680	9. 768490	*Oct.* 16. 3. 50
1618	♊ 16. 1. 0	37. 34. 0	♈ 2. 14. 0	♈ 6. 10. 00	35. 50. 0 A	37975	9. 579498	*Oct.* 29. 12. 23
1652	♊ 28. 10. 0	79. 28. 0	♈ 28. 18. 40	♊ 10. 41. 35	58. 14. 0 A	84750	9. 928140	*Nov.* 2. 15. 40
1661	♊ 22. 30. 30	32. 35. 50	♋ 25. 58. 40	♋ 21. 37. 30	17. 17. 0 B	44851	9. 651772	*Jan.* 16. 23. 41
1664	♊ 21. 14. 0	21. 18. 30	♌ 10. 41. 25	♌ 8. 40. 35	16. 1. 50 A	102575½	0. 011044	*Nov.* 24. 11. 52
1665	♏ 18. 02. 0	76. 05. 0	♊ 11. 54. 30	♉ 24. 6. 35	23 8. 0 B	10649	9. 027309	*Apr.* 14. 5. 15½
1672	♑ 27. 30. 30	83. 22. 10	♉ 16. 59. 30	♋ 9. 26. 00	69. 27. 40 B	69739	9. 843476	*Feb.* 20. 8. 37
1677	♏ 26. 49. 10	79. 03. 15	♌ 17. 37. 5	♋ 16. 21. 05	75. 44. 10 B	28059	9. 448072	*Apr.* 26. 00. 37½
1680	♑ 2. 2. 0	60. 56. 0	♐ 22. 39. 30	♐ 27. 26. 50	8. 11. 10 A	00612½	7. 787106	*Dec.* 8. 00. 6
1682	♉ 21. 16. 30	17. 56. 0	♒ 2. 52. 45	♒ 2. 0. 30	16. 59. 20 B	58328	9. 765877	*Sept.* 4. 07. 39
1683	♍ 23. 23. 0	83. 11. 0	♊ 25. 29. 30	♋ 10. 36. 55	82. 52. 00 B	56020	9. 748343	*Julii* 3. 2. 50
1684	♐ 28. 15. 0	65. 48. 40	♏ 28. 52. 0	♐ 15. 15. 25	26. 35. 20 A	96015	9. 982339	*Maii* 29. 10. 16
1686	♓ 20. 34. 40	31. 21. 40	♊ 17. 00. 30	♊ 16. 24. 00	31. 17. 35 B	32500	9. 511883	*Sept.* 6. 14. 33
1698	♐ 27. 44. 15	11. 46. 0	♑ 00. 51. 15	♑ 10. 47. 20	0. 38. 10 A	69129	9. 839660	*Oct.* 8. 16. 57

Hæc Tabula vix indiget explicatione, cum ex titulis ſatis pateat quid ſibi velint Numeri. Diſtantiæ autem periheliæ æſtimantur in ejuſmodi partibus quales media diſtantia Terræ à Sole habet centies millenas.

Tabula

Tabula Generalis pro Supputando Motu Cometarum in Orbe Parabolico.

Medius motus.	*Angulus à peribelio.*			*Logarith. pro diſtan. à Sole*	*Medius motus.*	*Angulus à peribelio.*			*Logarith. pro diſtan. à Sole.*
°	gr.	′	″		°	gr.	′	″	
1	1.	31.	40	0.000077	31	42.	55.	07	0.062400
2	3.	3.	15	0.000309	32	44.	3.	16	0.065835
3	4.	34.	43	0.000694	33	45.	10.	26	0.069316
4	6.	6.	0	0.001231	34	46.	16.	35	0.072839
5	7.	37.	1	0.001921	35	47.	21.	36	0.076396
6	9.	7.	44	0.002759	36	48.	25.	33	0.079984
7	10.	38.	2	0.003745	37	49.	28.	29	0.083604
8	12.	7.	53	0.004876	38	50.	30.	23	0.087249
9	13.	37.	17	0.006151	39	51.	31.	11	0.090912
10	15.	6.	6	0.007564	40	52.	30.	54	0.094594
11	16.	34.	20	0.009115	41	53.	29.	42	0.098298
12	18.	1.	54	0.010798	42	54.	27.	32	0.102019
13	19.	28.	47	0.012609	43	55.	24.	22	0.105752
14	20.	54.	53	0.014550	44	56.	20.	11	0.109490
15	22.	20.	14	0.016607	45	57.	15.	5	0.113240
16	23.	44.	43	0.018783	46	58.	9.	2	0.116995
17	25.	8.	22	0.021072	47	59.	2.	5	0.120756
18	26.	31.	7	0.023470	48	59.	54.	13	0.124518
19	27.	52.	55	0.025969	49	60.	45.	26	0.128278
20	29.	13.	52	0.028551	50	61.	35.	45	0.132035
21	30.	33.	39	0.031263	51	62.	25.	14	0.135792
22	31.	52.	31	0.034045	52	63.	13.	50	0.139541
23	33.	10.	23	0.036916	53	64.	1.	38	0.143288
24	34.	27.	12	0.039864	54	64.	48.	38	0.147029
25	35.	42.	59	0.042892	55	65.	34.	50	0.150762
	36.	57.	41	0.045989	56	66.	20.	14	0.154482
27	38.	11.	20	0.049154	57	67.	04.	51	0.158192
28	39.	23.	56	0.052383	58	67.	48.	22	0.161890
29	40.	35.	26	0.055668	59	68.	31.	51	0.165578
30	41.	45.	50	0.059010	60	69.	14.	16	0.169254

Tabula Generalis pro Supputando

Medius motus.	Angulus à perihelio.	Logarith. pro diſtan. à Sole.	Medius motus.	Angulus à perihelio.	Logarith. pro diſtant. à Sole.
°	gr. ′ ″		°	gr. ′ ″	
61	69. 55. 58	0.172914	91	86. 20. 34	0.274176
62	70. 36. 56	0.176557	92	86. 46. 20	0.277239
63	71. 17. 16	0.180188	93	87. 11. 43	0.280284
64	71. 56. 56	0.183803	94	87. 36. 45	0.283306
65	72. 35. 57	0.187404	95	88. 01. 27	0.286308
66	73. 14. 15	0.190978	96	88. 25. 49	0.289293
67	73. 51. 59	0.194540	97	88. 49. 48	0 292252
68	74. 29. 6	0.198085	98	89. 13. 32	0.295201
69	75. 05. 38	0.201614	99	89. 36. 54	0.298122
70	75. 41. 35	0.205122	100	90. 00. 00	0.301030
71	76. 16. 56	0.208612	102	90. 45. 14	0.306782
72	76. 51. 43	0.212080	104	91. 29. 18	0.312469
73	77. 25. 57	0.215529	106	92. 12. 14	0.318060
74	77. 59. 41	0.218963	108	92. 54. 4	0.323587
75	78. 32. 54	0.222378	110	93. 34. 52	0.329042
76	79. 5. 35	0.225769	112	94. 14. 40	0.334424
77	79. 37. 45	0.229142	114	94. 53. 30	0.339736
78	80. 9. 23	0.232488	116	95. 31. 22	0.344979
79	80. 40. 34	0.235809	118	96. 8. 22	0.350153
80	81. 11. 16	0.239127	120	96. 44. 30	0.355262
81	81. 41. 31	0.242416	122	97. 19. 48	0.360306
82	82. 11. 19	0.245684	124	97. 54. 17	0.365284
83	82. 40. 40	0.248933	126	98. 28. 00	0.370200
84	83. 9. 34	0.252159	128	99. 00. 57	0.375052
85	83. 38. 4	0.255366	130	99. 33 11	0.379842
86	84. 6. 8	0.258552	132	100. 4. 43	0.384576
87	84. 33. 49	0.261720	134	100. 35. 45	0.389252
88	85. 1. 5	0.264865	136	101. 5. 48	0.393868
89	85. 27. 58	0.267989	138	101. 35. 22	0.398428
90	85. 54. 27	0.271092	140	102. 4. 19	0.402930

Motu Cometarum in Orbe Parabolico.

Medius motus.	*Angulus à perihelio.*	*Logarith. pr. diſtan. à Sole.*	*Medius motus.*	*Angulus à perihelio.*	*Logarith. pr. diſtan. a Sole.*
o	gr. ′ ″		o	gr. ′ ″	
142	102. 32. 41	0.407380	204	113. 37. 25	0.523406
144	103. 00. 31	0.411784	208	114. 9. 52	0.529705
146	103. 27. 47	0.416132	212	114. 41. 23	0.535886
148	103. 54. 31	0.420430	216	115. 12. 02	0.541958
150	104. 20. 43	0.424676	220	115. 41. 51	0.547922
152	104. 46. 22	0.428866	224	116. 10. 52	0.553782
154	105. 11. 33	0.433012	228	116. 39. 7	0.559538
156	105. 36. 16	0.437110	232	117. 6. 38	0.565199
158	106. 00. 32	0.441164	236	117. 33. 27	0.570762
160	106. 24. 23	0.445178	240	117. 59. 35	0.576233
162	106. 47. 47	0.449144	244	118. 25. 5	0.581616
164	107. 10. 44	0.453060	248	118. 49. 57	0.586912
166	107. 33. 17	0.456936	252	119. 14. 14	0.592122
168	107. 55. 27	0.460772	256	119. 37. 56	0.597252
170	108. 17. 14	0.464208	260	120. 1. 6	0.602301
172	108. 38. 37	0.468318	264	120. 23. 44	0.607274
174	108. 59. 39	0.472030	268	120. 45. 52	0.612174
176	109. 20. 20	0.475705	272	121. 7. 30	0.616998
178	109. 40. 40	0.479340	276	121. 28. 39	0.621750
180	110. 00. 40	0.482937	280	121. 49. 22	0.626438
182	110. 20. 20	0.486498	284	122. 9. 38	0.631056
184	110. 39. 41	0.490022	288	122. 29. 28	0.635608
186	110. 58. 44	0.493512	292	122. 48. 54	0.640098
188	111. 17. 28	0.496965	296	123. 7. 57	0.644525
190	111. 35. 55	0.500384	300	123. 26. 36	0.648893
192	111. 54. 05	0.503769	310	124. 11. 40	0.659559
194	112. 11. 58	0.507121	320	124. 54. 36	0.669880
196	112. 29. 34	0.510441	330	125. 35. 34	0.679876
198	112. 46. 55	0.513729	340	126. 14. 44	0.689568
200	113. 4. 00	0.516984	350	126. 52. 12	0.698970

Tabula Generalis pro Supputando Motu Cometarum in Orbe Parabolico.

Medi. motus.	*Angulus a peribelio*			*Logarith. prodiſtan. a Sole.*	*Medius motus.*	*Angulns a peribelio.*			*Logarith. prodiſtan. a Sole.*
o	gr.	'	"		o	gr.	'	"	
360	127.	28.	6	0.708104	820	141.	49.	24	0.970836
370	128.	2.	33	0 716976	840	142.	10.	00	0.978397
380	128.	35.	38	0.725606	860	142.	29.	56	0.985771
390	129.	7.	27	0.734006	880	142.	49.	10	0.992970
400	129.	38.	4	0.742186	900	143.	7.	48	1.000000
410	130.	7.	34	0.750160	920	143.	25.	51	1.006871
420	130.	36.	2	0.757930	940	143.	43.	21	1.013586
430	131.	3.	30	0.765516	960	144.	00.	18	1.020155
440	131.	30.	2	0.772918	980	144.	16.	46	1.026583
450	131.	55.	41	0.780148	1000	144.	32	46	1.032876
460	132.	20.	30	0.787216	1500	149.	26.	8	1.158188
470	132.	44.	32	0.794122	2000	152.	26.	15	1.246058
480	133.	7.	50	0.800882	2500	154.	32.	20	1.313703
490	133.	30.	25	0.807494	3000	156.	7.	27	1.368678
500	133.	52.	20	0.813969	3500	157.	22.	49	1.414974
520	134.	34.	18	0.826522	4000	158.	24.	36	1.454950
540	135.	14.	0	0.838600	4500	159.	16.	36	1.490125
560	135.	51.	28	0 850187	5000	160.	1.	12	1.521521
580	136.	27.	6	0.861369	5500	160.	40.	5	1.549874
600	137.	00.	57	0.872155	6000	161.	14.	24	1.575718
620	137.	33.	13	0.882575	6500	161.	45.	00	1.599460
640	138.	3.	58	0.892649	7000	162.	12.	34	1.621417
660	138.	33.	21	0.902401	7500	162.	37.	34	1.641838
680	139.	1.	29	0.911866	8000	163.	00.	23	1.660922
700	139.	28.	25	0.921012	8500	163.	21.	20	1.678834
720	139.	54.	16	0.929907	9000	163.	40.	42	1.695708
740	140.	19.	5	0.938549	9500	163.	58.	38	1.711662
760	140.	42.	56	0.946951	10000	164.	15.	20	1.726784
780	141.	05.	55	0 955124	50000	170.	52.	0	2.197960
800	141.	28.	3	0 963082	100000	172.	45.	44	2.399655

Tabulæ Generalis Conſtructio & Uſus.

Ut Planetæ in Orbibus Ellipticis, ita Cometæ in Parabolicis Solem in Foco communi ſitum ambiunt; ea lege ut Areæ æquales æqualibus temporibus deſcribantur. Quoniam vero Parabolæ omnes inter ſe ſimiles ſunt, ſi determinata aliqua pars Areæ datæ Parabolæ dividatur in partes quotlibet; in omnibus Parabolis fiet ſimilis diviſio ſub iiſdem angulis, atque diſtantiæ erunt proportionales: ideoque una noſtra Tabula pro Cometis *Fig. 6.* omnibus ſufficiet. Calculi autem hujus Tabulæ hæc eſt ratio. In Schemate *Plate 4.* ſit *S* Sol, *POC* orbita Cometæ, *P* Perihelion, *O* Locus ubi Cometa quadrante diſtat a Perihelio, *C* Locus quivis alius. Junge *CP*, *CS*, ac fiant *ST*, *SR* æquales ipſi *CS*; ac ductis rectis *CR*, *CT* (quarum hæc Curvæ Tangens eſt, illa perpendicularis) in axem *PSR* demitte normalem *CQ*. Jam data quavis Area *COPS*, oportet angulum *CSP*, & diſtantiam *CS* inquirere. Quoniam ob naturam Parabolæ recta *RQ* ubique æqualis eſt ſemilateri recto, ponatur latus rectum$=2$, adeoque $RQ=1$: ac ſit recta $CQ=z$: erit itaque $PQ=\frac{1}{2}zz$, ac Segmentum parabolicum $COP=\frac{1}{12}zzz$. Triangulum autem *CSP* erit $=\frac{1}{4}z$: adeoque Area mixtilinea *COPS* erit $\frac{1}{12}z^3+\frac{1}{4}z=a$, ac $z^3+3z=12a$. Quare reſoluta hac æquatione Cubica, *z* ſive ordinatim applicata *CQ* innoteſcet. Proponatur jam Area *OPS* in partes centenas dividenda: hæc Area duodecima pars eſt quadrati lateris recti, adeoque $12a$ æquantur Quadrato illo$=4$: Si itaque ſucceſſive extrahantur radices æquationum $z^3+3z=$ 0,04: 0,08: 0,12: 0,16, *&c.* habebuntur totidem *z* ſive ordinatim applicatæ *CQ* reſpective; ac diviſa erit area *SOP* in partes centenas. Eodemque modo ultra locum *O* continuandus eſt calculus. Radix autem hujus æquationis, cum *RQ* ſit$=1$, Tangens eſt Tabularis anguli *CRQ*, ſive dimidii anguli *CSP*, adeoque angulus *CSP* datur. Ejuſdemque anguli *CRQ* ſecans *RC*, Media proportionalis eſt inter *RQ* ſive Unitatem & *RT*, quæ dupla eſt ipſius *SC*, ut ex Conicis notiſſimum eſt. Quod ſi *SP* ponatur 1, adeoque latus rectum$=4$ (ut in Tabula noſtra) ipſa *RT* erit diſtantia quæſita; duplum ſcilicet ipſius *SC* in priore Parabola. Ad hunc modum itaque præcedentem Tabulam elaboravi repræſentandis omnium Cometarum motibus inſervientem: hactenus enim nullus ex obſervatis Parabolæ leges reſpuit.

Reſtat jam præcepta Calculi tradere, modumque ſupputandi locum Cometæ viſum ex his Numeris exhibere. Cometæ autem in Parabola moventis Velocitas ubique eſt ad velocitatem Planetæ gyrantis in Circulo circa Solem, ad eandem a Sole diſtantiam, ut $\surd 2$ ad 1: ut conſtat *ex Principiis Phil. Nat. Math. Lib.* I. *Prop.* 16. *Corol.* 7. Si itaque Cometa in perihelio ad diſtantiam æqualem diſtantiæ Terræ a Sole ſupponatur, erit area diurna, quam deſcriberet Cometa, ad aream quam deſcribit Terra, ut $\surd 2$ ad 1. ac proinde tempus annuum, ad tempus quo Cometa talis deſcriberet Quadrantem Orbitæ ſuæ a Perihelio ut 3. 14159, *&c.* (hoc eſt ut area circuli) ad $\surd\frac{8}{9}$. Cometa igitur deſcriberet Quadrantem illum diebus 109. 14^h. $46'$. adeoque area illa Parabolica areæ *POS* analogia in centum particulas diſtributa, ſingulis diebus competunt particulæ 0,912280.

Cujus Logarithmus nempe 9,960128 in perpetuum uſum ſervandus eſt. Tempora autem, quibus Cometa in diſtantia majore vel minore Quadrantes ſimiles deſcriberet, ſunt ut Revolutiones in Circulis; hoc eſt in ſeſquiplicata ratione diſtantiarum: adeoque areæ diurnæ in partibus centeſimis Quadrantis æſtimatæ (quas medii Motus menſuras, inſtar Graduum, ponimus) ſunt in ſingulis in ſubſeſquialtera ratione diſtantiæ Periheliæ a Sole.

His neceſſario præmiſſis proponatur alicujus e Cometis noſtris Locum viſum ad datum tempus ſupputare. Primum itaque Solis Locus ab Æquinoctio in promptu ſit, ejuſdemque diſtantiæ a Terra Logarithmus. 2°. Capiatur intervallum Temporis inter Tempus Perihelii & Tempus datum, in diebus partibuſque diei decimalibus. Hujus numeri Logarithmo addatur Logarithmus conſtans 9,960128 ac complementum Arithmeticum ſeſquialterius Logarithmi diſtantiæ Periheliæ a Sole: ſumma Logarithmus erit Motus medii in prima Columna Tabulæ Generalis quærendi. 3°. Cum motu medio capiatur in Tabula correſpondens angulus a Perihelio, & Logarithmus pro diſtantia a Sole: ac in Cometis Directis adde, in Retrogradis ſubduc, ſi fuerit Tempus poſt Perihelium; vel in Directis ſubduc & in Retrogradis adde, ſi fuerit ante Perihelium, angulum ſic inventum a loco vel ad locum Perihelii. Et habebitur Locus Cometæ in orbita propria: Et ad Logarithmum pro Diſtantia ibidem inventum addatur logarithmus diſtantiæ Periheliæ, ſumma erit Logarithmus diſtantiæ veræ Cometæ a Sole. 4°. Cum Loco Cometæ in Orbita, dato Loco Nodi, capiatur diſtantia Cometæ a Nodo: ac data Inclinatione plani, dabuntur, Notiſſimis Trigonometriæ præceptis, Locus Cometæ ad Eclipticam reductus, cum Inclinatione ſive Latitudine Heliocentrica, ac Diſtantiæ Curtatæ Logarithmus. 5°. Ex his datis, iiſdem omnino regulis quibus loca Planetarum, ex dato Loco & Diſtantia Solis; obtinebitur Locus Viſus ſeu Geocentricus cum Latitudine viſa. Id quod exemplo uno vel altero operæ pretium erit illuſtrare.

Exemp. I. Quæritur Locus Cometæ Anni 166$\frac{4}{5}$ *Martii* 1°. 7^{h}. 00.
P. M. *Londini*. Hoc eſt 96d. 19^{h}. 8′. poſt Perihelion
ejus *Novemb.* 24°. 11^{h}. 52′. Celebratum.

Log. diſt. Perihel.	0. 011044	*Perihel.* ♌	10. 41. 25	*Log. pro diſt.*	0. 255369
Log. Seſquialt.	0. 016566	*Ang. Correſp.*	83. 38. 05	*Log. Perihel.*	0. 011044
Comp. Arith.	9. 983434	*Comet. in Orb.* ♉	17. 3. 20	*Co. ſin. Incl.*	9. 990754
	9. 960128	☊ ♊	21. 14. 00	*Log. diſt. Curt.*	0. 257167
Log. Temp.	1. 985862	*Com. a Nodo*	34. 10. 40	*Log. diſt.* ☉	9. 997918
Log. Med. Mot.	1. 929424	*Red. ad Eclip.*	32. 19. 05	☉ ♓	21. 44. 45
Medius Motus	85. 001	*Com. Helioc.* ♉	18. 54. 55	*Com. Viſus* ♈	29. 18. 30
		Incl. Bor.	11. 46. 50	*Lat. Viſa*	8. 36. 15. *Bor.*

Exemp. II. Quæritur Locus Cometæ Anni 1683 *Julii* 23° 13^{h}. 35′. P. M. *Londini*. Vel 13^{h}. 40′. T. æquat. hoc eſt 21d. 10^{h}, 50′. poſt Perihelion.

Log. diſt. Perihel.	9. 748343	*Perihel.* ♊	25. 29. 30	*Log. pro diſt.*	0. 111336
Log. Seſquialt.	9. 622514	*Ang. Correſp.*	56. 47. 20	*Log. Perihel.*	9. 748343
Comp. Arith.	0. 377486	*Comet. in Orb.* ♈	28. 42. 10	*Co. ſin. Incl.*	9. 913187
	9. 960128	☊ ♓	23. 23. 00	*Log. diſt. Curt.*	9. 772866
Log. Temp.	1. 310723	*Com. à* ☋	35. 19. 10	*Log. diſt.* ☉	0. 005104
Log. Med. Mot.	1. 648337	*Red. ad Eclip.*	4. 48. 30	☉ *Locus* ♌	10. 41. 25
Medius Motus.	44. 498	*Com. Helioc.* ♓	28. 11. 30	*Com. Viſus* ♋	5. 11. 50
		Incl. Bor.	35. 2. 00	*Lat. Bor.*	28. 52. 00

Momento autem primi Exempli, *Londini* obſervatum eſt Cometam applicari ad Stellam ſecundam *Arietis*; ita ut novem minutis illa borealior repertus ſit, ac tribus minutis orientalior: Obſervante D^{no}. *Roberto Hookio.* In ſecundo autem Exemplo ipſe, in vicinia *Londini*, inſtrumentis quibus olim Stellas Auſtrales obſervaveram, Cometæ locum deprehendi ♋ 5°. 11′. $\frac{1}{2}$, cum Latitudine Boreali, 28°. 52′, conſentiente ad amuſſim obſervatione *Grenovicenſi* eodem pene momento facta.

Cometa autem Anni 1680, qui pene Solem attigit, (non enim triente ſemidiametri corporis Solaris a ſuperficie ejus diſtabat in Perihelio) cum Latus rectum exiguum admodum ſit, Tabula Generali haud coerceri potuit, ob immanem Motus medii velocitatem: præſtat itaque in hoc, poſtquam inventus fuerit Motus medius, ex eodem, ope præcedentis æquationis $zzz+3z={}_{1}\frac{4}{00}$ *Mot. Med.* Tangentem dimidii anguli a Perihelio elicere, una cum Logarithmo pro diſtantia a Sole. Quibus datis iiſdem omnino regulis ac in cæteris procedendum eſt.

Ad hunc itaque modum Aſtronomico Lectori examinare licet numeros a me poſitos, quos ſumma cura ex obſervationibus quæ ſuppetebant exantlavi; neque enim, antequam probe ad incudem redacti fuerint, ac multorum annorum ſtudio quantum fieri poſſit politi, in publicum prodeunt. Hoc autem ſpecimen Aſtronomiæ Cometicæ, futuri operis Prodromum, editum eſſe volui; ne forte ſuperveniente fato perirent lucubrationes noſtræ, ob Calculi difficultatem non cuivis homini denuo ſuſcipiendæ. Monendus autem eſt Lector, quinque priores ordine Cometas, quorum tertius & quartus eſt a *Petro Apiano* obſervatus, quintus vero a *Paulo Fabricio*, uti & decimus a *Mæſtlino* (ni fallor) anno 1596 conſpectus, non eundem certitudinis gradum cum reliquis præ ſe ferre. Neque enim debitis organis nec cura ad hoc requiſita obſervationes ipſæ peractæ ſunt; adeoque inter ſe diſſidentes nullo modo cum computo regulari conciliari poſſunt. Cometam Anni 1684 unus vidit *Blanchinus* obſervator *Romanus*: ultimum vero Anni ſc. 1698 *Pariſienſes* ſoli conſpexerunt, ejuſque curſum inſolito modo deſignarunt. Obſcurus hic admodum, etiamſi velox ac terris ſatis vicinus, noſtros ſane oculos alioquin non incurioſos effugit. Inſignes autem duos hac noſtra ætate Cometas, alterum Anno 1689 Menſe *Novembri* ortum, alterum Menſe *Februario* Anni 1702, Catalogo ſubjungere non licuit, propter defectum obſervationum. Etenim verſus mundi plagas Auſtrales curſum dirigentes, ac in *Europa* vix conſpicui, contemplatores non habuere negotio pares. Quod ſi forſan ex partibus *Indicis* advectæ fuerint accuratæ obſervationum ſeries ad hoc neceſſariæ; lubens calculum repetere, horumque Orbitas, reliquorum ad modum, Numeris deſignandi laborem ſuſcipere non gravabor.

Anguſtia autem paginæ 328, factum eſt, ut omiſſa ſit neceſſaria illa Columella quæ oſtendat an directe vel retrograde moti fuerint Cometæ. Sciat itaque Aſtronomus undecim e noſtris Cometis directo curſu ſecundum ſeriem ſignorum proceſſiſſe, nempe illos annorum 1532, 1556, 1580, 1585, 1618, 1652, 1661, 1672, 1680, 1684 & 1686. Reliquos vero tredecim motu retrogrado contra ſeriem ſignorum curſum tenuiſſe. Quibus perpenſis, ac collatis inter ſe cæteris horum Cometarum motuum Elementis, videre eſt, nullo ordine diſpoſitos eſſe Orbitas; neque ipſos, Planetarum more, Zodiaco comprehendi poſſe, quaquaverſum tam Retrograde quam directe indifferenter latos; unde manifeſtum eſt eos motu vorticali nullo modo circumagi. Quinetiam diſtantiæ Periheliæ nunc majores nunc minores reperiuntur; unde pronum eſt ſuſpicari etiam multo plures eſſe Cometas, qui in partibus a Sole remotioribus, obſcuri caudaque deſtituti, adeoque nobis inconſpicui, præterlabi poſſunt.

Hactenus Cometarum Orbes conſideravimus ut perfecte Parabolicos; quo ſuppoſito conſequeretur Cometas, vi Centripeta verſus Solem impulſos, a ſpatiis infinite diſtantibus deſcendere, caſuque ſuo velocitatem tantam acquirere, ut iterum in ſpatia Mundi remotiſſima ſeſe abdere poſſint, perpetuo niſu ſurſum tendentes, ac ad Solem nunquam reverſuri. Cum autem ſatis frequentes ſint Cometarum adventus; ac eorum nullus reperiatur motu ferri Hyperbolico, ſeu velociore quam cadendo ad Solem acquirere debeat, credibile eſt potius in Orbibus valde Excentricis revolvi eos circa Solem, ac poſt longiſſimas periodos reverti. Sic enim Numerus eorum præfinitus eſſet, ac fortaſſe non uſque adeo magnus. Spatia autem inter Solem Fixaſque tanta ſunt, ut Cometæ revolventi cum Periodo quantumvis longa ſatis loci ſit. Latus autem rectum Ellipſis eſt ad Latus rectum Parabolæ eandem Periheliam diſtantiam habentis, ut diſtantia Aphelia in Ellipſi eſt ad Axem totum Ellipſis; Velocitates autem ſunt in dimidiata ratione eorundem: quapropter in Orbibus valde Excentricis ratio hæc accedit proxime ad rationem æqualitatis. Tantilla autem differentia, quæ intercedit ratione majoris in Parabola velocitatis, facillime in ſitu Orbis determinando compenſatur. Hujus itaque Tabulæ Elementorum Motuum uſus præcipuus eſt, atque etiam propter quem illam conſtruere operæ pretium duxi, ut, ſi quando novus Cometa emerſerit, poſſimus collatis elementis dignoſcere an poterit eſſe aliquis ex antiquis, necne; ac proinde Periodum Orbitæque Axem determinare, reditumque prædicere. Ac ſane multa me ſuadent ut credam Cometam anni 1531 ab *Apiano* obſervatum, eundem fuiſſe cum illo qui anno 1607 deſcriptus eſt a *Keplero* & *Longomontano*, quemque ipſe iterum reverſum vidi ac obſervavi anno 1682. Quadrant Elementa omnia, ac ſola inæqualitas periodorum adverſari videtur: hæc autem tanta non eſt ut cauſis Phyſicis non poſſit attribui. *Saturni* enim motus a cæteris, præſertim *Jove*, ita interturbatur, ut per aliquot dies integros incertum ſit hujus Planetæ tempus Periodicum. Quanto magis talibus erroribus obnoxius erit Cometa, qui quatuor pene vicibus altius excurrit *Saturno*, cujuſque velocitas, vel tantillum aucta, Orbem ab Elliptico in Parabolicum poſſit immutare? Confirmatur etiam eundem eſſe potuiſſe ex eo, quod anni 1456 æſtate, conſpectus fuerit Cometa eodem pene modo inter Solem & Terram tranſiens retrograde: quem, licet a nemine obſervatus fuerit Aſtronomice, ex periodo modoque tranſitus non diverſum a prædictis extitiſſe conjicio. Unde auſim ejuſdem reditum fidenter prædicere, anno ſcil. 1758. Quod ſi hoc evenerit, nulla amplius erit dubitandi cauſa, quin redire debeant cæteri. Habebunt ergo Aſtronomi in hac arena quo ſe exerceant per multa ſæcula, priuſquam tot tantorumque Corporum circa commune centrum Solis revolventium numerus cognoſcatur, ac motuum ſymptomata certis regulis coerceantur. Crediderim equidem Cometam etiam anni 1532, eundem fuiſſe cum illo, qui ab *Hevelio* obſervabatur ineunte anno 1661: ſed obſervationes *Apiani*, quas ſolas de primo habemus, nimis rudes ſunt, nec quicquam certi in re tam ſubtili ex iiſdem elici poteſt. Juſto volumine hæc omnia exequi mihi animus eſt, nec Aſtronomiæ promovendæ hac in re deero, ſi Deo O. M. viſum fuerit vitam facultateſque prorogare. Interim quicunque modum Conſtruendi Cometarum Orbes per tres obſervationes accurate habitas addiſcere cupit, ſub finem libri de Syſtemate Mundi, ſive tertii *Philoſophiæ Nat. princip. Math.* magni ipſius Inventoris methodum inveniet: Quam poſtea Digniſſimus Collega meus *D. Gregorius*, Lib. V. pereruditæ Aſtronomiæ ſuæ Phyſicæ & Geometricæ plene & luculenter illuſtravit.

Unicum autem non abs re erit nec injucundum, hic loci Lectorem monere Aſtronomum; nempe quod nonnulli ex his Cometis Nodos ſuos habeant adeo Orbi Terræ annuo vicinos, ut ſi forte acciderit, tempore reditus Cometæ, *Terram* occupare Loca in orbe ſuo Nodo proxima, dum Cometa incredibili cum Velocitate præterierit, Parallaxin etiam habiturus ſit valde obſervabilem, quæque fuerit ad Solis parallaxin in ratione data. Unde occaſione talium tranſituum oblata erit anſa, rara quidem ſed optima, determinandi Solis a *Terra* diſtantiam; quam hactenus non niſi mediante parallaxi *Martis* Acronychii, vel *Veneris* perigææ, triplo quidem ſolari majore, ſed quæ vix ullis inſtrumentis ſentiatur, laxe admodum concludere licuit. Quem Cometarum uſum ſuggeſſit Clariſſimus Geometra Ds. *Nic. Facio.* Cometa etenim anni 1472 parallaxin habuit pluſquam vigeſies Solari majorem. Ac ſi Cometa anni 1618 appuliſſet, juxta medium Menſis *Martii*, ad Nodum ejus Deſcendentem; vel ſi Cometa anni 1686 paulo citius ad Nodum Aſcendentem perveniſſet, profecto Terris admodum propinqui etiam adhuc magis notabiles habuiſſent parallaxes. Inter omnes vero nullus propiore appulſu Terris minatus eſt quam ille anni 1680: Hic inito Calculo non amplius ad Boream diſtabat

ab Orbe noſtro annuo, quam Semidiametro Solari (ſive Radio Lunaris Orbitæ, ut exiſtimo) idque Nov. 11. 1^{h}. 6′. *P. M.* Quo tempore, ſi Terræ quoad Longitudinem conjunctus fuiſſet, parallaxis ſane Lunari æqualis in Cometæ motu obſervari potuiſſet. Hæc Aſtronomis dicta ſunto. Quæ vero ab hujuſmodi allapſu vel contactu, vel denique colliſione corporum cœleſtium (quæ quidem omnino non impoſſibilis eſt,) conſequi debeant, rerum Phyſicarum ſtudioſis diſcutienda relinquo.

Obſervations of the Comet at Rome *in* 1664 *by Mr* Ray, *n.* 309. *p.* 2350.

XLI. *December* the 20th, 1664. *S. N.* About three a Clock this Morning, I obſerved the *Comet*; it was in the Conſtellation of *Hydra*, not far from the Foot of *Crater*. It appeared about the bigneſs of a Star of the firſt Magnitude, but nothing ſo lucid and bright. It had a very long Tail, which pointed almoſt directly towards the Heart of *Hydra*: The Tail ſhew'd ſomewhat like Rays of a Candle burning in a Miſt: The Figure of it was Conical; the length of it 5 or 6 Degrees; the breadth at the Baſe not above a Degree and half. The Body of this *Comet* was about three Degrees to the South-Eaſt of the moſt Southerly Star in the Foot of *Crater*; it ſtood very near in a Right Line with the two lowermoſt Stars in the Foot of *Crater*, which are common to it and *Hydra*. See the Figures.

Plate 6.

December 21. In the Morning, about the ſame Hour, it was removed about a Degree and half from the place where it ſtood, Weſtward, and a little to the South. The Tail pointed ſtill towards the Heart of *Hydra*, and appeared 10 Degrees long at the leaſt.

December 22. At the ſame time it was removed from the place where it ſtood the Day before, to the ſame Point, and about the ſame diſtance as the Night before. The Tail of it ſtill pointed to *Cor Hydræ*, or a little thought above it, as the two former Days, and was rather longer than ſhorter: It alſo to my thinking, appeared brighter and larger; the Body of it being bigger than any Fixt Star, except *Sirius*.

December 23. It was removed to the ſame Point, and about the ſame Diſtance as the Day before; the Tail of it was as long as ever, and the *Comet* brighter. The Tail pointed almoſt directly to *Cor Hydræ*.

December 24, 25, 26. All theſe Three Nights were cloudy.

December 27. We found it ſtrangely removed from the Place where it was: It was ſtill Weſtward, and a little to the South, as before. The Body of the Star was ſtill brighter, and the *Cauda* about it greater, and more buſhy, and yet as long as before; it pointed almoſt directly againſt *Canis major*. The Body of it was among the Stars of *Argo*.

December 28. The ſame time it was removed above 2 Degrees towards the ſame Point, and come within 4 or 5 Degrees of the moſt Eaſtern Stars in the bright Triangle in the Buttocks of *Canis major*. The *Moon* ſhining, we could not ſo well judge either of the Bigneſs of the Body, or the length and buſhineſs of the Tail.

December 29. It was ſtrangely removed, and got before, not the Eaſtern Star only of the mentioned bright Triangle, but alſo the moſt Northern. I think, at leaſt, in this laſt 24 Hours, it had moved 4 Degrees.

Decem: 20: 1664.

Dec: 21: 1664.

Part 3. page 338.

Decem: 22: 1664.

Decem: 23: 1664:

Dec: 27: 1664:

in
lumbis sir:

Dec: 28: 1664. Hoc Schema vero Situi Stellarū propius

Decembr. 29:

The Moon ſhining bright, the Tail could not well be obſerved, yet ſtill it ſeemed to point directly to *Canis minor.*

Obſervations on the Comet in 1680. by Mr. Gottf. Kirch, *with Remarks by*——*n.* 342. *p.* 170.

XLII. Cometa ille qui anno exeunte 1680 viſus eſt, plurimis de cauſis præcipuus merito habendus eſt; tam ob Curſum ejus quadrimeſtrem, quo novem integra Signa percurrit; quam ob immenſam Caudæ magnitudinem & claritatem: maxime vero ob inſignem Orbitæ Curvitatem, cujus ope tandem patefacta eſt Cometarum Theoria. Nam dum Aſtronomi omni adhibita diligentia motum ejus obſervationibus definiendo deſudarunt, conatus eorum ſecundavit * D. *Newtoni* in Geometricis vis pene divina, qui primus mortalium Cometas Orbes Parabolicis maxime affines deſcribere probavit, ac datis tribus locis accurate obſervatis eoſdem conſtruere docuit, remque hujuſce Cometæ exemplo illuſtravit. Accidit autem, neſcio quo fato, ut Cometa hic (quem veſpertinum tantopere profecuti ſunt Cœliſpices) antequam Solem attigerat matutinus, nec *Pariſiis* neque *Grenovici* ne ſemel quidem obſervatus ſit: quique eum viderunt & obſervarunt, incongrua & inter ſe pugnantia, ac pro rei ſubtilitate parum idonea prodidere: neque ante *Novemb.* 17. mane a quopiam Obſervatorum viſus eſt. Unde factum ut Orbitæ pars illa, qua ad Solem deſcendit Cometa, paulo incertius definiri potuit. Bona autem fortuna nuper incidimus in Librum meritiſſimi Aſtronomi Domini *Gottfried Kirch, Germani*, anno 1681 *Noribergæ* impreſſi, cui titulus **Neue Himmels Zeitung**, hoc eſt *Novus Nuncius cœleſtis*; ubi auctor diligentiſſimus nobis exponit, quo caſu ductus Cometam hunc nondum adnata Cauda obſcurum, ac vix nudis oculis conſpicuum detexerit; dum ſcilicet Lunam & Martem ei vicinum obſervaturiens circumluſtraret, die *Novemb.* 4. *S. V.* mane, idque *Coburgi Saxoniæ*, quod oppidum XI. Grad. *Londino* orientalius eſt, ſub altitudine poli 50° 20′ circiter. Excitatus autem, ut ait, rumoribus Cometæ in *Germania* viſi, vultu in Orientem verſo pernoctavit, ut cœlo tum forte perquam ſereno, ſi quid novi oriretur, ſitum ejus notaret. Luna vero jam ad ſtellam aliquam *Tychoni* incognitam applicata, (ſed quæ in Catalogo *Flamſteedii Britannico* habetur, eſtque numero 44ta *Leonis*) voluit dictæ ſtellæ locum ex circumvicinis Fixis determinare; dumque Tubum trium graduum capacem hinc inde circumrotat, incidit in Luculam nebuloſam, ſpeciem inſolitam præ ſe ferentem, quamque vel novum Cometam eſſe, vel Stellam nebuloſam ad inſtar ejus quæ in *Cingulo Andromedæ* eſt, ſtatim concluſit.

* *Princip. Philoſ.* L. III. *ſub. fin.*

Plate 4. *Fig.* 7.

Primo autem Cometam vidit, Hora $4\frac{1}{2}$ matutina, paulo altiorem duabus ſtellulis Teleſcopicis, *a* & *e* cum quibus tamen Hora Sexta viſus eſt in linea accurate recta; unde conſtabat eum moveri, idque motu directo. Inter horas vero 5. & 6. Tubo decempedali Phænomenon hoc contemplatus eſt, viditque duas alias ſtellulas contiguas prioribus minores, literiſque *e* & *d* notatas, & ſupra has tertiam *g*. Erat autem diſtantia Cometæ ab *e* paulo minor quam ab *a*, major vero diſtantia *d e*. 6^{h}. 38′. diſtantia Cometæ ab *e* dupla erat intervalli inter ipſas *d e*, ac linea *d e* producta reliquit Cometam * infra ſe, ſic tamen ut marginem ejus ſuperiorem

* *Tubo ſcilicet objecta invertente.*

riorem attingeret. 6h. 45'. Cometa jam ſenſibiliter remotior erat ab *e* quam ab *a*, diſtabatque ab *a* paulo plus quam dimidio diſtantiæ ſtellularum *a* & *g*. Notandum vero eſt Horologium totis 14 minutis cœlum anticipaſſe, uti ex altitudinibus *Cordis Leonis* tum captis patuit.

Nobilis ſane eſt hæc obſervatio, adeoque Stellularum Cometæ tunc adjacentium loca non una methodo perquiſivimus, exercitatiſſimam manum ſuam & inſtrumenta perelegantia præbente Rev. D. *Jac. Pound*, R.S.S. Unde conſtabat ſtellulas illas tum temporis infraſcriptos habuiſſe ſitus, nempe

	Long.				Lat.			
a	♌	29°.	54'	20''	1.	29.	20	Bor.
d		29.	27.	20	1.	8.	00	
e		29.	34.	30	1.	10.	45	

Circulus autem maximus per *a* & *c* ductus, deprehenſus eſt per *Ultimam Caudæ Urſæ majoris* tranſire, adeoque angulum cum circulo Longitudinis ad *a* eſſe 15° 36'$\frac{1}{2}$. Cumque diſtantia Cometæ ab *a* verſus *c* paulo major fuerat dimidio diſtantiæ *ag*, (quam Tubo ſedecim pedum & Micrometro deprehendimus 21 $\frac{2}{5}$ min.) ponamus eam fuiſſe duodecim minutorum; & ex datis proveniet Cometæ locus ♌ 29°. 51' cum Lat. Borea 1°. 17'$\frac{3}{4}$. Hora ſcilicet Horologii 6h, ſed *Londini* 5h 2'. Temp. Appar.

Deinde *Novemb.* 6. mane, 4h. 42', Tubo bipedali deprehendit Cometam omnino in linea recta inter *Martem* & Stellulam *N*; quæ quidem in *Catalogo Britannico* 45ta Leonis eſt, & tunc erat in ♍ 2° 42' cum Lat. Auſt. 0° 16'$\frac{1}{6}$. *Mars* autem tum temporis habuit (ex collatis obſervationibus paulo antea & poſt factis) ♍ 3° 46'$\frac{1}{2}$ cum Lat. Bor. 1° 56'. Unde, ob datam ejus viam, Cometa occupavit ♍ 3°. 23' cum Lat. Bor. 1° 6'. *Londini* Temp. App. 3h 58' mane.

Novembris quoque 11mo 5h 15' mane, Cometa æqualiter diſtabat a Stellis Leonis σ & τ *Bayero*, nondum vero attigit rectam eaſdem jungentem, ſed parum abfuit ab ea. In *Catalogo Britannico* σ tunc habuit ♍ 14° 15'. cum Lat. Bor. 1° 41' fere, τ vero ♍ 17° 3'$\frac{1}{2}$ cum Lat. Auſt. 0° 34'. Proinde Cometæ Latitudo paulo minor erat medio inter illas, nempe quam 0° 33'$\frac{1}{2}$ Bor. ac Longitudo quam ♍ 15°. 39'. Sed huic non utique fidendum, cum pendeat ab æſtimata æqualitate diſtantiarum, quæ res lubrica eſt. Cauda autem jam cœpta non niſi dimidio gradu longa Tubo decempedali viſa eſt.

A ſmall Teleſcopical Comet ſeen at London, *on the* 10*th* of June, 1717. *by Dr.* Halley, *n.* 354. *p.* 721.

XLIII. That the Number of Comets traverſing our Solar Syſtem is much greater than ſome, on account of the late rareneſs of their Appearance, have ſuppoſed it, may be collected from ſeveral ſmall ones which have within few Years been deſcribed in the Memoirs of the *French* Royal Academy of Sciences; thoſe diligent Obſervers aſſuring us that they diſcover'd one in *Sept.* 1698. another in *Febr.* 1699 a third in *April* 1702. and again a fourth in *Novemb.* 1707. none of which, as far as I can learn, were ever ſeen in *England*; all of them having been

very obſcure and without Tails, by means whereof Comets uſually firſt ſhew themſelves. And beſides theſe, two other Comets with remarkably long Tails, the one in *Novemb.* 1689. the other in *Feb.* 1702. paſt by unobſervable in theſe our Northern Climats, they having great South Latitude, and their Motions directed toward that Pole. Hence we may juſtly conclude, that the Returns of Comets are much more frequent than is vulgarly reckoned, and that it is only contingent that for theſe 35 Years no one of them has been ſeen and obſerved by our Aſtronomers.

But there may be ſtill a much greater Number of theſe Bodies, which by reaſon of their Smallneſs and Diſtance are wholly inviſible to the naked Eye, ſo that unleſs Chance do direct the Teleſcope of a proper Obſerver, almoſt to the very Points where they are (againſt which there are immenſe Odds) it will not be poſſible for them to be diſcovered, whereof take the following Inſtance. On *Monday*, *June* 10. in the Evening, the Sky being very ſerene and calm, I was deſirous to take a View of the Diſk of *Mars* (then very near the *Earth*, and appearing very glorious) to ſee if I could diſtinguiſh in my twenty four Foot Teleſcope, the Spots ſaid to be ſeen on him. Directing my Tube for that purpoſe, I accidentally fell upon a ſmall whitiſh Appearance near the Planet, reſembling in all reſpects ſuch a *Nebula* as I lately deſcribed * in *Philoſ. Tranſact.* N°. 347. but ſmaller. It ſeem'd to emit from its upper part a very ſhort kind of Radiation directed towards the Eaſt, but Northerly withal; which, conſidering its Situation, was nearly towards the Point oppoſite to the Sun. The great Light of the Moon, then very near it, and alſo very near the Full, hinder'd this *Phænomenon* from being more diſtinctly ſeen; but its Place in the Heavens was ſufficiently aſcertained from the Neighbourhood of *Mars*, from whom it was but about half a Degree diſtant towards the Southweſt, the difference of Latitude being ſomewhat more than that of Longitude; and *Mars* being at that time in ♐ 17°. 30′ with 3°. 48′. South Latitude, I concluded the place thereof in ♐ 17°. 12′ with 4°. 12′. Lat. South, or thereabouts; the which may yet be more ſecurely determined by help of two ſmall fixt Stars I found near it, the more northerly of which I judged to have the ſame Latitude with it, and to follow it at about the diſtance of ſix Minutes; the other Star was about one Minute in conſequence thereof; the Angle at the Northern Star was a little obtuſe, as of about 100 Degrees, and the diſtance of our *Nebula* from it *ſeſquialter* to the diſtance of the two Stars, or rather a little more. The Reverend Mr. *Moſes Williams*, Mr. *Alban Thomas*, and my ſelf, contemplated this Appearance for above an Hour, *viz.* from $10\frac{1}{2}^{h}$ to near twelve, and we could not be deceiv'd as to its Reality; but the Slowneſs of its Motion made us at that time conclude that it had none, and that it was rather a *Nebula* than a *Comet*.

However, ſuſpecting that it might have ſome Motion, I attended the next Night, *June* 11th, at the ſame Hours, and in the ſame Company, when with ſome Difficulty by reaſon of the Thickneſs of the Air, we found

* v. ſupr. p. 235.

found the two little Stars, but the *Nebula* could not at that time be ſeen, which we then imputed to the want of a clearer Sky. But on *Saturday*, *June* 15. the Moon being abſent, and the Air perfectly clear, we had again a diſtinct View of the two Stars, with an entire Evidence that there remained no Footſtep or Sign of it, in the place where we had firſt ſeen this *Phænomenon*, which we therefore now found to be a Comet, and that being far without the Orb of the Earth, and in it ſelf a very ſmall Body, it appeared only like a little Speck of a Cloud, ſuch as would ſcarce have been diſcerned in an ordinary Teleſcope, much leſs by the naked Eye.

A Comet at Berlin, *n.* 357. *p.* 820.

XLIV. 1. Cl. *Chriſtfridus Kirchius*, motus Corporum cœleſtium, ut munere ſibi a Societate Scientiarum Regia (*Berolini*) demandato recte fungeretur, ſedulo obſervans, a. d. XV. Kal. Febr. (*Jan.* 18. *ſt. n.*) 1717. veſperi dimidia Septima, verſus Septentiones fortuito Cometam conſpexit. Vicinus erat ad dextram (ſtellarum) γ & β *Bayeri* in *Urſa Minore*, nudoque oculo longe diſtinctius apparebat, quam β *Urſæ minoris*, licet ea inſignis ſit Stella ſecundæ magnitudinis, cum longe pallidior quidem majore tamen diametro, atque ſatis clara luce maxime circa centrum, conſpiceretur. Per Tubum viſus lucidam rotundamque referebat nubeculam: Caudæ autem nullum obſervari potuit veſtigium, neque Nucleus dignoſci. Motu celerrimo ab hora VII. ad XI. proceſſit, graduſque quatuor cum dimidio abſolvit, ut ex obſervationibus colligitur.

Die 19no & 20mo *Januarii* cœlum nubibus fuit obductum. Die vero 21mo Cometa longe receſſerat a loco ſuo nupero, atque in *Caſſiopea* deprehendebatur, ubi cum ſtellis ε & δ triangulum conficiebat (*an æquicrure?*) ſcil. Hora 5^{h} 45′ in 17° 34′ ♉, ſub latitudine Boreali 49° 54′ hærebat: Deinde 9^{h} 15, in 16° 38′ ♉ ſub Lat. Bor. 49° 2′ conſpiciebatur. Cæterum multum decreverat, atque a celeritate ſua remiſerat; præterquam enim quod pallidior quam ante apparebat, ſtellas etiam quartæ dignitatis magnitudine haud ſuperare nudo oculo conſpectus videbatur, inque orbita ſua, quatuor cum dimidia horis, non ultra ſeſquigradum proceſſerat: Tubi autem beneficio diameter ejus 7 min. inveniebatur.

Jan. 23. hora IV. mat. Cometa cum δ & φ *Caſſiopeæ* triangulum æquicrurum efficiebat, cum ab utraque 2° 41$\frac{1}{2}$ abeſſet. Hoc mane duarum horarum ſpatio vix dimidium gradum abſolvit; hora decima veſpertina cum δ *Caſſiopeæ* & φ *Perſei* in linea recta cernebatur, atque a priori 3° 38′, a poſteriori 3° 9′ diſtabat. Diameter ejus erat 5 min. nudoque oculo conſpectus ſtellam quintæ magnitudinis referebat.

Die 24to *Jan.* hora VI. mat. nondum attigerat φ *Perſei*, ſed cum *v* & *g* ejuſdem Aſteriſmi triangulum æquicrurum ſiſtebat, & ab utraque non plane 3$\frac{1}{2}$ grad. aberat.

A Remark by Dr. Halley, *ibid.*

2. *Deſiderantur Obſervationes Diei* 18væ, *cum Cometa velociſſime motus terræ proximus erat, unde certius de Via ejus tam vera quam apparente judicium ferre poſſemus. Manifeſtum autem eſt eum Polo Æquatoris Boreo viciniſſimum die* Januarii 19no *tranſiiſſe. Quod ſi cui libeat has Obſervationes*

ad examen revocare, calculoque accurato ſubjicere; in illius gratiam, loca Stellarum fixarum, quarum hic ſit mentio, ex Catalogo Britannico *excerpta, ſubnectuntur: Unde etiam patebit nonnulla in hac motus Cometæ deſcriptione haud rite ſe habere; quæ tamen, a Cl.* Kirchio *corrigi, in pleniori ejus quam promiſit hiſtoria, ſpes eſt.*

Stellarum fixarum Loca ineunte Anno 1718.

BAYERO		Long.				Lat. Bor.		
			°	′	″	°	′	″
Urſæ minoris	β	♌	9	18	0	72	58	10
	γ	♌	17	35	15	75	13	15
Caſſiopeæ	δ	♉	14	00	35	46	23	25
	ε	♉	20	50	8	47	31	50
	φ	♉	11	36	35	45	4	5
Perſei	υ	♉	8	32	0	35	23	45
	φ	♉	10	41	35	36	49	15
	g	♉	12	15	20	36	18	37

XLV. I. *Aſtronomical Obſervations at* Greenwich, *by Mr* Flamſteed, *n.* 337. *p.* 65.

Obſervationes SATURNI.

Temp. per Horolog.	Tempora correcta.	Die Veneris Januarii 5. 1711.	Diſtantiæ a Vertice.
10. 14. 26	10. 14. 41	Calx Caſtoris μ ♊ tranſiit	28. 50. 10
11. 11. 38	11. 11. 53	In Inguine Pollucis, δ tranſ.	28. 59. 00
12. 4. 45	12. 5. 00	Saturnus tranſit ——	30. 23. 20
		Aſcenſio recta ♄ 119. 01. 00	
		Diſt. a Polo Bor. 68. 55. 20	
		Die Solis Januarii 7.	
11. 2. 53	11. 0. 31	Geminorum δ tranſiit ——	28. 59. 20
11. 50. 36	11. 48. 14	In pede Bor. ♋ μ tranſiit ——	29. 5. 20
11. 55. 22	11. 53. 00	Saturnus tranſiit ——	30. 21. 50
12. 15. 41	12. 13. 19	Cancri η *Bayero* tranſit ——	30. 4. 30
12. 26. 18	12. 23. 56	Cancri γ, Aſellus Bor. tranſ. ——	28. 59. 50
		Aſcenſio Rect. ♄ 118. 52. 00	
		Diſt. a Polo Bor. 68. 53. 50	
		Die Lunæ, Januarii 8.	
10. 58. 35	10. 57. 49	Geminorum δ tranſiit ——	28. 59. 15
11. 50. 46	11. 50. 00	Saturnus tranſit ——	30. 20. 35
12. 11. 25	12. 10. 30	Cancri η tranſiit ——	30. 4. 25
		Aſcenſio Rect. ♄ 118. 46. 30	
		Diſt. a Polo Bor. 68. 52. 35	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Jovis, Januarii 25. 1711.	Diſtantia a Vertice.
9. 45. 53	9. 46. 26	Geminorum δ tranſiit	28. 59. 10
10. 32. 27	10. 33. 00	Saturnus tranſiit	30. 4. 00
11. 10. 54	11. 11. 27	Cancri δ, Aſellus Auſt. tranſ.	32. 17. 50
		Aſcenſio Rect. ♄ 117. 23. 00	
		Diſt. a Polo Bor. 68. 36. 00	
		Die Saturni, Januarii 27.	
9. 37. 25	9. 38. 2	Geminorum δ tranſiit	28. 59. 20
10. 13. 11	10. 13. 48	Sub latere Pollucis *l* tranſiit	30. 51. 50
10. 23. 23	10. 24. 00	Saturnus tranſiit	30. 2. 30
10. 40. 59	10. 41. 36	Poſt Caudam ♋ *d* tranſiit	32. 14. 10
10. 50. 16	10. 50. 53	Cancri η tranſiit	30. 4. 35
		Aſcenſio Rect. ♄ 117. 14. 00	
		Diſt. a Polo Bor. 68. 34. 30	
		Die Martis, Januarii 30.	
9. 29. 45	9. 26. 55	Geminorum δ tranſiit	28. 59. 40
10. 14. 50	10. 12. 00	Saturnus tranſiit	30. 0. 00
10. 33. 20	10. 30. 30	Cancri *d* tranſiit	32. 14. 10
10. 42. 35	10. 39. 45	Cancri η tranſiit	30. 4. 45
		Aſcenſio Rect. ♄ 117. 00. 30	
		Diſt. a Polo Bor. 68. 32. 00	
		Die Mercurii, Feb. 28.	
8. 18. 32	8. 15. 00	Saturnus tranſiit	29. 43. 00
8. 27. 11	8. 23. 39	In Boreo pede Cancri μ tranſ.	29. 5. 40
9. 2. 50	8. 59. 18	Aſellus Boreus Cancri γ tranſ.	29. 0. 2
10. 39. 59	10. 36. 27	Lucida Colli Leonis γ tranſ.	30. 12. 05
		Aſcenſio Rect. ♄ 115. 31. 30	
		Diſt. a Polo Bor. 68. 15. 00	
		Die Jovis, Mart. 1. 1711.	
8. 14. 5	8. 11. 00	Saturnus tranſiit	29. 42. 50
8. 22. 47	8. 19. 42	Cancri μ tranſiit	29. 5. 35
8. 58. 28	8. 55. 23	Aſellus Boreus tranſiit	29. 0. 15
10. 35. 36	10. 32. 31	Lucida Colli Leonis tranſiit	30. 12. 00
		Aſcenſio Rect. ♄ 115. 30. 30	
		Diſt. a Polo Bor. 68. 14. 50	
		Die Veneris, Nov. 9.	
17. 21. 12	17. 21. 23	Sequens ad π Cancri tranſiit	35. 21. 10
17. 29. 49	17. 30. 00	Saturnus tranſiit	34. 9. 50
18. 13. 25	18. 13. 36	Auſtrina Colli Leonis η tranſ.	33. 18. 50
		Aſcenſio Rect. ♄ 136. 58. 00	
		Diſt. a Polo Bor. 72. 42. 00	

Temp. per Horolog.	Tempora correcta.	Die Lunæ, Nov. 19.	Diſtantia a Vertice.
16. 46. 20	16. 41. 00	Saturnus tranſit ——	34. 8. 00
16. 59. 31	16. 54. 11	Teleſcopica *a* tranſit ——	33. 45. 30
17. 29. 59	17. 24. 33	Auſtrina Colli Leonis η tranſ.	33. 18. 30
		Aſcenſio Rect. ♄ 136. 58. 30	
		Diſt. a Polo Bor. 72. 40. 05	
		Die Jovis, Nov. 22.	
16. 24. 36	16. 19. 30	Cancri ſequens ad π tranſiit	35. 21. 30
33. 06	16. 28. 00	Saturnus tranſit —— ——	34. 6. 50
46. 27	16. 41. 21	Teleſcopica dicta tranſiit ——	33. 45. 10
17. 16. 51	17. 11. 45	Auſtrina Colli Leonis tranſiit	33. 18. 30
		Aſcenſio Rect. ♄ 136. 55. 45	
		Diſt. a Polo Bor. 72. 39. 00	
		Die Solis, Decembris 30. 1711.	
11. 9. 43	10. 52. 42	Lucidus pes ♊ γ tranſit ——	34. 51. 30
13. 29. 25	13. 12. 24	Cancri præced. ad o tranſit	35. 4. 00
13. 49. 01	13. 32. 00	Saturnus tranſit —— ——	33. 30. 00
14. 39. 50	14. 22. 49	Lucida Colli Leonis tranſiit	33. 18. 40
		Aſcenſio Rect. ♄ 135. 10. 00	
		Diſt. a Polo Bor. 72. 2. 5	
		ANNO MDCCXII.	
		Die Saturni, Januarii 12.	
11. 39. 52	11. 33 06	Ad caudam Cancri ζ tranſiit	32. 59. 10
12. 40. 46	12. 43. 00	Saturnus tranſit ——	33. 12. 00
12. 43. 16	12. 36. 30	Cancri ſequens ad π tranſiit	35. 22. 20
13. 36. 44	13. 29. 58	Cor Leonis tranſiit ——	38. 6. 55
		Aſcenſio Rect. ♄ 134. 12. 00	
		Diſt. a Polo Bor. 71. 44. 00	
		Die Saturni, Jan. 19.	
11. 45. 31	11. 36. 55	Aſellus Auſtrinus tranſiit	32. 17. 10
11. 58. 14	49. 38	Auſtrina ad o Cancri tranſiit	35. 3. 50
12. 11. 36	12. 3. 00	Saturnus tranſiit ——	33. 2. 5
12. 20. 00	11. 24	Teleſcopica *b* tranſiit ——	32. 33. 20
		Aſcenſio Rect ♄ 133. 37. 00	
		Diſt. a Polo Bor. 71 34. 10	
		Die Solis, Jan. 27.	
10. 57. 50	11. 4. 31	Aſellus Auſtrinus tranſiit ——	32. 17. 10
11. 10. 54	11. 17 35	Borea ad o Cancri tranſiit	34 48. 10
11. 21. 19	12. 28. 00	Saturnus tranſiit ——	32. 50. 20
11. 32. 20	39. 01	Teleſcopica *b* tranſiit ——	32. 33. 25
		Aſcenſio Rect. ♄ 132. 58. 00	
		Diſt. a Polo Bor. 71. 22. 20	

Temp. per Horolog.	Tempora correcta.	Die Lunæ, Martii 31. 1712.	Diſtantia a Vertice.
7. 8. 46	7. 7. 41	Aſellus Auſtrinus tranſiit	32. 16. 50
7 20. 35	7. 19. 30	Saturnus tranſiit	32. 03. 30
7. 36. 50	7. 35. 45	Cancri præced. ad π tranſiit	35. 19. 10
7. 39. 36	7. 38. 31	Sequens ad π tranſiit	35. 21. 20
8. 31. 51	8. 30. 46	Auſtrina colli Leonis tranſ.	33. 18. 40
		Aſcenſio recta ♄ 130 02. 00	
		Diſt. a Polo Bor. 70. 35. 30	
		Die Veneris, Novemb. 7.	
18. 17. 13	18. 22. 00	Saturnus tranſiit	37. 57. 30
18 26. 13	18 31. 00	Leonis 40 *Catal. Brit.* tranſ.	41. 14. 10
18. 28 35	18. 33. 22	Ejuſdem 41 tranſiit	40. 15. 20
		Aſcenſio Rect. ♄ 150. 15. 30	
		Diſt. a Polo Bor. 76. 29. 40	
		Die Lunæ, Nov. 17.	
17. 33. 19	17. 34. 59	Cor Leonis tranſiit	38. 7. 20
17. 42. 20	17. 44. 00	Saturnus tranſiit	38. 00. 50
17. 57. 48	17. 59. 28	In Axilla Leonis ρ tranſiit	40. 41. 45
18. 14. 19	18. 15. 59	In ventre Leonis *l* tranſiit	39. 24. 50
		Aſcenſio Rect. ♄ 150. 31. 00	
		Diſt. a Polo Bor. 76. 33. 00	
		Obſervationes JOVIS.	
		Anno MDCCXI.	
		Die Saturni, Maii 26. 1711.	
12. 44. 28	12. 39. 44	Serpentarii 58 *Cat. Br.* tranſ.	76. 12. 00
52. 40	47. 56	Sagittarii Nebuloſæ *a* tranſiit	75. 39. 20
13. 8. 44	13. 4. 00	Jupiter tranſiit	74. 37. 00
15. 43	10. 59	Sagittarii 11 tranſiit	72. 5. 20
		Aſcenſio Rect. ♃ 270. 19. 00	
		Diſt. a Polo Bor. 113. 11. 50	
		Die Solis, Maii. 27.	
10 47. 24	10. 43. 2	Media frontis ♏. δ tranſ.	73. 12. 00
12. 57. 46	53. 24	Teleſcopica *c* Jovem præced.	75. 7. 50
13. 4. 22	13. 00. 00	Jupiter tranſiit	74. 36. 50
11. 53	7. 31	Sagittarii 11 tranſiit	72. 5. 20
		Aſcenſio Rect. ♃ 270. 11. 00	
		Diſt. a Polo Bor. 113. 11. 40	
		Die Solis Junii 3.	
10. 15. 41	10. 10. 45	Media frontis Scorpii tranſiit	73. 12. 5
12. 14. 11	12. 9. 15	Nebuloſæ Sagittarii *b* tranſiit	75. 10. 20
12. 17. 09	12. 12. 13	Ejuſdem Nebuloſæ *a* trauſiit	75. 39. 30

Temp.

Temp. per Horolog.	Tempora correcta.	Die Solis Junii 3.	Diſtantia a Vertice.
12. 28. 56	12. 24. 00	Jupiter tranſiit	74. 38. 00
13. 8. 36	13. 03. 40	Sagittarii in Oculo præced. tranſ.	74. 29. 15
		Aſcenſio Rect. ♃ 269. 15. 00	
		Diſt. a Polo Bor. 113. 12. 50	
		Die Lunæ, Junii 4. 1711.	
10. 11. 51	10. 7. 19	Media frontis Scorpii tranſiit	73. 12. 00
12. 10. 18	12. 5. 46	Nebuloſæ Sagittarii *b* tranſ.	75. 10. 20
12. 24. 32	12. 20. 00	Jupiter tranſiit	74. 38. 10
13. 4. 45	13. 0. 13	Præced. ad ν Sagittarii tranſ.	74. 29. 10
13. 5. 41	13. 1. 09	Sequens ad ν tranſiit	74. 24. 50
		Aſcenſio Rect. ♃ 269. 07. 00	
		Diſt. a Polo Bor. 113. 13. 00	
		Die Saturni, Junii 9.	
9. 52. 16	9. 49. 5	Media frontis Scorpii tranſ.	73. 11. 55
11. 50. 44	11. 47. 33	Nebuloſæ *b* tranſ.	75. 10. 15
12. 2. 11	11. 59. 00	Jupiter tranſiit	74. 38. 25
12. 45. 8	12. 41. 57	Præced. ad ν Sagittarii tranſ.	74. 29. 10
		Aſcenſio Rect. ♃ 268. 25. 00	
		Diſt. a Polo Bor. 113. 13. 15	
		Die Solis, Junii 10.	
9. 48. 22	9. 44. 38	Media frontis Scorpii tranſiit	73. 12. 00
11. 46. 51	11. 43. 07	Nebuloſæ *b* tranſiit	75. 10. 15
11. 57. 44	11. 54. 0	Jupiter tranſiit	74. 38. 35
12. 41. 14	12. 37. 30	Præcedens ad ν Sagitt. tranſ.	74. 29. 15
		Aſcenſio Rect. ♃ 268. 16. 45	
		Diſt. a Polo Bor. 113. 13. 25	
		Die Saturni, Julii 14.	
8. 58. 10	8. 50. 4	Serpentarii 48, *C* tranſiit	75. 7. 20
9. 10. 32	9. 2. 26	Ejuſdem 54 ſive *D* tranſiit	72. 56. 00
9. 21. 6	9. 13. 00	Jupiter tranſiit	74. 37. 50
9. 40. 49	9. 32. 43	Sagittarii in arcu μ tranſiit	72. 31. 30
		Aſcenſio Rect. ♃ 264. 12. 30	
		Diſt. a Polo Bor. 113. 12. 40	
		Die Solis, Julii 15.	
8. 54. 8	8. 46. 24	Serpentarii *C* tranſiit	75. 7. 20
9. 6. 28	8. 58. 44	Serpentarii *D* tranſiit	72. 56. 00
9. 16. 44	9. 9. 00	Jupiter tranſiit	74: 37. 50
9. 36. 46	9. 29. 2	Sagittarii μ tranſiit	72. 31. 35
		Aſcenſio Rect. ♃ 264. 07. 45	
		Diſt. a Polo Bor. 113. 12. 40	

Temp. per Horolog.	Tempora correcta.	A N N O MDCCXII. Die Jovis, Julii 3.	Diſtantia a Vertice.
12. 36. 3	12. 25. 14	Capric. ſub Oculo σ tranſiit	7 . 24. 40
12. 44. 6	12. 33. 17	Capric. in roſtro π tranſiit	70. 33 00
12 45. 42	12. 34. 53	Capricorni ρ tranſiit	70. 09 50
12 57. 49	12. 47 00	Jupiter tranſiit	7 . 26. 20
13. 16. 24	13. 5. 35	Capric 20 *Catal. Brit* tranſ.	71. 33. 10
		Aſcenſio Rect. ♃ 306. 09. 20	
		Diſt. a Polo Bor. 110. 0. 30	
		Die Martis, Julii 15.	
11. 39. 34	11. 39. 33	Capricorni σ tranſiit	71. 24. 45
55. 1	11. 55. 00	Jupiter tranſiit	71. 49. 20
12. 19. 53	12. 19 52	Capricorni 20 tranſiit	71. 33. 10
24. 38	12. 24. 37	♑ni in medio corpore η tranſ.	72. 23. 25
		Aſcenſio Rect. ♃ 304. 34. 35	
		Diſt. a Polo Bor. 110. 23. 30	
		Die Mercurii, Sept. 17.	
7. 32. 37	7. 31. 56	Teleſc. Jovem præced. tranſ.	72. 49. 40
7. 38. 41	7. 38. 00	Jupiter tranſiit	72. 51. 10
7. 42. 28	7. 41. 37	Teleſc. Jovem ſequens tranſ.	73. 14. 40
		Aſcenſio Rect. ♃ 299. 43. 00	
		Diſt. a Polo Bor. 111. 25. 30	
		Die Veneris, Septembris 19.	
7. 30. 55	7. 32. 00	Jupiter tranſiit	72. 50. 45
8. 19. 47	8. 20. 52	Capricorni η tranſiit	72. 23. 40
		Aſcenſio Rect. ♃ 299. 45. 00	
		Diſt. a Polo Bor. 111. 25. 05	
		Die Lunæ, Octobris 6.	
6. 25. 35	6. 31. 30	Jupiter tranſiit	72. 39. 50
6. 36. 22	6. 42. 17	Capricorni in roſtro ο tranſ.	70. 56. 25
6. 46. 36	6. 52. 31	Capricorni in Cervice υ tranſ.	70. 33. 30
		Aſcenſio Rect. ♃ 300. 39. 00	
		Diſt. a Polo Bor. 111. 14. 10	

Obſervationes MARTIS.

Anno MDCCXI.

		Die Solis Jan. 7.	
12. 40. 35	12. 38. 56	Cancri Auſt. ad ο tranſiit	35. 04. 00
12. 40. 56	12. 39. 17	Cancri Bor. ad ο tranſiit	34. 48. 00
12. 58. 45	12. 57. 6	Cancri ſequens ad π tranſiit	35. 21. 20

Temp.

Temp. per Horolog.	Tempora correcta.	Die Solis Jan. 7.	Diſtantia a Vertice.
13. 27. 15	13. 25. 36	Leonis ↓ tranſiit	36. 8. 50
13. 50. 52	13. 49. 13	Auſtrina Colli Leonis *n* tranſ.	33. 19. 00
14. 00. 39	13. 59. 00	Mars tranſiit	34. 51. 30
14. 30. 14	14. 28. 35	Borea Ventris ♌, *k* tranſiit	35. 45. 40
		Aſcenſio Rect. ♂ 150. 20. 00	
		Diſt. a Polo Bor. 73. 23. 35	
		Die Saturni, Januarii 27. 1711.	
10. 40. 59	10. 41. 30	Cancri 20, prima ad *d* tranſiit	32. 14. 10
10. 50. 16	10. 50. 47	Cancri *n* tranſiit	30. 4. 35
12. 8. 29	12. 9. 00	Mars tranſiit	32. 14. 00
12. 25. 27	12. 25. 58	Auſtrina Colli Leonis *n* tranſ.	33. 19. 00
12. 37. 59	12. 38. 30	Lucida Colli Leonis γ tranſiit	30. 11. 50
		Aſcenſio recta ♂ 143. 37. 00	
		Diſt. a Polo Bor. 70. 36. 00	
		Die Lunæ, Januarii 29.	
12. 1. 50	11. 59. 00	Mars tranſiit	31. 58. 50
12. 21. 58	12. 19. 08	Leonis *n* tranſiit	33. 19. 05
12. 34. 31	12. 31. 41	Lucida Colli Leonis γ tranſiit	30. 11. 50
		Aſcenſio Rect. ♂ 142. 49. 30	
		Diſt. a Polo Bor. 70. 30. 50	
		Die Martis, Jan. 30.	
10. 33. 30	10. 30. 25	Cancri 20, prima ad *d* tranſ.	32. 14. 10
10. 42. 35	10. 39. 30	Cancri *n* tranſiit	30. 4. 45
11. 56. 5	11. 53. 00	Mars tranſiit	31. 51. 35
12. 17. 49	12. 14. 44	Leonis *n* tranſiit	33. 19. 5
12. 30. 20	12. 27. 15	Leonis γ tranſiit	30. 11. 50
		Aſcenſio Rect. ♂ 142. 25. 00	
		Diſt. a Poloa Bor. 70. 23. 35	
		Die Mercurii, Feb. 28.	
8. 27. 11	8. 23. 58	In pede Bor. Cancri μ tranſ.	29. 5. 40
9. 2. 50	8. 59. 37	Aſellus Boreus γ tranſ.	29. 00. 20
9. 31. 13	9. 28. 00	Mars tranſ.	30. 5. 10
10. 27. 28	10. 24. 15	Leonis *n* tranſ.	33. 19. 55
10. 39. 39	10. 36. 26	Leonis γ tranſ.	30. 12. 5
		Aſcenſio Rect. ♂ 133. 45. 00	
		Diſt. a Polo Bor. 68. 37. 10	
		Die Jovis, Martii 1. 1711.	
8. 22. 47	8. 19. 27	Cancri μ tranſ.	29. 5. 35
8. 58. 28	8. 55. 8	Aſellus Boreus tranſ.	29. 00. 15
9. 26. 20	9. 23. 00	Mars tranſ.	30. 6. 20

Temp.

Temp. per Horolog.	Tempora correct.	Die Jovis Martii 1. 1711.	Diſtantia a Vertice.
10. 23. 04	10. 19. 44	Leonis η tranſ.	33. 19. 50
10. 35. 36	10. 32. 16	Lucida Colli Leonis tranſ.	30. 12. 00
		Aſcenſio Rect. ♂ 133. 37. 30	
		Diſt. a Polo Bor. 68. 38. 20	
		ANNO MDCCXII.	
		Die Lunæ, Novemb. 17.	
18. 21. 7	18. 19. 52	Leonis 55 *Catal. Brit.* tranſ.	43. 45. 40
18. 25. 53	18. 24. 38	Sub Ventre Leonis *c* tranſ.	43. 49. 40
18. 30. 14	18. 28. 59	Leonis χ tranſiit	42. 35. 10
18. 43. 15	18. 42. 00	Mars tranſiit	43. 7. 25
		Aſcenſio Rect. ♂ 165. 48. 00	
		Diſt. a Polo Bor. 81. 39. 45	
		Die Jovis, Novemb. 20.	
18. 13. 8	18. 6. 40	Leonis 55 tranſ.	43. 45. 40
18. 17. 53	18. 11. 25	Leonis 58, *c* tranſ.	43. 49. 50
18. 22. 14	18. 15. 46	Leonis χ tranſ.	42. 35. 15
18. 38. 18	18. 31. 50	In Poplite Leonis σ tranſ.	43. 52. 10
18. 40. 28	18. 34. 00	Mars tranſ.	43. 36. 55
		Aſcenſio Rect. ♂ 167. 6. 30	
		Diſt. a Polo Bor. 82. 9. 15	

Obſervationes SOLIS.

Anno MDCCXI.

Die Saturni, Januarii 6.	
Solis centro per planum Arcus Meridionalis tranſeunte, limbus ejus remotus & Auſtrinus diſtabat a Vertice. —— ——	72. 31. 10
Die Veneris, Januarii 26.	
Solis Meridiani limbus proximus a Vertice ——	66. 35. 10
Die Martis, Januarii 30.	
Solis limbus proximus a Vertice —— ——	65. 38. 40
Die Lunæ, Junii 4.	
Solis limbus remotus, &c. ——	28. 24. 18
Die Saturni, Julii 14.	
Solis limbus remotus ——	31. 55. 20
Die Martis, Novembris 20.	
Solis limbus remotus ——	73. 30. 40
Die Martis, Decembris 4.	
Solis limbus remotus a Vertice ——	74. 59. 30

Anno MDCCXII.	Diſtantia a Vertice.
Die Mercurii, Januarii 2.	
Solis limbus remotus a Vertice	73. 17. 30
Die Saturni, Januarii 12.	
Solis limbus remotus	71. 18. 00
Die Veneris, Martii 7.	
Solis limbus remotus	52. 31. 00
Die Veneris, Maii 9.	
Solis limbus remotus	31. 40. 50
Die Martis, Octobris 7.	
Solis limbus remotus a Vertice	61. 29. 50

Obſervationes LUNÆ.

Anno MDCCXI.

Temp. per Horolog.	Tempora correcta.	Die Saturni, Maii 19. 1711.	Diſtantia a Vertice.
10. 22. 12	10. 18. 32	Scorpii *γ* ſive Libræ 15 tranſ.	75. 32. 50
10. 50. 55	10. 47. 15	☽ Limbus tranſ. centro a Vert.	75. 38. 40
10. 52. 10	10. 48. 30	☽ Centrum tranſiit	75. 22. 00
11. 8. 30	11. 4. 50	Scorpii *b* tranſiit	76. 15. 20
11. 11. 10	11. 7. 30	Scorpii *A* tranſiit proximo	75. 50. 55
		Aſcenſio Rect. ☽ 229. 18. 30.	
		Diſt. a Polo Bor. 114. 13. 40	
		Die Lunæ, Novemb. 19.	
16. 41. 28	16. 36. 8	Lunæ centrum tranſ. remoto	37. 6. 30
17. 42. 16	16. 36. 56	Lunæ limbus tranſiit, centro	36. 52. 30
16. 46. 20	16. 41. 00	Saturnus tranſiit	34. 8. 00
16. 59. 31	16. 54. 11	Stella Teleſcopica *a* tranſiit	33. 45. 30
17. 29. 53	17. 24. 33	Auſtr. Colli Leonis *η* tranſ.	33. 18. 30
		Aſcenſio Rect. ☽ 135. 41. 40	
		Diſt. a Polo Bor. 75. 23. 20	

Anno MDCCXII.

		Die Saturni, Januarii 12. 1712.	
7. 40. 50.	7. 34. 00	Medium Eclipſis Lunaris, quo tempore Chorda partis in Luna deficientis erat 24. 30; maximus autem defectus 8. 30 a parte Borea. Lunæ Diam. 30. 48.	
11. 39. 52	11. 33. 6	Cancri *ζ* *Bayero* tranſiit	32. 59. 10
12. 13. 46	12. 7. 00	☽ centrum tranſiit, remoto	34. 14. 10

Temp.

Temp. per Horolog.	Tempora correcta.	Die Saturni, Januarii 12.	Diſtantia a Vertice.
12 40. 46	12. 34. 90	Saturnus tranſiit	33. 12. 00
43. 16	12. 36. 30	Cancri ſequens ad * tranſiit	35. 22. 20
13. 36. 44	13. 29. 58	Cor Leonis tranſiit	38. 6. 55
		Aſcenſio Rect. ☽ 127. 25. 30	
		Diſt. a Polo Bor. 72. 31. 00	
		Die Jovis, Martii 6	
7. 39. 8	7. 38. 8	♊ 80 *Catal. Brit.* tranſiit	30. 29. 30
7. 49. 42	7. 48. 42	Geminorum 86 *l* tranſiit	30. 51. 45
7. 54. 45	7. 53. 45	Lunæ limbus tranſiit, centro	30. 50. 40
7. 55. 48	7. 54. 48	☽ centrum tranſiit, proximo	30. 35. 30
8. 1. 41	8. 0. 41	Nona Cancri μ tranſiit	29. 4. 45
		Aſcenſio Rect. ☽ 116. 14. 40	
		Diſt. a Polo Bor. 69. 22. 50	
		Die Mercurii, Maii 7.	
9. 47. 38	9. 46. 2	Virginis 70 *Catal. Brit.* tranſiit	66. 9. 50
9. 48. 48	9. 47. 12	Virginis 71 tranſiit	65. 45. 30
9. 52. 51	9. 51. 15	Ejuſdem 75 tranſiit	68. 7. 40
10. 9. 36	10. 8. 00	Lunæ limbus tranſ. centro	69. 12. 30
10. 10. 43	10. 9. 7	Lunæ centrum tranſ. remoto	69. 28. 35
		Aſcenſio Rect. ☽ 208. 4. 30	
		Diſt. a Polo Bor. 107. 45. 50	
		Die Jovis, Maii 8. 1712.	
10. 52. 2	10. 50. 00	Libræ 8 *Catal. Brit.* tranſiit	74. 51. 50
11. 1. 35	10. 59. 33	Scorpii γ ſive Libræ 15 tranſ.	75. 32. 30
11. 8. 2	11. 6. 00	Lunæ limbus tranſiit, centro	73. 42. 30
11. 09. 16	11. 7. 14	Lunæ centrum tranſ. remoto	73. 58. 20
11. 50. 33	11. 48. 31	Scorpii *A* tranſit	75. 51. 10
11. 57. 34	11. 55. 32	Media frontis ♏ δ tranſit	73. 12. 10
		Aſcenſio recta ☽ 223. 45. 00	
		Diſt. a Polo Bor. 112. 16. 10	
		Die Saturni, Maii 10.	
12. 18. 05	12. 16. 30	Cor Scorpii *Antares* tranſiit	77. 10. 00
13. 3. 48	13. 2. 13	Serpentarii A tranſit	77. 32. 50
13. 18. 13	13. 16. 38	Lunæ centrum tranſ. remoto	78. 1. 30
13. 19. 30	13. 17. 55	Lunæ limbus tranſiit, centro	77. 46. 05
13. 35. 25	13. 33. 50	Prima Sagittarii *p* tranſiit	79. 4. 20
14. 16. 8	14. 14. 33	Sagittarii λ tranſiit	76. 57. 00
		Aſcenſio Rect. ☽ 258. 03. 20	
		Diſt. a Polo Bor. 116. 20. 15	

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Obſervationes SATURNI.

Temp. per Horolog.	Tempora correcta.	Die Solis, Januarii 25. 1713.	Diſtantia a Vertice.
8. 30. 15	8. 28. 5	Geminorum προπους tranſiit	28. 13. 20
8. 41. 7	38 57	Pes Caſtoris η tranſiit	28. 54. 50
8. 49. 8	8. 46. 58	Calx ejuſdem ſive μ tranſiit	28. 50. 40
12. 10. 49	12. 8. 39	Leonis ψ *Bayero* tranſiit	36. 9. 30
12. 25. 24	23. 14	Leonis ν tranſiit	37. 40. 30
12. 34. 21	32. 11	Saturni centrum tranſiit	36. 51. 45
12. 35. 44	12. 33. 34	Cor Leonis tranſiit	38. 7. 00
		Aſcenſio Rect. ♄ 147 55 10	
		Diſt. a Polo Bor. 75 23 55	
		Longitudo ♌ 25 8 15	
		Latitudo Bor. 1 31 27	
		Die Jovis Februarii 5.	
11. 20. 52	11. 14. 10	Leonis ξ *Bayero* tranſiit	38. 55. 20
30. 9	23. 27	Leonis ο tranſiit	40. 17. 30
35. 18	28. 36	Leonis 16*ta Cat. Brit.* tranſ.	38. 21. 20
47. 8	40. 26	Leonis ν tranſiit	37. 40. 30
52. 42	46. 00	Saturni centrum tranſiit	36. 32. 50
11. 57. 25	50. 43	Cor Leonis tranſiit	38. 7. 5
12. 0. 35	53. 53	Leonis 31*ma* tranſiit	36. 42. 50
5. 40	11. 58. 58	Leonis 34*ta* tranſiit	36. 19. 30
12. 10. 50	12. 4. 8	Leonis 38*va* tranſiit	35. 3. 50
		Aſcenſio recta ♄ 147. 4. 45	
		Diſtant. a Polo 75. 5 00	
		Longitudo ♌ 24. 14. 8	
		Latitudo Bor. 1. 32 16	
		Die Veneris Februarii 6:	
8. 22. 53	8. 14. 43	Lucidus pes Pollucis, ♊ γ tr.	34. 52. 00
11. 29. 22	11. 21. 12	Leonis ψ tranſiit	36. 9. 15
11. 43. 58	11. 35. 48	Leonis ν tranſit	37. 40. 30
11. 49. 10	11. 41. 0	Saturni centrum tranſiit	36. 31. 00
		Aſcenſio recta ♄ 146. 59. 0.	
		Diſtantia a Polo 75. 3. 10	
		Longitudo ♌ 24. 8. 17	
		Latitudo Bor. 1. 32. 8	

Temp. per Horolog.	Tempora correcta.	Die Mercurii Febr. 18.	Diſtantia a Vertice.
11. 0. 15	10. 53. 0	Saturni centrum tranſiit	36. 12. 00
11. 8. 53	11. 1. 38	Cor Leonis tranſiit	38. 7. 5
11. 17. 7	11. 9. 52	Leonis 34*ta Cat. Brit.* tranſ.	36. 19. 35
11. 22. 16	11. 15. 1	Ejuſdem 38*va* tranſiit	35. 3. 55
		Aſcenſio Rect. ♄ 146. 5. 00	
		Diſtantia a Polo 74. 44. 10.	
		Longitudo ♌ 23. 12. 47	
		Latitudo Bor. 1. 32. 41	
		Die Lunæ Martii 2. 1713.	
10. 6. 30	9. 52. 3	Leonis ψ tranſiit	36. 9. 30
19. 27	10. 5 00	Saturni centrum tranſiit	35 55. 20
31. 21	16. 54	Cor Leonis tranſiit	38. 7. 5
34. 31	20 4	Leonis 31*ma* tranſiit	36. 42. 50
10. 39. 35	10. 25. 8	Leonis 34*ta* tranſiit	36. 19. 40
		Aſcenſio Recta ♄ 145. 16. 00	
		Diſtant. a Polo 74. 27 30	
		Longitudo ♌ 22. 22. 40	
		Latitudo Bor. 1. 32. 52	
		Die Martis Aprilis 7.	
7. 43. 1	7. 41 15	Leonis ψ tranſiit	36. 9. 25
50. 46	7. 49 00	Saturni centrum tranſiit	35. 31. 10
57. 36	55. 50	Leonis ν *Bayero* tranſiit	37. 40. 20
8. 6. 39	8. 4. 53	Leonis in collo η tranſiit	33. 19. 25
16. 7	14. 21	Ejuſdem 34*ta Cat. Br* tranſ.	36. 19. 30
8. 21. 17	8. 19. 31	Ejuſdem 38*va* tranſiit	35. 3. 50
		Aſcenſio recta ♄ 143. 57. 45	
		Diſtantia a Polo 74. 3. 15	
		Longitudo ♌ 21. 3. 32	
		Latitudo 1. 31. 20	
		Die Mercurii Aprilis 8.	
7. 47. 43	7. 46. 00	Saturni centrum tranſiit	35. 31. 15
54. 34	52. 51	Leonis ν tranſiit	37. 40. 20
8. 3. 37	8. 1. 54	Leonis η tranſiit	33. 19. 20
13. 4	11. 21	Leonis 34*ta* tranſiit	36. 19. 35
8. 18. 14	8. 16. 31	Leonis 38*va* tranſiit	35. 3. 45
		Aſcenſio Recta ♄ 143. 57. 30	
		Diſtantia a Polo 74. 3. 20	
		Longitudo ♌ 21. 3. 20	
		Latitudo 1. 31. 10	
		Saturno pene ſtationario.	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Jovis Novemb. 5. 1713.	Diſtantia a Vertice.
18. 22. 00	18. 15. 37	Leonis ϖ in genu ſeq. tranſ.	42. 4. 00
30. 8	23. 45	Cor Leonis tranſiit	38. 7. 5
54. 35	48. 12	Leonis in Axilla ς tranſiit	40. 42. 0
19. 11. 5	19. 4. 42	Leonis in ventre *l* tranſiit	39. 25. 15
19. 26. 23	19. 20. 00	Saturni centrum tranſiit	42. 10. 40
		Aſcenſio Rect. ♄ 162. 23. 20	
		Diſt. a Polo 80. 43. 00	
		Longitudo ♍ 10. 13. 40	
		Latitudo Bor. 1. 39. 37	

Obſervationes JOVIS.

Anno MDCCXIII.

Temp. per Horolog.	Tempora correcta.		Diſtantia a Vertice.
		Die Solis Auguſti 9.	
12. 40. 4	12. 37. 27	Aquarii λ in effuſione Aquæ tranſit	60. 32. 50
12. 48. 37	12. 46. 0	Jovis centrum tranſiit	60. 48. 35
12. 52. 36	12. 49. 59	Aquarii 73*ta Cat. Brit.* prima ad *b* tranſit	60. 46. 20
13. 4. 21	13. 1. 44	Aquarii in aqua χ tranſiit	60. 49. 10
		Aſcenſ. Rect ♃ 341. 33. 5	
		Diſt. a Polo Bor. 99. 21. 40	
		Longitudo ♓ 9. 26. 00	
		Latitudo Auſt. 1. 25. 8	
		Die Lunæ Auguſti 10.	
12. 36. 21	12. 33. 55	Aquarii λ tranſiit	60. 32. 50
12. 44. 26	12. 42. 00	Jovis centrum tranſiit	60. 52. 00
12. 48. 53	12. 46. 27	Aquarii 73*ia* tranſiit	
		Aſcenſ. Rect. ♃ 341. 26. 5	
		Diſtant. a Polo 99. 25. 5	
		Longitud. Jov. ♓ 9. 18. 17	
		Latitudo Auſt 1. 25. 40	
		Die Lunæ Octobris 26.	
7. 29. 16	7. 28. 42	Aquarii in Clune σ tranſiit	63. 34. 40
7. 36. 34	7. 36. 0	Jovis centrum tranſiit	63. 00. 5
8. 14. 34	8. 14. 0	Aquarii 80*ma* prima ad ψ tranſ.	62. 5. 20
8. 17. 45	8. 17. 11	Aquarii 84*ta* ſeq. ad ψ tranſ.	62. 37. 5
		Aſcenſio Rect. ♃ 335. 41. 30	
		Diſtantia a Polo 101. 33. 20	
		Longitudo Jovis ♓ 3. 16. 00.	
		Latitudo Auſt. 1. 19. 8	

Temp. per Horolog.	Tempora correcta.	Die Martis Octob. 27.	Diſtantia a Vertice.
7. 25. 40	7. 23. 34	Aquarii Clunis σ tranſiit	63. 34. 35
7. 33. 6	7. 31. 00	Jovis centrum tranſiit	62. 59. 15
8. 11. 00	8. 8. 54	Aquarii prima ad ↓ tranſiit	62. 5. 15
8. 14. 10	8. 12. 4	Sequens ad ↓ tranſiit	62. 37. 10
		Aſcenſio Rect. ♃ 335. 43. 20	
		Diſt. a Polo 101. 32. 30	
		Longitudo ♓ 3. 17. 58	
		Latitudo Auſt. 1. 19. 00	
		Die Jovis Octob. 29.	
7. 18. 29	7. 15. 19	Aquarii σ tranſ.	63. 34. 40
26. 10	7. 23. 00	Jovis centrum tranſiit	62. 57. 20
8. 3. 47	8. 0. 37	Prima ad ↓ tranſiit	62. 5. 20
6. 55	8. 3. 45	Sequens ad ↓ tranſiit	62. 37. 10
		Aſcenſio Rect. ♃ 335. 47. 45	
		Diſt. a Polo 101. 30. 35	
		Longitudo Jov. ♓ 3. 22. 41	
		Latitudo Auſt. 1. 18. 49	

Obſervationes MARTIS.
Anno MDCCXIII.

		Die Mercurii Feb. 18. 1713.	
12. 28. 38	12. 21. 20	Leonis in poplite τ tranſiit	47. 2. 15
12. 51. 6	12. 43. 48	In ancone Alæ ♍ β tranſ.	48. 5. 40
13. 10. 21	13. 3. 3	Virginis 10*ma Catal. Brit.* r tranſ.	47. 57. 40
13. 13. 18	13. 6. 00	Martis centrum tranſiit	47. 2. 5
13. 21. 8	13. 13. 50	In cervice Viginis *c* tranſiit	46. 33. 30
		Aſcenſio Rect. *Martis* 179. 29. 20	
		Diſt. a Polo 85. 34. 35	
		Longitudo ♍ 27. 46. 00	
		Latitudo Bor. 3. 51. 10	
		Die Martis Martii 3.	
11. 55. 52	11. 53. 2	In vertice Virginis ν tranſ.	43. 20. 00
12. 4. 50	12. 2. 00	Martis centrum tranſ.	45. 14. 20
10. 54	8. 4	In vultu Virginis π tranſ.	43. 15. 25
20. 8	17. 18	Undecima Virginis *s* tranſ.	44. 3. 30
12. 30. 25	27. 35	Virginis 16 in Cervice *c* tranſ.	46. 33. 25
12. 32. 37	12. 29. 47	Virginis 17*ma Cat. Br.* tranſiit	44. 33. 45
		Aſcenſio recta *Martis* 175. 1. 15	
		Diſt. a Polo Bor. 83. 46. 45	
		Longitudo ♍ 22. 57. 33	
		Latitudo Bor. 3. 43. 37	

Temp.

Temp. per Horolog.	Tempora correcta.	Die Martis Aprilis 7. 1713.	Diſtantia a Vertice.
9. 4 47	9. 3. 10	Sub Ventre Leonis χ tranſ.	42. 35. 25
9. 17 37	9. 16. 00	*Martis* centrum tranſ.	42. 42. 50
9. 38. 10	9. 36. 33	Prima Virg. *Cat. Br.* ω tranſ.	41. 44. 50
9. 45. 00	9. 43. 23	Borea in Vertice ♍ ξ tranſ.	41. 37. 15
		Aſcenſio Rect. *Martis* 165. 45. 40	
		Diſt. a Polo 81. 15. 10	
		Longit. *Martis* ♍ 13. 30. 40	
		Latitudo Bor. 2. 26. 31.	
		Die Mercurii Aprilis 8.	
9. 1. 44	9. 0. 29	Leonis χ tranſiit	42. 35. 30
9. 14. 15	9. 13. 00	*Martis* centrum tranſ.	42. 43. 40
9. 35. 7	9. 33. 52	Virginis ω tranſiit	41. 44. 55
9. 41. 58	9. 40. 43	Virginis ξ tranſiit	41. 37. 15
		Aſcenſio Rect. *Martis* 165. 41. 00	
		Diſtantia a Polo 81. 16. 00	
		Longitud *Martis* ♍ 13. 26. 45	
		Latitudo Bor. 2. 23. 58	
		Die Veneris Maii 1.	
7. 55. 9	7. 50. 00	*Martis* centrum tranſ.	44. 17. 30
8. 18. 12	8. 13. 3	In Vertice Virginis ν tranſ.	43. 20. 00
8. 33. 14	8. 28. 5	In Vultu Virginis π tranſ.	43. 15. 30
		Aſc. Rect *Martis* 166. 59. 40	
		Diſt. a Polo 82. 49. 50	
		Longitudo ♍ 15. 15. 00	
		Latitudo Bor. 1. 27. 40	
		Die Saturni Maii 2.	
7. 52. 45	7. 47. 00	*Martis* centrum tranſ.	44. 24. 20
8. 15. 7	8 9. 22	Virginis ν tranſiit	43. 20. 5
8. 30. 8	8. 24. 23	Virginis π tranſiit	43. 15. 25
		Aſc. Rect. *Martis* 167. 10. 00	
		Diſtantia a Polo 82. 56. 40	
		Longitudo ♍ 15. 27. 5	
		Latitudo Bor. 1. 25. 20	

Obſervationes LUNÆ.

Anno MDCCXIII.

		Die Solis Januarii 25.	
8. 9. 33	8. 7. 23	Teleſcopica *a* tranſ.	28. 26. 20
8. 15. 5	8. 12. 55	Tauri 123*ia Catal. Brit.* tranſiit	27. 2. 30
8. 20. 20	8. 18. 10	Lunæ limbus præced. tranſiit, centro a Vertice }	27. 32. 40.

Temp.

Temp. per Horolog.	Tempora correcta.	Die Solis Januarii 25. 1713.	Diſtantia a Vertice.
8. 21. 23	8. 19. 13	Lunæ centrum tranſiit limbo remoto a Vertice	27. 47. 40
8. 22. 52	8. 20. 42	Lunæ cuſpis Bor. a Vertice	27. 17. 40
8. 30. 15	8. 28. 5	Geminorum προπὸς tranſiit	28. 13. 20
8. 41. 7	8. 38. 57	Pes Caſtoris ν tranſiit	28. 54. 50
8. 49. 8	8. 46. 58	Calx ejuſdem μ tranſiit	28. 50. 40
		Aſc. Rect. Cent. ☽ 84. 26. 55	
		Diſt. a Polo viſa 66. 4. 40	
		Sed adhibit. Paral. 65. 39. 50	
		Longit. Lunæ ♊ 24. 56. 30	
		Latitudo Bor. 0. 57. 00	
		Die Lunæ Januarii 26.	
8. 26. 41	8. 24. 36	Propus tranſiit	28. 13. 30
8. 37. 31	8. 35. 26	Pes Caſtoris ν tranſiit	28. 54. 50
8. 45. 31	8. 43. 26	Calx Caſtoris μ tranſiit	28 50. 40
9. 9. 43	9. 7. 38	Lunæ limbus præcedens tranſiit, centro a Vertice	28. 45. 00
9. 10. 50	9. 8. 45	Lunæ centrum tranſ. limbo remoto a Vertice	29. 0. 5
9. 12. 30	9. 16. 25	Lunæ cuſpis Bor. a Vertice	28. 30. 10
9. 27. 50	9. 25. 45	♊orum 46*ta Bat. Brit.* tranſ.	28. 26. 10
9. 42. 44	9. 40. 39	♊orum in Inguine δ tranſ.	28 59. 30
		Aſc. Rect. cent. ☽ 97. 43. 50	
		Diſt. a Polo viſa 67. 17. 5	
		Adhibita Parallaxi 66. 51. 15	
		Longit. Lunæ ♋ 7. 6. 18	
		Latitudo Auſt. 0. 8. 48.	
		Obſervationes SATELLITUM JOVIS.	
		Die Veneris Octob. 30.	
6. 56. 30	6. 52. 35	Quartus Satelles viſus eſt emergens ab umbra, diametro Jovis diſtans a *tertio* ei proximo ad dextram, Tubo ſcil. octo pedum.	
7. 4. 00	7. 00. 00	Clare explenduit, & linea ducta a proximo illo per centrum Jovis emergentem reliquit ad Auſtrum, *ſitu ſcilicet inverſo.*	
7. 36. 31	7. 32. 30	Pegaſi μ tran. per planum Arcus meridionalis	
		Die Saturni Novemb. 7.	
7. 13. 2	7. 5. 00	Secund. Satelles emergebat, vel potius emergere incipiebat Tubo octo pedum.	
9. 5. 11	8. 57. 00	Piſcium δ in Lino auſtrali tranſiit.	

N. B.

A Remark by Dr. Halley, *ibid. p.* 294.

N. B. Stella illa Teleſcopica *a* quæ die Januarii 25° Lunam præceſſit, Aſcenſionem rectam tunc habuit 81° $28'\frac{1}{2}$, & diſtabat a Polo 66° 58′ 20″, unde fit Longitudo ejus ♊ 22° $9'\frac{1}{2}$ cum Latitudine Auſtrali 0° $13'\frac{1}{2}$. Hæc autem eſt ea ipſa ſtella ad quam applicabatur Jupiter in Statione ſecunda, anno 1634. Februarii 6, eamque non niſi tribus ſui corporis diametris ad Auſtrum reliquit, obſervante *Gaſſendo*: ut habetur inter Obſervata ejus p 174. Et ad eandem *Mars* obſervatus eſt Septembris 6to anno 1644. mane, ut videre eſt in Prolegomenis Selenographiæ *Hevelianæ* p 65. & Fig 1

An Occultation of Jupiter, *by the Moon at* Wanſtead, *by Mr.* Pound, *n.* 347. *p* 401.

2. Having after Midnight carefully corrected the Clock by no leſs than ten Obſervations of the Altitude of the *Lucida Arietis*, the Error thereof was found 5′. 13″. too faſt, the extreams not differing above 6″: And in the Morning about 7^h, by as many Altitudes of the *Sun*, with a like Agreement, the ſame Error was found 5′. 14″, to be deducted from the Times ſhewn by the Clock.

Julii 13°. 1715. P.M.N.	Time by the Clock.			Time correct.		
	h.	′	″	h.	′	″
The third Satellite of *Jupiter* was hid by the Moon	13	27	33	13	22	20
The firſt Satellite was hid	13	32	35	13	27	22
The ſecond Satellite was hid	13	34	25	13	29	11
The firſt Contact of the Limbs of ♃ and ☾	13	34	54	13	29	41
Jupiter wholly hid	13	36	23	13	31	10
The third Satellite came out from behind the dark ſide of the Moon	14	7	25	14	2	12
The firſt Satellite	14	12	25	14	7	12
The ſecond Satellite	14	14	38	14	9	25
The firſt Limb of *Jupiter* came out	14	14	45	14	9	32
The following Limb of *Jupiter*, or laſt Contact	14	16	15	14	11	2
The fourth Satellite emerged	14	18	49	14	13	36

Jupiter and the Satellites were to the Northward of the viſible way of the Moon's Center.

This Occultation was obſerved through a Teleſcope, in which the focal length of the Object Glaſs was $4\frac{1}{2}$ Feet, and of the Eye Glaſs $2\frac{1}{2}$ Inches. And the Aperture of the Object was $1\frac{1}{10}$ Inch.

I could perceive no Colours on *Jupiter*'s Limb, either at his Immerſion or Emerſion, when the Axis of the Tube was directed to him.

Obſervations by the ſame, n. 350. p. 506.

3. Anno 1715. *Auguſti* 21°. 8^{h}. 25′ ½ Temp. æq. *Mars* præcedebat ſcil. Aſcenſione recta, Mediam frontis Scorpii (*Bayero* δ) 6′. 54″. Borealior Fixa 9′. 47″.

Sept. 18°. 7^{h}. 30′. *Mars* præcedebat Claram in pede Serpentarii (*Bayero* θ) 17′. 48″. & eandem habuit Declinationem accurate.

Novemb. 30. 18^{h}. 8′. *Saturnus* præcedebat γ, ſive ſecundam Alæ Virginis 23′. 19″. & erat Fixa Auſtralior 25′. 3″. *Decembris* autem 4to 17^{h}. 25′. præcedebat eam 10′ 50″. & Auſtralior erat 29′. 00″.

Anno autem 1716. *Feb.* 22°. 7^{h}. 23′. T. æq. *Mars* præcedebat ζ Piſcium, ſive Sequentem trium clariorum in Lino Auſt. Piſcium 3′. 35″. eademque Auſtralior erat 1′. 23″. quam proinde obtegere debuit ante bihorium, forſan corporaliter.

Junii 22°. 8^{h}. 52′. T. æq *Venus* ſequebatur Cor Leonis 34′. 50″. & fixa Auſtralior erat 7′. 23″.

Aug. 14°. 15^{h}. 00′. *Jupiter* præcedebat *Propoda* uno tantum minuto cum Declinatione Bor. minore 14′. 26″.

Aug. 19°. 13^{h}. 2′. *Jupiter* præcedebat fixam Teleſcopicam, quæ vocetur *b*, 50′, 08″. eandem habens Declinationem accurate.

Aug. 24°. 12^{h}. 19′. *Jupiter* Micrometro diſtabat a prædicta *b*, 5′. 54″. ſimulque ab alia Fixa clariore *a* 7′. 17″. Diſtantia fixarum 12′. 31″. Tunc minor *Jovis* diameter 0′. 38″.

Sept. 12°. 17^{h}. 00. *Venus* recens a Statione ſecunda ſequebatur Teleſcopicam 17′. 40″, eaque Auſtralior erat 5′. 30″. Hæc autem Fixa tunc occupavit ♌ 27°. 44′ ½ cum Lat. Auſt. 5°. 39′.

Octob. 15°. 17^{h}. 12′. ½ *Venus* diſtabat Microm. a Fixa τ in Crure Leonis 27′. 55″.

Novemb. 20°. 6^{h}. 18′ ½ *Jupiter* regreſſus eſt ad ſtellas *a* & *b*, ad quas obſervatus eſt *Aug.* 24°. & diſtabat a *b* 6′. 21″. ab *a* vero 11′. 36″.

Novemb. 21. 7^{h}. 38′. *Jupiter* diſtabat a *b* 9′. 19″, & ab *a* 3′. 48″. Fixæ inter ſe 12′. 30″. *Jovis* diameter minor ſive Axis 0′. 44″. Deinde hora 18^{h}. 50′. viſa eſt ſtella *a* limbo *Jovis* quaſi adhærere, eratque quaſi ⅔ ſemidiametri vel 0′. 15″. centro *Jovis* Borealior. Juxta has autem obſervationes conſtat medium Occultationis Fixæ, interpoſito *Jovis* corpore, contigiſſe *Nov.* 21°. 19^{h}. 55′. vel proxime. Deinde

Nov.	30.	5^{h}.	41′.	♃. præced. *Propoda*	12′.	36″	*Auſtralior*	7′. 36″
Dec.	4.	6.	0.	♃. Sequeb. eam	22.	49	———	7. 47
Dec.	5.	6.	0	Repet. ———	31.	35	———	7. 50
Dec.	6.	6.	0	Repet. ———	40.	30	———	7. 52
Dec.	7.	6.	0	Iterum ———	49.	15	———	7. 54

Ex his ultimis Obſervationibus liquet *Jovem* & *Propoda* eandem habuiſſe Longitudinem *Dec.* 1°. 15^{h}. 29′. quo tempore *Jupiter* Auſtralior erat ſtella 7′. 40″. Ex iiſdem etiam conſtabit *Jovem* in oppoſito Solis fuiſſe, quoad Longitudinem, *Dec.* 6°. 12^{h}. 46′.

N. B. Stellas illas Teleſcopicas *a* & *b* vocatas, haberi in Catalogo Fixarum Britannico D. *Flamſteedii*, ubi ipſi *a* Locus datur, ad annum ſcil. 1690 ineuntem, ♊ 27°. 54′. 29″ cum Lat. Auſt. 0 21′. 55″; alteri vero

b ♊

b ♊ 28°. 5′. 24″ cum Lat. Auſt. 28′. 5″. Neque aliam novimus Fixam a corpore *Jovis* occultatam & ab invento Teleſcopio obſervatam, præter jam dictam Stellam *a*; ad quam olim arctiſſime applicabatur *Jupiter*, ante annos 83. *Decembris* nono St. nov. Anni 1633. Veſp. cum *Gaſſendus Diniæ* vidit *Jovem* huic Fixæ conjunctum, nec niſi quinque ſemidiametris corporis ſui ſuperiorem. Unde calculo debite inito, conſtabit Nodos hujus Planetæ Planumque Orbis ejus ſitum in Sphæra Fixarum ſervare immobilem, vel ſaltem lentiſſimo motu * cieri.

* *Gaſſend.* Obſer. Tom. IV. p. 162.

4. Anno 1717, *Aprilis* 15°. 9^{h}. 49′. T. æq. obſervavit D. *Pound* apud *Wanſted*, *Jovem* jam reverſum ad ſtellam illam, quam *Novemb.* 22°. 1716. mane corpore ſuo texerat. *Jovis* autem centrum tum temporis diſtabat ab ea Stella (quæ tertia eſt Geminorum in *Catalogo Britannico*) 23′. 22″. boream verſus; ſimulque ab alia vicina, quæ quarta eſt Geminorum in dicto Catalogo, 27′ 11″. atque huic fere conjunctus erat planeta.

Aſtronomical Obſervations, com. to the R. S. n. 357. p. 847.

Aprilis 25.° ſequente, eodem obſervatore ac loco, 10^{h}. 3′. T. æq. *Jupiter* apud quatuor Fixas exiguas viſus eſt, eas omnes præcedens, & in ipſo quaſi principio Cancri. Centrum autem planetæ diſtabat ab *e* 13′ 00″. ab *h* 11′. 32″. ab *f* 19′. 53″. & a *g* 9′. 27″.

Poſtridie vero *Apr.* 26°. 9^{h}. 7′. *Jovis* centrum diſtabat ab *e* 8′. 35″, ab *f* 9′. 00″, a *g* 4′ 5″, & ab *h* 13′. 50″. Jamque præterierat omnes præter *f* ad quam tendebat, quamque parum admodum die craſtino infra ſe relinquere debuit.

Eodem fere momento, hora ſcil. nona, *Londini* viſa eſt ſtella *g* in vertice Trianguli Iſoſcelis ac fere Iſopleuri cum *Jovis* centro ac tertio Satellite, tum ſex *Jovis* diametris ad occaſum diſtante, niſi quod parum admodum baſe longiora erant crura; ac intra quadrantem horæ, angulus ad *Jovis* centrum, qui prius major erat angulo ad Satellitem, factus eſt eo ſenſibiliter minor.

Tres autem Stellæ *h*, *g*, *e*, ſunt 10ma, 11ma, & 12na, Geminorum in *Catal. Britan.* juxta quem tum temporis ſitum habuere, *h* in ♋ 0° 22′. 55″. cum Latit. Borea 0° 11′. 25″. Et *g* in ♋ 0°. 28′. 25″. Lat. Bor. 0°. 3′. 40″. *e* vero in ♋ 0°. 29′. 20″. cum Lat. Auſt. 0°. 8′. 05″. Diſtat autem quarta *f* a Stella *g* 11′. 40″, ab *e* 12′. 50″. ac denique ab *h* 20′. 36″, unde conſtabit locus ejus. Ex his manifeſtum eſt *Jovem* Latitudinem habuiſſe parvam admodum Borealem, nec ſemiminuto majorem, ſaltem ſi dictis ſtellarum locis habenda fides. Hæc poſteris uſui eſſe poſſunt definiendo Nodorum *Jovis* motu, ſi quem habeant.

Ejuſdem anni *Junii* 18vo 10^{h}. *Londini*, in ædibus Societatis Regiæ, viſus eſt *Saturnus* Stellæ fixæ Teleſcopicæ admodum propinquus, a qua vix diſtabat ad Auſtrum una Annuli diametro, & normalis in lineam Anſarum a Stella demiſſa incidebat in medium Anſæ orientalis. Fixa hæc parvula nullique Catalogo adſcripta tunc habuit ♎ 12° 58′½ cum Lat. Bor. 2°. 33′. proxime; comitemque habet ſibi adjunctam ac luce æqualem, quatuor minutis ad ortum diſtantem, ac paulo auſtraliorem, unde facile dignoſci poterit, locuſque ejus ſi cui libeat verificari.

Eadem nocte 10^h. 30′ *Mars* viſus eſt prope Stellam quæ præcedit 35. *Scorpii*, a qua diſtabat Tubo 24 pedum menſurata 7′. 16″; idque in recta per claram in pede Ophiuchi θ & dictam Stellam producta. Hæc autem Stella præcedit 35. *Scorpii* 30′. 27″. Aſc. Rect. eaque Auſtralior eſt 2′. 28″, unde fit locus ejus tum temporis *Sagitt.* 15°. 24′. 20″. Lat. Auſt. 3°. 59′. 25″. Sed θ Ophiuchi tunc habuit *Sagitt.* 17°. 28′, & Lat. Auſt. 1°. 47′. 38″. *Mars* itaque Stellam præcedebat Longitudine 4′. 58″, auſtralior ea 5′. 30″

Deinde *Sept.* 13°. 8^h. 5′. T. æq. *Mars* viſus eſt a Dom. *Pound* præcedere claram in humero *Sagittarii* σ 11′. 54″. Aſc. Rect. ſimulque borealior erat Stella 22′ 56″. Hora 8^h. 25′. erat diſtantia Planetæ a Stella 25′. 00″. accurate.

Decemb. 5. 18^h. 30′. T. æq. conſenſu ſæpius repetitarum obſervationum, invenit D. *Pound Saturnum* præcedere Teleſcopicam claram ſibi vicinam 27′. 19″. Aſc. Rect. Stellaque auſtraliorem eſſe 1′. 59″. Simul *Saturnus* præcedebat κ in Syrmate *Virginis* 1°. 25′. 21″, eaque auſtralior erat 4′. 05″. Hinc *Saturni* locus *Libra* 29°. 16′. 21″. Lat. Bor. 2°. 22′. 21″. Teleſcopica autem tunc habuit *Libr.* 29°. 40′. 56″. Lat. Bor. 2°. 33′. 43″.

Anno 1718. *Jan.* 7. 5^h. 30′. T. æq. *Venus* apud duas Stellas in *Catal. Britan.* omiſſas obſervata eſt. Erat autem Planeta utraque Fixa Borealior, diſtans a præcedente 32′. 30″, a ſequente 17′. 30″. Stella præcedens tunc habuit *Piſc.* 14°. 42′. 20″, cum Lat. Auſt. 0°. 40′. 10″; altera vero ſequens *Piſc.* 15°. 21′. 55″. Lat. Auſtral. 0°. 27′. 15″. prout ex obſervationibus D. *Flamſtedii* colligere licet.

Jan. 15. 8^h. 00′, T. æq. *Jupiter* præcedebat η in pectore *Cancri* 3°. 30′. 50″. Aſc. Rect. fixaque Auſtralior erat 14′. 15″. Hinc provenit *Jovis* locus *Canc.* 28°. 20′. cum Latitudine Borea 0°. 36′. 45″.

Martii 11. 10^h. 36′ T æq. *Saturnus* præcedebat κ in Syrmate *Virginis* 18′. 51″, eaque Fixa auſtralior erat 5′. 23″. Hinc fit Locus *Saturni Scorp.* 0. 18′. 34″. cum Lat. Bor. 2. 44′. 8″. Poſito ſcilicet, juxta *Catal. Britan.* κ *Virginis* occupare ♏ 0. 34′. 10″, cum Lat. 2. 55′. 40″. Eadem nocte 17^h. 00′. *Weſtmonaſterii* obſervarunt DD. *Deſaguliers* & *Gray Saturnum* præcedere Stellam 19′. 00″ cum declinatione majore in Auſtrum 4′. 45″.

April 8. 11^h. 30′. *Londini* viſus eſt *Saturnus* nuper Acronychus parum admodum occidentalior Teleſcopica clara, eademque 5 minutis borealior. Unde Fixæ locus *Libræ* 28. 18′. 30″. Lat. Bor. 2. 41′. Circulus autem magnus per hanc Stellam & Saturnum ductus dirigi videbatur ad Stellam $5^{\text{tæ}}$ magnitudinis in *Catal. Brit.* omiſſam, ſed quæ *Hevelio* eſt *in cuſpide Alæ Boreæ Virginis*, cuique locum aſſignat *Libr.* 26. 10′, cum Lat. 14. 43′. Bor.

Eadem nocte 13^h. 20′, apud *Wanſted*, perpendiculum a dicta Stella Teleſcopica in lineam Anſarum *Saturni* demiſſum præcedebat centrum planetæ quaſi ſeſquialtera diametro annuli; aberat autem Stella ad Auſtrum ab Anſarum axe 4′. 30″. Simul Anſæ orientalis extremitas deprehenſa eſt in linea recta inter hanc Stellam & aliam eidem quaſi longitudine conjunctam, quæ tunc a *Saturno* diſtabat 24′. 48″. verſus Boream. Locus autem prioris Stellæ tunc fuit *Libr.* 28. 18′. 30″. cum Lat. Bor. 2. 41′. proxime.

Sept. 7. circa meridiem incidit conjunctio *Jovis* & *Veneris* arctiſſima, cujus quidem ſpectaculum Aſtronomis noſtris invideruut Nubes. Die autem ſexto præcedente mane, vel 5ᵈ 22^{h}. 57′. 30″. T. æq. apud *Wanſted*, *Venus* occidentalior diſtabat a *Jove* 1. 3′. 28″. Die autem 7 17^{h}. 21′, *Venus* jam facta orientalior a *Jove* aberat 43′. 8″; ac 17^{h}. 34′, *Venus* auſtralior erat *Jove* differentia declinationum 14′ 23″. Et 17^{h}. 39′. capta eſt diſtantia Planetarum 44′. 4″. Hinc calculo accuratiſſimi Obſervatoris conjuncti ſunt *Sept.* 7. 0^{h}. 9′. T. æq. *Veneris* centro tum *Jovis* auſtraliore non niſi 1′. 42″.

Denique *Sept.* 18. mane, apud *Wanſted*, *Jupiter* viſus eſt prope *Cor Leonis*, quocum die præcedente conjunctus fuerat. *Sept.* 17. 16^{h}. 51′. T. æq. *Jovis* centrum aberat a *Corde Leonis* 24′. 22″; & 17^{h}. 6′. 20″. erat diff. Declin 12′. 43″. Dein poſt Horam, nempe 17^{h}. 54′, facta eſt diſtantia 24′. 44″; ac 18^{h}. 7′. differentia Declinationum inventa eſt 12′. 35″. Hinc ſupputante Dom. *Pound*, fit *Sept.* 17. 18^{h}. 00′. T. æq. *Jovis* locus ♌ 26. 11′. 7″. cum Lat. Bor. 45′. 39″.

1718. *Octob.* 10°. mane, applicabatur *Jupiter* ad Fixas Teleſcopicas, quarum loca, occaſione primæ apparitionis Cometæ anni 1680. (de qua vide *Phil. Tranſ.* * N°. 342) ſedulo inquiſivit Rev. D. *Pound*, ac nuper verificata nobiſcum communicavit, una cum accurata obſervatione tranſitus Jovis juxta eas hac vice, ac deinde altera *Febr.* 11°. ſtatim ab oppoſitione Solis & Jovis. Ineunte autem *Januario* 1719. loca ſtellarum ſic ſe habuere.

*n.*363.*p.*1109.

* *v. ſupr. p.* 340.

	Long.	Lat. Bor.		Long.	Lat. Bor.
d ♌	29°. 59′. 43″	1. 7. 50	*a* ♍	0°. 25′. 41″	1°. 28′. 54″
e ♍	0. 6. 13	1. 10. 18	*x* ♍	0. 5. 43	0. 51 56
c ♍	0. 3. 13	0. 32. 50			

Ubi notandum ſtellas *d* & *e* eandem præciſe hoc ſeculo ſortiri declinationem, *x* vero exiguam eſſe ſtellulam in priore deſcriptione ob parvitatem omiſſam.

Jam *Octob.* 9°. 17^{h}. 50′. T. æq. Jovis limbus orientalis attigit lineam ſtellas *e* & *c* jungentem, ſimul centrum ejus diſtabat ab *e* 21′. 20″. & a *c* 16′. 25″. ſtatimque aberat a *d* 19′. 35″. Parvula *x* Jovi proxima latuit, luce ejus obumbrata.

Decemb. 11°. 18^{h}. 30. T. æq. *Saturni* centrum diſtabat a μ *Libræ Bayero*, 28′. 32″, & fixa Borealius erat 4′. 31″. Hinc concluſit D. *Pound* Obſervator *Saturni* locum ♏ 10° 41′. 10″, cum Lat. Boreal. 2°. 16′ 43.

1719 *Feb.* 11°. 6^{h}. $56'\frac{1}{2}$ T. æq. *Jovis* retrogradi centrum diſtabat a ſtella *d* ſuperius deſcripta —— —— 10′. 42″

6. $58\frac{1}{4}$ Idem centrum diſtabat ab *e* —— —— 6 7

9. $37\frac{1}{3}$ Iterum diſtantia capta a *d* —— —— 10. 9

9 $43\frac{1}{2}$ Iterum ab *e* —— —— 6 11

9. $49\frac{1}{2}$ Jovis centrum diſtabat ab *a* —— —— 25. 21

9. $58\frac{1}{2}$ Idem centrum a parvula *x* —— —— 24 38

Circa Horam ſeptimam *Jovis* limbus orientalis attigit lineam per *x* & *e* productam; *Jupiter* itaque tunc habuit ♏ 0° 6 cum Latitudine Boreal. 1° 16′ 30″ Deinde,

Feb. 13° 8ʰ 0′ T. æq. Declinatio centri Jovis, Micrometro menſurata, Borealior erat ea ſtellæ utriuſque *d* & *e* 11′ 37″, & 8ʰ 20′ eadem differentia inventa eſt 11′ 36″. Hora vero 8ʰ 48′ centrum Jovis diſtabat ab *e* 17′ 40′.

Apr. 22° 10ʰ 45′ T. æq. *Saturni* centrum ſequebatur *μ Libræ* 4″½ Temp. ſive 1′ 8″ Aſc. Rectæ. Micrometro autem Borealior inventus eſt fixa 35′ 25″ Stella autem in *Catalogo Britannico* tunc habuit ♏ 10° 16′ 8″ Lat. Bor. 2° 3′ 54″

Maii 16° 8ʰ 00′ T. æq. ♃ ſequebatur *Cor Leonis* 1° 34′½ Aſcenſionis rectæ; Borealior autem erat ſtella illa 0′. 41″½ Temporis, hoc eſt, 10′ 7″ Arcus cœleſtis.

Eadem nocte, 15ʰ 18′ T app. Obſervavit D. *Stephanus Grey Martem*, ratione Aſcenſionis rectæ, ſequi ſtellam *in Cauda Capricorni* orientalem 16′ 15″; ſimul non niſi 0′ 11″ auſtralior erat quam Fixa.

Junii 7° 10ʰ 15′. T. app. *Jupiter* directus iterum reverſus eſt ad ſtellas Teleſcopicas prædictas, & tum ſequebatur ſtellam *d* 0′ 35″ Aſcenſionis rectæ, & 10ʰ 30′ diſtabat fixa a limbo *Jovis* proximo 4′ 18″.

Poſtridie *Junii* 8°. 10ʰ 20′, *Jupiter* ſequebatur ſtellam alteram *e* 1′ 30″ Aſcenſionis rectæ, ac ſtatim diſtantia limbi *Jovis* proximi a ſtella capta eſt Micrometro 7′ 30″.

Julii 5. 8ʰ 26′. T. app. Conjungebantur arcte *Jupiter* & *Venus*, quæ tum Borealior præcedebat *Jovem* ſecundum Aſcenſionem rectam 1′. 20″: Centrorum autem diſtantia ex decies repetitis media, capta eſt 13′ 36″. Hæc tria **Londini** obſervata communicavit harum Scientiarum eximius Cultor D. *Martinus Folkes*, R. S. Soc.

Aug. 3. 12ʰ 20′ T. æq. *Mars* pene Acronychus ſequebatur ſtellam τ *Aquarii Bayero* 10′ 58″ Temporis, ſive 2° 44′ 57″ Aſcenſionis Rectæ. Erat autem fixa *Mars* Borealior 0′ 36″ tantum; unde conceſſo loco ſtellæ *Britannico* fit locus *Martis* obſervatus ♓ 7° 10′ 10″ cum latitudine Auſtrali 6° 38′ 10″.

Aug. 10° 11ʰ 50 T. æq. *Mars* ſequebatur fixam minorem quæ præcedit τ *Aquarii* 1° 39′ 30″ ratione Aſcenſionis rectæ; Auſtralior vero quam fixa 10′ 42″.

Aug. 16° 7ʰ 18′ T. æq. *Spica Virginis* præcedebat *Veneris* centrum 5″¼ ſecundis temporis, ſive 1′ 20″ Aſcenſionis rectæ, auſtralior Planeta 18″½ temp. ſive 4′, 35″.

Aug. 17° *Mars* pridie Acronychus ac Terris proximus obſervatus eſt ad duas ſtellulas contiguas, Parallaxis ejus inveſtigandi gratia, juxta methodum a D. *Caſſino*, in libro de Cometa anni 1680. exhibitam. Harum ſtellularum borea tum temporis locum habuit ♓ 3° 5′. 50″ cum Latitudine auſtrali 6° 6′¼: altera vero Auſtralior habuit ♓ 3° 5′ 30″, cum Lat. Auſt. 6° 10′¼ proxime. Hora vero 10ʰ 40′ T. æq. Auſtralem ſequebatur *Mars* 41 min. 40″ Aſcenſionis rectæ, eaque adhuc Auſtralior erat 7′ 50″.

Sept. 18 9^h 20′ T. æq. *Mars* viſus eſt præcedere ſtellam in *Catalogo Britannico* Aquarii 53^{iam}. 3′ 45″ temporis, ſive 56′ 24″ Aſcen. Rectæ; ſimulque Stella Borealior erat limbo Martis boreo, non niſi una Planetæ diametro. Locus ſtellæ ♒ 29′ 57″$\frac{1}{2}$ Lat. Auſt. 4°. 48$\frac{1}{2}$.

Octob. 30. Veſperi 5^h 45″ T. app. *Mars* proximus ſtellis duabus contiguis ad *b* ♒ *Bayero*, quæ ſunt ♒ 73^{ia} & 74^{ta} *Catal. Brit.* Præterierat rectam per eaſdem ductam, eratque angulus ad *Martis* centrum ad ſenſum rectus: Borea vero ſtellarum eandem habuit declinationem cum limbo Planetæ auſtrino. 5^h 53′ diſtantia ſtellæ a centro *Martis* 2′ 30″ 5^h 56′ centrum *Martis* diſtabat a tertia & Auſtraliore ad *b*, ſive 75^{ta} aquarii, 17′.04″. 6^h 18′ diſtantia centri a Borea ſive 73^{ta} erat 3′ 5″. Hinc concludere licet *Martem*, hora 3^h 30′ proxime, ſtellæ Boreæ conjunctum fuiſſe, eamque uno tantum minuto ad Boream reliquiſſe. Fixæ autem locus e *Catalogo Britannico* tunc erat ♓ 10° 29′ 00″ cum Lat. Auſt. 1° 40′$\frac{1}{4}$. 74^{ta} vero habuit ♓ 10°. 29′ 50″ cum Lat. Auſt. 1° 44′$\frac{1}{4}$.

Novemb. 16° 19^h 18′ T. æq. *Venus* præcedebat *Lancem Libræ* Auſtrinam 3′ 13″ Temp. ſive 48′ 23″ Aſcen. Rect. ſimulque fixa borealius erat centrum *Veneris* 7′ 45″. *Venus* quaſi Stationaria apud Nodum ejus aſcendentem.

Dec. 3. 19^h T. æq. Saturnus præcedebat tertiam ad ζ Libræ, ſive Libræ 29. *Cat. Brit.* 0′ 46″ Temp. ſive 11′ 32″ Aſc. Rect. Erat autem fixa Auſtralior 15′ 29″ differentia per Micrometrum capta. Unde Saturni locus ♍ 20° 25′ $\frac{1}{4}$ cum Lat. Bor. 2° 5′$\frac{1}{4}$.

Lunar Obſervations and Eclipſes. n. 357. *p.* 852.

5. Anno 1717. *Jan.* 12. *Weſtmonaſterii* obſervavit Dom. *Stephanus Gray* Lunæ appulſum ad quatuor Stellas contiguas ſub cornu Auſtrino *Tauri*, apud quas obſervata eſt Luna Anno 1683. *Mart.* 23. ſt. v. ab *Hevelio* & *Flamſtedio.* Itaque 9^h 45′ T. app. Luna gibba viſa eſt quaſi conjuncta cum Stella e quatuor præcedente, quæ eſt *Tauri* 107. *Catal. Brit.* quæque tunc Auſtralior erat Lunæ limbo Auſt. ſeſquialtero minuto. 11^h 29′ altera, quæ minor eſt, & ideo in Catalogo omiſſa, occultabatur paulo infra medium obſcuri limbi. Ad 12^h 24′ Tertia & clarior (110. *Tauri*) in ipſa fere conjunctione ſex minutis diſtabat a limbo boreo. Denique 12^h 54′ ſequens e quatuor (111 *Tauri*) limbo Boreo ſuperior erat 3′ 30″. Locus autem præcedentis, ſive 107 *Tauri*, ex dicto Catalogo tunc erat *Gemini* 18. 12. Lat. Auſt. 5^h. 18′; *Tauri* autem 110 habuit *Gem.* 19. 26′$\frac{1}{4}$ cum Lat. Auſt. 4° 44′: Sequens vero ſive 111 *Tauri*, erat in *Gem.* 19 45′ Lat. Auſt. 4. 48′$\frac{1}{6}$. Secunda parvula, ut ex aliis obſervationibus conſtat, Locum tunc habuit *Gem.* 19 17′ Lat. 5. 5′ fere.

Eodem anno *Mart* 16. mane, erat Eclipſis Lunæ partialis, apud nos ob cœlum nubilum inconſpicua. At apud *Cambridg Nov-Anglorum*, Dom. *Robie* Aſtronomiæ peritiſſimus vidit Eclipſeos initium circa horam nonam. Finem vero, juxta *Paludem Mæotida*, ad 11^h 42′ 30″ ſat accurate. Eſt autem *Cambridg* ſub altitudine Poli 42^h 25′, *Londino* 71 grad. ſive 4^h 44′ occidentalior, ut ex pluribus antea obſervatis conſtat. Apud *Limam Peruviæ*, obſervante D. *Petro Peralta*, Mathematico Regio, Initium

n. 363. *p.* 1113.

hujus Eclipſis fuit 8^{h} 41 8″. Finis autem 11^{h} 19′ 55″. Et ad inſulam quam *Virgine Gorda* vocant, obſervante D. *Candler* Navarcha Regio, deſiit Eclipſis 12^{h} 13′. *P. M.* fine per cœlum ſudum diſtincte viſo. Denique, *Pariſiis* obſervabant etiam D. *Caſſini*, & D. *De la Hire*. Hic Initium æſtimavit 13^{h} 54′. Finem vero certius 16^{h} 38′ 10″. At ille initium 13^{h} 55′. Finem 16^{h} 38′ 25′. Maxima obſcuratio huic 7$\frac{1}{3}$ Dig. illi 7$\frac{1}{2}$ Dig.

Hinc ex Fine, in ſingulis locis ut videtur accuratius ſumpto, proveniunt Longitudinum differentiæ inter *Pariſios & Limam* 5^{h} 18′ 20″, Inter *Pariſios* & *Cambridg* 4^{h} 55′ 50″ Inter *Pariſios* & Inſulam *Virgine Gorda* 4^{h} 25′ 20″. E quibus ſi 9′ 40″ ſubduxeris, provenient Longitudines ad occaſum *Londini*, nempe *Limæ* 77° 10′. *Cambridg Nov-Anglorum* 71°$\frac{1}{2}$, ac denique inſulæ *Virgine Gorda* 63° 55′. unde Inſularum adjacentium ſitus Geographici certo corrigi poterint.

n. 357. p. 852. *Sept.* 9. veſperi, in ædibus Societatis Regiæ *Londini*, obſervarunt nonnulli e Sociis finem Eclipſeos Lunaris 7^{h} 26′. Luna autem orta eſt juxta medium Eclipſeos, nec niſi paulo ante finem e nubibus horizontem obſidentibus
n. 363. p. 1113. ſeſe extricaverat. *Pariſiis* vero D. *Caſſino* Finis 7^{h} 34′ 50″, D. *Maraldo* 7^{h} 35′ 30″, & Dno *De la Hire* 7^{h} 34′ 15″. Simul D. *Wurtzelbaur Noribergæ* eundem Finem vidit 8^{h} 10′ 45′. Hinc confirmantur Meridianorum differentiæ *Londinum* inter & *Pariſios*, præſertim ex obſervatione D. *Maraldi*, nempe 9′ 30″; uti & inter *Londinum* & *Noribergam* 44′ 45″, quantam ſæpius olim experti ſumus.

n. 357. p. 853. *Sept.* 14. Veſperi, hac prima vice poſt longum intervallum rediit Luna ad occultandum *Palilicium*. Favit autem admodum cœlum *Londini* præter ſolitum purum, ita ut Luna & Stellæ exorientes in ipſo quaſi Horizonte ſimul conſpicerentur. Incidit Immerſio Stellæ 9^{h} 6′ 20″, Luna nondum 3° alta, in ipſo quaſi medio Limbi orientalis, e regione ſcilicet Boreæ partis maculæ illius exiguæ quam *Hevelius* Stagnum *Mœridis* vocat, quamque *Ricciolus* ſui ipſius nomine inſignivit. Emerſit autem paulo infra medium limbi obſcuri ad 9^{h} 58′ 20″, in ictu oculi tota ſua claritate effulgens; unde etiam in tam illuſtri Stella quaſi nullitas diametri demonſtratur.
n. 363. p. 1113. *Pariſiis*, obſervantibus ſigillatim DD. *Maraldo* & *Deliſle* Juniore, Evanuit ſtella e regione Maculæ *Grimaldi* ſive *Paludis Mareotidis*, Hora 9^{h} 11′ 35″. Emerſit autem e limbo Lunæ obſcuro 10^{h} 3′ 55″.

n. 357. p. 853. *Septembris* 23. veſperi, incidit Eclipſis Solis vix ullibi in *Europa* conſpicua. Ex *America* autem noſtra duplicem obtinuimus ejus obſervationem; alteram ex literis illuſtris Viri D. *Keith* Provinciæ *Penſylvaniæ* Præfecti digniſſimi, qui *Philadelphiæ*, ſub altitudine Poli 40° 00′ fere, vidit Eclipſin jam cœptam (ſed quæ ante minutum temporis nondum inceperat) ad 11^{h} 55′. Circa medium Digiti erant quaſi decem. Finis autem viſus eſt accurate ad 2^{h} 46′ 35″.

Altera autem hujus obſervatio habita eſt ad *Cambridg Novæ Angliæ* Academiam, a Dom. *Robie*, de quo ſupra: Initium Eclipſeos ibi obſervatum eſt 0^{h} 23′ 00″ poſt meridiem. Ad 1^{h} 47′ defecere IX Digit. Ad 3^{h} 5′ 10″ deſiit Eclipſis, Sole integro per Tubum 24 pedum conſpecto. Hæc ex literis accurati Obſervatoris communicavit D. *Guil. Derham*.

Dec. 5. Luna paulo ſupra *Palilicium* invecta eſt: Tranſitum autem ſatis arctum obſervavit D. *Jac. Bradley,* A. M. eruditus Juvenis, ingenio ſimul & induſtria pollens. Hic, cum Luna jam propemodum plena eſſet, Stellam contulit cum inſigni illa Macula quam *Ricciolus Tychonem, Hevelius Sinam* appellat, & ex pluribus æqualibus diſtantiis Micrometro ante & poſt captis, Stellam dictæ maculæ centro proximam apparuiſſe concluſit ad $11^h\ 15'\ 8''$ T. æq. apud *Wanſted.* Ad $11^h\ 15'\ 42''$ diſtabat Palilicium a limbo Lunæ proximo & Auſtrino $5'\ 55''$. Macula autem *Tycho* ab eodem limbo aberat $4'\ 16''$. Ad $11^h\ 18'\ 42''$ Stella erat in linea recta cum maculis *Tychonis* & *Copernici,* ſive *Sinæ* & *Ætnæ*; & $11^h\ 25'\ 27''$ T. æq. erat in recta cum *Tychone* & *Keplero.* Inter hæc obſervata eſt Lunæ diameter $32'\ 45''$.

Anno 1718. *Jan.* 29. veſperi, DD. *Deſaguliers* & *Gray, Weſtmonaſterii* alteram *Palilicii* Occultationem præſtolabant; ſed nubium interventu impediti, viderunt ſaltem quod $5^h\ 52'$ nondum immerſerat Stella; attenuatis autem poſtea nubibus concluſa eſt Emerſio ad $7^h\ 20'$, e regione Promontorii *Sarmatiæ Aſiaticæ Hevelii.*

Feb. 19. mane. Iidem obſervatores ibidem varie cum nubibus colluctati Eclipſin Solis ægre conſpexerunt: Hora tamen 6. 59′ viſi ſunt deficere duo Digiti, & poſt unum temporis minutum chorda inter Cuſpides viſa eſt æqualis ſemidiametro Solis.

Apud *Wanſted* autem D. *Pound* notavit ad $6^h\ 54'\ 7''$ T. app. chordam inter Cuſpides $18'\ 30''$. Ad $7^h\ 17'\ 00''$ erat $10'\ 18''$. Ad $7^h\ 19'\ 30''$ eadem inventa eſt $8'\ 05''$ Deſiit autem Eclipſis ad $7^h\ 23'\ 20''$.

Feb. 25. veſperi $6^h\ 44'$ T. app. *Weſtmonaſterii,* Stella prima *Hyadum* in Naribus *Tauri* (γ *Bayero*) viſa eſt in recta per cuſpides Lunæ, adeoque propemodum conjuncta; diſtabat autem a limbo Lunæ Auſtr. $5'\ 51''$. Diameter Lunæ $31'\ 45''$ menſurata Micrometro.

Feb. 28. $8^h\ 36'$ T. app. etiam *Weſtmonaſterii,* viſa eſt immerſio Stellæ in poplite Pollucis (γ Geminorum *Bayero*) ſub limbi Lunæ obſcuri ea parte, quæ paulo Borealior erat, ea macula quam Hevelius Cretam vocat. Emerſio ipſa ob cœlum minus purum non conſpecta eſt: ſed ad $9^h\ 51'$ egreſſa erat Stella e limbo lucido, a quo diſtabat $3'$ circiter, e regione Boreæ partis Inſulæ Majoris Caſpii.

Aug. 8. Luna orta eſt paulo infra Palilicium, cum quo tamen ob nubes conferri non potuit. Apud *Wanſted* autem $13^h\ 2'\ 00''$. T. app. viſa eſt præcedens contiguarum, ad σ Tauri *Bayero,* in linea recta per cuſpides Lunæ, diſtans ab Auſtrino $4'\ 36''$ Ad $13^h\ 7'\ 25''$ Stella *p* ejuſdem Catalogi emerſit paulo intra medium obſcuri limbi. Ad $13^h\ 9'\ 4''$ emerſit ſequens contiguarum dictarum, tantum diſtans a Cornu Auſtrino quantum contiguæ illæ inter ſe, *h. e.* 7 Min.

By Mr. James Bradley, *ibi.* p. 858.

Sept. 5. Mane, Sole jam fere 30 gr. alto, vidi apud *Wanſted* arctiſſimum Lunæ infra Palilicium tranſitum, cujus diſtantiam, a limbo proximo, ad $7^h\ 59'\ 00''$ T. æq. Micrometro, inveni $5'\ 38''$. Ad $8^h\ 17'\ 5''$ diſtabat a limbo $1'\ 25''$. Stella autem ad $8^h\ 33'\ 15''$ erat in linea recta, per Lunæ cuſpides tum obtuſiuſculas, nec niſi $0'\ 13''$ diſtabat a

Borea. 8^h 41′ 00″ jam Cuſpidem illam reliquerat 3′ 42″. Et 8^h 45′ 37″ ab eadem diſtabat 5′ 36″. Lunæ diameter ad 8^h 58′ capta eſt 31′ 7″.

XLVI. *Papers Omitted.*

n.291.p.1594. 1. Obſervatio Eclipſis Lunæ *Dec.* 12. 1703. a *J. Hodgſon, Londini.* The ſubſtance of this Tract is repeated in *Engliſh*, p. 285.

n.354. p 692. 2. An Advertiſement to Aſtronomers of the Advantages that may accrue from the Obſervations of the Moon's frequent Appulſe to the *Hyades* during the Years 1718, 1719 and 1720.

XLVII. *Accounts of Books.*

n.283.p.1312. Aſtronomiæ Phyſicæ & Geometricæ Elementa. Auctore Davide Gregorio, *M. D.* Aſtr. Prof. Savil. & R. S. S. *Oxon.* 1702.

CHAP. IV.

Mechanicks, Acouſticks.

1. *Account of Dr.* Hook's *Marine Barometer, by Dr.* Halley, *n. 269.p.791.*

THE Mercurial Barometer requiring a perpendicular poſture, and the Quick-ſilver vibrating therein with great Violence upon any Agitation, is therefore uncapable of being uſed at Sea, (though it hath lately been contrived to be made portable) ſo it remained to find out ſome other Principle, wherein the Poſition of the Inſtrument was not ſo indiſpenſably neceſſary. It is about forty Years ſince, that the *Thermometers* of *Robt. de Fluctibus*, depending on the Dilatation and Contraction of included Air by Heat and Cold, have been diſuſed, upon diſcovery that the Airs preſſure is unequal; that inequality mixing it ſelf with the Effects of the warmth of the Air in that Inſtrument. And inſtead thereof was ſubſtituted the ſeal'd Thermometer, including Spirit of Wine (firſt brought into *England*, out of *Italy*, by Sir *Robert Southwell*) as a proper Standard of the Temper of the Air in relation to Heat and Cold; that ætherial Spirit being of all the known Liquors the moſt ſuſceptible of Dilatation and Contraction, eſpecially with a moderate degree of either Heat or Cold. Now this being allowed as a Standard, and the other Thermometer that includes Air being graduated with the ſame Diviſions, ſo as at the time when the Air was included, to agree with the Spirit-Thermometer in all the degrees of Heat and Cold, noting at the ſame time the preciſe height of the Mercury in the common Barometers: It will readily be underſtood that whenſoever theſe two Thermometers ſhall agree, the preſſure of the Air is the ſame it was, when the Air was in-

included and the Inſtrument graduated: That if in the Air-Thermometer the Liquor ſtand higher than the Diviſion marked thereon, correſponding with that on the Spirit-Glaſs, it is an Indication that there is a greater preſſure of the Air at that time, than when the Inſtrument was graduated. And the contrary is to be concluded when the Air-glaſs ſtands lower than the Spirit, *viz.* that then the Air is ſo much lighter, and the Quick-ſilver in the ordinary Barometers lower than at the ſaid time of Graduation.

And the ſpaces anſwering to an Inch of Mercury will be more or leſs, according to the quantity of Air ſo included, and the ſmallneſs of the Glaſs Cane, in which the Liquor riſes and falls, and may be augmented almoſt in any proportion, under that of the ſpecifick gravity of the Liquor of the Thermometer to Mercury. So as to have a Foot or more for an Inch of Mercury, which is another great Convenience.

It has been obſerved by ſome, that in long keeping this Inſtrument, the Air included either finds a means to eſcape, or depoſits ſome Vapours mixt with it, or elſe for ſome other Cauſe becomes leſs Elaſtick, whereby in proceſs of Time it gives the height of the Mercury ſomewhat greater than it ought; but this, if it ſhould happen in ſome of them, hinders not the uſefulneſs thereof, for that it may at any time very eaſily be corrected by Experiment, and the riſing and falling thereof are the things chiefly remarkable in it, the juſt height being barely a Curioſity.

In theſe parts of the World, long Experience has told us, that the riſing of the Mercury forebodes fair Weather after foul, and an Eaſterly or Northerly Wind; and that the falling thereof, on the contrary, ſignifies Southerly or Weſterly Winds, with Rain, or ſtormy Winds, or both; which latter it is of much more conſequence to provide againſt at Sea than at Land; and in a Storm, the Mercury beginning to riſe is a ſure Sign that it begins to abate, as has been experienced in high Latitudes both to the Northwards and Southwards of the Equator.

The Form of this Inſtrument is ſhown in the Cut: wherein, *Plate 4. Fig. 9.*

A B repreſents the Spirit-Thermometer, graduated from 0, or the freezing Point, through all the poſſible degrees of the heat or cold of the Air, at leaſt in theſe Climates.

C D is the Air-Thermometer, graduated after the ſame manner, with the like degrees.

E F is a Plate applyed to the ſide of the Thermometer *C D*, graduated into Spaces anſwering to Inches and Parts of an Inch of Mercury, in the common Barometers.

G. A Hand ſtanding on the Plate at the height of the Mercury thereon, as it was when the Inſtrument was graduated, as ſuppoſed here at $29\frac{1}{2}$ Inches.

L M a Wire on which the Plate *E F* ſlips up and down, parallel to the Cane of the Thermometer *C D*.

K, Any point at which the Spirit ſtands at the time of Obſervation; ſuppoſe at thirty eight on the Spirit-Thermometer; ſlide the Plate *EF* till the Hand *G* ſtand at 38 on the Air-Thermometer, and in caſe the Liquor therein ſtand at 38 likewiſe, then is the preſſure of the Air the ſame as at the time of Graduation; *viz.* 29, 5; but if it ſtand higher, as at 30 at *I*, then is the preſſure of the Air greater; and the diviſion on the ſliding Plate againſt the Liquor ſhews the preſent height of the Mercury to be 29 Inches 7 Tenths.

I had one of theſe Barometers with me in my late Southern Voyage, and it never failed to prognoſticate and give early notice of all the bad Weather we had, ſo that I depended thereon, and made Proviſion accordingly; and from my own Experience I conclude that a more uſeful Contrivance hath not for this long time been offered for the benefit of Navigation.

A new Baroſcope, by Mr. Caſwell, n. 290 *p.* 1597. *Fig.* 10. *Plate* 4.

II. Suppoſe *ABCD* is a Bucket of Water, in it the Baroſcope *xrezyosm* which conſiſts of a Body *xrsm*, and a Tube *ezyo*, the Body and Tube are both concave Cylinders communicating with each other, and made of Tin (for want of Glaſs;) the bottom of the Tube *zy* has a Lead-weight to ſink it, ſo that the Top of the Body may juſt ſwim even with the Surface of the Water by the Addition of ſome Grain-weights on the Top. The Water when the Inſtrument is forc'd with its Mouth downwards gets up into the Tube to the height *yu*. There is added on the top a ſmall concave Cylinder, which I call the Pipe, to diſtinguiſh it from the bottom ſmall Cylinder, which I call the Tube: This Pipe is to ſuſtain the Inſtrument from ſinking to the Bottom; *md* is a Wire; *ms*, *de* are two Threads oblique to the Surface of the Water, which Threads perform the Office of Diagonals: For that while the Inſtrument ſinks more or leſs by the Alteration of the Gravity of the Air, there where the Surface of the Water cuts the Thread, is form'd a ſmall Bubble, which Bubble aſcends up the Thread while the Mercury of the common Baroſcope aſcends.

The Circumference of the Body is 21 Inches, therefore its Area$=35$: the Altitude $ms=4$, therefore the Bodies Solidity$=140$, each Baſe *xm*, *rs*, has a Convexity whoſe Altitude is 6.5, therefore the Conoid on each Baſe is nearly$=11\frac{1}{2}$, therefore *d* the whole Body is $=(140+11\frac{1}{2}+11\frac{1}{2}=)$ 163, and *b* the entire Altitude of the Body $=(4+.65+65=)5.3$. The Inner Circumference of the Tube is 5.014, therefore its Area $n=2$ the length of the Tube$=4.5$, therefore the Tube's Capacity$=9$, therefore *c*, the Content of the Body and Tube$=163+9=172$ Cubic Inches, that is almoſt $2\frac{1}{2}$ Quarts.

Suppoſe the Air's Preſſure when greateſt$=30.5$ Inches of ☿$=(30.5\times 14=)$ 427 of Water, and $f=427$, therefore $fc=73444$. Put *a* for the Depth *ou*, of the Air in the Tube when the Body is juſt all immers'd, the Air in the Inſtrument on Immerſion contracts ſomewhat by the cold of the Water; this Contraction I find is nearly as much as would be produc'd duc'd by an Addition of 1 Inch to the Atmoſphere's Altitude 427, this in cold Weather, but in warm Weather 'tis probably twice as much: but we will now ſuppoſe it$=1$, therefore the Depth of the Surface of the Water in the Tube below the Surface of the outer Water is $=b+a$, therefore the Preſſure on that inner Surface is as the Altitude of the Atmoſphere above it$=f,+b+1+a=F+a$ (putting $F=f+b+1$.) Then for that the Spaces into which the Air is contracted, are reciprocal to their reſpective Preſſures, and for that while the Inſtrument is out of the Water the Preſſure f anſwer'd to the Space c, therefore $F+a:f::c:\frac{fc}{F+a}$=Space which the Air takes up in the Inſtrument under Water; therefore, $\frac{fc}{F+a}-d$ =that part of the Tube which is poſſeſs'd by Air$=an$ (ſuppoſing the Tube's Area $2=n$). Therefore $fc-Fd-ad=Fan+aan$. Therefore $aa+F+\frac{d}{n}\times a=\frac{fc-Fd}{n}$. Put $F+\frac{d}{n}=2g$, therefore $aa+2ga=\frac{fc-Fd}{n}$ therefore $a=\sqrt{:\frac{fc-Fd}{n}+gg}.-g$

Then ſuppoſe the Atmoſphere's Gravity leſs ſo much as to ſink the ☿ $\frac{1}{10}$ Inch $=1.4$ of Water, and therefore putting $\varphi=F-1.4$, and in the laſt Equation α inſtead of a, and γ inſtead of g, you have $\alpha=\sqrt{\frac{fc-\varphi d}{n}+\gamma\gamma}:-\gamma$. Thus I find $\left.\begin{array}{l}a=2.72\\ \alpha=2.94\end{array}\right\}$ and therefore $\alpha-a=.22$, which $.22\times n$ gives .44 Cubic Inches, and (ſuppoſing a Cube-Inch$=253$ Grains) $.44\times253=111$ Grains weight of Water that was gotten up into the Tube in the 1ſt Caſe more than in the 2d, and therefore the Baroſcope requires an Addition of 111 Grains on its Top to ſink it with the Level of the Water in the 2d Caſe more than in the 1ſt, and this upon the ſinking of the ☿ in the common Baroſcope only $\frac{1}{10}$ Inch: Now 1 Grain in this new Baroſcope is nearly as diſcernable as $\frac{1}{10}$ Inch in the common, and therefore this new Baroſcope is more exact than the common 111 Times.

Put $f=427$. $c=172$. $d=163$. $n=2$ as above, only change F, put F 437.3, that is, ſuppoſe the Body ſunk in Water 4 Inches lower; in this Caſe $\alpha=2.08$, therefore $a-\alpha=.64$ which multiplyed into $\varphi n=1{,}28$ Cubic-Inches, which $\times$ 253 gives 324 Grains, and ſo much the Body's top xm being ſunk 4 Inches under Water, the Body becomes heavier, than while xm was at the Surface of the Water. Therefore this 1.28 divided by the aforeſaid Depth 4 gives .32 the Area of the top Pipe ſuch as would ballance or buoy up the Body at any Depth. Strictly ſpeaking, the Pipe ſhould be gradually bigger upward in order to ſuſtain the Inſtrument at any Depth, but as to Senſe 'tis Cylindrical, and its Circumference$=2.005$. But for that the leaſt Alteration of the Air would make the Body's top xm in that Caſe paſs through the 4 Inches (which 4 Inches I ſuppoſe all the Variety of Depth that the Inſtrument has room given it in the Bucket to aſcend or deſcend) therefore the Pipe is made a ſmall matter bigger, (*viz.*) its Circumference is 2.14; whereby the Pipe, according as the Body ſinks more, gives more reſiſtance to the deſcending Body. The Pipes Area is .3643: Therefore the Capacity of the Pipe in 4 Inches Altitude is $=1.457$. But as abovesaid to give juſtly no Reſiſtance, its Capacity ſhould be 1.28. Therefore this 1.28 taken from 1.457, leaves .177 the actual Reſiſtance in 4 Inches depth, *viz.* $(.177 \times 253 =)$ 44 Grains.

But this Reſiſtance will not be the ſame in all Weathers, in order therefore to calculate what it will be when the ☿ of the common Baroſcope is very low, for example, but 28 Inches high$=392$ of Water; f muſt be ſuppos'd$=392$, therefore $F=f+b+1=398.3$, and the reſt as before; *viz.* $d=163$, $fc=67424$. $Fd=649229$. Thence by the aforeſaid Equation $\left.\begin{array}{l} a=2.59 \\ \alpha=2.84 \end{array}\right\}$ Therefore $\alpha - a = .25$, which $\times\, n$ gives .50 Cubic Inches, which $\times 253 = 126$ Grains. So that this Baroſcope when the ☿ is loweſt, is more exact than the common 126 Times, ſuppoſing the Body immers'd afreſh when the ☿ is ſo low.

Next while the ☿ is ſo very low, ſuppoſe the top of the Body depreſs'd four Inches under Water; therefore $\varphi = F + 4 = 402.3$, the reſt are as before, *viz.* $fc = 67424$, then α will be 1.9: but before, while the top of the Body was at the Surface, α was 2.59. Therefore the Difference $.69 \times$ Tube's Area 2, gives 1.38 Cubic-Inches, which $\times 253$ gives 349 Grains, and ſo much the Baroſcope is heavier when the top xm is 4 Inches under Water, or which comes to the ſame, ſuppoſing that ☿ at 28, and xm at the Surface; this Baroſcope by the ☿'s aſcending $\frac{4}{14}$ Inch will become heavier 349 Grains. The Pipe's Capacity in 4 Inches Altitude was 1.457, from which take the aboveſaid 1.38, the reſidue $=.077$, which $\times 253$ gives 19 Grains in 4 Inches; ſo that the Pipe will ſuſtain the Baroſcope, and alſo 44 when the ☿ is $30\frac{1}{2}$ high, and but 19 Grains when the ☿ is 28 high. The fewer Grains difference there are in its ſinking, through 4 Inches, the more nice the Baroſcope will be.

There where the Thread cuts the Surface of the Water, is form'd a Bubble; therefore this Bubble while the Inſtrument ſinks in Water 4 Inches, which is all the room that I give it, moves on the 2 Diagonal Threads twenty Inches; it follows therefore that 120 Grains difference would make the Bubble walk over 120 Inches, if the Threads were ſo long; but, as it has been above calculated, about 120 Grains difference of Weight of the Inſtrument is produc'd by ſo much of the Alteration of the Air, as would make the ☿ of the common Baroſcope $\frac{1}{10}$ Inch: therefore when the ☿ aſcends $\frac{1}{10}$ Inch, the Bubble of this new Baroſcope aſcends 120 Inches; therefore this new Baroſcope is more exact than the common Baroſcope by about 1200 times.

Obſervations with this Baroſcope.

1. While the ☿ of the common Baroſcope is often known to be ſtationary 24 Hours together, the Bubble of the new Baroſcope is rarely found to ſtand ſtill one Minute.

2. Suppoſe the Air's Gravity encreaſing, and accordingly the Bubble aſcending, during the Time that it aſcends 20 Inches, it will have many

ſhort deſcents, of the quantity of ½ Inch, 1, 2, 3, or more Inches, each of which being over it will aſcend again. Theſe retroceſſions are frequent, and of all Varieties in quantity and duration, ſo that there is no judging of the general courſe of the Bubble by bare Inſpection though you ſee it moving, but by waiting a little time.

3. A ſmall Blaſt of Wind will make the Bubble deſcend; a Blaſt that can't be heard in a Chamber of the Town, will ſenſibly force the Bubble downward. The Blaſts of Wind ſenſible abroad cauſe many of the aboveſaid Retroceſſions, or Accelerations in the general Courſe; as I found by carrying my Baroſcope to a place where the Wind was perceptible.

4. Clouds make the Bubble deſcend. A ſmall Cloud approaching to the Zenith works more than a great Cloud near the Horizon. In cloudy Weather the Bubble deſcending, a break of the Clouds (or clear place) approaching to the Zenith, has made the Bubble to aſcend; and after that break had paſs'd beyond the Zenith a conſiderable Space, the Bubble again deſcended.

5. All Clouds (except one) hitherto by me obſerv'd, have made the Bubble to deſcend. But the other Day the Wind being *North*, and the courſe of the Bubble deſcending, I ſaw to the Windward a large thick Cloud near the Horizon, and the Bubble ſtill deſcended, but as this Cloud drew near the Zenith it turn'd the way of the Bubble making it to aſcend, and the Bubble continued aſcending till the Cloud was all paſs'd, after which it reſumed its former deſcent. It was a Cloud that yielded a cold Shower of ſmall Hail.

An Improvement of the Heſſian *Bellows, by Mr.* Papin, *n.* 300. *p.* 1990. *Plate* 4.

III. The ſhape of the *Tympanum* was Cylindric, as may be ſeen Fig. 11. where *D A F C* is the Circumference: *C P*, *D P*, *A P*, are the *Radii* which bear the Wings *Cm*, *D n*, *A o*: *B e* is the aperture through which the Wind muſt be driven in the Direction of the Tangent *C B*: And it may be obſerv'd, that when the Engine is working, every Wing from the end of the aperture *e*, till it comes to the beginning of the ſame aperture *C*, drive always the ſame Air, with the ſame Swiftneſs, and at the ſame diſtance from the Center: So that in peruſing all that Circumference, the Air doth find reſiſtance by Friction, and gets nothing at all. I do therefore now make the Circumference of the *Tympanum* in a ſpiral ſhape, which is to be ſeen Fig. 12. where the ſpiral Circumference is *A F G B*, the *Radii* are *A P*, *C P*, *D P*, *&c.* The Wings are *A M*, *C N*, *D O*, *&c.* The aperture is *A B*. And it is to be obſerved, that every Wing in going round drives new Air, becauſe the Air which is firſt in Motion, finds place to recede from the Center towards the ſpiral Citcumference; and ſo it gives room to new Air to come to the Wing: and when the Wings come near to the aperture, they drive their new Air into the aperture without any Friction; and the Air which hath been firſt driven and removed from the Wing, cannot loſe its Swiftneſs, becauſe the Wings which continually follow, do continually drive new Air, which keeps that which is before always in the ſame Swiftneſs. This new ſhape of the *Heſſian* Bellows affords alſo another advantage; becauſe the Air in going round follows the ſpiral Line, which is nearer to the ſtraight Line than a circular Circumference; and when the Air comes to the Aperture, it gets into it without any loſs of ſubſtance; but in the Cylindrical Machine, Fig. 12. the air doth always go round in a circular Circumference, and when it comes to the aperture, the Wind is driven directly in the direction of the Tangent but juſt in the beginning at C; and afterwards the Impulſion is oblique: and this obliquity is always increaſing until the Wing comes to the *Punctum A*: Now it is known how much Diminution ſuch an obliquity can make to the ſtrength. I believe therefore that this ſpiral Figure is a good Improvement to this Engine. And indeed I have made ſuch Bellows where the *Radius A P* is but $10\frac{1}{2}$ Inches, the Wing *A m* 2 Inches broad and 9 Inches high; becauſe the *Tympanum* is alſo ſo high, or a little more; the aperture *A B* is alſo 9 Inches, or a little more, ſo that it makes a ſquare hole. When I work this Engine with my Foot, it makes ſuch a Wind, that it may raiſe up 2 Pounds weight; and without doubt a ſtronger Man could do much more: But this is more than ſufficient for our purpoſe, ſince we muſt but drive air enough for the Reſpiration of ſuch Men that can work in the Mine; and we may eaſily with Boards make wooden Pipes, to carry the Wind to the very bottom: So that the Air within will be continually renewed as well as without.

Of the Motion of Sound, by Mr. Derham, *n.* 313. *p.* 2.

IV. Diſſenſus inter authores celeberrimos de Soni Velocitate facili intuitu in ſequente Tabella conſpici poteſt: in qua (Pedibus Anglicanis) Spatium exhibetur quod Sonorum Progreſſui in uno Minuto Secundo Temporis aſcribunt.

	Pedes,	
D. Iſ. Newton, Eq. Aur.	968	Prin. Ph. Nat. Math. L. 2. Prop. 50.
Nobilis D. Roberts	1300	Philoſ. Tranſact. N. 209.
Nobilis D. Boyle	1200	Eſſay of Languid Motion p. 24.
D. Walker	1338	Philoſ. Tranſact. N. 247.
Merſennus	1474	Baliſtic. Prop. 39.
D. Flamſteed & Halley	1142	
Florentini celebres	1148	Exp. per Acad. del Cimen. p. 141.
Galli celebres	1172	Du Hamel Hiſt. Acad. Reg.

Inter ultimum & penultimum diſſenſus non eſt magnus, & *Gallorum* non multo major; cæterorum vero magnus eſt. Et Ratio manifeſte hæc eſt; vel ſcilicet ab Inſtrumenti defectu; vel a Diſtantia; vel a Ventis.

1 *Inſtrumentum*, quo nonnulli virorum horum inclytorum dimenſi ſunt, non fuit *Automaton*; ſed *Bolis funipendula*, quæ Minuta Secunda vibrat. Sed omnibus, in hiſce rebus exercitis, manifeſtum eſt, Bolidem multo minus commodam eſſe, nec tam accuratam ac Automaton; quoniam neceſſarium eſt, Oculum primo occupatum eſſe in obſervando Coruſcationem, deinde ad Bolidem, ſive Pendulum reſpicere: quod tempus conterit, & confuſionem creat. Hoc autem, una cum Senſuum, & Captus ſive atten-

tionis noſtræ tarditate, magnum errorem efficere poteſt; uti bene notum eſt iis qui Experimenta de his fecerunt. Præſertim ſi

2. *Intervallum* inter rem ſonantem, & Obſervatorem parvum fuerit. At vero manifeſtum eſt, quod plerique iſtorum laudatorum Virorum Experimenta ſua fecerunt ad intervallum tantum paucorum Pedum, & per Soni reditum, ſive Echo dimenſi ſunt. Horum enim nonnulli vix ultra 6 vel 700 pedes menſurationem extendebant, aliique non ultra Milliare unum. Sed ſemper obſervavi ambiguitatem oriri in tam parva diſtantia, quamvis optimum adhiberetur Inſtrumentum. Errorque leviſſimus in tantillis diſtantiis, magnus eſt habendus. Nam Pendulum forſan dimidium ſui diadromi, ſive arcus, præteriit ab ultima pulſatione, cum Sonus primo fuerit emiſſus: Sed nos iſtum Pulſum numeramus, ac ſi Vibratio fuiſſet tota & completa; vel forſan Vibrationem anticipamus. Et poſtquam Sonus nos pertigit, forſan plus vel minus quam par eſt numeramus.

Vel ſi Diſtantia ſat fuerit longa, tamen error exinde poteſt oriri, ſi

3. *Ventorum* ratio non ſit habita. De quo in ſequentibus.

Hæc ſunt certa, inevitabilia, & perpetua incommoda, quæ Menſurationem progreſſus Sonorum comitantur: quæ in parvis intervallis (ut dixi) præſertim ſi Inſtrumenta mala ſint, magnos errores producere poſſunt: & ſine dubio maxima fuere cauſa tanti inter tantos authores diſſenſus.

Sed obſervari poteſt, quod Spatia a tribus ultimis in Tabella Obſervatoribus aſſignata, quam proxime conveniunt. Quod proculdubio hinc provenit, quia nempe bonis Automatis inſtructi fuerunt. In quorum uſu, *Auris* ſola occupatur in Vibrationibus Penduli excipiendis, dum *Oculus* attendit Coruſcationem, ſive aliquam aliam Soni emiſſionem. Hæ quoque Obſervationes intervallis longinquis factæ fuere, in quibus error puſillus non magni erit. Dominorum enim celeberrimorum *Flamſteedii* & *Halleii* Obſervationes factæ ſunt ad intervallum trium fere milliarium (paucis Perticis plus vel minus exceptis) ab Obſervatorio Regio, ſuper Collem *Shooterianum*: & Sonus advenit in $13\frac{1}{2}$ Secundis Temporis. Nobiles iſti *Florentini* & celeberrimi ex *Acad. del Cimento* ad idem fere intervallum Experimenta ſua fecerunt; & quædam ad intervallum unius tantum Milliaris. Et denique celeberrimi D. D *Caſſini*, *Picard*, & *Roëmer* ad intervallum 1280 Hexapedarum Gallicarum, quod eſt plus quam $1\frac{1}{2}$ Milliare Anglicanum.

Ut veritas inter prædicta diſſidia innoteſcat, experimenta plurima ad varia intervalla feci; Scil. ab uno, ad 12 milliaria, & plura. Et ad Tempus dimetiendum, habeo accuratiſſimum *Automaton portabile*, cum Pendulo Semiſecunda vibrante.

Tutius autem ut procederem, ſequentes Quæſtiones a meipſo diſcutiendas propoſui.

1. Quantum Spatium Sonus percurrit in Secundo Minuto Temporis, vel alio Temporis intervallo?

2. An Sclopus verſus Obſervatorem diſploſus, in eodem temporis intervallo Sonum mittit, ac cum in contrariam partem diſplodatur?

3. An in quolibet Atmoſphæræ ſtatu, cum Mercurius in Barometro aſcendit vel deſcendit, Soni percurrunt idem Spatium in eodem Temporis intervallo?

4. An Soni velocius Die quam Nocte moventur?

5. An Ventus favens Sonum accelerat, & adverſus retardat? Sive an, & quomodo Venti Sonum afficiunt?

6. An tranquillo Cœlo Sonus velocius movetur, quam Vento flante?

7. An vehemens Ventus in tranſverſum flans accelerat, an retardat motum Soni?

8. An Soni eundem habent motum Æſtate ac Hyeme, Die ac Nocte?

9. An etiam in Nivoſo, ac Sudo Cœlo?

10. An Sonus magnus & exiguus eundem habent motum?

11. An in omnibus Sclopeti elevationibus, viz. Horizontali, 10 gr. 20 gr. ad 90 gr. Sonus in eodem temporis intervallo Obſervatoris aurem pertingit?

12. An omnimodi Soni, Sclopetorum, Campanarum, Malleorum, &c. eundem habent motum?

13. An variæ Pulveris Pyrii vires motum Soni variant?

14. An in Culminibus Montium altorum, & Vallibus; ſive in ſummis Atmoſphæræ partibus, & imis, Soni idem percurrunt Spatium in eodem Temporis intervallo?

15. An Sonus acclivis & declivis eundem habet motum? Sive an a Jugo Montis deſcendit in Radicem eodem paſſu, ac a Radice aſcendit in Jugum?

16. An Sonus principio velocius, & in fine tardius movetur, ut in plurimis aliis motibus violentis accidit?

17. Annon potius ſit æquabilis? Nempe Annon in dimidio Temporis, dimidium Spatii; in quarta parte Temporis, quartam partem Spatii, &c. movetur?

18. An in omnibus Regionibus, Septentrionalibus, & Auſtralibus, in *Anglia*, *Gallia*, *Italia*, *Germania*, &c. eundem habent motum?

19: An Sonus recta, ſive breviſſimo itinere, a loco in locum tranſit; an ſecundum ſuperficiem interjacentis telluris?

Ad hæc determinanda Amicos Generoſos mihi vicinos petii (quorum beneficia hic gratiſſime agnoſco) ut Sclopos ex Turribus, aliiſque locis eminentibus diſploderent, ad intervallum 1, 2, 3, uſque ad 8 millia paſſuum (quod maximum eſſe intervallum reperi, ex quo Sclopi Sonum audire potui in his partibus, arboribus, &c. obſitis.) Hæc Sclopeta magno mihi fuerunt uſui. Sed Tormenta, quæ maxime propoſito inſerviebant, bellica illa fuerunt apud *Blackheath* [*Sakers* vocata] quæ exercentur in educandis Tyronibus Tormentariis Inclytiſſimæ noſtræ Reginæ miniſtraturis. Horum Tormentorum micantes flammulas ex Eccleſiæ meæ turre videre, & fragorem audire potui in omnibus fere cœli tempeſtatibus; etiam interdiu, ope Teleſcopii. Ideoque cum omni cura & diligentia me ad horum Tormentorum obſervationem accinxi, uſque a Februario 170^{4}.

Poſt paucas obſervationes inter eorum diſploſiones factas, ſpeciale quoddam experimentum faciendum comparavi, benignitate nuperi D. Baronis

Granville tunc Præfecti, & cæterorum Virorum clarorum qui in *Turre Londinenſi* Rei Tormentariæ Regiæ miniſtrant (quorum beneficia hic gratiſſime agnoſco.) Duo Tormenta bellica (*Sakers* vocata) juxta ſe ſita ſunt, adverſo unius ore, averſo alterius. Hæc duo Tormenta Feb. 13. 170$\frac{4}{5}$. diploſa fuere, unaquaque Semihora ab Hora ſexta pomeridiana ad mediam noctem, leni aura directe adverſus Sonum ſpirante. Temporis intervallum inter Coruſcationem ſinguli Tormenti (quam nudo oculo videre potui) & Soni adventum, ſemper fuit circiter 120 vel 122 Semiſecunda Temporis. Dixi 120 vel 122, quoniam Sonus duplicatus advenit; ſcil. prior Sonus intra 120 Semiſecunda (qui languidior) ſecundus intra 122 (qui intentior.) Et eodem modo, per totum obſervationis tempus, ſingulorum Tormentorum fragor advenit, nempe duplicatus.

Hæc Reduplicatio Soni mihi videtur Echo, repercuſſa, ut opinor, a *Molendino Blackheathenſi*, vel Domibus juxta ſitis. De quo nullam habeo Dubitandi rationem, præter ſententiam contrariam Amici cujuſdam docti, & ſagacis Philoſophi; qui credit nullam Echo audiri, niſi quæ facta eſt per Objecta Phonocamptica non procul ab Obſervatore, non per ea prope Vocale, ſive Sonorum, vel alia longinqua Objecta.

§. 2. *De Sonis longe repercuſſis, ſive Echo longinqua.*

Pro Digreſſione forſan habebitur hæc Diſquiſitio: ſed quoniam ad Soni ſubjectum attinet, ideo paucas de hac re Obſervationes ingenioſis non fore ingratas ſpero.

Et primo hoc Legibus Echus non contrarium credo. Deinde notandum eſt, hunc duplicem Sonum directe a *Blackheath* veniſſe: neque enim prior Sonus inde veniebat, & alter (Echus more) aliunde; nempe ultra me, vel a dextra, vel ſiniſtra, vel ab ulla alia parte. Idemque ſæpius obſervavi, cum Tormenta magna e Navibus diſploderentur in *Fluvio Thameſi* (præcipue ſi aer fuerit ſerenus & tranquillus veſperi & mane, cum Tormenta Vigilaria (Anglice *Watch-guns*) exonerarentur. Poſtquam Fragor Tormenti aurem pertigit, audivi eum longe percurrentem ſecus Fluvium, & a Ripa, Montibus, & Scopulis (juxta littus *Cantianum* confertim ſitis) per plura Milliaria reboantem.

Hæc omnia, inquit Amicus, a Repercuſſione Domuum, &c. prope Te proveniunt. Sed nequid de Debilitate Soni dicam, poſtquam plurima Milliaria percurrit, & de ejuſdem incapacitate, ſi tam procul veniſſet, ut repelleretur per objecta Phonocamptica juxta Obſervatorem, potius quam per Objecta Phonocamptica juxta Sonorum ſita; (ut nihil de his dicam) exemplum unum vel alterum dabo, unde plane conſtabit, quod Echo facta per Objecta Phonocamptica prope Rem vocalem ſive ſonantem poteſt per plura milliaria audiri, æque ac primarius Sonus, aliquando etiam eodem intentior.

Sæpe obſervavi Tormenta magna bellica e Navibus in *Thameſi Fluvio* veſpere diſploſa circa loca vocata *Deptford* & *Cuckolds-Point*, plerumque fragorem edere duplicatum, triplicatum, quadruplicatum, vel adhuc amplius multiplicatum; & quod Fragores poſteriores ſunt magis Sonori. Et cum hinc & illinc Stadium, etiam quadrantem vel dimidium Milliaris in tranſverſum ivi, Sonus tamen idem fuit. Memini quod octavo Martii noviſſime elapſi plurimæ Bombardæ magnæ diſploſæ fuere alicubi inter *Deptford* & *Cuckolds-Point* prædictas, e Nave quam in *Thameſi* ex mea Eccleſia proſpexi. Harum fragor quinquies vel ſexies hoc modo repetitus eſt. Inter Coruſcationem & Sonum 122 Semiſecunda numeravi Vento in tranſverſum flante. Tunc temporis ideo Tormenta a me diſtabant pluſquam 13 millia paſſuum. Duo primi crepitus languidiores erant quam tertii; ſed crepitus ultimi omnium maxime Sonori. Et cum a dextra quadrantem milliaris tranſiviſſem, idem fuit multiplicatus Sonus: & cum a ſiniſtra, idem. Et præterea in aliquibus meis Stationibus, præter multiplicatum Sonum, plane audivi languidam Echo ab Eccleſia mea, ſive Domibus adjacentibus repercuſſam: quod ſæpenumero tunc obſervavi, quoties Tormenta diſploſa fuere.

Alia hujus generis Obſervatio fuit quodam die Dominico, circa biennium vel triennium abhinc, ex Sono Tormenti magni militaris diſploſi alicubi in *Thameſi* fluvio cis vel trans Oppidum *Graveſend* vocatum. Hujus Tormenti fragor fuit multiplicatus ad minimum octies, novies, vel decies, ſecundum hanc temporis menſuram Hunc multiplicatum Sonum plurimi (ad Dei cultum tunc temporis accedentes) putabant eſſe Fragores multorum Tormentorum e Nave dimicanti: (Sed ut opinor, nil aliud fuit niſi Echo polyphona, ex unius vel alterius & alterius Tormenti diſploſi Sono, a Navibus pluribus vel Littore juxta ſitis repercuſſo) Quod autem pro me facit, hoc eſt, quod non ipſe ſolum (dum in meo Horto deambularem) audivi, ſed etiam multi alii qui procul diſtabant. Item D. *Barret* R. S. S. domi ſuæ eundem repetitum Sonum audivit, ad intervallum 4 fere milliarium ab *Upminſter*, ubi ipſe audivi.

Ex quibus omnibus luculenter conſtat ſententiam prædicti Amici (plurimis nominibus colendi) eſſe falſam.

§. 3. *De Echo, ſive Repercuſſione Sonorum in Aere.*

His de *Echo* dictis, ſpero non ingratum fore exemplum adjicere de *Repercuſſione Soni ab aereis particulis*; quod poteſt confirmare quæ dicta ſunt.

Cum audivi Fragores magnorum Tormentorum bellicorum, præcipue in tranquillo & ſereno Cœlo, ſæpenumero obſervavi *Murmur*, excelſe in aere, præceſſiſſe Crepitum. Et in *Nebula tenui*, ſæpe Bombardarum Sonum audivi in Sublime, ſupra caput, in aere, per plurima Milliaria percurrentem; adeo ut Murmur iſtud per 15″ temporis perduraverit. Diutinum hoc Murmur, meo judicio, provenit a Particulis Vaporoſis in atmoſphæra ſuſpenſis, quæ curſui Undulationum Soni oppugnant, eaſque Obſervatoris auribus reverberant, indefinitarum Ἠχῶν more: quas *Murmur in aere* vocamus.

His rite perpenſis, manifeſtum erit Echo longinque factam poſſe audiri; & Reduplicationem iſtam prædictam fragoris Tormentorum *Blackheathenſium* proculdubio veniſſe ab ipſo *Blackheath*, prout modo aſſerui.

§. 4.

§. 4. *De Sonis Sclopetorum omnifariam diſploſorum*, &c.

Ut autem redeam a Digreſſione de Repercuſſione Sonorum, pergam ad Obſervationes meas de eorum *Progreſſu*, quas ex plurimis Experimentis feci. Et quod jamjam de Sono Bombardarum *Blackheathenſium* ſuggeſſi, in omnibus aliis reperi, viz. Motum Soni nec velociorem nec tardiorem eſſe, ſive Tormentum verſus Obſervatorem, ſive a contrario diſplodatur. In omnibus item *Sclopi poſitionibus*, Horizontali, Erecta; & in omnibus ejuſdem elevationibus, 10 gr. 20 gr. &c. nulla eſt variatio Soni. Adeo vera eſt illa de hac re obſervatio Nobilium & Inclytorum iſtorum Virorum ex *Academia del Cimento Florentiæ*.

Pulveris Pyrii quoque Vis, ſive ſit fortis, ſive debilis, ejuſque major vel minor Quantitas, licet augeat, vel minuat Sonum, non tamen accelerat, aut retardat ejuſdem Motum.

§. 5. *De Motu Soni in qualibet Cœli, & Anni tempeſtate.*

Kircherus dicit *ſe ſemper diverſam Soni celeritatem inveniſſe, diverſis temporibus, mane, meridie, veſperi, noctu.* Sed ego (meliore Chronometro fretus, & commodiore diſtantia) nunquam reperi celeritatem Soni eſſe diverſam his temporibus. In omni autem tempeſtate, ſive Cœlum ſit *Sudum* & *Serenum*, ſive *Nubiloſum* & *Turbidum*; ſive *Nix* decidit, ſive *Nebula* (quæ ambæ fortiter retundunt Soni Audibilitatem) ſeu *Tonat*, aut *Fulgurat*; ſive *Æſtus* vel *Frigus* adurit; ſive *Dies*, vel *Nox* ſit, *Æſtas* vel *Hyems*; ſive *Mercurius* in *Barometro* aſcendit vel deſcendit: verbo dicam, in omnibus quibuſcunque Atmoſphæræ mutationibus (Ventis tantum exceptis) Motus Soni nec velocior nec tardior eſt; tantum magis vel minus clarus eſt ex illa Medii variatione. Quod forſan *Kircherum* ſagacem decepit.

Phonurg. l. 1. c. 1. *Præluſ.* 3. *Sect.* 2.

Hinc ſequitur Concluſiones *D. Walker* a Doctoris *Plot*, *Kircheri*, & ſuis ingenioſis Obſervationibus deductas erroneas fuiſſe.

Philoſ. Tranſ. N. 247. *Abr. Vol.* I. *p.* 506.

§. 6. *De Motu Soni intenſi & languidi, & diverſorum corporum Sonantium.*

Licet *Kircherus* contra ſentiat, attamen non dubito quin omnium corporum Strepitus, Sclopetorum, Campanarum, Malleorum, *&c.* eandem Velocitatem habeant. Anno 1704. Pulſationes Mallei & Fragorem Sclopeti comparavi, ad Milliaris intervallum (maximum ſpatium ad quod Mallei ſonum audire potui) & reperi utrorumq; Sonum in eodem tempore adveniſſe: & quod $\frac{3}{4}$, $\frac{1}{2}$, & $\frac{1}{4}$ ejuſdem Spatii pertranſiverunt in $\frac{3}{4}$, $\frac{1}{2}$ & $\frac{1}{4}$ ejuſdem temporis.

Ibid.

Quod ad Sonos *Intenſos* & *Languidos* attinet, non dubito quin idem Spatium in eodem Temporis intervallo percurrant. Ut ex his Experimentis aliqua ex parte manifeſtum erit.

Jan. 13. 170$\frac{4}{4}$. Archibombardarius Arcis *Tilburienſis* meo hortatu diſploſit unum & alterum Sclopetum, & Tormentum magnum Militare, in quod injectum pulverem pyrium bene fiſtulaverat. Horum omnium Strepitus in eodem tempore ad me, tria circiter milliaria diſtantem, pervenit.

Archibombardarius quoque *Angliæ* Sept. 11. 1705. poſt Solis occaſum, in mei gratiam, diſploſit ſuper *Blackheath* nonnullos Sclopos (Anglice *Muskets,*) Tormenta magna bellicoſa (*Sakers* vocata,) & Pyrobola (Anglice *Mortars*). Sclopos exaudire non potui, propter magnam diſtantiam, vel quia aer non ſat ſerenus fuit. Sed Tormentorum & Pyrobolorum Sonos in eodem Temporis intervallo exaudivi, licet Fragor Pyroboli fuit multo torpidior & remiſſior, quam Tormentorum.

§. 7. *De Æquabilitate Motus Soni.*

Hanc talem eſſe deprehendi, qualem *Academia del Cimento* illuſtris dudum præfinivit. Soni quippe progrediuntur Dimidium Spatium in Dimidio Temporis intervallo ; Quartam Spatii partem in Quarto Temporis intervallo ; & ſic deinceps. Quod ex exemplis in ſequenti Tabella conſtabit.

Locus quo diſploſio facta fuit.	Penduli Vibrat. numerus.	Diſtantia Locorum.		Ventorum Tendentia.
		Trigonometrice.	Per Sonum	
		Milliaria.	Milliaria.	
Hornchurch Eccleſia	9	0, 9875		tranſverſo.
Okendon Bor. Eccleſ.	$18\frac{1}{2}$	2, 004	2, 0	tranſverſo.
Mola Upminſterienſ.	$22\frac{1}{2}$	2, 4	2, 4	favent.
	23		2, 48	nive: tranſv.
Warley parvæ Eccleſ.	$27\frac{1}{2}$	3, 0	2, 97	forte faven.
Rainham Eccleſ.	$33\frac{1}{4}$	3, 58	3, 59	tranſverſo.
Mola Alveleientis	33	3, 58	3, 57	tranſverſo.
Dagenham Eccleſ.	35	3, 85	3, 78	faventi.
Weal Auſtrin. Eccleſ.	45	4, 59	4, 86	tranſverſo,
ThorndonOrient.Ecc.	$46\frac{1}{2}$	5, 09	5, 03	paulo fav.
Barking Eccleſia	$70\frac{1}{2}$	7, 7	7, 62	favente
TormentaBlackheath	116	12, 5	12, 55	tranſverſo.

Intervalla Locorum ab *Upminſter* (ubi obſervavi) in hac Tabella notata, quanta potui accuratione dimenſus ſum vel Virga Menſuratoria, vel Arte Trigonometrica. Et ex magna conſonantia inter intervalla hoc modo, eademque per Soni motum dimenſa, cum Inſtrumentorum meorum præſtantia, tum Obſervationum & Calculorum veritas patet. Differentia enim inter Intervalla dimenſa, & eadem Sono capta, aut prorſus nulla eſt, aut tantum paucularum centeſimarum partium, niſi cum Ventus fuerit ſecundus (Eccleſia *Weal Auſtrinæ* excepta, de quo poſthac.) Ita nempe in Obſervationibus ex Eccleſiis *Dagenhamenſi*, *Warleienſi*, *Thorndonienſi*, & *Barkingenſi* factis, diſtantiæ per Sonum notatæ paulo breviores viſæ ſunt ; quia Ventus Sonum acceleravit. At in conficienda hac Diſtantiarum per Sonum Columna, nihil propter Ventorum accelerationem conceſſi ; ſed numerum Vibrationum, ſive Semi-ſecundorum tantum diviſi per 9¼, vel 9,25 (numerum Semi-ſecundorum in quibus Sonus Mille paſſus tranſiit.)

Æquabilitas quoque motus Soni ex hac Tabella manifeſta eſt; prout patebit ex collatione Vibrationum & Diſtantiarum: ſive ex ſola Columna Diſtantiarum per Sonum.

Ut autem nihil deeſſet in harum rerum confirmationem, iter feci ad *Arenas Foulneſianas* in littore noſtro *Eſſexiano.* Hæ Arenæ (Maris quotidiano Æſtu allutæ, & obtectæ) faciunt magnam & exactam Planitiem multorum milliarium. Super hanc Planitiem tantum ſex milliaria dimenſus ſum, quia neque Maris æſtus, neque mora mea, ut majus intervallum dimetirer, permitterent. Ad cujuſque fere Milliaris finem experimenta feci per Sclopetorum Exploſionem, non ſine magno Vitæ periculo, ex Maris Fluxu, & Noctis tenebris. Ex quibus Experimentis comperi Obſervationes meas priores omnes juſtiſſimas, & veras fuiſſe, ſcilicet Sonum unum Milliare pertranſire in 9¼ Semi-ſecundis: duo Milliaria in 18½ Semi-ſecundis: tria Milliaria, in 27¾ Semi-ſecundis, & ſic deinceps.

§ 8. *De Acclivi & Declivi Sonorum Motu: ſive De eorundem Aſcenſu & Deſcenſu.* Item an *recta, vel ſecundum interjacentis telluris ſuperficiem a loco in locum tranſeunt?*

Quod ad 15 & 19 Quæſita attinet; ingenue fateor me nunquam ullis quæ feci hactenus Experimentis mihimet ipſi ſuper his rebus ſatisfeciſſe.

De progreſſu Soni per breviſſimam viam, ratio dubitandi fuit diſcrepantia inter Spatium *Weal* Villæ & *Upminſter* Trigonometrice, & per Sonum dimenſum; prout in Tabella præcedenti exhibetur. Menſuratio Trigonometrica tot modis, & tam bonis Angulis capta eſt, ut de ea nullus dubitem. Sed quoniam per Soni motum diſtantia major videtur, & Superficies interjacentis Soli hujuſmodi formam induit, qualis in figura exhibetur; ideo ſubdubitavi annon paululum tortuoſe Sonus vagatur? *Fig.* 13. ſive annon Acclivitas illa interjacens in *(A)* Soni Undulationibus oppugnando retundit, eaſque tardat?

Ut nodum hunc quodammodo ſolverem, Experimentum fieri curavi, Sono Sclopi a Cacumine *Collis Langdonienſis* in Vallem ſubjacentem, ad intervallum 3,79 millia paſſuum. Intervallum Trigonometrice, ex Angulis & Baſi ſat magnis, bene dimenſum eſt; & Experimentum factum, cum lenis aura paululum Sono oppoſuerit. Inter Coruſcationem & Crepitum 35½ Semi-ſecunda numeravi: Qui numerus ad intervallum adeo quadrat, & cum cæteris experimentis tam proxime convenit, ut non dubitandum ſit quin Sonus a Cacumine in vallem recta (per aerem) deſcenderit, & non juxta Superficiem curvatam interjacentis Soli.

Errorem igitur aliqualem fuiſſe credo in Obſervationibus *Wealienſibus* prædictis, quoniam nec in Experimento noviſſimo *Langdonienſi*, neque in ullis aliis tale aliquid obſervavi.

§. 9. *De Sonorum tranſlatione ſive Motu in Italia.*

Ingenioſus *Richardus Townleius* Armiger (Literis ad me datis Anno 1704.) ſignificaverat, ‘ Sonos raro exaudiri *Romæ* tam longe ac in *Anglia*,

‘ noſtriſque *Borealibus Regionibus.* Speciatim vero aiebat, Se, cum *Romæ* commoratus eſt, dum Bombardæ quædam *Caſtelli S. Angelo vocati*, ob læta nuncia diſploderentur, atque ipſe ſuper Montem *Trinitade* dictum ſtaret, obſervaſſe Sonum multo languidiorem eo loci fuiſſe, quam in ullo alio ad eandem diſtantiam ſito’. Et poſt ejus mortem Frater ejuſdem mihi Scripto retulit, quod Anno 1688, ‘ Cum relicta *Roma* ad *Caſtellum Gondolfo* (eminentiorem quendam locum prope *Lacum Albanum*, duodecim circiter Italica Milliaria a *Roma*) ut bonas horas contereret, ſe contulerit, animadvertiſſe ſonum Bombardarum magnarum a Caſtello (prædicto) *St Angelo* obſtrepentium, ſibi tamen imminutum & debilem videri. Alio quoque tempore, cum Curru circa prædicti Caſtelli mænia veheretur, Bombardæque ingentes exinde boarent, nec talem tantumve Sonum ibi loci ac alibi emittere videbantur.

Cl. D. Doctor *Newton*, *Florentiæ* Ablegatus *Britannicus*, ſuper hac re rogatus, narrat quod in itinere a *Bononia*, *Florentiam* verſus, Bombardarum exoneratarum ſtrepitum ad *S. Michaelis in Boſco* urbem (in *Bononiæ* vicinia) exaudiverit, quæ tamen Bombardæ *Mirandulæ* diſploſæ ad 40 millia paſſuum diſtabant; quem locum *Gallorum* acies obſidione tum cinxit. Ac nocte inſequenti eundem Sonum cum in *Appenninis* pernoctaret (20 millia paſſuum longius remotus) exaudiverit.

Obſervationes vero & Experimenta, quæ idem Vir inſigniſſimus ab aliis fieri curavit, ſpecialem locum merito ſibi vendicant.

‘ In inferiori *Florentiæ* arce, Bombarda *Colubrina* inter horas primam & tertiam Noctis crebro diſplodebatur; Virique quidam *Ligurni* aſſervabantur, qui diligenter obſervare juſſi ſunt, an ejuſdem crepitum exaudire poſſent. Quorum nonnulli qui ad *Lanternam*, & *Marzocco* poſiti erant, nullum audiebant, (forte quia Maris fremitus Sonum infuſcabat:) alii vero qui ſtabant ſuper *Veteris Arcis* munimenta (quæ *Donjon* appellant) quique ad *Montem Rotonda* dictum (qui quinque circiter milliaria a *Ligurno* abeſt verſus *Montem Nero*) miſſi, auribus exceperunt: Et quotieſcunque exonerabatur, toties ejus fragor iiſdem in locis clare exauditus eſt. Hujus autem *Arcis Florentiæ* diſtantia a *Monte Rotondo* recta linea vix minor 55 milliaribus cenſetur. Et notatu dignum eſt, quod interjacentia rura pleriſque collibus obſita ſunt, qui paulo impeditiorem Soni viam reddant neceſſe eſt. His accedit, quod eodem veſpere ventus quidam Occidentalis leniter ſpirabat, qui (cum *Ligurnus* ſitus ſit ad Libonotum reſpectu *Florentiæ*) liberiorem Soni expanſionem aliquatenus præpedire merito credatur’.

‘ Quo autem locus apertus & undique patens haberetur, ſeligebatur tractus ille Maris qui *Ligurnum* & *Portum*, *Ferraio* dictum, interjacet, cujus diſtantia ſecundum peritiſſimorum Nautarum calculum 60 milliarium eſſe deprehenditur. Tormentorum autem militarium fragor a *Ligurno* ad prædictum *Portum Ferraio*, locaque vicina haud raro pervenit. Nec aſpirantium Ventorum auxilio opus eſt, ad promovendum huncce Soni progreſſum, quo nempe exaudiatur. Imo vero ventus quilibet, ſive ſit ſecundus, ſive adverſus, eidem impedimento eſt, ipſumque Sonum minus ſonorum reddit: forteaſ, quia Maris hinc agitati fremitus magis obeſt, quam Aeris eodem confluentis curſus prodeſt. Proinde tunc ſolum exauditur Sonus, cum Ventus prorſus ſilet, vel tantum leniſſime ſuſurrat, cum Aer ſerenus eſt, & Mare tranquillum. Neque tum quidem ab omnibus locis indiſcriminatim exauditur, ſed ab iis ſolis quæ paulo eminentius ſita ſunt; cujuſmodi ſunt duo iſta Propugnacula, quæ *Stella* & *Falcon* nominantur & Locus *Mulini* dictus. Præterea requiritur ut ipſe Obſervator quam attentiſſimum ſe præbeat, nec ulla obſtrepentium voce aut clamore impediatur, & infeſtetur. Tum vero interdiu æque ac noctu (modo Atmoſphæra ſit ſerena & tranquilla) exaudiat; niſi quod nocturno tempore fortior & acutior aliquanto videatur Sonus, cum nulli occurrunt Strepitus, qui luci ſæpius aures offendere ſolent.’

‘ Porro nobis nunciatum fuit a Teſtibus fide digniſſimis, quod pluribus abhinc annis, cum graſſaretur *Seditio Meſſanenſis*, ipſaque Urbs obſidione premeretur, Tormentorum bellicorum fragor *Auguſtæ* & *Syracuſanorum* Incolarum aures percelleret.’

‘ Item cum *Galli* Tormentis muralibus *Genuam* concuterent, conſtat quod eorundem crepitus ad *Montem* uſque *Nigrum*, qui *Ligurno* ſupereminet, pertigerit.’

‘ Ex hiſce Obſervationibus proclives ſumus ad credendum nullum eſſe ſuper hac re diſcrimen inter *Italiam* & *Plagas Boreales*.’

‘ Quod autem ad alterum Quæſitum attinet; utrum Ventus directe vel adverſe ſpirans, Sonum accelerat vel retardat? Eidem hactenus certo reſponderi haud poteſt. Neque enim ipſa quæ adhibuimus Experimenta, quibus veritatem indagatam fore ſperavimus, Quæſtioni dirimendæ ſufficiunt. Quippe æſtivo tempore (quo plerumque interdiu venti ſpirant a Mari, & Occidente; cum autem adveſperaſcit, ſilere ſolent) defuere nobis commodiſſimæ occaſiones hanc rem ſæpius & certius experiundi. Speramus tamen, inclinante Anno, poſtquam alia ſucceſſerit Tempeſtas, opportuniora nanciſci tempora, quibus hujuſmodi Experimenta felicius & iterato, & cum majori ακριβείᾳ inſtitui & comprobari poſſint. Impræſentiarum autem referre ſufficiat quid nobis evenerit 10 Auguſti poſtremo elapſi, cum quæ ſequuntur experimenta capere licuerit.’

‘ *Coleurina* quædam (60) ſuper Cortinam inferioris Propugnaculi *Florentiæ* adducebatur, ibique ſic poſita ut Os ejuſdem verſus *Artemino* ſpectaret (quod eſt Ruſticanum *Magni Hetruriæ Ducis* Palatium, Colle quodam altiore ſitum, prædictique Propugnaculi latus Occidentale reſpiciens, a quo etiam circiter 12 millia paſſuum diſtat.) Diem quendam ſelegimus cum Ventus occidentalis aliquanto fortius ſpiraret, ut Soni motus contrario vento repelleretur. Hoc autem parum juvabat: quippe ſub veſperam, cœlum omnino tranquillum erat, vel ſaltem adeo tenui aura agitabatur, ut candelæ flammam haud disjiceret.

‘ Hic Loci relictis quibuſdam harum rerum peritis, quibus antea quæ potiſſimum curarent in mandatis dedimus, ad prædictum *Palatinum Artemino* conceſſimus. Prout juſſimus, inter Horas primam & tertiam Noctis, *Coleurina* ſæpius exonerata eſt; & 49 Secunda Mi-

nuta

‘ Minuta inter ejuſdem Coruſcationem & Fragorem jugiter numeravimus. Nos etiam in *Artemino* Bombardas quaſdam accendimus; atque inter harum coruſcationem & fragorem prædicti Spectatores (quos in Arce reliquimus) tantum 48 Minuta Secunda numeraverunt. Unde conſtabat Sonum unius tantum Minuti Secundi intervallo velocius ab *Artemino* ad *Florentiam*, quam retrorſum ferri.’

‘ Haud adeo noſtræ obſervationi confidimus, ut minutulum hoc Velocitatis diſcrimen ad Venti conſpirantis aut renitentis vim referre audeamus. Quippe ipſius Obſervatoris error, qui Penduli vibrationes numerabat, huic fortean occaſionem dederit. Quod ſane facile fieri poſſit. Sæpius enim eveniat neceſſe eſt, ut emicantem flammam non niſi poſt inceptam Penduli Vibrationem videat, Sonituſque fragorem nondum terminata Vibratione exaudiat: adeo ut ipſe Calculum una Vibratione auctiorem, quam par eſt, hoc pacto faciat, dum interea Temporis ſpatium ſit utrinque par & idem.’

‘ Sperabamus autem proximo mane Ventum forte contrarium exoriturum, (Sæpius enim hic loci, primo ſaltem diluculo, Ventus ab Oriente ſpirare ſolet) qui inceptis Experimentis magis inſerviret. *Colubrinam* igitur rurſus, cum illuxerit Dies, exonerari juſſeramus: Ventus autem nec Votis nec Operi favebat; quippe qui paululum tantum ad Borealem plagam ſe convertiſſet. Adeo ut variatio Temporis, & Velocitatis Soni, in tantilla Venti mutatione, vix perciperetur. Solitas proinde 49 Penduli Vibrationes, ut prius, numerabamus. Interea temporis hæc eadem Experimenta expendere ſperamus, quamprimum tempeſtas magis idonea occurret, Ventorumque crebriores mutationes, commodiores occaſiones dabunt eadem melius experiendi, unde tandem pleniſſime nobis ſatisfiat.’

Quod ad Spatium attinet, quod Soni quovis aſſignato tempore percurrunt, de eodem nondum inter ſe conſtat; ſed ab experimentis quibuſdam conjiciebant rem ita ſe habere, prout Experimenta *Academiæ del Cimento* teſtabantur.

Ex his Obſervationibus, aliiſque nobiſcum communicatis abunde patet, Multo longius exaudiri poſſe Sonos in *Italia*, quam prædictus Amicus ingenioſus nos docuit. Ipſe enim eximius *Ablegatus* Bombardarum ingentium ſtrepitum ad 60 Milliarium diſtantiam auribus percepit. Quæ etiam ejuſdem ſuaſu *Florentiæ* exonerabantur, eædem 55 Millia paſſuum audiebantur. Tormenta militaria *Ligurni* diſploſa ad 60 Milliarium intervallum aures feriebant. Quæ *Meſſanæ* exonerabantur (ut ex Tabulis Geographicis patet) eorem aures, qui centum fere Italica Milliaria ſemoti ſunt, percellebant. Quæ denique in concutienda *Genua* diſploſa ſunt, eorundem fragor plus quam 90 Millia paſſuum Italicorum (ut ex Mappis) pervenit.

Quibus omnibus in mentem revocatis, & ſerio perpenſis, vix poſſum quin credam non minus late propagari Sonos in omnibus *Meridionalibus*, quam in hiſce *Borealibus* Terrarum plagis. Quamvis haud deſunt exempla longioris Sonorum progreſſus in quibuſdam Septentrionalibus Terræ partibus.

bus. Generoſus quidam *Danus* (inſigniſſimi noſtri *Danici Principis* Famulus) mihi inter confabulandum narravit ſe, cum in *Dania* vitam ageret, Bombardarum *Carolſcroomiæ* diſploſarum crepitum, 80 Milliaria Anglicana (ni fallit memoria) remotum clare exaudiviſſe. *Doctor Hearn* (Regis *Sueciæ* Medicus) narrationem quandam ad *Regalem* noſtram *Societatem* miſit, de Bombardis *Holmiæ* exploſis, cum exequiæ unius ex regiis Principibus celebrarentur, A. D. 1685; quarum fragor 30 Suevorum Milliarium intercapedinem percurrit, quæ 180 Milliaria Anglicana fere exæquant. Navali etiam illa Pugna quæ geſta eſt *Angliam* inter & *Hollandiam* A. D. 1672. Tormentorum bellicorum Strepitus plus quam ducentis Milliaribus interjacentium attonitas aures percuſſit; quippe qui trans *Inſulam noſtram* ad *Salopiam* uſque & *Walliam* pertingebat.

Quod proinde ambo Fratres *Townleii* obſervarunt, idem prædicto *Caſtello S. Angelo*, vel *Romæ* ſaltem, proprium omnino eſt, & peculiare. Neque enim perſpicax eorundem ingenium, fidamve curam male ſuſpicari licet. Iſta igitur Soni diminutio, quam iidem animadvertebant (niſi male auguror) vel ad prædicti Caſtelli Situm, vel ad interjacentes Domos (paſſim & ubique in iſta confertiſſima Urbe ſurgentes) vel ad Strepitus ejuſdem undique perſonantes, vel ad Ventos adverſos, vel demum aliam conſimilem cauſam referendum eſt: Vel forſan hi Viri prædictas ſuas Obſervationes fecerunt eo Aeris ſtatu, quo Soni, quamvis maxime ſecundos habeant Ventos, multo tamen languidiores ſunt, quam aliis temporibus, cum prorſus adverſi ſunt.

§. 10. *De varia Sonorum Remiſſione & Intenſione* (*ſive Audibilitate*) *pro diverſo Atmoſphæræ ſtatu.*

Sæpius *Æſtate*, cum jam incaluerit aer, obſervavi Sonos ſupra modum languidiores videri, debileſque admodum ad aures ferri; cum alia tempeſtate, præſertim *Hyeme*, ſi forte gelaſcit, multo magis argutos & ſtridulos eoſdem fuiſſe, fortiuſque aures perculiſſe. Spirante etiam *Borea* vel *Euro* (quantumvis adverſe) Sonos clariores, magiſque ſtridulos eſſe ſenſi, quam ſi ex contrariis plagis Venti ſpirarent: ut *Kircherus* quoque *Romæ* obſervavit. Hoc autem non conſtans & perpetuum eſt.

Neque quid magis certum ex *Mercurii in Barometro* aſcendentis vel deſcendentis inſpectione colligerem, quod tamen credulus autumabam. Sonos enim aliquando maxime claros & argutos, aliquando maxime debiles & languidos cum ad ſummum aſcenderet; e contra aliquando maxime ſtridulos, interdum maxime deficientes, cum Mercurius ad imum deſcenderet, comperi.

Pariter etiam incerte ſe res habet quoad Serenum & Nebuloſum aerem. Tempore pluvioſo & humido ſæpe obſervavi Sonos obtundi, & "Poſt "imbres vehementiores plurimum virium acquirere, ut *Kircherus Romæ*. Sed contrarium quoque ſæpe evenit. Maii 31. A. D. 1705. Aer hic loci magis ſerenus, Vaporumque expers fuit, quam unquam antea me vidiſſe memini. Tam purum etenim liquidoque ſerenum erat Cœlum, ut objecta longiſſime remota clare facileque proſpicerem. Sed tamen Bombardas in agro *Blackheath* tunc temporis exploſas exaudire non potui (ſi unam excipias, cujus fragorem jam prorſus languentem auribus perceperim) quamvis omnium eminus micantem flammulam clare cernerem. Eodemque tempore Nubium & Venti motus cum ſono conſpirabat; Aura etiam leniſſima tunc ſpirabat, quæ compoſitos crines vix moveret; & omnia denique ad Soni vim motumque promovendum neceſſaria concurrere videbantur. E contra vero, cum prorſus mutatus fuerit Aeris & Cœli ſtatus, cum omnia turbida viderentur, & Atmoſphæra vaporibus plena, ſæpe ſtridulos Sonos, nec minus crebro eoſdem hebetes & remiſſos exaudivi.

Nebulas ſpiſſas certum eſt Sonos quam maxime hebetare. Soni enim tunc admodum languidi & obtuſi plerumque videntur. Quod ab interpoſitis vaporibus, & ſpiſſis particulis, quæ Nebulam conſtituunt, certiſſime provenit. Idem etiam de *Nivoſo Cœlo* obſervavi. Cum enim Nix recens in terram decidit, protinus hebeſcunt Soni. Cum vero glaciata fuerit ejus ſuperficies, Soni repente acutiores fiunt, Campanaſque & Bombardas tinnientes & reboantes eo uſque exaudivi, ac ſi Nix humum non conſperſerat. *Townleius* prædictus haud ita pridem ſe obſervaſſe aiebat (cui non abſimile egomet expertus ſum) dum per oppidum quoddam equo veheretur, Campanarum (quæ tum haud ita procul pulſabantur) Sonum ad aures vix poſſe pervenire, ſi quando Domus Nive tecta occurreret interjacens. Adeo ut ipſe, oppidulum ingreſſus, plurimum miratus ſit, Campanas tam ſubito ſilere, dum primas interjectas ædes prætergrederetur; deinde repente reſonare, cum proximum vacuum intervallum præteriret. Quod quidem per totum viæ curſum in eodem oppido obſervavit, Campanarum nempe Sonum ad aures pertingere, vel non; prout ædificia nive obſita occurrerent interpoſita, vel non.

§ 11. *De Ventorum vi, ſive influentia in Soni Motum.*

Illuſtriſſima *Academia del Cimento* ab experimentis invenit Sonorum motum nec ab adverſis Ventis retardari, nec a ſecundis accelerari: Sed utcunque ſpirarent Venti, ſemper idem Spatium in eodem tempore percurrere. In iſta ſententia fuit *Gaſſendus*, cæterique fere omnes qui antea vel poſtea Philoſophati ſunt. Quoniam vero contrarium hujus patet ab ipſa Experientia, erroris coarguendi ſunt. In quem ideo incidiſſe videntur, quod ad nimis breve intervallum Experimenta ſua inſtituerentur. Omnino enim veriſimile eſt hoſce Philoſophos ad unius tantum, vel ad ſummum, duorum, triumve Milliarium diſtantiam obſervationes ſuas feciſſe. Quas proinde vitioſas eſſe haud miror. Sin autem ad 10 aut 12 Millia paſſuum, accuratis Inſtrumentis adhibitis, rem tentaſſent (quod ipſe ſæpius feci) errorem facile agnoſcerent.

Quem communem errorem Egomet (horum Virorum authoritate fretus) diu admiſi; donec tandem Bombardarum in agro *Blackheath* obſervatione triennali, & amplius, eundem feliciter detexi. Cum autem primum Sonos aliquando citius, aliquando tardius ad aures pertigiſſe ſenſi, erroris cujuſdam a me facti ſuſpicio animum ſubiit, vel quod Automati vibrationes minus recte numeravi, vel coruſcantem Bombardæ flammulam

mulam male obſervavi; vel in alium conſimilem errorem haud attentus inciderim. Poſtquam vero Bombardæ de induſtria in mei gratiam exonerarentur ſingulis ſemihoris, ab hora ſexta veſpertina uſque ad mediam noctem, Sonumque perpetuo ſine ulla notabili varietate, 120 vel 122 ſemiſecundorum ſpatio pervenire ſenſi, quamvis Ventus directe adverſus fuiſſet; aliis autem temporibus, cum Ventus ſecundus ſpiraret, ſive e directo, ſive ex tranſverſo, aut obliquo, earundem Bombardarum Sonum 111, 112, 113, 114, 115, 116 vel ad ſummum 117 Semiſecundorum ſpatio advenire deprehendi; tum demum me certiſſime perſuaſum habui, reale aliquod diſcrimen fuiſſe, quod iſtam in Obſervationibus varietatem perperiſſet.

Neque ſolum *Secundi* aut *Adverſi Venti Sonorum motum accelerant aut tardant*, ſed etiam *pro graduum varietate, quo vehementius aut lenius ſpirant, eo magis minuſve eundem promovent aut impediunt.* De quibus omnibus in majorem certitudinem, ſpeciales quaſdam obſervationes in ſequenti Tabella ſubnectam; poſtquam prænotavi Bombardas in agro *Blackheath* circiter 60 gradus a Meridie diſtare, hoc eſt ad Plagam a S W b W aliquanto remotiorem vergere.

Tabella Sonorum Bombardarum in Agro *Blackheath*, pro Ventorum, Viriumque quibus agitantur, varietate.

Dies Menſis & Anni.	Hora Diei.	Numerus Vibrationum.	Ventorum Plaga.	Nubium Plaga.	Altitudo ☿.
1704.					
Febr. 13	6 h. ad med. noc.	120 122	NEbE 1	NEbE	29 99
11	11 ½ mane	119	E 2	E	30 22
1705.					
Mar. 30	10 mane	113	SW 7	SW	29 30
Apr. 2	8 ½ p. M.	114½	SbW 1		
3					
3	10 mane	116½	S 4	Inferior S Sup. Wb. N	29 80
5	1 p. M.	111	SWbW 7		
13	8 ½ mane	120	NbE 2	SWbW	29 70
24	5 p. M.	116	SWbW 0		29 26
Sept. 11	6 ½ p. M.	115	W 2	NW	29 59
	7 p. M.	115½	WbN 2	WbN	Saker. Mortar.
29	10 ½ mane	112	SSW 6	SSW	29 38
Octob. 6	10 mane	117	E SE 1 & 2	S E	29 34
Nov. 30	meridie	115	SSW 4	SSW	29 10
Febr. 15	11 mane	116	SbW 1	S W	29 60
1706.					
Nov. 29	11 ½ mane	116	S W 0	SWbW	30 06
	meridie	118	SWbS 1		
Febr. 7	meridie	113	SWbW 4	W	29 83

Ab Experimentis Ap. 5. & Sept. 29. factis, patet Ventos vehementiores urgere & maturare Sonorum motum. Quinto enim Aprilis, cum prope conſpiraret Venti Soniquemotus, fortior etiam aliquanto idem Ventus fuerit (prout figura [7] annexa denotat, pariter ac Cifra [0] tranquillum Cœlum; & Figuræ 1, 2, 3, 4, &c. varias Ventorum vires ſignificant) tunc temporis, inquam, Sonus 111 Semiſecundorum ſpatio iter confecit. Aprilis autem 24, cum Ventus ab eadem Plaga ſpiraret, & Aer tranquillus eſſet, idem itineris ſpatium non niſi 116 Semi-ſecundorum intervallo Sonus peragraſſet. Ita etiam Feb. 7. 1706, cum ab eodem Cardine ſpiraret Ventus, & ſecum deferret Sonum, viribus autem jam dimidio minoribus, 113 Semi-ſecunda elapſa ſunt priuſquam Sonus aſſuetum confecit iter. Ita demum Sept. 29. 1705. Vento vehementiore ſpirante, & minus ſecundo, Sonus intra 112 Semi-ſecunda progreſſum abſolvit. Ex quibus, aliiſque in Tabella exemplis liquido conſtat, *Ventos Fortiores Soni propagationem adjuvare, Leniores autem eandem minus promovere.*

Idem etiam conſtat de iis Ventis, ſive Aeris torrentibus, qui Soni progreſſui e directo favent, vel obſtant; eos nimirum *ejuſdem motum celeriorem vel tardiorem reddere.* Quique *Intermedii* volvuntur Atmoſphæræ fluxus, eos *Intermedium pariter Soni progreſſum*, ſive Penduli Vibrationum numerum efficere.

Maximum diſcrimen, quod in Soni progreſſu per 13 fere milliarium ſpatium hactenus animadverti, novem circiter aut decem Semiſecunda exæquat, cum nempe Venti fortes promovent, & lenes tantum impediunt Sonum. Cum vero tenues ſolum aut prorſus nulli obſtant, vel adjuvant eundem, tum quidem differentia duo vel tria Semi-ſecunda haud ſuperat.

§. 12. *De Ventorum Velocitate.*

Ut quantum ſpatium quovis aſſignato tempore perflent Venti pro certo ſcirem, leviuſcula quædam corpora in parandis experimentis adhibui. Cujuſmodi ſunt Pappus, Plumæ leves, &c. quæ propoſito melius inſervire videbantur, quam Inſtrumentum illud quod in * *Actis Philoſophicis*, No. 24. deſcriptum habemus; vel etiam illud alterum magis commodum Molæ alatæ figuram referens, ab acutiſſimo D. *Doctore Hook*, ni fallor, excogitatum.

* *Abr. Vol.* II. p. 42.

Ex plurimis quæ feci experimentis, leviuſculorum corporum ope, cum variæ Ventorum vires fuerint, deprehendi, Ventum vehementiſſimum vix 60 Millia paſſuum horæ ſpatio percurrere. Exempli gratia; Aug. 11. 1705. tantam Procellam excitavit Venti vehementia, ut ipſam Molam Pneumaticam, juxta locum quo obſervationes meas feci, pene ſubverteret. [Ventorum Vires (uti modo diximus) hiſce Characteribus plerumque notavi; 0, 1, 2, 3, 4, 5, 6, uſque ad 10, 15, aut plures gradus.] Prædicti autem Venti vires 12 circiter aut 14 graduum horum reſpondere æſtimavi: & a quamplurimis iteratis experimentis animadverti, Turbinem iſtum circiter 33 pedes, ſpatio Semi-ſecundi Minuti percurrere, ſive 45 Milliaria in Hora. Unde colligo Ventum concitatiſſimum & maxime nimboſum (illo vehementiſſimo, qui Menſe Novembris 1703 furebat, haud excepto) non plus quam 50 aut 60 millia paſſuum horæ ſpatio prætervolare.

Poſtquam rapidorum Ventorum Velocitatem dimenſi ſumus, quæ ſit minus rapidorum celeritas conjicere haud difficile eſt. Horum enim curſus pariter notavi, variiſque ab Experimentis edoctus ſum, horum nonnullos 15, quoſdam 13, alios multo plura, alioſque multo pauciora milliaria horæ ſpatio conficere: quoſdam autem tam lento motu ferri, ut vix unum milliare in hora peragrent. Alii porro Venti ſunt adeo tardigradi, ut eoſdem aliquis, equo vel pedibus iter faciens, facile prævertat. Quod Senſibus patet; quoties gradum ſiſtimus, lenem auram nos placide ventilantem percipimus: ſin autem cum eodem pergimus, nullam prorſus ſentimus: ſi vero celerius pedem movemus, comitantis & conſpirantis Auræ loco, adverſantem, & in ora vultuſque ſpirantem aerem perſentimus. Ita, quieſcente prorſus Atmoſphæra, & ſtagnante, ſi forte ambulamus, aut equitamus, lenem Auram nos tunc prementem ſentimus, tantarum nempe virium, quantæ motui quo ferimur reſpondent. Eodemque Motus gradu, ſive Velocitate, fertur Venti aura, ſive Aeris fluxus, cum pari impetu nos morantes aut ceſſantes premat.

Hoc unum inſuper quoad Sonos obſervabo: nempe, cum eorum Motus Vento celerior ſit, patet quod iſtæ Atmoſphæræ partes quibus imprimuntur, aut deferuntur Soni, non ſunt eædem ac illæ ex quibus conflantur Venti, ſed quædam aliæ magis æthereæ & volatiles, quantum divinare licet. *Venti* enim celerrimi haud plus quam 60 milliaria Horæ ſpatio prætervolant: *Soni* vero plus quam 700 millia paſſuum eodem tempore percurrunt.

Sin autem objiciatur quod Venti Sonos celeriores aut tardiores reddunt: Reſponderetur; Hoc non a ſolo proprioque ventoſarum particularum fluxu, ſeu tendentia proficiſci, ſed potius ab omnium Atmoſphæræ particularum, cum craſſiorum, tum ætherearum conjuncto & conſpirante motu. Quæ Curſus, ſive Motus directio, ſi Sonorum Undulationibus faveat, Sonorum appulſum exinde accelerari; ſin adverſetur, retardari, omnino veriſimile eſt.

§. 13. *De Sonorum Velocitate.*

Ex quamplurimis quæ prænotavimus, firmiſſime concludo, Sonos hoc Velocitatis gradu propagari, Nempe, *ad Milliaris intervallum* (*ſive* 5280 *Pedum Anglicanorum*) *ſpatio* $9\frac{1}{4}$ *Semi ſecundorum percurrere*: Vel (quod eodem redit) Pedes 571 unius Semi-ſecundi, vel 1142 Pedes unius Secundi Minuti Temporis ſpatio.

Hoc autem prædictum ſpatium pertranſeunt Soni, ſi tranſverſus Atmoſphæræ fluxus intercurreret, & eſt *Medius* eorum *Progreſſus* ſive *Motus*. Sin autem Ventus Soni rapiditatem augeat, poſſibile eſt ut pluſquam 600 Pedes Semi-ſecundi ſpatio prætereat. Vel e contra, ſi moram ei innectat, haud plus quam 560 Pedes eodem temporis intervallo progrediatur.

Prædictæ Obſervationes, & Experimenta non parum conducere videntur,

1. *Philoſopho*; qui vel hinc aliqua ex parte inſtructior ſit ad arcanam Sonorum Naturam inveſtigandam; & eorundem plurima Phænomena abſtruſa explicanda.

2. *Nautæ*; qui hinc diſcat quanto intervallo abſunt Naves, quas procul fluctuantes, vel ad anchoras ſtantes cernit: quam longe item Tellus, aut optata Arena, eminus conſpecta, diſtat. Quæ ex Sclopetis de induſtria exploſis, ſigno quodam dato, facile certoque innoteſcant.

3. *Militi*: ad inveniendum quam procul Hoſtis Caſtra locavit; ad quam diſtantiam ſita eſt Urbs obſeſſa, Arx, Armamentarium, &c. ad Tormenta muralia libranda, & dirigendos Pyrobolos, glandeſque ignivomas.

4. *Geographo*: ad Locorum diſtantias facilius & certius menſurandas. Quivis enim intra horam unam aut alteram, parva pulveris pyrii copia

inſtructus, totam fere Regionem Tabula accuratiſſime deſcriptam hoc pacto exhibeat. Sclopeta enim diſploſa Diſtantias (prout diximus) oſtendent: & quodvis Inſtrumentum Mathematicum quo metiuntur Angulos, vel Inſtrumentum illud vulgare quo Decempedatores utuntur (*the Plain Table* vocatum,) vel ſola Regula Pinacidiis inſtructa, variorum locorum Situs indicabit; quæ deinde delineare haud difficile eſt.

Hac etiam ratione in Mapparum rectitudinem, & veritatem quis facile inquirat; & ſiquos habeant errores, corrigat.

Hæc demum (Soni) obſervandi ratio, Locorum inacceſſorum, præſertim vero latiſſimorum Fluviorum, & ejuſmodi locorum haud aliter menſurabilium, diſtantiis dimetiendis magnopere inſerviat.

5. *Echometræ.* De hoc ludicro & jucundo Soni Phænomeno (ſcil. *Echo*) licet plurimi docti viri olim & poſtea ſollicite quæſiverint, de plurimis tamen ad idem ſpectantibus non bene inter ſe convenit: ſpeciatim de ſpatio Loci ad repetitionem 1, 2, 3 vel plurium Syllabarum neceſſario; vel (quod eidem redit) de ſpatio ab *Echo* peragrato in certo quodam temporis intervallo. *Merſennus* —— paſſus ad Vocem Monoſyllabam repetendam concedit; *Blancanus* 24 paſſus (cui aſtipulatur noſtras celeberrimus Dr. *Plott*,) ſed *Ath. Kircherus* aſſerit nihil omnino certi de eo definiri poſſe, quod nempe Ventorum variatio, viriumque Soni intenſio & remiſſio, & multa alia immenſam variationem pariunt.

Rationem autem hujus diſſenſus inter laudatos hoſce viros reddere haud difficile eſt. Ex plurimis enim cauſis oriri poteſt; ex tarditate nimirum & diverſa noſtrorum Senſuum diſpoſitione; vel ex varia Sonorum audibilitate; ex Syllabarum ipſarum gravi vel acuto ſono ſive earundem contracta vel producta pronunciatione; vel ex qualibet alia cauſa temporis intervallum protrahente. Nullus enim dubito (Exempli gratia) quin ſi Objectum aliquod Phonocampticum repercutere potuerit omnes Syllabas hujuſce ſequentis carminis, viz.

Vocalis Nymphe, quæ nec reticere loquenti,

Quod haud valeret repercutere omnes Syllabas ſequentis carminis, quoniam paulo productior eſt ejus pronunciatio,

Corpus adhuc Echo, non vox erat, & tamen uſum:

Et multo minus repetere valeret aſperas omnes, & productas Syllabas ſequentis carminis licet numero pauciores,

Arx, tridens, roſtris, ſphinx, præſter, torrida, ſeps, ſtrix.

Verum a præcedentibus de Soni motu Obſervationibus concludere licet, Quod uti Soni, ita Ἠχὼ certa & determinata ſpatia in certo quodam præſcriptoque tempore percurrunt. Quod ipſum ab experientia ſæpius edoctus ſum, ſcil. Echo redire in duplo temporis intervallo quo Vox primaria Objectum Phonocampticum pertingebat, ex. gr. Echus regreſſus in eodem temporis intervallo fuerit, in quo Primarius Sonus duo Stadia percurriſſet ſi non repercuſſus fuiſſet.

Et hoc in dimetiendis Locorum diſtantiis magno mihi uſui ſæpe fuit. Exempli gratia: Cum in *Thameſis* fluvii ripa ſtarem, Villæ *Woolwich* oppoſita, monoſyllabæ vocis Echo a Domibus adverſis repercuſſa fuit in ſex Semi-ſecundis Minutis temporis. Unde colligo Latitudinem fluvii *Thameſis*, eo loci a margine unius Ripæ ad marginem alterius, 1712 pedes Anglicanos eſſe, ſive ſupra quadrantem milliaris. Nam ut 9,25 (Semi-ſecunda): ad 5280 (pedes in milliari uno): Ita 6 (Semi-ſecunda): ad 3424, 8 pedes. Cujus dimidium eſt 1712,4 pedes.

Denique hoc pacto *Intonantium Nubium Altitudo*, & ipſius *Tonitru, Fulguriſque* Diſtantia facile innoteſcant.

Of the Nature and Property of Sounds, by Dr. Grandi *n.* 319. *p.* 270.

V. Elegantiſſimum Præſulis * Armachani commentariolum de Sono ſumma animi voluptate perlegi, at circa Semiplani figuram in calæ ejus ſcripti commemoratam, vereor, ut ſatis aſſequi mentem Auctoris valuerim.

Comparat doctiſſimus Præſul ſcientiam auditus cum Theoria viſionis, atque ut hæc in directam, reflexam, & refractam dividitur, ita illam pari ratione trifariam diſtribuit, ut non modo ſonos directos, & reflexos (quod dudum in uſu fuit) ſed & refractos conſideret; quem-admodum autem eximiis inventis opticis, catoptricis, & dioptricis viſionem a majoribus noſtris magna jam ex parte perfectam fuiſſe animadvertit, ita compluribus inſtrumentis Acuſticis, Catacuſticis, & Diacuſticis, ſive Phonicis, Cataphonicis, & Diaphonicis (utrovis enim modo denominat) auditum, tam ex objecti, quam medii, vel organi parte perfici poſſe non dubitat, eoque ſpectantia problemata proponit, quæ tamen in hoc ſcripto, nedum abſque demonſtratione, ſed & abſque determinatione, aut conſtructione ulla exhibentur, unde non major ad ipſorum ſolutionem lux nobis affulget, quam quæ, ante inventa a M. Galilæo ſcientiæ motus principia, haberi potuiſſet ad enodationem problematum circa determinandam projectorum ſemitam, vel aquarum ex data altitudine deſcendentium velocitatem, propoſitorum. Neque enim affirmare verebor, perinde ignota nunc eſſe Acuſticæ doctrinæ fundamenta, certe nondum paſſim vulgata, aut inter eruditos recepta; licet fortaſſe laudatiſſimo huic Præſuli innotuiſſe videantur, ſi quæ ab illo indicata, & promiſſa ſunt, attendamus, quorum quidem uberiori expoſitione, atque aperta demonſtratione totam ſibi literariam Rempublicam demeruiſſet; cum vix credendum ſit, omnes ſimplici illa Opticorum, & Acuſticorum comparatione fore contentos, quæ vix ultra ſatis latam analogiam extenditur, ob tot diſcrimina, quibus propagatio Lucis a diffuſione Soni ſecernitur: inter quæ illud palmarium eſt, quod Lux per lineam rectam ſemper exporrigitur, dum ſonus etiam per curvas, & inflexas utcunque

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Vid. Phi. Tran. n 156. *Abrid. p.* 508.

ſemitas quaquaverſum ſpargitur, atque, intercepto cujuſvis opaci corporis obice, ſenſibilis redditur.

Et vero hæc ipſa, quæ de Soni diffuſione doctiſſimus Auctor noſter ediſſerit, ejus differentiam a lucis propagatione manifeſtant: docet ſiquidem, ſonum ſecus parietes, aut fornices lævigatiſſimos, elliptica, vel cycloidali, potius quam circulari flexura donatos, blando quodam, & expeditiſſimo lapſu feliciter excurrere, necnon per mollem aquæ ſuperficiem, ſonoris tremoribus, quibus aer criſpatur, obſequentem validius promoveri: quæ vereor, ut in luminis propagatione adeo generatim obſerventur; nam de Ellipſi quidem hoc tantum habemus ex catoptrica demonſtratum, quod radii lucis ex altero ejus foco *D* emanantes, & in ellipticam curvam *A B C* impingentes, inde reflexi in altero foco *E* colligentur; at ſi ex **Fig. 14.**
alio quovis puncto *G*, præter focos, exeant radii, non omnes amplius in unum punctum coibunt, ſed ita reflectentur, ut curvam cauſticam *f F f* contactu ſuo efforment, ſupra cujus convexitatem exiſtentes, uno aut altero reflexo radio, non pluribus gaudere poterunt, in ipſa vero curva poſiti aliquot ex maxime vicinis participabunt; atqui intra cavitatem ejuſdem verſabuntur, ab omni reflexorum radiorum illapſu immunes erunt, adeo nullum inde ſibi emolumentum obventurum ſperabunt.

Cycloidem quod attinet, oſtendit quidem Cl. V. Joannes Bernoullius in Actis Lipſiæ 1697. Lucis radium, ſi per media tranſiret, quorum raritates in quolibet puncto juxta rationem ſubduplicatam altitudinum variarent, ita continuo flexu refrangendum, ut in curvam cycloidis ſinuaretur: at ſeu reflexione, ſeu directa per idem medium propulſione, quid figura cycloidis ad feliciorem luminis diffuſionem conferret, plane non **Fig. 15.**
video; hæc enim curva focis omnino caret, adeo ut in nullo puncto radios recolligere poſſit, ſed in curvas irregulares abeunt radii ab ipſa reflexi, niſi quod ubi ad axem *K L* paralleli radii *P M*, *Q N* in Cycloidem *E M K N H* inciderent, tunc linea cauſtica per contactum reflexorum radiorum *M R*, *N S*, efformata, ex binis cycloidibus *E R L*, *H S L*, circulo ſubduplæ diametri generatis, componeretur, radioſque reflexos confertiſſimos circa utriuſque confinium *L*, ad medium baſis reflectentis cycloidis exhiberet: cæterum tam in his quam in aliis cauſticis ex qualibet luminoſi puncti radiorumque poſitione reſultantibus, eædem obſervationes locum haberent, quas ſub finem præcedentis paragraphi cauſticis per ellipſim efformatis competere diximus.

De plana aquarum ſuperficie nihil eſt quod addam, cum pateat, lucis radios per illam aut omnino refractos tranſire, aut ordinata reflexione in adverſam partem remitti, perinde ac e chryſtalli ſolidioris ſuperficie; imo ab hac aliquanto fortius, quam ab illa, tantum abeſt, ut per illam facillime repentes in directum expeditius promoveri poſſint, illumque blandum progreſſum obtinere, quem tremoribus harmonicis, per mollem aquarum ſuperficiem, criſpatione ſua ipſorum flexui ſe accommodantem, ſerpentibus Auctor tribuit: immo & dubitare licet, an lævigatiſſimæ ſpeculorum ſuperficies, perinde ac luminis, ſic ſoni reflexionibus valde conducerent, cum Echo ipſa ſpeluncarum receſſus aſperrimos, magis quam politos,

litos, ac tenui gypſo incruſtatos parietes habitare videatur, ab incultis vallibus, ab anfractuoſis antris, atque e veterum ædificiorum ruderibus frequentius reſpondens.

Jam Acuſticum ipſum ſeu ſpheram Phonicam ab eodem propoſitam, divinando magis, quam interpretando, exponere utcunque aggrediar, verbis illius primum adductis, ut cum meis conjecturis mox ſubjiciendis conferri queant, & quam exacte iiſdem reſpondeant cujuſlibet poſſit arbitrio judicari.

Addam hoc loco, inquit Auctor, *Semiplani Acuſtici, ſeu Sphera Phonica figuram, quaſi tentamen ad magnum ſcientiæ hujus principium explicandum, quod in ſonorum progreſſione conſiſtit. Conſideretis oportet rude hoc ſemiplanum velut horizonti parallelum, nam ſi eidem perpendiculare fuerit, ſuppono extremitatem illius ſuperiorem non amplius circularem futuram, ſed hyperbolicam, partem vero inferiorem æqualem fore uni ex maximis terræ circulis: adeo ut univerſa ſphera Phonica, ſi ita appellare liceat, ſit quadam hyperbola ſolida, ſuper concava baſis ſphericæ ſuperficie erecta.* Porro Diagramma Londino tranſmiſſum hujuſmodi erat, nullis præterea notis ad ejus illuſtrationem facientibus inſtructum. Fig. 16.

Fig. 17. Ipſe igitur, hoc alio Schemate ſubſtituto, mentem Auctoris aperire ſatagam. Sit terræ globus *C G F E*, atque in puncto *C* ejus ſuperficiei ſonus aliquis excitetur. Hic per terram ipſam nec non per aerem circumquaque propagabitur, adeo ut quo tempore ad maximum terræ circulum polo *C* deſcriptum, nempe ad Peripheriam *G B E* aut reipſa (licet fortaſſe inſenſibiliter) pervenit, aut ſaltem (ſi validior eſſet) perveniret, per aerem diffuſus, quoddam ſpatium repleret, pro varia tranſitus facilitate, non prorſus ſpherice, ſed inæqualiter exporrectum, & a perimetro hyperbolæ *G L A K E*, circa axem *C A O* ſonoro corpori *C* perpendicularem poſitæ, circumſcriptum: imo vero a ſuperficie conoidis hyberbolicæ, quam hyperbola *A L G* circa ſuum axem rotata generat, definitum. Itaque univerſa ſphera phonica, per quam dato tempore ſonus extenditur, erit ſolidum ſpatium comprehenſum ab hyperbolica conoide *G A E B*, quæ maximo terræ circulo *G B E* inſiſtit, & concava ſuperficie hemiſpherica *G C E B* inferius terminatur: quod quidem ſpatium plano ad horizontem parallelo ubilibet ſectum, exhibebit ſemicirculum *L I K*, qualem oſtendit Auctoris figura, quem & ſemiplanum appellat, eo quod ipſius diagrammatis proſpectus alteram ſolum medietatem ejus exhibeat, dimidia reliqua trans hyperbolam verticalem (quæ & ipſa phonicam ſpheram per axem bifariam ſecat) inconſpicua manente. Verum quæ ſit hujuſmodi hyperbolæ ſpecies, aut quibus principiis doctrina hæc fulciatur, nec Auctor indicat, nec mihi ſuppetit unde hac de re quidpiam certi conjiciam.

Quod unum ſupereſt, adnitar, ut inverſo veſtigandi ordine procedens, detegam primo per quod linearum genus tremores ſonoros diffundi oporteret, ut in ejuſmodi hyperbolam dato tempore expanderentur; ſecundo, quæ raritatis variatio foret in variis aeris altitudinibus ſupponenda, ut (ſtante refractionis communi lege, qualem radii lucis obſervant) ſonorum directiones juxta inventam linearum ſpeciem flectere poſſet; ac tertio, quæ viciſſim dicenda ſit lex refractionis, quam ſonori tremores in ejuſmodi

modi curvarum genus abeuntes ſequuntur, ſuppoſita raritatis aeris variatione tali, qualem plerique Philoſophorum, & Mathematicorum in illo agnoſcunt juxta reciprocam rationem ponderis atmoſphæræ incumbentis, & inferiores partes gravantis, quam experimentis congruere teſtantur.

Pro quo conſideremus, corpus ſonorum C tremores ſuos per directiones Cn, Cm, Cb, quaquaverſus communicare, aut certe juxta eas lineas, per quas impulſum fuerat, ſe reſtituendo repellere aerem, eumdemque frequentiſſimis oſcillationibus protrudere, quibus criſpatur, atque ad motum tremulum juxta eaſdem directiones diffuſum ſollicitatur; hi ergo tremores quodam minimo tempore perveniſſe concipiantur ad puncta n, m, b, unde iter ſuum proſequentes, ſucceſſive poſt aliud datum tempus ſimul propagabuntur, primus ad punctum N, ſecundus ad M, tertius ad H; iterumque poſt aliud datum tempus ſimul progredientur, prior ad G, alter ad L, poſtremus ad A. Nunc igitur lineas quidem $CnNG$, $CmML$, $CbHA$, per quas quilibet tremor ſucceſive diffunditur, voco *Radios ſonoros*; lineas vero n, m, b, NMH, GLA, quas prædicti ſonori radii, omneſque alii ſynchroni iis intermedii dato quolibet tempore ſimul attingunt, *Undas ſonoras* appello. Fig. 18.

Et quidem in medio prorſus ubilibet uniformi, ceſſante cauſa, quæ tremores ſonoros a ſua directione in hanc, vel illam partem deflectere cogat, patet ſonoros radios ſemper rectos procedere, ſeu per viam breviſſimam ab uno ad alium terminum directe progredi, atque undas penitus circulares ſonoro corpori concentricas efficere, quia cum non majorem hic, quam alibi tranſitus difficultatem inveniant, utique ad pares diſtantias ſinguli dato quovis tempore elongabuntur: Secabit autem radius quilibet undam ſuam perpendiculariter, atque undæ quælibet concentricæ & ſimiles erunt, ut conſtat ex elementis.

At in medio difformis denſitatis, velut in aere terræ circumfuſo, qui diverſam pro varia altitudine (nam caloris, frigoris, humiditatis, & ſiccitatis vices, quæ ad certam legem revocari non poſſunt, pro nunc ſeponimus) raritatem obtinet, ſolus radius CHA perpendiculariter trajiciens omnes aereas lamellas, ſive ſuperficies terræ concentricas, irrefractus tranſibit, ac rectus manebit; cæteri vero iiſdem ſuperficiebus oblique impingentes continuo quodam flexu in quolibet puncto refringentur, & in curvas $CmML$, $CnNG$ ſinuabuntur; nec non pro varia tranſitus facilitate non ad eandem ubivis diſtantiam dato tempore progredientur, quare puncta A, L, G, aut H, M, N, quæ eodem momento ſonus per quoſlibet radios emiſſus attingit, inæqualiter a ſonoro C remota erunt, adeoque undæ ALG, HMN, bmn, omnino circuli non erunt corpori ſonoro concentrici, ſed alterius generis curvæ, quas tamen oportet invicem ſimiles eſſe, ac ſimiliter poſitas: quare in hypotheſi noſtri Auctoris, qui extremam illam undam ALG, ultimos globi terraquei fines lambentem hyperbolicam voluit, neceſſe eſt quaſvis alias undas intermedias HMN, bmn, eſſe hyperbolas ſimiles, ac ſimiliter poſitas, diverſis quidem verticibus, A, H, b, ſed eodem centro, ad eumdem axem, & ſub ſimilibus laterum figuris deſcriptas; nam quæcumque ſit ratio, quæ oſtendat, ob ſimultaneum appulſum ſoni ad puncta A, L, G, per vias ſynchronas CHA, CML, *C N G*, unde *A L G* faceſſere in curvam talis ſpeciei (puta hyperbolicam;) eadem prorſus iiſdem fundamentis evincet, ob ſimultaneum appulſum ſoni etiam ad puncta *H*, *M*, *N*, per ſynchronas lineas *C h H*, *C m M*, *C n N*, undam *H M N*, in curvam ejuſdem ſpeciei (nempe hoc caſu in hyperbolam ſimilem, ac ſimiliter poſitam) pariter abire, ut de ſe conſtat. Nec dubium inſuper, ſonoros radios *C H A*, *C M L*, *C N G*, ſemper undas illas ſimiles *A L G*, *H M N*, *h m n* debere perpendiculariter, ſive ad rectos angulos ſecare, ut in circularibus undis contingit; quod cum in ſimili propoſito de lucidis undis oſtenderit jam Vir Cl. Chriſt. * Hugenius, non eſt cur in hac obſervatione, pluribus momentis confirmanda, tempus teratur.

Itaque inveſtigatio viæ, per quam radii ſonori, juxta hypotheſim Auctoris noſtri, propagantur, ad hoc pure geometricum Problema reducitur, ut inquiratur natura curvarum, quaſlibet hyperbolas ſimiles, & circa eumdem axem, eodem centro ſimiliter deſcriptas, perpendiculariter ſecantium. Sint hyperbolæ ſimiles *A L G*, *H M N*, *h m n*, aliæque innumeræ intermediæ, aut ſupra, vel infra ipſas ſimiliter poſitæ, idem commune centrum *O* habentes, eodemque axe *O A H*, cui alter *O S* conjugatur, deſcriptæ: ducenda eſt per punctum *C* curva *C m M L*, aut *C n N G*, propoſitas omnes hyperbolas perpendiculariter ſecans. Deſcribatur per datum punctum *C*, inter aſymptotos *O A*, *O S*, hyperbola *C m M L* talis naturæ, ut poſita ratione tranſverſi lateris priorum hyperbolarum *A L*, *H M*, &c. ad latus rectum earumdem æquali rationi *t* ad *r*, poteſtates ordinatarum *L Q* denominatæ ab exponente *r* ſint reciproce proportionales poteſtatibus abſciſſarum a centro *O Q* denominatis ab exponente *t*, nempe facta $OQ = x$,

Fig. 19. & $QL = y$, ita ut $y^r = \frac{1}{x^t}$; ſive, ducta qualibet alia ordinata *m i*, *M I*, ita ut ratio diſtantiarum a centro *O Q*, *O I* ſit reciproce tam multiplicata rationis applicatarum *I M*, *Q L*, quam multiplex eſt fractio $\frac{r}{t}$ unitatis. Dico hanc ſatisfacere quæſito; ducta enim cujuſvis hyperbolæ *A L* tangente *L P* in puncto, ubi a curva *C M L* ſecatur, necnon *S L R* tangente ipſius hyperbolæ *C M L* in eodem puncto, patet ex his quæ in Theorematum Hugenianorum demonſtratione, cap. 7. n. 9. oſtendimus, fore *O Q* ad *Q R*, ut exponens poteſtatis diſtantiarum *O Q* ad exponentem poteſtatis ordinatarum *Q L*, nempe ut *t* ad *r*; ſed ut *t* ad *r*, nempe ut tranſverſum latus ad rectum, ita per 37. l. Conic. eſt rectangulum *O Q P* ad quadratum *Q L*; igitur ut *O Q* ad *Q R*, ſive ſumpta communi altitudine *Q P*, ut rectangulum *O Q P* ad rectangulum *P Q R*, ita rectangulum *O Q P* ad quadratum *Q L*, quod ideo æquabitur rectangulo *P Q R*; quare angulus *P L R* rectus erit: unde curva *C M L* perpendiculariter occurret in puncto *L* hyperbolæ *A L G*, eodemque modo aliis hyperbolis *H M N*, *h m n*, in punctis *M*, *m*, in quibus illas ſecat, perpendicularis eſſe oſtendetur; quod erat, &c.

Hinc primo colligitur, quod ſi hyperbola determinans ſphæram Phonicam Auctoris noſtri, nempe *A L G*, aliæque ſimiles concentricæ *H M N*, *h m n*, fuerint æquilateræ, tunc propter æqualitatem laterum *t*, & *r*, hyperbola *C M L* erit & ipſa hyperbola Apolloniana, & quidem pariter æquilatera, ejus enim æquatio ſuperius allata transformabitur in hanc $y=\frac{1}{x}$ ubi ratio ordinatarum ſimpliciter reciproca erit rationis diſtantiarum a centro, itaque radii pariter ſonori æque ac ſonoræ undæ, juxta hanc hypotheſin forent hyperbolæ ejuſdem ſpeciei, diverſa duntaxat poſitione collocatæ: Memini porro Illuſtriſſimum Equitem Iſaacum Newtonum Opticæ ſuæ, lib. 3. p. 287. Obſerv. 10. oſtendere, quod & radii lucis trans duorum cultrorum acies in obſcurum cubiculum admiſſos, in hyperbolicas fimbrias, qualis eſſet *C M L*, pariter ſinuari, cujus phænomeni ſi ratio phyſica afferri poſſet, eadem hyperbolicos pariter ſoni radios, quales Armachani Præſulis ſyſtema invehere videtur, fortaſſe perſuaderet.

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* In Tractatu Gallico, de Lumine, p. 44.

Secundo obſervandum eſt, quod ſi plures ejuſmodi curvæ, ſeu radii hyperbolici *m M L*, *n N G*, &c. ſecantes undas hyperbolicas *A L G*, *HMN*, &c. perpendiculariter deſcribantur, non in unum exacte punctum *C* poterunt convenire, tametſi propius & propius coeant ad partes *C*, atque ad intervallum pervenire poſſint, minus quolibet dato intervallo; quare concipiendi erunt radii illi hyperbolici a corpuſculo *C* alicujus extenſionis procedere, non ab aliquo mathematico puncto, quod ipſum convenientiſſimum eſt; ſonus enim ex colliſione corporum naſcitur, non ex unius rigoroſi puncti, ſeu termini extenſionis tremore produci poteſt.

Imo cum omnes undæ a ſonoro corpore propagatæ eſſe debeant, ut ſupra vidimus, hyperbolæ ſimiles; congruum eſt, ut concipiamus, corpus ſonorum *C* quaſi fibrillam minutiſſimam frequentiſſime oſcillantem, cujus minima, & veluti initialis unda infinite propemodum exigua 2 3 4, & ipſa revera hyperbolica ſit, ſeu potius apex phyſicus alicujus hyperbolæ; ita ut nimirum fibrilla oſcillatoria corporis ſonori *C*, dum pulſatur, ex ſitu directo 2 C 4 detruſa in ſitum concavum 2 5 4, vi percuſſionis adigatur, tum vehementiſſimi elateris ſui, nec non propriæ tenſionis vi, reſtituta in convexam hyperbolam 2 3 4 intumeſcat, ac rurſus reducta alternis vibrationibus, fluctuans hinc inde, ſuos tremores in hyperbolicas undas, ipſimet initialibus 2 3 4, 2 5 4 perpetuo ſimiles, ſurſum, ac deorſum ſuapte natura, & in medio utrinque libero expandat, ſed obice terreſtris globi *C E* (cujus centrum *T*) impedita fortaſſe, hyperbolicas undas ſuas dumtaxat ſurſum propaget, & Phonicam Sphæram ab Auctore noſtro excogitatam deſcribat, hemiſpherio terreſtri ab inferiori parte interruptam, ac definitam. Quod ſi vera eſſet *P. Pardies* doctrina Artic. 81. ſuæ ſtaticæ propoſita, quod chordæ extenſæ reipſa hyperbolicam figuram, qualis eſſet 2 5 4, cujus centrum ſit idem quod centrum terræ, vi proprii ponderis aſſumant, nemo non videt eam ipſam confirmando Auctoris noſtri ſyſtemati fore congruentiſſimam, hinc enim ratio haberetur, cur fibrilla quævis ſonori corporis, *C*, dum ad vibrationes harmonicas ſollicitatur, in hyperbolam 2 5 4 excurreret centrum habens in centro terræ *T*, ſimiliterque in aliam æqualem 2 3 4 aſſurgeret, indeque per alias ampliores hyperbolas tremorem diffunderet, quarum omnium centrum eſſet *O* æque diſtans a ſonoro corpore *C*, ac ſonorum corpus illud remotum ſit ab ipſomet centro terræ;

quare diſtantia *C O* æqualis ſemidiametro Globi Terraquei limitem definiret, ultra quam nulla ſonora unda propagaretur, nulluſque poſſet ſonus audiri, & linea *O S*, utpote aſymptotos quorumvis hyperbolicorum radiorum, per quos defertur ſonus, confinium beatæ illius regionis conſtitueret, in qua ab omni terrenarum rerum ſtrepitu ſecuris in ſumma tranquillitate philoſophari liceret.

Porro ne quis ſpeculationem hanc eo nomine contemnendam putet, quod fibrilla quævis ſonori corporis, cum breviſſima ſit, ac valide diſtenta, ſemper in ſitu recto 2 *C* 4, manere videatur, nec poſſe in concavas, aut convexas hyperbolas 2 5 4, 2 3 4 ſinuari, conſiderandum inſuper eſt, hyperbolas quo majoribus axibus præditæ fuerint, eo magis ampliari, & ad lineam rectam accedere; itaque ob ingentem diſtantiam centrorum *T*, vel *O*, ſicut lineæ quas gravia cadentia deſcribunt, licet in centrum *T* collimantes pro parallelis habentur, & arcus circuli horizontalis cum recta ejus tangente confunditur, ita & initiales illas hyperbolas 2 5 4, 2 3 4 ferme coincidere dicendæ ſunt cum recta 2 *C* 4, unde ſenſibilis non eſt incurvatio fibrillarum oſcillantium in ſonoro corpore, nec ſe prodit undarum hyperbolicarum ſpecies, niſi ubi in amplius ſpatium *G L A L G* dilatatæ fuerint centro ſuo propius accedentes.

Animadvertendum adhuc tamen, his principiis poſitis, conſequens fore, ut ſonus hinc inde ad latera non excurreret ultra ſpatium ab hyperbolicis extremis radiis 2 9 8 *g*, 4 7 6 *g* comprehenſum, quas tangerent rectæ *T* 2, *T* 4, a centro terræ per terminos fibræ oſcillantis adductæ; ac revera fibræ illius tremores juxta aliam directionem non procederent, quam per *T* 2, *T* 3, *T* 4, aliaſque intermedias angulo 2 *T* 4 comprehenſas, ſingulis particulis fibræ ejuſdem correſpondentes, itaque ſpatium extra dictas hyperbolas 2 9 8 *g*, 4 7 6 *g* poſitum ab omni tremore harmonico vacaret, nec poſſet juxta ſenſum Auctoris phonica ſphæra ad integrum terræ hemiſpherium exporrigi; itaque oportet, nunquam reipſa unicam aliquam ſonori corporis fibrillam tremere, quin terminos aliarum fibrarum, quibus connectitur, & inter quos diſtenditur, eo ipſo trahat, & ad harmonicum tremorem pariter ſollicitet, quæ rurſus alias, quibus implicantur, abducunt, & ad tremorem exſtimulant, quemadmodum tenſa chorda muſica ligneo inſtrumento, cui alligatur, tremores ſuos evidenter communicat; itaque harmonicæ oſcillationes in alia corpora, quibus mediate, vel immediate connectitur, percuſſa fibra ſonori corporis, ſubinde transfunduntur, licet magis magiſque ſemper debilitatæ ac demum inſenſibiles redditæ per hemiſpherii terreſtris ſuperficiem ſparguntur, & longius ac longius ſerpentes obrepunt (quod auris ipſa terræ applicata, & magnos ſaltem fragores in maxima diſtantia excitatos diſcernens teſtari poteſt) itaque ex aliis etiam locis emergunt alii ſonori radii hyperbolici per totum terræ hemiſpherium, a quibus Phonica ſphera Præſulis Armachani ſatis impleri poſſit.

Secundam quæſtionem generalius ſolvere conabor, ut major inde fructus elici queat; intelligatur quivis radius ſeu lucidus, ſeu ſonorus *N n G*, in cujuſvis naturæ curvam, continua ſui refractione, mutatus: quæritur qua lege variari ſupponenda ſit denſitas, aut raritas medii in ejus diverſis altitudinibus, ut ſtante refractionis theoria, quæ ſinum refractionis ſemper raritati medii refringentis proportionalem ſupponit, radius ille in talis naturæ curvam obire potuerit? Sit axis curvæ $Nn\,G$, quam radius refractus efficit, recta CO, in qua ſumpto quolibet puncto C, radio quovis CL deſcribatur quadrans circularis LPp, ductaque ubilibet refracti radii tangente NR, nr, agatur ex C radius dictæ tangenti parallelus, occurrens circulo in P, ductaque PF axi parallela, occurrat ordinatæ NQ ad axim perpendiculari in puncto F; dico inde ortam curvam FfF exprimere ſuis ordinatis FQ, fq raritates medii in variis ejus altitudinibus; nam quia CP eſt parallela ipſi RN, erit angulus PCB æqualis angulo, quem radius refractus Nn in puncto N efficit cum perpendiculo; & ideo BP, ſive FQ erit ſemper ſinus refractionis, poſito CP ſinu toto; quare cum ſuppoſita ſit lex ea refractionis, ut ſinus ejuſdem proportionalis ſit raritati medii; utique eadem FQ exprimet medii raritatem ad altitudinem Q, ſive ad æque altum punctum N, per quod radius tranſit. Quod erat, &c.

In noſtro autem propoſito, ubi $QN = \frac{1}{X^{\frac{t}{r}}}$, propter $Y^r = \frac{1}{X^t}$, ſi FQ exponens raritatem aeris vocetur Z, erit $Z = \frac{t}{\sqrt{X^{\frac{2r+2t}{r}} + tt}}$; aut ſumpta etiam r, & CP pro unitate, fiet $Z = \frac{t}{\sqrt{X^{2+2t}+tt}}$, atque in caſu quod unda hyperbolica fuerit æquilatera, adeoque & radius hyperbola ſimilis æquilatera, $y = \frac{1}{X}$ propter $t = 1$, fiet $Z = \frac{1}{\sqrt{X^4+1}}$.

Quoniam vero tum † Jacobus Hermannus tum David * Gregorius oſtendunt Curvam, quæ determinat gradus raritatum aeris eſſe logarithmicam, adeo ut altitudines OQ, oq, ſive X ſint logarithmi numerorum exponentium aeris raritates in punctis Q, q: patet radii continue refracti Nn, NG curvaturam ea lege procedere, ut ſinus complementi incidentiæ, & refractionis ad poteſtatem $\frac{r}{r+t}$, elevati rationem habeant compoſitam ex ratione ſinuum rectorum ad ſimilem poteſtatem evectorum, & ex ratione quam habent logarithmi raritatum.

† *Act. Lipſ.* 1706.

* *Aſtron. Lib.* 5.

Cæterum etſi conſenſerim, ordinariam legem refractionis lucis dare ſinus incidentiæ & refractionis proportionales raritatibus mediorum, non diſſimulo tamen id fortaſſe non adeo exactum eſſe, cum ratio ſinuum in refractione ex aere in vitrum ſit circiter ſeſquialtera, aer vero pluſquam millies vitro ſit rarior. Sed cum viderent Geometræ majorem fieri ſinum refractionis in tranſitu ad aliud medium pro majori facilitate qua illud lux penetrat in communi hypotheſi, vel pro majori difficultate juxta Carteſium, qui ſupponit e contrario lucem magis refringi ob majorem difficultatem in rariori medio quam in denſiori (ut gravia corpora ob majorem difficultatem penetrandi denſiora corpora, in hi magis refringuntur, reſiliendo a perpendiculari) & utramque legem in eo convenire, quod pro majori

medii raritate, major fieret refractio: hinc invaluit, ut ſinus proportionales dicerentur, non quidem facilitati, aut difficultati tranſitus, quarum alterutra ab aliis in dubium vocatur, ſed raritati medii, in qua omnes conveniunt, licet vera proportio illi non prorſus reſpondeat in eadem geometrica ratione; itaque ubicunque raritatis mentio facta eſt, ſubroganda eſt fortaſſe facilitas tranſitus in communi, & difficultas in Cartheſiana hypotheſi, præterquam ubi diximus, raritatem ex pondere aeris incumbentis variatam reſpondere altitudinibus ut numeri logarithmis ſuis reſpondent; hoc enim exacte veriſſimum eſt.

Experiments concerning Sound, by Mr. Hawksbee. *Actual Sound not to be tranſmitted through a Vacuum, n. 321. p. 367.*

V. 1. I took a ſtrong Receiver, arm'd with a Braſs-hoop at bottom, in which I included a Bell as large as well it could contain. This Receiver I ſcrew'd ſtrongly down to a Braſs-Plate, with a wet Leather between, and was full of common Air, which could no ways make its eſcape. Thus ſecur'd, it was ſet on the Pump, where it was cover'd with another large Receiver. In this manner, the Air contain'd between the outward and inward Receivers was exhauſted.

Now here I was ſure, when the Clapper ſhould be made to ſtrike the Bell, there would be actual Sound produc'd in the inward Receiver, the Air in which was of the ſame denſity with common Air; and could ſuffer no Alteration by the *Vacuum* on its outſide, ſo ſtrongly was it ſecur'd on all ſides. And as I ſaid before, that if the ſonorous Body ſhould ſuffer in any meaſure, by being in a very rare *Medium*, ſo as to contribute to the loſs of its Sound, that this Method ſeem'd probable to diſcover it.

Thus all being ready for Trial, the Clapper was made to ſtrike the Bell; but I found that there was no Tranſmiſſion of it through the *Vacuum*, tho' I was ſure there was actual Sound produc'd in the Receiver.

This plainly ſhews, and ſeems poſitively to confirm, That Air is the only *Medium* for the Propagation of Sound.

The Propagation of Sound, paſſing from the ſonorous Body into the common Air, in one Direction only, ib. p. 369.

2. To try whether that Sound, which ſhould be propagated in a Receiver, having a Communication with the open Air at one ſmall Aperture only, but otherwiſe intirely ſurrounded by a *Vacuum*; whether I ſay, that Sound would be increas'd, or continu'd Sounding longer, at each Stroke that ſhould be given the Bell, than it would do, were not its Body encompaſs'd by ſuch a *Medium*: The Bell was included as in the pre-mention'd Experiment; only, to the upper part of its Receiver, was ſcrew'd a Box with Collars of Leather; and on the Top of the outward Receiver, was laid a Braſs-Plate with a wet Leather between: In the middle of which Plate, was likewiſe ſcrew'd another Braſs Box with Collars, as before. Theſe Receivers when plac'd on the Pump, had their Boxes ſtanding directly one over the other. Through both of them in that Poſition, I paſt a hollow Braſs Tube, which exactly fitted their Perforations: thus the inward Receiver had a Communication with the outward Air, and the outward Receiver thereby was ſecured from the Ingreſs of the Circumambient *Medium*. Now when the Air contained between the Receivers was pretty well exhauſted, and the Bell ſtruck, the Sound was ſenſibly very

vigorous, and (I think) very nearly as great as before any Air was taken away at all; yet if ones Finger was apply'd to the Apperture of the hollow Braſs Tube, the Sound would be ſo much diminiſh'd, as but juſt to be diſtinguiſh'd. By this we ſee, that ſince the Sound in that ſtate cannot be tranſmitted through the Receiver that includes it, by means of the ſurrounding *Vacuum*, yet the Receiver is certainly ſtruck with it; but finding no conveyance that way reverberates and makes its Paſſage where it finds leaſt Reſiſtance. Nor did I obſerve, that although the Sound had but one Paſſage from its Receiver, and that but a ſmall one, that it continued any longer from the Stroke, than if it had been made in the open Air.

Of the Propagation of Sound through Water, ib. p. 371.

3. The Receiver which contain'd the Bell was ſcrew'd down to a Braſs-plate, with a Leather between, as in the former Experiment; This Receiver with its Bell, was ſuſpended in a large Glaſs-Veſſel, by four Twine-threads to the top, and as many to the bottom: whereby it remain'd in the middle between both, concluding that theſe Threads would ſo abſorbe the Water when it ſhould come to be put in, that there could be no Apprehenſion, that any Sound ſhou'd be convey'd by them from the ſounding Body, any more than if they were intirely Water. Thus provided, the Clapper was made to ſtrike the Bell, whoſe Sound was ſomething leſs by the Interpoſition of the Glaſs, than it would be, had it been made in the open Air; however it was very audible, and might be heard at a conſiderable diſtance; It appear'd to the Ear to be very harſh, in reſpect to the Tone it afforded us. But now, when the Water came to be pour'd in, and the inward Receiver ſurrounded by it, at leaſt an Inch and an half from the neareſt part of the outward Glaſs, the Clapper again was made to give the Sound; which it did, ſeemingly, very little leſs, in reſpect to its Audibility; but much more mellow, ſweet, and grave at leaſt two or three Notes deeper than it was before.

VI. *A Paper of leſs general uſe Omitted*, viz.

A Letter from Mr. *Joſeph Williamſon* Watchmaker to the Publiſher; *n.* 363. *p.* 1080. wherein he aſſerts his Right to the curious and uſeful Invention of making Clocks to keep Time with the Sun's apparent Motion, againſt the Pretenſions of a *French* Watchmaker, who claim'd the ſaid Invention.

CHAP. V.

Hydroſtaticks, Hydraulicks.

I. *The weight of common Water under different Circumſtances, by Mr.* Hawksbee, *n.* 318. *p.* 221.

FIrſt, I took a Glaſs of common Water, and having weigh'd nicely a Glaſs-Bottle in it, whoſe Bulk was equal to the Bulk of 575 Grains of the ſame Fluid; then I caus'd ſome of the ſame Water to be boyl'd over the Fire, and after that, it was included in *Vacuo*, and there remained till it became of the ſame Temperature (as to Coolneſs) with common Water. Thus to the utmoſt of my Power, I endeavour'd to extricate all the Air out of the Water, thinking in that State it would become more denſe than when I weigh'd my Bottle firſt in't; but contrary to my Expectation, I found that the Bottle had juſt the ſame weight in it, as before, which ſeems to confirm the Impoſſibility to compreſs Water by force into a leſſer ſpace than it naturally poſſeſſes; for if upon the removal of ſuch a quantity of Air from out of its Body, the Parts do not ſlide any cloſer together, how ſhould a Weight laid upon its Surface, when its Interſtices ſeem to be replete with Air, make any Impreſſion on it. The Body which was forc'd out of the Water by the prementioned means, I call Air, ſince, for any thing to the contrary that I can diſcover, it is ſubject to all the ſame Laws with it; but that the Water upon its Abſence ſhould not unite more cloſely than before, ſeems very ſurprizing to me; for I cannot conceive what Matter muſt ſupply the Vacancies, ſince the Particles of Water themſelves remain at the ſame Diſtances as if the Air was not withdrawn, otherwiſe the Water of neceſſity muſt become more denſe. But to proceed, I caus'd ſome Water to be heated about Blood warm, when weighing my Bottle in it, I found the Bulk of Water equal to the Bulk of the Bottle, which was about three Grains leſs than when cold; which ſhews, that the component parts of the Water are eaſily ſeparated by Heat, and the Matter lodg'd in its Interſtices, capable of Dilation: Then I took that Water that I had purg'd of all its Air (as near as I could,) and gave it a degree of Heat, not exceeding luke-warm; upon weighing the premention'd Bottle in it, I found, that although the Heat it had receiv'd was very inconſiderable, yet the Bulk of the Water, in that State, equal to that of the Bottle, was now diminiſh'd two Grains: which plainly ſhews, That notwithſtanding the Water contain'd no Air that I could diſcover, yet there ſeems a Matter latent in it capable of Intumeſcence.

Of the weight of Bodies in Water, by the ſame, n. 320. *p.* 306.

II. I took a piece of Sheet Braſs (which I take to be more cloſe and ſolid than that which is caſt) of an exact ſquare Inch, weighing juſt 482 Grains; I then cut as many Inches of Braſs Tinſel, as were equal to the ſame Weight. The Number of theſe ſquare Inches were 255. Now theſe being of an equal weight with the other ſingle piece in common Air, I concluded from the inequality of their Surfaces that a diſproportion conſiderable in their ſpecifick Gravities would enſue, by weighing them in Water; the Water in one touching ſo many more parts of the Superficies than the other; and 'twas from what is generally aſſerted, that the ſmaller Bodies are, ſo the diſproportions of their Bulks to their Superficies encreaſe; and that ſuppoſing them infinitely ſmall or as Gold diſſolv'd in *Aqua Regis*, or Silver in *Aqua Fortis* muſt be, then their Superficies being touch'd by ſo many parts by the including *Menſtruum*, which is in ſuch a diſproportion to their Diameters or Bulks of Matter, as diſpoſes them to remain ſuſpended in it. This I take to be the general Solution of that Phænomenon, and theſe Conſiderations gave birth to that Experiment, yet when I came to bring it to the Teſt, I found to my great ſurprize, (being prepoſſeſt to the contrary) but two Grains difference; the ſingle piece weigh'd in the Water about 422 Grains; all the other Bodies together, hadly two Grains leſs: And this upon two or three Tryals ſucceeded much the ſame, notwithſtanding they were made with all the Caution imaginable. Now ſince ſo ſmall an Inequality is the Matter of Fact, between Bodies of the ſame Species weigh'd in Water, whoſe diſproportions of Surfaces are, as 1 to 255, (for I reckon the Sides of all the Tinſel Bodies to be equal to the ſides of the ſingle Braſs piece,) I muſt conclude, That thoſe Bodies muſt be infinitely ſmall, whoſe inequality of their Surfaces to their Bulks does exceed thoſe in this Experiment: For ſuppoſing one of theſe thin Squares ſhould be wrought into the form of a Globe, I am very apt to think, That the Diſproportion then of its Surface to its Bulk of Matter, would not be ſo great as its preſent form renders it.

Moreover, That although the Diſproportions of the Surfaces of Bodies to their bulk of Matter be very great; yet, that that is the only Reaſon why a Metallick Body ſhould be ſuſpended in a *Menſtruum* ſpecifically lighter than it ſelf, is very doubtful: For certainly if it was ſo, we might reaſonably have expected to have met with a much greater difference in the Bodies made uſe of in the newly recited Experiment: For there it ſhould ſeem neceſſary, that where we had ſo great a difference in point of Superficies, there we ſhould alſo have had a difference ſomething proportional in point of weight; which did not happen. I think therefore that there muſt be ſome other Agent or Quality, not only to aſſiſt, but to govern in the Caſe. And what we call a corroding *Menſtruum*, I take to be a Fluid adapted to attract ſuch, or ſuch a Body, (as we find no one of them to operate alike on all;) but, as I ſaid before, *Aqua Regis* for ſeparating the parts of Gold, *Aqua Fortis* for Silver: Now this Separation of their Parts by Attraction, ſeems to proceed from the *Menſtruum*'s Affection to the Body immers'd, and the Body reciprocally to the *Menſtruum*, and both to act on one another with greater Vigour, than either of their own Particles do upon their contiguous Fellows; by which means a Separation of Parts muſt (I think) conſequently follow. Thus being at liberty, they with the *Menſtruum* become as one Body, and remain ſuſpended in any part of it by their mutual Attraction. And that one *Menſtruum* in this Caſe ſhould affect one Body more than another, is no more than why the Magnet ſhould affect Iron only.

Different Denſities of common Water, by the ſame, n. 319. p. 267.

III. I caus'd a Quart of Water to be heated near ſcalding hot, and then put it into a convenient Glaſs with my Thermometer, the Spirit in which ſoon aroſe into the Ball a-top, where it remain'd till the Water cooling caus'd it to deſcend: by this time the Spirit in the Thermometer and the Water were become of an equal Temperature; and when it had deſcended to 130 Degrees above the Freezing Point, I began my Obſervations; which take as follows. I weighed a ſmall Bottle in't, and found the Bulk of Water equal to it in that State was 574 Grains. When the Spirit had deſcended to 80 Degrees above the Freezing Point, the Bulk of Water equal to the Bottle then weigh'd three quarters of a Grain more than before. At thirty degrees above the Freezing Point, the quantity of Water equal to the Bulk of the Bottle was again increaſed about three quarters of a Grain. At the Freezing Point, it weigh'd ſtill ſomething more; in all about two Grains from 130 Degrees above the Freezing Point, to that very Point. Which to me ſeems conſiderable, and ought to be taken notice of by ſuch Gentlemen, who judge of a Mineral or any other Water by its weight, when they have not an opportunity of making the Experiment at the Fountain-head; for there I ſuppoſe the Water is at the ſame Degree of Temperature at all Seaſons.

Now according to this Experiment, I find, that Water is condenſable by Cold one 28th part of the whole, from the greateſt Degree of Heat in this Climate. Suppoſing then, that the Water in the Sea ſhould ſuffer the ſame Alterations by the change of the different Seaſons, (as I ſee no reaſon but very nearly it muſt) it would be eaſy to compute, that a Ship which ſhould draw two Fathoms, or 12 Feet Water, in ſuch Weather as is underſtood by the greateſt Degree of Heat, would draw about half an Inch leſs from the greater Denſity of the Fluid, when reduc'd to the premention'd Degree of Cold; and conſequently would Sail better at that time.

Of the ſeeming ſpontaneous Aſcent of Water by the ſame, n. 319. p. 258.

IV. 1. *Exp.* I. The ſeveral Experiments, which already have been made in relation to the ſpontaneous Aſcent of Water in ſmall Tubes, not only by my ſelf, but ſeveral others, with much the ſame Succeſs, gave me the occaſion of thinking, whether the Figure of the Veſſel might, or might not, contribute to the Oddneſs of the Appearance. And to give my ſelf that Satisfaction, (an Account of which I thought would not be unacceptable to the Society) I proceeded as follows.

I procured a Couple of Glaſs Planes, about ſeven Inches long, and one and a half broad; theſe Planes were part of a broken Looking Glaſs; and notwithſtanding when clapt together, they ſeem'd to touch one another in ſo many parts, yet when they came to be immerſed in a Liquid, the Liquid would aſcend between them; but ſo thin and colourleſs it was, that it could not without difficulty be diſcerned, but upon the Separation of them, when they would be found wet on all their parts: therefore to make it more obvious, I put a ſmall piece of thin Paper on each corner; by which means, when laid one on the other, they became ſeparated by ſuch a diſtance, as is equal to the thickneſs of the Paper. In this manner I plung'd one end under ſome ſtrongly ting'd Liquor; where it no ſooner arriv'd, but the Water run (but not with that Velocity as in a ſmall Tube) gradually, ſometimes higher in one part than in another, ſhooting it ſelf very pleaſingly into Branches divers ways, and ſo would continue till it had arriv'd to its greateſt height; but that would be according to the diſtance the Planes were plac'd aſunder: for if, inſtead of one, two pieces of Paper were laid on each corner of the Planes, the Water then would not aſcend ſo high between them, as when they were ſeparated only by a ſingle one. And then, if the Planes were any ways declin'd, the Water would ſtill ſpread it ſelf farther and farther, agreeable to the degree of Declination: and this on ſeveral Tryals ſucceeded much the ſame.

Exp. II. Having ſeen the Succeſs of the former Experiment in the open Air, I was willing to try what appearance it would afford in *Vacuo*; accordingly I fixt the two Planes ſo to a Braſs Wire, (which paſt through the Cover of a Receiver) that I could make them deſcend at Pleaſure. In this manner, with a Diſh of ting'd Liquor, I convey'd them within the Receiver; which having plac'd on my Pump, I proceeded to exhauſt its contain'd Air, which the Gage, in a little time, diſcover'd to be pretty nicely done. Then I plung'd the Planes (ſeparated by Pieces of thin Paper as before) into the Water, where, as in the open Air, it aroſe between them; only with this difference, that there appear'd more Intervals, or Spaces, between the Branches of the aſcending Liquid, than in the former Experiment: but when I came to let in the Air, thoſe Intervals vaniſh'd, and an intire Body of the Liquid ſucceeded; yet the exact form of the upper parts of it remain'd unalter'd.

Exp. III. By the foregoing Experiments I found, that neither the Figure of the Veſſel, nor the Preſence of the Air, did any ways aſſiſt in the Production of the forementioned Appearance. To try therefore whether a quantity of Matter would help to unriddle the Miſtery; I produc'd two Tubes of an equal Bore, as near as I could, but of very unequal Subſtances, one of them being at leaſt ten times the thickneſs of the other; yet when I came to plunge them into the prementation'd Liquid, the Aſcent of it ſeem'd to be alike in both. Now ſince the form of the Veſſel, the preſence of the Air, or the quantity of Matter that compoſes the Veſſel, do not any thing contribute to the Production of the Phænomenon, it may not be amiſs, to inquire a little into the Nature and Property of ſome other Body, that operates with equal Vigour, under the pre-mentioned Circumſtances; and by a Compariſon of one with the other, we may at length arrive nearer to account for the ſame.

What I ſhall now uſe to compare with theſe Experiments, is the Magnet.

Firſt, A Magnet of any form will attract Iron.

So by the firſt Experiment, the Figure of the Veſſel ſeems no ways to contribute to the Aſcent of the Water.

Secondly, The Magnet is no ways leſſen'd in its vigour of Attraction, even in ſo thin a Medium as a *Vacuum*.

So by the ſecond Experiment we find the preſence of the Air to be no ways neceſſary to aſſiſt in the Aſcent of the Water, in ſmall Tubes, or between the Planes.

Thirdly, The Magnet, as ſuppoſe one of a Pound weight, that will take up or ſuſpend a piece of Iron of the like weight, and no more, (ſuppoſing it to be in every part of equal vertue) when ſeparated and broke into a number of ſmall parts, (imagining them not to weigh above half a Grain each) and theſe dreſt, and arm'd according to Art, will then be capable to ſuſpend fifty, nay perhaps a hundred times more the weight of Iron amongſt them now ſeparate, than they could when all of one Maſs; which appears to me, that the Attractive Quality of the Stone ſeems to be increas'd in Proportion as its Superficies is to its Bulk of Matter.

So by the third Experiment, I found that the Quantity of Matter, that was us'd to compoſe one Veſſel more than the other, ſignify'd nothing to the Aſcent of the Water, which ſeem'd wholly to depend on the largeneſs, or the ſmallneſs of their Cavities, as to the height it would ariſe in them; and as their Cavities are leſſen'd, ſo the Diſproportions of their inward Surfaces to their Cavities are increas'd.

And as the Magnet, when ſeparated into the prementioned number of ſmall Parts, will attract more than when united in one, and is no more than ſeparating or working the premention'd thick Body of Glaſs into a number of ſmall Tubes, that is multiplying the Surfaces; the Water then would ariſe in each of them ſingly, as it would when all in one Body, its Cavity being the ſame with the others; by which means, the quantity of Water aſcending in them is augmented from the ſame Quantity of Matter.

To conclude: There ſeems to be ſuch an agreeableneſs of the Qualities or Diſpoſitions of one with the other, that I ſee no reaſon why the Facts proceed not from one and the ſame Cauſe; for as the inward Surfaces of the Tubes are made ſmaller and ſmaller, ſo the Power of their Attraction (as is viſible by the higher Aſcent of the Water in them) is greater and greater, and is moſt demonſtrable by the Experiments of the Planes; for their inward *Area* being always the ſame, ſo that as they are placed nearer and nearer to each other, the Cavity or Space between them becomes leſs and leſs, and conſequently the Diſproportions are increaſed, whereby the Power of their Attraction is augmented.

Exp. VI. This Experiment I take to be very Analogous to thoſe lately made on the ſeeming ſpontaneous Aſcent of Water between Glaſs, Marble, and Braſs Planes, as alſo with thoſe made in Capillary Tubes;

ſince it ſeems to proceed from the ſame Principle, and ſubject to the ſame Laws, as appears by matter of Fact; which take as follows. I took a Glaſs Tube about 32 Inches long, the Diameter of its Cavity near three quarters of an Inch: This, when I had ty'd a piece of Linnen Cloth at one end, (to prevent the Aſhes from falling out) I proceeded to fill with Aſhes at the other: the Aſhes were fifted through a pretty fine Searſe. At every ſmall Portion I put in, I ramm'd them ſtrongly down with a Rammer, whoſe Baſis was very little leſs than the Bore of the Tube; by which means, I laid, or rather crouded them as cloſe together as poſſible. When the Tube was become full, I ty'd over that end of it by the Neck a ſmall and limber Bladder, having firſt expreſt all the Air out of its Body, in order to receive that Air, which I expected would be forc'd through the Aſhes upon the Aſcent of the Water. In this manner I plung'd the end of the Tube, to which I had ty'd the Linnen, (as it was,) under the ſurface of Water in a Glaſs, and found the Water preſently begin to Aſcend in it: It aroſe a pretty pace at firſt; for in 16 Minutes time it had aſcended near an Inch and three quarters: but as it aroſe higher, ſo its Progreſs became ſlower; for at the end of 24 Houts, the Water had attain'd but to 16 Inches; the Bladder at the top being near half fill'd with that Air which had deſerted the Aſhes as the Water aſcended in them. At the ſame time I found the upper part of the Tube, to which the Bladder was ty'd, to be crack'd round, and ſoon after drop'd off. However, I had the Satisfaction deſir'd. And ſo continuing the Experiment, I found at 24 Hours diſtance from the laſt Obſervation, the Water had aſcended in the Aſhes 6 Inches higher, which was very diſcernible by the change of Colour it gave them, diſtinct from thoſe that were dry.

Again, at the like diſtance of time from the laſt notice, the Water had ariſen 4 Inches and a half, and ſomething better. On the 4th Day, at the uſual time of Obſervation, it had aſcended 3 Inches higher: and when the following 24 Hours were finiſh'd, the Water reach'd within half an Inch of the top, by its aſcent of 2 Inches. About 10 Hours after, it had compleatly reach'd the Extremity of the Tube. Then deſiring to know what Quantity of Water the Aſhes had Abſorb'd, I weigh'd a Glaſs of Water nicely, part of which I pour'd into the Glaſs, in which the Tube had all along been kept, till it reach'd the Mark the Surface of the Water ſtood at, when the Tube was firſt plung'd into it; and found the quantity to be equal to the weight of 1792 Grains, which is nearly the Bulk of 7 Cubical Inches; the Capacity of the whole Tube, in which it aroſe, was equal but to about 13 Inches of the ſame Denomination. Now this Experiment to me ſeems ſurprizing enough from the following Obſervations.

Firſt, That the Water not only aſcended in the Aſhes, as between the premention'd Planes, and in the ſmall Tubes, contrary to its natural Gravitation; but with ſuch a Power too, as to force, and put to flight pretty ſtrongly impriſon'd Air, which was contain'd in the Interſtices of the arm'md Aſhes.

Secondly, That the removal of this impriſon'd Air could not be done without a Power ſurmounting its Reſiſtance, which muſt be great, ſince upon endeavouring to force Air through the Body of Aſhes by the ſtrength of my Breath, when the Tube was not above half filled, it prov'd unſucceſsful. Not but that I believe, if the ſame force had been continu'd for ſome time, it would have found its way through.

Thirdly, That the Water aſcends faſteſt at firſt, when there is a larger quantity of Interſtitial Air to remove, (if I may call it ſo,) than when the Column of the dry Aſhes grows ſhorter, by the higher Aſcent of the Water in them.

Fourthly, That notwithſtanding the Tube was rammed as full as it could with Aſhes, yet their Interſtices were ſo many, as to receive, or imbibe another Body, equal in bulk to above half the Content of the whole.

Fifthly, That the Water aroſe, not only in the Aſhes adjoyning to the inward Surface of the Tube, but equally in the whole Body of it, as I found upon Examination.

Sixthly, That the Air lodg'd in the Interſtices of the Aſhes, was protruded through them as the Water aſcended, was manifeſt by the Intumeſcence of the Bladder: And notwithſtanding the Accident of the Bladders falling off, I cannot but conclude, that the Quantity of it muſt be equal to the like bulk of Water which ſupplyed its place.

I repeated the ſame Experiment in *Vacuo*, in a Tube much about the ſame Diameter of the other, but not above 10 Inches in length: This Tube, being fill'd with Aſhes as before, was plac'd in *Vacuo*, where it remain'd ſome time, to give liberty for the Air contain'd in them to get away. Then plunging the lower end of the Tube under ſome Water, I found (as I expected,) that the Water aroſe faſter in the Aſhes in that Medium, than in common Air; for in about 4 Hours time, it had reach'd the Extream of its height; which plainly ſhews, that the Preſence of the Air is ſo far from being neceſſary in the Production of this odd Phænomenon, that it is a manifeſt Impediment to it.

ib. p. 265.

2. The Aſcent of Water in Capillary Tubes has been taken notice of ſome Years ago, but that it ſhould ariſe between two Glaſs Planes, whoſe Sides lie open to the Air, I had not ſo much as received a hint of before I firſt diſcover'd it. And I find that this *Phænomenon* is not to be ty'd up to Glaſs Bodies alone; for Stone, or Braſs, and for ought I know, moſt other Bodies that have ſmooth Surfaces, or that their Surfaces may become nearly contiguous to one another, may give the like Appearance; as is plain by the following Experiments. I procur'd a pair of Marble Planes, that were ground as true as the Workman could make them: Theſe when I had joyn'd together dry, without any thing between, I plung'd the edge of them about a quarter of an Inch under the Surface of the Water, and continu'd them ſo for ſome Minutes of time: then taking them out, I found I could not eaſily part them without ſliding them one from off the other; which when I had done, 'twas eaſily diſcoverable how far the

Water had made its way between them, which, upon divers Tryals, I have found different; but at all times, when I had newly rubbed over the Planes with Wood Aſhes, the Water would aſcend higheſt. Now whether the ſmall Duſt of the Aſhes adhering to the Planes may contribute any thing towards it; or that they better clear them from an oily or viſcous Matter, that may be communicated to them from our Hands, I cannot yet determine: However, whatever the occaſion is, the Matter of Fact is true. Then I took a pair of round Braſs Planes, and ordered them as before; the Succeſs of which was very agreeable with the former.

There is one thing I forgot to take notice of in a former Experiment; which is the Aſcent of Spirit of Wine, or Oyl of Turpentine between two Glaſs Planes, without any thing to ſeparate them. It cannot be imagined but that theſe Planes muſt touch each other in a multitude of Parts; yet for all that, and notwithſtanding they are held forcibly together, the Spirit of Wine will inſinuate, and aſcend ſeemingly in an entire Body, between all the contiguous parts of them, as before and after their Separation nothing appears to the contrary.

To the prementioned Experiments give me leave to add what I have ſince obſerved, in plunging the Planes in Spirit of Wine, Oil of Turpentine, and common Oil: That all theſe different Fluids aroſe between as the tinged Water; only with this difference, the common Oil very ſluggiſhly; it was near an Hour ariſing ſo high between them, as the other Liquids would in leſs than half a Minute. They all aroſe in an intire Body from ſide to ſide of the Planes, without thoſe Intervals or Spaces, which generally happen on the Aſcent of the Water. I likewiſe took a couple of round Glaſs Planes, and having laid them one on another, without Paper, or any thing elſe between to keep them ſeparate; in this manner I plung'd one edge juſt under the Surface of the tinged Liquor, and found the Water almoſt inſtantly had reach'd the Extreams of them in all parts: By which we find, that the Water not only aſcends directly upwards, but runs ſideways, obliquely, or in any direction.

Conf. Part III. *p.* 181. & *ſeqq. infra.*

Of the Motion of running Waters, by Dr. Jurin, n. 355. *p.* 748.

V. Aquæ Motum ex imi vaſis foramine defluentis ſæpe videmus, tum in ipſa re Hydraulica, tum in ejus Principiis ad Oeconomiam Animalem applicandis, aliis cum Potentiis comparari. Cujus Motus quantitatem cum hactenus nemo, quod ſciam, recte determinaverit, uſurpare ſolent ejus loco ſcriptores Hydraulici Columnæ aqueæ pondus foramini incumbentis. Quod qui faciunt, id ſane neutiquam animum advertunt fieri omnino non poſſe, ut Motus aliquis cum pondere quieſcente conferatur. Poterit autem Aquæ defluentis Motus facili opera definiri hunc in modum.

Plate 4. *Fig.* 21.

Sit $SHAHS$ Aquæ ſuperficies infinita, CC foramen circulare in fundo factum, AB recta perpendicularis per foraminis centrum ducta, $SGCCGS$ Columna ſive Cataracta Aquæ per foramen CC decurrentis, SGC

Curva, cujus rotatione circa Axem *A B* generatur Solidum, ſive Catacta, *S G C C G S.* Aqua enim cum libere, & motu accelerato deſcendat ad normam corporum omnium gravium, neceſſario in minorem amplitudinem contrahitur, prout majorem velocitatem acquirit inter cadendum, & profluit ex foramine *C C* ea cum velocitate, quæ cadendo ab altitudine *A B* comparatur.

Velocitas autem corporis gravis cadendo genita, ex *Galilæi* demonſtratis, rationem obtinet ſubduplicatam altitudinis, unde cecidit. Quare, ſi ducatur ad Curvam *S G C* Ordinata quævis *D E*, atque ipſa *D E* vocetur y, & *A D* x, exponetur velocitas Aquæ in ſectione *E E* per $\sqrt{x}$, & Factum ex ea velocitate ducta in ipſam ſectionem per $\sqrt{x} \times y^2$.

Quod Factum eſt ut moles Aquæ dato temporis ſpatio per eam ſectionem tranſeuntis; cumque eadem Aquæ moles dato tempore per ſingulas Cataractæ ſectiones tranſeat, proinde Factum iſtud perpetuo ſibi conſtabit, eritque $\sqrt{x} \times y^2 = 1$, & $x y^4 = 1$.

Quæ eſt Æquatio Curvæ *S G C*, cujus partem, intra datum vas comprehenſam, delineavit, ejuſdemque Æquationem non obſcure indicavit Magnus *Newtonus*, *Prop.* 36. *Libr.* 2. *Princip.* qui primus omnium veram Aquæ effluentis velocitatem, ex genuinis Principiis deductam, Orbi Literato expoſuit.

Eſt autem ipſa Curva Hyperboloeides quarti Ordinis, cujus altera Aſymptotos eſt recta *A S* ad Horizontem parallela, altera *A B* eidem perpendicularis.

Hujus Poteſtas eſt Quadrato-Cubus Ordinatæ *F G*, ductæ ad punctum *G*, ubi recta *A G*, biſecans angulum ab Aſymptotis comprehenſum, Curvæ occurrit.

Spatium *S A D E S*, inter Curvam *S G E*, Ordinatam *D E* & Aſymptotos *A D*, *A S* incluſum, æquale eſt quatuor partibus tertiis Rectanguli *H D*, ſub Abſciſſa *A D* & Ordinata *D E* contenti. Eſtque proinde Spatium *S H E* pars tertia ejuſdem Rectanguli.

Solidum *S G E E G S.* convolutione ſpatii *S A D E S*, circa Axem *A D*, generatum, duplum eſt Cylindri incumbentis ſectioni *E E.* Unde Solidum cavum, quod gignit converſio ſpatii *S H E G S*, circa eundem Axem, Cylindro incumbenti æquale eſt. Quæ omnia facili calculo inveniuntur per Methodum Fluxionum inverſam.

Theorema I. Aqua ex vaſe amplitudinis infinitæ, per foramen circulare in fundo factum, decurrente, Motus totius Cataractæ aqueæ Horizontem verſus æqualis eſt Motui Cylindri aquei, ſub ipſo foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis; vel æqualis eſt Motui molis Aquæ, quæ dato quovis tempore effluit, cujus ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale altitudini Aquæ.

Demonſtratio primæ partis. Ducatur ad Curvam *S G C* alia Ordinata *d e*, priori *D E* quamproxima.

Curva circa Axem *A B* converſa, generabunt Ordinatæ *D E*, *d e*, Circulos duos, quibus intercipitur Solidum naſcens *E E e e*. Id ſolidum æquale eſt Facto ex altitudine Dd ducta in ſectionem EE, & Motus ejus æquatur Facto ex ipſo ſolido ducto in velocitatem ejuſdem, ſive Facto ex altitudine Dd, ſectione EE, & velocitate Aquæ in ea Sectione. Cumque ſupra oſtenſum ſit, Factum ex quavis Sectione Cataractæ & velocitate Aquæ in ea Sectione, quantitatem eſſe conſtantem, erit proinde Motus totius Cataractæ æqualis Facto ex quantitate illa conſtante ducta in Summam omnium altitudinum Dd, ſive in ipſam AB, hoc eſt, Motui Cylindri ſub ipſo foramine & altitudine Aquæ, cujus velocitas æquet velocitatem Aquæ per foramen effluentis. *Q. E. D.*

Corol. 1. Data altitudine Aquæ, erit Motus Cataractæ in ratione foraminis.

2. Dato foramine, erit Motus Cataractæ in ratione ſeſcuplicata altitudinis, ſive in ratione triplicata velocitatis, qua Aqua per foramen exit.

3. Dato Motu Cataractæ, erit foramen reciproce in ratione ſeſcuplicata altitudinis, vel reciproce in ratione velocitatis triplicata.

Demonſtratio ſecundæ partis. Moles Aquæ dato tempore effluentis eſt ad Cylindrum ſub ipſo foramine & altitudine Aquæ, ut longitudo quam Aqua effluens æquabili velocitate dato iſto tempore percurſura ſit, ad altitudinem Aquæ. Cumque velocitas, quæ tribuitur moli Aquæ effluentis, ſit ad velocitatem Cylindri reciproce in eadem ratione, erunt Motuum quantitates utrinque æquales. *Q. E. D.*

Corol. 1. Data altitudine Aquæ & mole effluente, Motus Cataractæ eſt in ratione inverſa temporis quo iſta moles effluit.

2. Data altitudine & tempore, Motus Cataractæ eſt ut moles Aquæ tempore iſto effluentis.

3. Dato tempore & mole Aquæ effluentis, erit Motus Cataractæ in ratione altitudinis.

4. Dato Motu Cataractæ & altitudine, moles effluens eſt in ratione temporis.

5. Dato Cataractæ Motu & mole Aquæ effluentis, altitudo eſt ut tempus.

6. Dato tempore & Motu Cataractæ, erit Aquæ effluentis moles reciproce ut altitudo.

Theorema II. *Fig.* 22. Si capiatur BA, quæ ſit ad BD, ut DG^4 ad $DG^4 - BC^4$; Aqua decurrente ex dato vaſe Cylindrico ſemper pleno $GGEE$, per foramen circulare CC in fundo medio factum, Motus Cataractæ aquæ Horizontem verſus æqualis erit Motui Cylindri ſub foramine & altitudine AB, cujus velocitas æquet velocitatem Aquæ per foramen exeuntis; vel erit æqualis Motui molis Aquæ quæ dato quovis tempore effluit, cujuſque ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale altitudini AB.

Demonſtratio primæ partis. Ducatur AS ipſi DG parallela, & Aſymptotis AS, AB, per puncta G, C deſcripta concipiatur Curva *Newtoniana* SGC.

Ut conſtet Aquæ altitudo, ſupplendus eſt exeuntis locus Cylindro aqueo $ggGG$, deſcendente cum ea velocitate uniformi, quæ acquiritur cadendo ab A ad D, quemadmodum docet Vir incomparabilis Propoſitione prædicta.

Motui hujus Cylindri æquatur, per Theorema ſuperius, Motus Cataractæ *S S G G*. Ergo Motus Aquæ deſcendentis, cum ſit compoſitus ex Motu Cylindri aquei *g g G G*, & Motu Cataractæ *G G C C*, æqualis eſt Motui Cataractæ integræ *S G C C G S*, *h. e.* per Theorema primum, Motui Cylindri aquei ſub foramine & altitudine *A B*, cujus velocitas æqualis ſit velocitati Aquæ per foramen decurrentis. *Q. E. D.*

Pars ſecunda ſequitur ex priore.

Corol. 1. Oriuntur hinc omnia Propoſitionis præcedentis Corollaria, ſubſtituendo altitudinem *A B*, pro Aquæ altitudine

2. Si vas alia figura fuerit, atque Cylindrica; aut foraminis figura pro circulari fuerit quadrata, triangularis, vel qualiſcunque; aut ipſum foramen non ſit in medio fundo ſitum, vel etiam in latere vaſis factum; idem erit Motus Cataractæ, ſcilicet æqualis Motui Priſmatis aquei ſub foramine & altitudine *A B*, cujus velocitas par ſit velocitati Aquæ effluentis. Nam eadem Aquæ moles, cum eadem velocitate atque in priori Hypotheſi, tum per ipſum foramen, tum per ſingulas Cataractæ ſectiones tranſibit.

3. Si vaſis Diameter permagnam rationem obtineat ad Diametrum foraminis, negligi poterit altitudo *A D*, & vaſis ipſius altitudo pro altitudine Cylindri, vel Priſmatis aquei, uſurpari.

Hactenus caſum illum particularem, quo Aqua, Gravitatis vi, ex vaſe defluit, ſeorſum conſideravimus. Id eo fecimus lubentius, tum quod illum fere ſolum adhibere ſoleant Mathematici, quoties agitur de Fluidorum impetu, tum quod Curvæ Hyperbolicæ ſupra expoſitam proprietatem, qua Cataractam Aquæ deſcendentis format, non indignam cenſeamus contemplatione Geometrarum. Alioqui potuiſſet iſte caſus nullo negotio deduci ex Theoremate generali, quod proximo loco proponemus.

Theorema III. *Fig.* 23. Aqua fluente per Canalem plenum quemcunque *A B C D* ſecundum lineam *E F*, cui ſit perpendiculare utrumque Canalis orificium *A B* & *C D*, Motus Aquæ verſus Orificium *C D*, ſive Motus impedimenti, quod in ipſo orificio oppoſitum ſiſtat Motum totius Aquæ, æqualis eſt Motui Priſmatis aquei ſub qualibet Sectione Canalis *C H* & linea directionis, ſive longitudine Canalis *E F*, quod moveatur eadem cum velocitate, qua Aqua fluit per iſtam Sectionem: ſive æqualis Motui molis Aquæ, quæ dato quovis tempore effluit ex Canali, cujuſque ea ſit velocitas, qua percurratur eodem dato tempore ſpatium æquale longitudini Canalis.

Caſ. 1. Sit linea directionis recta quævis *E F*.

Facile demonſtratur pars prima eodem modo, quo Theorema primum. Eſt enim Factum ex quavis ſectione Canalis *C H*, & Aquæ velocitate in ea Sectione, quantitas conſtans.

Pars ſecunda ſequitur ex prima.

Caſ. 2. *Fig*: 24 Si linea directionis *A B C D E*, ex pluribus rectis *A B*, *B C*, *C D*, *E F*, ad ſeſe invicem inclinatis ſit compoſita, idem erit Aquæ Motus. Nam Motus Aquæ in toto Canali compoſito *A B C D E*, conficitur ex Motibus Aquæ in partibus Canalis *A B*, *B C*, *C D*, *D E*, additis ſibi invicem. Statuimus autem aquam fluentem ſecundum rectam *A B*, mutata iſta directione in aliam, qua feratur ſecundum rectam *B C*, nihil ex Motu deperdere. Leges enim illas, quæ in motu corporum ſolidorum obſervantur, quoties eorundem directio mutatur, fluida non ſequuntur. Alioqui fluidum, mutata directione in aliam priori perpendicularem, penitus ſiſteretur, quod Experimentis neutiquam deprehenditur. Aqua porro ex Vaſis foramine exiliens, ſive deorſum, ſive ſecundum Horizontis planum, ſive recta ſurſum feratur, eandem obtinet velocitatem. Quod ſi aliquando vel ratiocinio ſubtiliori, vel Experimentis innoteſcet, aliquam Motus imminutionem ex mutata directione proficiſci, erit ejuſdem ratio habenda.

Si Curva fuerit linea directionis *A B*, referetur ad hunc Caſum, quippe quæ ex pluribus rectulis confecta concipi queat. *Fig.* 25.

Caſ. 3. *Fig.* 26. Si diviſus fuerit Canalis *A B* in plures ramos *B C*, *B D*, *B E*, longitudine æquales, eadem ratione invenietur Aquæ Motus, uſurpando pro linea directionis longitudinem *A B D*, compoſitam ex longitudine Canalis principis *A B*, & longitudine cujuſvis rami *B D*. Perinde autem eſt, ſive Aqua a Canali principe verſus ramos, ſive a ramis fluxerit verſus principem Canalem. Quod ſi rami fuerint inæquales, inveniendus eſt Motus Aquæ in ſingulis ramis, adhibendo pro linea directionis longitudinem confectam ex longitudine cujuſque rami, & longitudine principis Canalis.

Nullo negotio deducitur ex Caſu ſecundo.

Caſ. 4. *Fig.* 27. Si rami æquales, in quos diſtributus eſt Canalis *A B*, iterum in Canalem unicum *F G* colligantur, ad Motum Aquæ inveniendum adhibenda eſt pro linea directionis longitudo integra *A B D F G*, confecta ex longitudine principis Canalis *A B*, rami cujuſvis *B D F*, & Canalis recompoſiti *F G*. Si Rami ſint inæquales, inveniendus eſt in ſingulis Aquæ Motus, & eorum Motuum Summa Motui Aquæ in Canali recompoſito addendus. Sequitur ex Caſu 2, & 3.

Corol. 1. Data longitudine Canalis, & qualibet Sectione ejuſdem, erit Motus Aquæ in ratione velocitatis, qua Aqua fluit per iſtam Sectionem.

2. Data quavis Sectione, & velocitate Aquæ Sectionem iſtam præterfluentis, erit Motus Aquæ ut longitudo Canalis.

3. Data Canalis longitudine, & velocitate Aquæ in quavis Sectione, erit Aquæ Motus in ratione illius Sectionis.

4. Dato Motu Aquæ, & aliqua Sectione, erit longitudo Canalis in ratione inverſa velocitatis.

5. Dato Aquæ Motu, & longitudine Canalis, erit Sectio quævis reciproce ut velocitas.

6. Data velocitate in qualibet Sectione, & Motu Aquæ, erit iſta Sectio in ratione reciproca longitudinis.

7. Data longitudine Canalis, & mole Aquæ certo quovis tempore effluentis, erit Aquæ Motus reciproce ut iſtud tempus.

8. Data Canalis longitudine, & tempore, erit Aquæ Motus ut moles effluens.

9. Dato tempore, & mole Aquæ effluentis, erit Motus ut longitudo Canalis.

10. Dato Motu Aquæ, & longitudine Canalis, moles effluens eſt in ratione temporis.

11. Dato Aquæ Motu, & mole effluente, erit tempus ut longitudo Canalis.

12. Dato tempore, & Motu Aquæ, erit moles effluens reciproce ut longitudo Canalis.

13. Si binæ moles Aquæ motu contrario in directum occurrant, & pares ſint utrinque tum ſuperficies quibus in ſe invicem impingant, tum velocitates quibus iſtæ ſuperficies in adverſum moveantur, fuerit autem altera moles Aquæ guttulæ uni æqualis, altera Aqua omnis Oceano contenta, vel etiam quantitas Aquæ infinita; fieri poteſt ut una iſta guttula Aquam omnem Oceani, vel quantitatem Aquæ infinitam, non ſolum ſuſtineat, ſed poſt occurſum, eadem ac prius velocitate, ipſa in plagam eandem moveri pergat, eadem illam in partes contrarias repellat. Quod eſt mirabile paradoxon in re Hydraulica.

14. Si certa moles Aquæ, per canalem ex tubis duobus cylindricis, Diametro inæqualibus, compoſitum, a tubo ampliore verſus anguſtiorem fluat, & motus Aquæ neque minuatur inter fluendum neque augeatur, ſimul ac prima pars Aquæ tubi minoris initium ingreſſa fuerit, ſtatim tardius fluere incipiet, & continuato effluxu ex tubo latiore in anguſtiorem, gradatim magis retardabitur Aqua in tubo anguſtiore, uſque dum tota in eum tubum pervenerit Contrario modo res eveniet, fluente aqua a tubo minore verſus ampliorem. Quod eſt alterum Paradoxon in re Hydraulica. Ponitur autem aqua ubique ſibi cohærere.

Oriuntur bina iſta Corollaria ex Caſu 1.

15. Ex Caſu ſecundo datur Methodus æſtimandi Motum Sanguinis in qualibet arteria.

16. Datis quibuſcunque arteriis binis, æqualem Sanguinis molem tranſmittentibus, major eſt impetus Sanguinis in arteria a Corde remotiore quam in propiore. Quod eſt Paradoxon notatu dignum in Oeconomia animali.

17. Ex Caſu tertio oritur alterum Paradoxon in Oeconomia animali, nempe majorem eſſe Sanguinis motum ſive impetum, in arteriis omnibus Capillaribus ſimul ſumptis, quam in ipſa aorta. Item, major eſt in Capillaribus Venis, quam arteriis.

18. Ex Caſu quarto deducitur Methodus definiendi motum Sanguinis in quavis Vena.

19. Ex eodem deducitur tertium in Oeconomia Animali Paradoxon, nempe majorem eſſe Sanguinis impetum in Vena quavis, quam in arteria ei Venæ reſpondente, & proinde majorem eſſe in Vena Cava, quam in aorta.

Problema I. Invenire motum Aeris ex Pulmone effluentis.

Sit $l =$ Longitudo totius ductus aerei, ab Ore & Naribus ad extremos ramos Trachææ.

$q =$ Quantitas Aeris mediocri exſpiratione ex Pulmone emiſſa.
$Q =$ Aeris copia validiſſima exſpiratione expulſi.
$t =$ Tempus mediocris exſpirationis.
$T =$ Tempus exſpirationis fortiſſimæ.

Inde, per Theorema 3, *Caſ.* 3. Motus Aeris ex Pulmone effluentis, in exſpiratione mediocri $= \frac{q\,l}{t}$

fortiſſima $= \frac{\mathfrak{Q}\,l}{T}$.

Hoc eſt, Motus Aeris ex Pulmone exeuntis æqualis eſt motui molis Ae is, quæ unica exſpiratione emittitur, cujus ea ſit velocitas; qua percurratur tempore exſpirationis longitudo totius Canalis Aerei $\mathfrak{Q}$. *E. I.*

Aeris quantitatem exſpiratione mediocri emiſſam Vir Clariſſimus, *Alphonſus Borellus*, facto Experimento 18 circiter, vel 20 unciis cubicis definit. Eſt autem diverſa, non ſolum in diverſis Hominibus, ſed etiam temporibus diverſis, in Homine eodem. Ipſe Experimentum in hunc modum inſtitui.

Veſicæ madefactæ a parte inferiore pondus appendebam, & aptato eidem ſuperius tubo vitreo Diametro circiter unciali, naribus obturatis Aerem veſicæ leniter inſpirabam, per ſpatium trium minutorum ſecundorum, pondere interim in menſa quieſcente. Poſtea Veſicam cum Aere incluſo & pondere appenſo, ſub aquam in vaſe Cylindrico contentam, demergebam, notata diligenter altitudine, ad quam aqua attollebatur. Deinde, Aere ex veſica expreſſo, iterum eandem cum pondere in Aquam immittebam. Quod cum eſſet factum, facile inveniebatur aquæ moles, quæ vaſi infuſa altitudinem prius notatam conficeret. Experimento decies repetito, & additis ſibi invicem quantitatibus ſingulis inventis, earum decima, ſive media moles aquæ vaſi infuſa, reperiebatur 35 unciis cubicis æqualis. Quæ moles eſt Aeris veſica contentæ; & adjecta circiter parte duodecima, ſeu 3 unciis cubicis, ob Aeris condenſationem a frigore Aquæ factam, cum tempeſtas fuerit hyemalis, efficiuntur 38 unciæ cubicæ. Præterea addendum eſt tantillum, tum propter Aquæ preſſionem in veſicam, tum ob Vaporem qui cum halitu emittitur in humorem coactum; quod fiat neceſſe eſt ex frigore Aquæ, & veſicæ madidæ contactu. Æſtimavi igitur Aeris copiam, leni exſpiratione emiſſam tempore trium minutorum ſecundorum, numero rotundo 40 unciarum cubicarum.

In exſpiratione validiſſima exſpirabam uncias cubicas 125, tempore minuti ſecundi unius.

Hujuſmodi autem exſpiratione, cum vehementi Pulmonis contentione ad ſtrangulatum fere continuata, 220 uncias cubicas ex Pectore emittebam. Unde patet, ut id obiter moneam, multo plus Aeris in Pectore ſupereſſe, quam unica exſpiratione mediocri emitti.

Si ergo ponatur $l = 2$ pedes

$q = 40$ unciæ cubicæ

$\mathfrak{Q} = 125$ unciæ cubicæ

$t = 3''$

$T = 1''$

Aeris Gravitas Specifica ad Gravitatem Aquæ, ut 1 ad 1000.

Pes Aquæ cubicus $= 1000$ unc. *Avoird.*

Erit Motus mediocris Aeris Pulmone exeuntis æqualis motui ponderis Scrupulorum 4 & Granorum 9, quod percurrat unciam unam minuto ſecundo; vel motui ponderis Grani $1\frac{1}{3}$, quod eodem tempore conficiat longitudinem 5 pedum & 7 unciarum. Quæ eſt velocitas Aeris per Laryngem effluentis, poſita Laryngis Sectione $= \frac{1}{5}$ unciæ quadratæ.

Motus maximus Aeris Pectore expulſi æquatur motui ponderis unciæ $1\frac{3}{4}$ circiter, percurrentis unciam unam minuto ſecundo; ſive motui ponderis grani $1\frac{1}{3}$ percurrentis eodem tempore 52 pedes. Quæ eſt velocitas Aeris in fortiſſima exſpiratione per Laryngem erumpentis.

Corol. 1. Data Aeris copia & longitudine Canalis aerei, motus Aeris eſt in ratione inverſa temporis exſpirandi.

2. Data mole Aeris & tempore, erit motus in ratione directa longitudinis.

3. Data longitudine & tempore, motus eſt ut Aeris copia.

4. Dato motu & Aeris copia, erit longitudo in ratione directa temporis.

5. Dato motu & longitudine, erit Aeris moles directe ut tempus.

6. Dato motu & tempore, erit Aeris moles reciproce ut longitudo Canalis Aerei.

7. Motus Aeris eſt in ratione compoſita ex ratione quadruplicata Diametri cujuſvis homologæ ipſius animalis, & ratione inverſa temporis exſpirandi; vel in ratione compoſita ex ratione ponderis totius animalis, ratione ejuſdem ponderis ſubtriplicata, & ratione temporis reciproca.

Nam pondus animalis, Diametri cujuſvis homologæ Cubus & moles Aeris expulſi ſunt in eadem ratione. Ponitur autem Corpora Animalium Machinas eſſe ſimiliter factas.

Scholium. Longitudinem hic uſurpatam, vel ipſam eſſe concipies Canalis aerei longitudinem, ſi Rami omnes Trachææ longitudine æquales ponantur; vel mediam inter longitudines diverſas, ſi Rami ſint inæquales.

Problema II. Determinare impetum, ſive impreſſionem quam excipit interna Pulmonum ſuperficies ab Aere exſpirando.

Cum actioni æqualis & contraria ſit reactio; neceſſe eſt, ut, quanto motu urgetur ab interna Pulmonum ſuperficie Aer exſpirandus, tanto viciſſim ab Aere repellatur ſuperficies Pulmonum.

Unde, per Problema ſuperius, impetus dictus in exſpiratione me-

$$\text{diocri} = \frac{ql}{t}$$

$$\text{fortiſſima} = \frac{Ql}{T}\ Q.\ E.\ I.$$

Hinc poſitis iiſdem quæ in ſuperiore ponuntur, impetus mediocris Aeris in Pulmones æqualis eſt motui ponderis drachmæ circiter $1\frac{1}{2}$, quod minuti ſecundi ſpatio percurrat unciam unam; vel motui ponderis 19 librarum, conficientis eodem tempore $\frac{1}{1642}$ unciæ, quæ eſt velocitas Aeris in contactu ſuperficiei Pulmonis internæ. Ponimus autem cum Viro Doctiſſimo *Jacobo Keilio* ſuperficiem Pulmonis internam 21900 circiter unciis quadratis æqualem.

Impetus vero maximus Aeris in Pulmones æquatur motui ponderis unciæ circiter $1\frac{3}{4}$ moti unciam unam minuto ſecundo; vel motui ponderis 19 librarum, quod partem $\frac{1}{75}$ unciæ conficiat eodem tempore. Quæ eſt Aeris velocitas ad ſuperficiem Pulmonis in exſpiratione vehementi.

Corol. 1 Sequuntur ex hac Propoſitione Corollaria præcedenti ſubjuncta.

2. Impetus mediocris incumbens in partem ſuperficiei Pulmonis, quæ ſit ipſi Laryngis Sectioni æqualis, eſt motus ponderis $\frac{1}{21}$ grani, conficientis unciæ ſpatium minuto ſecundo, vel motus grani $1\frac{1}{3}$ quod eodem tempore percurrat unciæ partem $\frac{1}{1642}$. Impetus autem maximus in parem ſuperficiem eſt motus ponderis $\frac{1}{1321}$ partis grani quod unciam unam; vel motus ponderis grani $\frac{1}{3}$ quod $\frac{1}{175}$ unciæ ſingulis minutis ſecundis conficiat.

3. Impetus Aeris in mediocri exſpiratione in Pulmones impreſſus, æquatur motui Columnæ aqueæ percurrentis unciam unam minuto ſecundo, cujus Columnæ baſis eſt ipſa Pulmonum ſuperficies, interna altitudo autem eſt $\frac{1}{68658}$ unciæ. Eſtque Columnæ altitudo pars $\frac{1}{7303}$ unciæ, in exſpiratione omnium vehementiſſima.

4. Impetus incumbens in ſuperficiem parem circulo maximo Globuli Sanguinei in leni exſpiratione, eſt pars $\frac{1}{14}$ ponderis Globuli Sanguinei; in exſpiratione vehementi $\frac{2}{3}$ ejuſdem ponderis, moti unciam unam minuto ſecundo. Qua autem ratione Diametros Globulorum Sanguinis dimenſus ſim, cum uſui eſſe queat ad aliorum Objectorum minimorum magnitudines definiendas, libet obiter exponere. Capillum tenuem, & ſatis longum aciculæ pluries circumvolvi, ut omnes convolutiones ſeſe invicem accurate contingerent, quod admotum ſubinde Microſcopium luculenter oſtendebat. Deinde cum intercapedinem inter extremas utrinque circumvolutiones Circino cepiſſem, eandem Scalæ, quam vocant, Diagonali applicabam, ſpatiumque in Scala repertum per convolutionum numerum dividebam. Unde inventa eſt unius convolutionis latitudo, ſive ipſa Capilli Diameter. Poſtea Capillum eundem, in Segmenta minutula diviſum, plano Microſcopii, cui Sanguinis parum erat illitum ut Globuli conſpicerentur diſtincti, ſuperinſpergebam. Ea cum Microſcopio contuerer, reperiebam aliquibus in locis Capilli Segmenta ita commode diſpoſita, ut numerare liceret, quot Globuli Diametro Segmenti opponerentur. Erant autem Segmenta Diametro inæqualia, quod Capillus tenuior verſus extremum fuerit, quam propius a Radice, adeo ut jam 7, vel 8, jam 12, 13ve Globuli tranſverſæ Sectioni Capilli reſponderent Utroque autem Experimento ſæpius iterato, æſtimavi tandem mediam Capilli Diametrum parte $\frac{1}{324}$ unciæ, & Diametrum Globuli Sanguinei parte decima Diametri Capilli, ſive parte $\frac{1}{3240}$ unciæ.

5. Impetus, quem patitur interna Pulmonum ſuperficies ab Aere exſpirando, minor eſt Motu leniſſimi roris e Cœlo decidentis.

Scholium. Neglecta eſt in ſolutione Problematum duorum præcedentium impedimenti conſideratio, quod Aeri egredienti objicitur ex affrictu laterum Arteriæ Trachææ, ejuſque ramorum; cum perparvum ſit, neque ullo experimento ſatis accurate æſtimari poſſe videatur. Nec fuimus admodum ſolliciti de rationibus numerorum exquiſite ſervandis, cum id unum nobis propoſitum fuerit, ut methodum exponeremus æſtimandi, aliquanto certius quam antehac factum, vires eas, quibus agit Aer inter exſpirandum in vaſa ſanguinea ſuperficiem Pulmonis internam perreptantia. Unde dignoſci poteſt, utrum pares ſint hæ vires effectis iſtis producendis, qui iiſdem a Doctiſſimis quibuſdam Scriptoribus Medicis tribuuntur. Quod liberum eſto Lectoris Scientia Mechanica & Anatomica inſtructi Judicium.

Problema III. Definire impetum Sanguinis in Vena Cava prope dextram Auriculam Cordis; ſive motum Sanguinis per omnes Arterias & Venas fluentis, præter Pulmonares.

Sit $q=$ Quantitas Sanguinis una Cordis Syſtole in Aortam projecti.
$l=$ Longitudo media ductus integri Arterio-Venoſi ratione habita ramorum longiorum & breviorum.
$t=$ Temporis ſpatium inter binos pulſus interceptum.

Inde, per Theorema 3. Caſ. 4. impetus quæſitus $=\frac{ql}{t}$.

Hoc eſt, Impetus Sanguinis in Vena Cava æquatur motui molis Sanguineæ, quæ una Syſtole in Aortam projicitur, cujus ea ſit velocitas, qua percurri queat integra Arteriarum & Venarum longitudo temporis ſpatio inter binos Pulſus intercepto. *Q. E. I.*

Si in Corpore Humano ponantur
$q=2$ unciæ *Avoird.*
$l=6$ pedes
$t=\frac{3''}{4}$

Erit impetus Sanguinis in Vena Cava æqualis motui ponderis 12 librarum, quod unciæ unius longitudinem conficiat ſingulis minutis ſecundis; ſeu motui ponderis 2 librarum, quod pari temporis ſpatio percurrat pedem $\frac{1}{2}$. Quæ eſt fere Sanguinis velocitas in Cava fluentis. Ponimus autem, ex dimenſione Viri Doctiſſimi ſupradicti, Cavæ Sectionem dodrantem eſſe unciæ quadratæ.

Corol. Oriuntur ex hoc Problemate mutatis mutandis omnia Problematis primi Corollaria.

Problema IV. Determinare motum abſolutum Sanguinis in Vena Cava; ſive motum Sanguinis, per omnes Arterias & Venas fluentis præter Pulmonales, ſublata Vaſorum reſiſtentia.

Sit velocitas Sanguinis Naturalis, ad eam velocitatem qua Sanguis flueret, dempta omni reſiſtentia, ut 1 ad x. Cumque per *Corol.* ſuperioris Problematis, & *Corol.* 1. *Probl.* 1. Motus Sanguinis ſit in ratione velocitatis, erit inde motus quæſitus $=\frac{xql}{t}$. *Q. E. I.*

Quod ſi proportio per Experimentum a Viro Clariſſimo ſupra laudato inſtitutum inventa, ut veræ propinqua, admittatur, erit $x=2.5$.

Unde poſitis iiſdem quæ in ſuperiore ponuntur, motus abſolutus Sanguinis in Vena Cava æquatur motui ponderis 30 librarum, quod minuto ſecundo longitudinem uncialem percurrat; ſive motui ponderis 2 librarum percurrentis eodem tempore pedem $1\frac{1}{4}$. Qua fere velocitate Sanguis, omni reſiſtentia liber, per Cavam deferretur.

Problema V. Motum Sanguinis invenire in Vena Pulmonali prope ſiniſtram Cordis Auriculam; ſive motum totius Sanguinis per Pulmonem fluentis.

Præter notulas in *Probl.* 3. uſurpatas, ſit λ=Canalis Arterio-Venoſi Pulmonici media longitudo.

Unde, per *Theor.* 3. *Caſ.* 4. invenitur motus quæſitus$=\frac{q\lambda}{t}$.

Hoc eſt, motus Sanguinis per Pulmonem fluentis æqualis eſt motui molis Sanguineæ, quæ una Syſtole in Arteriam Pulmonalem projicitur, obtinentis eam velocitatem, qua percurratur longitudo Arteriarum ac Venarum Pulmonalium, tempore inter duos Pulſus intercepto. *Q. E. I.*

Si ponatur in Corpore Humano $\lambda = 1\frac{1}{2}$ pes.

Erit motus Sanguinis in Pulmone æqualis motui ponderis 3 librarum, percurrentis unciale ſpatium minuto ſecundo.

Problema VI. Definire momentum Sanguinis abſolutum in Vena Pulmonali.

Eodem Argumento, quod in *Probl.* 4. uſurpatum eſt, invenitur motus quæſitus $=2.5\times\frac{q\lambda}{t}$. *Q. E. I.*

Poſitis vero iiſdem quæ ſupra ponuntur, motus abſolutus Sanguinis Pulmonem præterfluentis æquatur motui ponderis $7\frac{1}{2}$ librarum, quod ſingulis minutis ſecundis unciæ unius ſpatium percurrat.

Scholium. Experimento *Keiliano* definita eſt proportio, quam obtinet Sanguinis per Aortam ejuſque ramos fluentis velocitas naturalis ad eam velocitatem qua Sanguis per eoſdem flueret, ſublata reſiſtentia Arteriarum & Sanguinis præcedentis. Eam nos proportionem ad Sanguinem per Arteriam Pulmonalem fluentem transtulimus. Quia vel ſublata vel imminuta ſecundum quamvis rationem reſiſtentia, quæ Sanguini per utramque Arteriam fluenti objicitur, neceſſario Sanguis pariter acceleratur in utraque Arteria. Id enim niſi fiat, bini Cordis Ventriculi aut eodem tempore non contrahentur, aut eandem Sanguinis quantitatem non ejicient. Quorum utrumvis, abſque ſumma totius Machinæ perturbatione & discrimine, fieri omnino non poteſt.

Corol. Ad tria Problemata præcedentia.

Sequuntur hinc Corollaria Problemati quinto ſubjuncta, mutatis mutandis.

Scholium ad quatuor Problemata ſuperiora.

Notandum Sanguinis velocitatem, tum per Pulmonem, tum per reliquum Corpus fluentis, cum reipſa æquabilis non ſit, hic tamen talem fingi, ut motus Sanguinis medius inveniatur.

The Art of living under Water, by Dr. Halley. *n.* 349. *p.* 492.

VIII. There have been many Methods propos'd, and Engines contriv'd for enabling Men to abide a comperent while under Water; and the Reſpiring freſh Air, being found abſolutely neceſſary to maintain Life, ſeveral ways have been thought of for carrying this *Pabulum Vitæ* down to the Diver, who muſt, without being ſomehow ſupply'd with this, return very ſoon or periſh.

We have heard of the Divers for Spunges in the *Archipelago* helping themſelves by carrying down Sponges dipt in Oyl in their Mouths; but conſidering how ſmall a Quantity of Air can be ſuppos'd to be contain'd

in the Pores or Interſtices of a *Spunge*, and how much that little will be contracted by the Preſſure of the *incumbent Water*, it cannot be believed that a Supply, by this means obtained, can long ſubſiſt a Diver. Since by Experiment it is found that a *Gallon* of *Air*, included in a *Bladder*, and by a Pipe reciprocally inſpired and exſpired by the *Lungs* of a Man, will become unfit for any further Reſpiration, in little more than one Minute of Time; and though its Elaſticity be but little altered, yet in paſſing the *Lungs*, it loſes its *vivifying Spirit*, and is rendred effete, not unlike the *Medium* found in *Damps*, which is preſent Death to thoſe that breath it; and which in an inſtant extinguiſhes the brighteſt *Flame*, or the ſhining of glowing *Coals* or red hot *Iron*, if put into it. I ſhall not go about to ſhew what it is the *Air* loſes by being taken into the *Lungs*, or what it communicates to the *Blood* by the extreme ramifications of the *Aſpera Arteria*, ſo intimately interwoven with the Capillary *Blood-Veſſels*; much leſs to explain how 'tis performed, ſince no diſcovery has yet been made to prove that the ultimate Branches of the *Veins* and *Arteries* there, have any *Anaſtomoſes* with thoſe of the *Trachæa*; as by the *Microſcope* they are found to have with one another. But I rather chooſe to leave this Enquiry to the Curious *Anatomiſt*, to whom the Structure of the *Lungs* is better underſtood; and ſhall only conclude from the aforeſaid Experiment, that a naked Diver, without a *Spunge*, may not be above a couple of Minutes encloſed in Water, (as I once ſaw a *Florida-Indian* at *Bermudas*) nor much longer with a *Spunge*, without Suffocating; and not near ſo long without great Uſe and Practice: ordinary Perſons generally beginning to ſtifle in about half a Minute of Time. Beſides if the Depth be conſiderable, the preſſure of the *Water* on the Veſſels is found by Experience to make the Eyes Blood-ſhot, and frequently to occaſion ſpitting of Blood.

When therefore there has been occaſion to continue long at the *Bottom*, ſome have contrived double flexible Pipes, to circulate *Air* down into a Cavity encloſing the *Diver* as with Armour, to bear off the preſſure of the *Water*, and to give leave to his Breaſt to dilate upon *Inſpiration*: the freſh *Air* being forced down by one of the Pipes with *Bellows* or otherwiſe, and returning by the other of them; not unlike to an *Artery* and *Vein*. This has indeed been found ſufficient for ſmall Depths, not exceeding twelve or fifteen Foot: but when the Depth ſurpaſſes three Fathoms, Experience teaches us that this Method becomes impracticable: for though the Pipes and the reſt of the *Apparatus* may be contrived to perform their Office duly, yet the *Water*, its weight being now become conſiderable, does ſo cloſely embrace and claſp the Limbs that are bare, or covered with a flexible Covering, that it obſtructs the Circulation of the *Blood* in them; and preſſes with ſo much force on all the Junctures, where the *Armour* is made tight with *Leather Skins*, or ſuch like, that if there be the leaſt defect in any of them, the whole Engine will inſtantly fill with Water, which will ruſh in with ſo much violence, as to endanger the Life of the Man below, who may be drown'd before he can be drawn up, Upon both which Accounts, the Danger encreaſes with the Depth. Beſides a Man thus ſhut up in a weighty *Caſe*, as this muſt needs be, cannot but be very unweildy and unactive, and therefore unfit to do what he is deſigned to do at the Bottom.

To remedy theſe Inconveniencies, the *Diving-Bell* was next thought of; wherein the *Diver* is ſafely conveyed into any reaſonable Depth, and may ſtay more or leſs time under *Water*, according as the *Bell* is of greater or leſſer Capacity. This is moſt conveniently made in form of a *Truncate Cone*, the ſmaller *Baſis* being cloſed, and the larger open; and ought to be ſo poiſed with *Lead*, and ſo ſuſpended, that the Veſſel may ſink full of *Air*, with its greater or open *Baſis* downwards, and as near as may be in a Situation parallel to the *Horizon*, ſo as to cloſe with the Surface of the Water all at once Under this Couvercle the Diver ſitting, ſinks down together with the included *Air* into the Depth deſired; and if the Cavity of the Veſſel may contain a Tun of *Water*, a ſingle Man may remain therein at leaſt an Hour, without much Inconvenience, at five or ſix Fathoms deep. But this included Air, as it deſcends lower, does contract it ſelf according to the Weight of the Water that compreſſes it; ſo as at thirty three Foot deep or thereabouts, the Bell will be half full of Water, the Preſſure of it being then equal to that of the whole Atmoſphere: and at all other Depths, the ſpace occupied by the compreſt Air in the upper part of the Bell, will be to the under Part of its Capacity fill'd with Water, as thirty three Feet to the Depth of the Surface of the Water in the Bell below the common Surface thereof. And this condenſed Air, being taken in with the Breath, ſoon inſinuates it ſelf into all the Cavities of the Body, and has no ſenſible Effect, if the Bell be permitted to deſcend ſo ſlowly as to allow time for that purpoſe. The only Inconvenience that attends it, is found in the Ears, within which there are Cavities opening only outwards, and that by Pores ſo ſmall as not to give admiſſion even to the Air itſelf, unleſs they be dilated and diſtended by a conſiderable Force. Hence on the firſt Deſcent of the Bell, a Preſſure begins to be felt on each Ear, which by degrees grows painful, as if a Quill were forcibly thruſt into the Hole of the Ear; till at length, the Force overcoming the Obſtacle, that which conſtringes theſe Pores yields to the Preſſure, and letting ſome condenſed Air ſlip in, preſent Eaſe enſues. But the Bell deſcending ſtill lower, the Pain is renewed, and again eaſed after the ſame manner. On the contrary, when the Engine is drawn up again, the condenſed Air finds a much eaſier Paſſage out of thoſe Cavities, and even without Pain. This Force on the auditory Paſſages might poſſibly be ſuſpected to be prejudicial to the Organs of Hearing, but that Experience teaches otherwiſe But what is more inconvenient in this Engine is the Water entring into it, ſo as to contract the Bulk of Air (according to the aforeſaid Rule) into ſo ſmall a ſpace, as that it ſoon heats and becomes unfit for Reſpiration, for which reaſon it muſt be often drawn up to recruit it; and beſides, the Diver, being almoſt covered with the Water thus entring into his Receptacle, will not be long able to endure the Cold thereof.

Being engaged in an Affair that required the Skill of continuing under Water, I found it neceſſary to obviate theſe Difficulties which attend the uſe of the common Diving-Bell, by inventing ſome means to convey Air down to it, whilſt below; whereby not only the Air included therein would be refreſh'd and recruited, but alſo the Water wholly driven out, in whatever Depth it was. This I effected by a Contrivance ſo eaſy, that it may be wondred it ſhould not have been thought of ſooner, and capable of furniſhing Air at the bottom of the Sea in any quantity deſired. The Deſcription of my *Apparatus* take as follows.

The Bell I made uſe of was of Wood, containing about 60 Cubick Foot in its Concavity, and was of the form of a Truncate Cone, whoſe Diameter at the Top was three Foot, and at Bottom five. This I coated with Lead ſo heavy that it would ſink empty, and I diſtributed the weight ſo about its bottom, that it would go down in a perpendicular Situation and no other. In the Top I fixed a ſtrong but clear Glaſs, as a Window to let in the Light from above; and likewiſe a Cock to let out the hot Air that had been breathed; and below, about a Yard under the Bell, I placed a *Stage* which hung by three Ropes, each of which was charged with about one hundred Weight, to keep it ſteddy. This Machine I ſuſpended from the Maſt of a Ship, by a *Spritt* which was ſufficiently ſecured by *Stays* to the Maſt-head, and was directed by *Braces* to carry it over-board clear of the Ship-ſide, and to bring it again within-board as occaſion required.

To ſupply Air to this Bell when under Water, I cauſed a couple of Barrels of about 36 Gallons each, to be caſed with Lead, ſo as to ſink empty; each having a Bung-hole in its loweſt Parts to let in the Water, as the Air in them condenſed on their Deſcent; and to let it out again when they were drawn up full from below, And to a Hole in the uppermoſt part of theſe Barrels I fixed a Leathern Trunk or Hoſe, well liquored with Bees-Wax and Oyl, and long enough to fall below the Bung-hole, being kept down by a Weight appended; ſo that the Air in the upper Part of the Barrels could not eſcape, unleſs the lower ends of theſe Hoſe were firſt lifted up.

The Air-Barrels being thus prepared, I fitted them with Tackle proper to make them riſe and fall alternately, after the manner of two Buckets in a Well; which was done with ſo much eaſe, that two Men with leſs than half their Strength, could perform all the Labour required: and in their Deſcent they were directed by Lines faſtned to the under edge of the Bell, the which paſt through Rings placed on both ſides the Leathern *Hoſe* in each Barrel; ſo that ſliding down by thoſe Lines, they came readily to the Hand of a Man, who ſtood on the Stage on purpoſe to receive them, and to take up the ends of the *Hoſe* into the *Bell*. Through theſe Hoſe as ſoon as their ends came above the Surface of the Water in the Barrels, all the Air that was included in the upper Parts of them was blown with great force into the Bell, whilſt the Water entred at the Bung-holes below, and fill'd them: and ſo ſoon as the Air of the one Barrel had been thus received; upon a Signal given, That was drawn up, and at the ſame time the other deſcended; and by an alternate Succeſſion furniſhed Air ſo quick, and in ſo great Plenty, that I my ſelf have been One of Five who have been together at the Bottom, in 9 or 10 Fathoms Water, for above an Hour and half at a time, without any ſort of ill Conſequence: and I might have continued there as long as I pleaſed, for any thing that appeared to the contrary. Beſides the whole Cavity of the Bell was kept entirely free from Water, ſo that I ſat on a Bench, which was diametrically placed near the Bottom, wholly dreſt with all my Cloaths on. I only obſerved, that it was neceſſary to be let down gradually at firſt, as about 12 Foot at a time; and then to ſtop and drive out the Water that entred, by receiving three or four Barrels of freſh Air, before I deſcended further. But being arrived at the Depth deſigned, I then let out as much of the hot Air that had been breathed, as each Barrel would repleniſh with Cool, by means of the Cock at the Top of the Bell; through whoſe Aperture, though very ſmall, the Air would ruſh with ſo much Violence, as to make the Surface of the Sea boyl, and to cover it with a white Foam, notwithſtanding the great Weight of Water over us.

Thus I found I could do any thing that was required to be done juſt under us; and that, by taking off the Stage, I could, for a ſpace as wide as the Circuit of the Bell, lay the Bottom of the Sea ſo far dry, as not to be over-ſhoes thereon. And by the Glaſs-Window, ſo much Light was tranſmitted, that, when the Sea was clear, and eſpecially when the Sun ſhone, I could ſee perfectly well to write or read, much more to faſten or lay hold on any thing under us, that was to be taken up. And by the Return of the Air-Barrels, I often ſent up Orders written with an Iron Pen, on ſmall Plates of Lead, directing how to move us from Place to Place as occaſion required. At other times, when the Water was troubled and thick, it would be as dark as Night below; but in ſuch Caſes I have been able to keep a Candle burning in the Bell, as long as I pleaſed, notwithſtanding the great Expence of Air neceſſary to maintain Flame.

This I take to be an Invention applicable to various Uſes; ſuch as Fiſhing for Pearl, Diving for Sponges, Coral, and the like, in far greater Depths than has hitherto been thought poſſible. Alſo for the fitting or plaining the Foundations of Moles, Bridges, *&c.* upon Rocky Bottoms; and for the cleaning and ſcrubbing of Ships Bottoms, when foul in calm Weather at Sea. I ſhall farther intimate that by an additional Contrivance, I have found it not impracticable for a Diver to go out of an Engine to a good Diſtance from it, the Air being convey'd to him with a continu'd Stream, by ſmall flexible Pipes, which Pipes may ſerve as a Clew, to direct him back again when he would return to the Bell.

IX *An Account of a Book omitted*

Joh. Poleni in Gymnaſio Patavino Phil. Ord. Prof. & Scient. S. R. Lond. & Berol. Sod. De Motu Aquæ Mixto, Libb. duo, *&c.* Patavii. 1717

CHAP. VI.

Geography, Navigation and Muſick.

Of drawing a Meridian Line thro France; by Monſ. Caſſini. *n.* 278 *p.* 1097.

I. MOnſieur *Caſſini* opened the Aſſembly of the *Academy of Sciences* held at *Paris*, *Nov.* 12. 1701. with a Diſcourſe containing the Obſervations he had made in his laſt Voyage, with a Deſign to determine the Paſſage of a Meridian Line (taken from a Point in the *Obſervatory* at *Paris*) from one End of *France* to the other. In the firſt Part of this Diſcourſe he went back to the moſt ancient Aſtronomers, and recounted their Opinions of the Spherick Figure of the Earth, and their Methods to know its Dimenſion, of which the two moſt Famous are, firſt, that of *Eratoſthenes* the *Cyrenian*, who lived in the Reign of *Ptolomie Euergetes*, King of *Ægypt*: The ſecond that of *Poſſidonius* of *Rhodes*, who lived in the Time of *Pompey the Great*. After having enlarged on the Methods uſed by theſe two ancient Philoſophers, he proceeded to thoſe of the Moderns *Johannes Fernelius*, and ſome others; And in the laſt place he related the Method of the late Monſieur *Picard*, of the *Academy Royal*, as the moſt exact. Then he ſpoke of his own Obſervations on the ſame Subject, of the uſe he had made of the *Satellites* of *Jupiter*, more fit for this than the Eclipſes of the Moon, in that they are more frequent; and ſaid that his Obſervations had been confirmed by the like made in *China*. He ſhewed the Method he took to determine the Paſſage of the *Meridian* taken from a Point in the Obſervatory at *Paris*. By the means of Triangles which he made through the whole Courſe of his Journey, and very exact Calculations, he determined the Place of this Meridian, and marked all the conſiderable Places through which it paſſed, from *Paris* to the higheſt Mountains of the *Pyreneans*, which ſeparate *Rouſſillon* from *Catalonia*; among theſe Mountains he obſerved one of a prodigious height, it being 1440 Toiſes high. But the moſt extraordinary Obſervation was that of the Inequality of the Degrees of the Meridian on the Earth; which is ſuch, that Monſieur *Caſſini* found that going Southward one degree ſurpaſſed another an 800th part, which may give great Reaſon to doubt of the exact Roundneſs of the Earth. Upon this occaſion he reported two different Opinions: the one Meſſieurs *Huygens* and *Newton*, the other of a Mathematician of *Strasbourg*, named *Eiſenſchmidius*. The two former hold that the Earth is flatted towards the Poles, ſo that it is ſomething of the Shape of an *Holland* Cheeſe; which they both conclude by Phyſical and Algebraical Deductions, from an Obſervation made at Cape *Verd*; that the *Pendulums*, though of the ſame Length, make their Vibrations there much ſlower than in the Northern Countries. The other Mathematician holds that the Figure of the Earth is Elliptick, ſo that it is ſtretch'd out towards the Poles, and has the Form of an Egg. Monſieur *Caſſini* left the Queſtion undecided. The Cities through which he obſerv'd the Meridian of *France* to paſs are *Dunkirk*, *Amyens*, *Aubigney*, *Bourges*, *Aurillac*, *Rodez*, *Alby* and *Carcaſſone*. Monſieur *l' Abbe Bignon* ſaid that the Meridian at *Paris* would be obſerv'd round the whole World, with the ſame Exactneſs as had already been begun to be done from one End of *France* to the other; and that there were Perſons of Courage ſufficient to undertake ſo painful a Voyage; and that nothing hindred them but the War, which began to be kindled every where. He meant by this Perſon Monſieur *de Chaſel*, who having already made ſeveral Voyages into the *Mediterranean*, in quality of the King's Engineer, and collected many Obſervations, was then making a true and exact Chart thereof.

To divide mechanically the Nautical meridian Line, by Mr. Peiks, *n.* 345 *p* 316

II. The moſt uſeful Projection of the Spheric Surface of Earth and Sea for Navigation, is that commonly call'd *Mercator*'s; tho' its true Nature and Conſtruction is ſaid to be firſt demonſtrated by our Countryman Mr. *Wright*, in his *Correction of the Errors of Navigation*. In this Projection the Meridians are all parallel Lines, not divided *equally*, as in the common plain Chart (which is therefore erroneous) but the Minutes and Degrees (or ſtrictly, the *Fluxions of the Meridian*) at every ſeveral Latitude are proportional to their reſpective *Secants*. Or a Degree in the projected Meridian at any Latitude, is to a Degree of Longitude in the Equator, as the *Secant* of the ſame Latitude is to *Radius*.

The Reaſon of which Enlargement of the Elements of Latitude is, to counterbalance the Inlargement of the Degrees of Longitude. For in this Projection the Meridians being all Parallel, a Degree of Longitude at (ſuppoſe) 60 Deg. Lat. is become equal to a Degree in the Equator, whereas it really is (on the Globes Surface) but *half* as much, the Radius of the Parallel of 60 Deg. (that is its *Coſine*) being but *half* the Radius of the Equator. Therefore to proportion the Degrees of Latitude to thoſe of Longitude, a Degree (or Elemental Particle) in the Meridian, is to be as much greater than a Degree (or like Particle) in the Equator, as the Radius of the Equator is greater than the Radius of the Parallel of Latitude, *viz.* its *Coſine*.

In *Fig.* 29. let the Radius CD repreſent the half of the Equator, DM an Arc of the Meridian; MS its Sine, CE its Secant; then is CS equal to its *Coſine*: and $CS : CM :: CD \ (= CM) : CE$, that is, as Coſine : to Radius : : ſo is Radius : to Secant. The Coſines being then, in this Projection, ſuppos'd all equal to Radius, or (which comes to the ſame) the Parallels of Latitude being all made equal to the Equator, the Radius of the Globe, at every point of Latitude (by the precedent Analogy) is ſuppoſed equal to the Secant of Latitude; and conſequently the Elements (Minutes, &c.) of the Meridian muſt be proportional to their reſpective Secants.

Plate 4.

The Way Mr. *Wright* takes for making his Tables of *Meridional Parts*, is by a continual Addition of Natural Secants, beginning at 1 Minute, and ſo proceeding to 89 Deg. Dr. *Wallis* (in * *Phil. Tranſ.* No. 176.) finds the Meridional Part belonging to any Latitude by this *Series*, putting S for its Natural Sine, *viz.* $S + \frac{1}{3}S^3 + \frac{1}{5}S^5 + \frac{1}{7}S^7 + \frac{1}{9}S^9$, &c. which gives the

* *Abr. Vol.* I. p. 572.

the *Meridional part* required. How to find the ſame Mechanically by means of an eaſily conſtructed Curve Line, is what I ſhall now ſhew.

1. Prepare a Rular AB (*Fig.* 28.) of a convenient Length, in which let Bo be equal to the Radius of the intended Projection. To the Point o, as a Center (on the narrower Edge of the Rular) faſten a little Plate-wheel wh tight to the Rular, and of a Diameter a little more than the Thickneſs of the Rular. Let KR (*Fig.* 29.) repreſent another long Rular to which AR is a perpendicular Line. Place the Rular AB upon the Line AR, with the Center of the Wheel at A Then with one Hand holding faſt the Rular KR, with the other Hand ſlide the end B of the Rular AB by the Edge of KR; ſo will the little Wheel wh deſcribe on the Paper a Curve Line ACB, to be continued as far as is convenient.

2. Having drawn the Curve ACB, draw a ſtreight Line KR by the Edge of the Rular KR: which Line is the *Meridian* to be divided, and alſo an Aſymptote to the Curve ACB.

3. In this Meridian, (accounting R to be the Point of its Interſection with the Equator,) the Point anſwering to any Degree of Latitude is thus found. In the perpendicular AR, make RG equal to the *Coſine* of Latitude (Radius being AR,) and from G draw GC parallel to KR, and interſecting the Curve in C. With Center C and Radius $CM = AR$, ſtrike an Arc cutting the Meridian at M; ſo is M the Point deſir'd.

4. In the Curve AC, let c be a Point infinitely near to C, and cm, ($= CM$,) a Tangent to the Curve at c, making the little Angle MCm, to which let the Angle RAr be equal: ſo is $Rr = Md$ (a Perpendicular from M to cm.) Draw CD equal and parallel to AR, interſecting KR in S. With Center C and Radius CD draw the Arc DM, and its Tangent DE and Secant CE.

5. Becauſe of the like Triangles CDE, Mdm; $CD : CE :: Md : Mm$, that is, as Radius to Secant of the Arc DM, (whoſe Coſine is $CS = GR$,) :: ſo is Md ($= Rr$ a Degree or Particle of the Equator:) to Mm the Fluxion or correſpondent particle of the Meridian Line RM. Whence, and from what is premiſed concerning the Nature of this Nautical projection, 'tis evident that RM is the *meridional Part* anſwering to the Latitude whoſe Coſine is GR. Or thus; with Center R and Radius AR deſcribe the Quadrant $A\kappa\alpha$, in which let the Arc $A\kappa$ be equal to the given Latitude. From κ draw κC parallel to KR and interſecting the Curve in C, ſo is $C\kappa$ the Meridional part deſir'd, being equal to RM, as is eaſy to ſhew.

6. As to the other properties of this Curve, 'tis evident, from its Conſtruction, that its *Tangent* (as CM) is a *Conſtant Line* every where equal to AR; the Curve being generated by the Motion of the Wheel at the End of the Rular which is its Tangent. And from hence the Curve ACB may, for Diſtinction, be call'd the *Equitangential Curve*.

7. The Fluxion of the Area $ARMC$ is the little Sector or Triangle MCd, which ſame is alſo the Fluxion of the Sector CDM: whence the Areas $ARMC$, CDM are equal, and the whole Area ACB, &c. KMR, being infinitely continued, is equal to the Quadrant $AR\alpha$.

8. To find the Radius of Curvature of any particle, as Cc; from C draw

an indefinite Line CT perpendicular to CM, (on the concave ſide of the Curve) and from c another Line perpendicular to cm, which Lines, (becauſe of the Inclination of CM to cm,) will ſomewhere meet as at T, making an Angle $CTc = MCm$. Theſe Angles being equal, their Radii are proportional to their Arcs: therefore, $Md : Cc :: MC : CT$. But $Cc = dm$ (becauſe of $CM = cm$) ſo that $Md : dm$ ($:: CD : DE$) $:: CM : CT$. But $CD = CM$, therefore $CT = DE =$ Tangent of the Arc DM

9. So that ſuppoſing ATt a Curve Line in which are all the Centers of Curvature of the particles of ACB, any point as T being found as before, the Length AT (by the Nature of *Evolution of Curves*,) is every where equal to the *Tangent* of its correſpondent Circular Arc DM. The point T is alſo found by making MT perpendicular to RM. and equal to the Secant CE: for ſo is the Angle $CMT = MCD$; and the Triangle MCT equal to the Triangle CDE.

10 Let AHb be an Equilater Hyperbola whoſe Semiaxis is AR and Center R. In the Meridian let RP be equal to the Tangent DE. Join AP, and draw $PH = AP$ and parallel to AR. Compleat the parallelogram $HNRP$, ſo will the poinr H be in the Hyperbola, and its ordinate HN ($= RP = DE = CT$) be equal to the Curve ATt. From whence, and from *Prop.* 3. *Coroll.* 2. of Dr. *Gregory's Catenaria* (*Phil.* * *Tranſ.* 231.) it appears that the Curve ATt is that call'd the *Catenaria* or *Funicularia*, viz. the Curve into whoſe Figure a *ſlack Cord or Chain* naturally diſpoſes its ſelf by the Gravity of its particles.

‘ 11. Hence we have another property of the *Catenaria* not hitherto ta-
‘ ken Notice of (that I know of,) *viz.* that ſuppoſing AR ($= a$, the
‘ conſtant Line in Dr. *Gregory*) equal to the Radius of the Nautical pro-
‘ jection, and RN the Secant of a given Latitude, then is NT the *Cate-*
‘ *naria's* Ordinate at N, equal to RM the Meridional part anſwering to
‘ the Latitude whoſe Secant is RN.

12. That TA is the *Catenaria* is alſo demonſtrable from Dr. *Gregory's* firſt *Prop.* Let Tu be the Fluxion of the Ordinate NT; and tu ($= Nn$) the Fluxion of the Axe AN. Then becauſe of like Triangles TCM, Tut, $CM : CT$ ($= TA$) $:: Tu : ut$, that is, as CM a conſtant Line to TA the Curve :: ſo is the Fluxion of the Ordinate, to that of the Axe ($\dot{y} : \dot{x}$) according to *Prop.* 1. *Catenariæ*.

13. From the Premiſes the Conſtruction and ſeveral Properties of the *Catenaria* are eaſily deducible; one or two of which I'll ſet down.

1. The Area $ATMR$ is equal to $AOPR$ a Rectangle contained by Radius AR and RP the Tangent anſwering to Secant $HP = TM$. For becauſe of the like Triangles CMm, CEe; $CM : CE :: Mm : Ee$, that is, putting r, s, t, m for Radius, Secant, Tangent and Meridional part RM) $r : s :: \dot{m} : \dot{t}$ whence $r\dot{t} = s\dot{m}$, and all the $r\dot{t} =$ all the $s\dot{m}$, that is, $AOPR = ATMR$, which agrees with Dr. *Gregory's* *Cor.* 5. of *Prop.* 7.

14. Suppoſing the former Conſtruction, let be added the Line RH, including the *Hyperbolick Sector* ARH. I ſay the ſame Sector is equal to half the Rectangle $ARMQ$ contained by Radius AR and the Meridional part RM, ($= \frac{1}{2} rm$) For the Sector $ARH =$ Triangle RNH wanting the

* *Abr. Vol.* I. *p.* 39.

Semiſegment *ANH*. The Fluxion of the Triangle *RNH* is $\frac{s\dot{t}+t\dot{s}}{2}$ The Fluxion of *ANH* is $t\dot{s}$. So the Fluxion of the Sector *ARH* is $\frac{s\dot{t}+t\dot{s}}{2}-t\dot{s}=\frac{s\dot{t}-t\dot{s}}{2}$ 'Tis found before (*Sect.* 13.) that $r : s\ (s : \frac{ss}{r}) :: \dot{m} : \dot{t}$; whence $s\dot{t}=\frac{ss}{r}m$. And becauſe of the like Triangles *CDE*, *Efe*, $CD : DE :: Ef : fe$. But $Ef = Mm = m$, becauſe both *Ef* and *Mm* are to *Md* in the ſame Reaſon, *viz.* as *s* to *r*; therefore $r : t\ (t : \frac{tt}{r}) :: \dot{m} : \dot{s}$: whence $t\dot{s}=\frac{tt}{r}\dot{m}$, and $\frac{s\dot{t}-t\dot{s}}{2}=\frac{ss-tt}{2r}\dot{m}=\frac{rr}{2r}\dot{m}=\frac{1}{2}r\dot{m}$, = the Fluxion of the Hyperbolick Sector *ARH*, whoſe flowing Quantity is therefore equal to $\frac{1}{2}rm=\frac{1}{2}ARMQ$. *Q.E.D.*

15. This ſhews another property of the *Catenaria, viz.* that it ſquares the Hyperbola; for *RM* is equal to *NT* the Ordinate of the *Catenaria.*

16. In *Fig.* 30. Let *AR* be Radius, *ACB* the Equitangential Curve; *MRN* its Aſymptote, in which let *M*, *N*, be any two points equally diſtant from *R*. Upon *M* draw *ML* parallel to *AR* and equal to the *Difference* of the Secant and Tangent of that Latitude whoſe Meridional part is *RM* (by §. 3, 4.) Upon *N* draw *NO* parallel to *AR*, and equal to the *Sum* of the foreſaid Secant and Tangent. Do thus from as many points in the Aſymptote as is convenient, and a Curve drawn equably through the points *L* - - - *A* - - - *O* - - &c. will be a *Logarithmic Curve*, whoſe *Subtangent* (being conſtant) is equal to Radius *AR*.

17. Let *no* be an Ordinate infinitely near and parallel to *NO*. $OP = Nn$ the Fluxion of the Aſymptote; *OT* the Tangent, and *TN* the Subtangent to the Logarith. Curve in *O*. Then $op : pO :: ON : NT$. But $ON = s+t$; therefore $op = \dot{s}+\dot{t}$. $pO = m$ (the Fluxion of the Meridian or Aſymptote.) So the Analogy is $\dot{s}+\dot{t} : \dot{m} :: s+t : NT$. By *Sect.* 13, 14, $\dot{s} : \dot{m} :: t : r$. alſo, $\dot{t} : \dot{m} :: s : r$. and thence $\dot{s}+\dot{t} : \dot{m} :: t+s : r$. wherefore *NT* (the Subtangent to *LAO*) is equal to Radius *AR* a conſtant Line, and conſequently the Curve *LAO* is the Logarithmic Curve and its Subtangent known.

18. The ſame Demonſtration ſerves for *LM*, (any Ordinate on the other Side of *AR*) only changing the Sine + into —; and then it agrees with Mr. *James Gregory*'s *Prop.* 3. *pag.* 17 of his *Exercitations*, viz. *That the Nautical Meridian is a Scale of Logarithms of the Differences whereby the Secants of Latitude exceed their reſpective Tangents, Radius being Unity.* So here *RM* is the Logarithm of *ML*, the Difference of the Secant and Tangent of the Latitude whoſe Meridional part is *RM*.

19. Suppoſing the precedent Conſtruction, if through any point *C* of the Curve *ACB* be drawn a right Line *GCW* parallel to *MR*, terminated with the Logarithmic Curve in *W* and the Radius *AR* in *G*: I ſay that the ſame right Line *WG* is equal to the intercepted part of the Curve Line *AC*.

20. Let $w\,g$ be a Line infinitely near and parallel to WG, and terminated by the ſame Lines; and CS, $W\sigma$, perpendicular to the Meridian; CS interſecting wg in z, and $W\sigma$ in y. Let CM be a Tangent to AC in C; $W\tau$ a Tangent to AW in W; ſo is $CM = \sigma\tau$. Becauſe of like Triangles Czc, CSM; and Wyw, $W\sigma\tau$; $CS : CM :: Cz : Cc$: alſo $W\sigma : \sigma\tau :: Wy : yw$. But $W\sigma = CS$; $\sigma\tau = CM$; $Cz = Wy$; therefore is yw the Fluxion of GW, equal to Cc the Fluxion of the Curve AC. Conſequently $GW = AC$. *q. e. d.*

21. It may be noted that this Equitangential Curve gives the Quadrature of a Figure of Tangents ſtanding perpendicular on their Radius. In *Fig.* 29. let $A\gamma\Gamma$ be a Curve whoſe Ordinates as $g\gamma$, $G\Gamma$, are equal to the Tangents of their reſpective intercept Arcs Ak, $A\kappa$. Let ΓG be produced to touch the Curve AC in C; then is the Area $A\Gamma G =$ to the Rectangle contained by Radius AR and GC the produced part of the Ordinate; or $A\Gamma G = AR \times GC$. The Demonſtration of which, and of the following *Section* I for Brevity omit.

22. If we ſuppoſe the Figure ACB, &c. KR (*Fig.* 29.) infinitely continued, to be turned about its Aſymptote RK as an Axe, the Solid ſo generated will be equal to a rectangled Cone whoſe Altitude is equal to AR. And its Curve Surface will be equal to half the Surface of a Globe whoſe Radius is AR. So that if the Curve be continued both ways infinitely (as its Nature requires) the whole Surface will be equal to that of a Globe of the ſame Radius AR.

The Deſcription of the Rular and Wheel, *Fig.* 28. is ſufficient for the Demonſtration of the Properties of the Curve: but in order to an actual Conſtruction for Uſe, I have added *Fig.* 31. where AB is a braſs Rular; wh the little Wheel, which muſt be made to move freely and tight upon its Axe (like a Watch-Wheel) the Axe being exactly perpendicular to the Edge of the Rular. s repreſents a little Screw-pin to ſet at ſeveral Diſtances for different Radii, and its under End is to ſlide by the Edge of the other fix'd Rular. p is a Stud for convenient holding the Rular in its Motion.

The Theory of Muſick reduc'd to Arithmetical and Geometrical Proportions; by Mr. **Tho. Salmon.**
n. 302. *p.* 2072.

III. Having had the Honour laſt Week of making the Tryal of a Muſical Experiment before the Society, it may be neceſſary to give a farther Account of it; that the Theory of Muſick, which is but little known in this Age, and the Practice of it, which is arriv'd at a very great Excellency, may be fix'd upon the ſure Foundation of Mathematical Certainty. The Propoſitions on which the Experiments were admitted, were; that Muſick conſiſted in proportions, and the more exact the proportions, the better the Muſick: That the proportions offer'd were the ſame that the ancient *Grecians* us'd: That the Series of Notes and half Notes was the ſame our Modern Muſic aim'd at: which was there exhibited upon Finger-boards, calculated in Mathematical proportion. This was demonſtrated upon a Viol, becauſe the Strings were of the greateſt length, and the proportions more eaſily diſcern'd; but may be accommodated to any Inſtrument, by ſuch mechanical Contrivances as ſhall render thoſe Sounds, which the Muſick requires.

To prove the foregoing propoſitions, two Viols were Mathematically ſet out, with a particular Fret for each String, that every ſtop might be in a perfect exactneſs: Upon theſe. a Sonata was perform'd by thoſe two moſt eminent Violiſts, Mr. *Frederick* and Mr. *Chriſtian Stefkins*, Servants to Her Majeſty; whereby it appear'd, that the Theory was certain, ſince all the ſtops were owned by them to be perfect. And that they might be prov'd agreeableto what the beſt Ear and the beſt Hand performs in modern practice, the famous *Italian*, Signior *Gaſperini*, plaid another Sonata upon the Violin in Conſort with them, wherein the moſt compleat Harmony was heard.

The full knowledge and proof of this Experiment may be found in the two following Schemes, wherein Muſick is ſet forth, firſt Arithmetically and then Geometrically: The Mathematician may, by caſting up the proportions, be ſatisfied, that the five ſorts of Half-Notes here ſet down, do exactly conſtitute all thoſe Intervals, of which our Muſick does conſiſt. And afterwards he may ſee them ſet forth upon a Monochord, where the meaſure of all the Notes and Half-Notes comes exactly to the middle of the String. The Learned will find that theſe are the very proportions which the old *Greek* Authors have left us in their Writings, and the Practical Muſician will teſtifie, that theſe are the beſt Notes he ever heard.

Figure the 1ſt, containing the Proportions ſet out Arithmetically.

An Eighth
1/2
A Seventh
8/15
A Sixth
3/5
A Fifth
2/3
A Fourth
3/4
A Greater Third
4/5
8/9 Tone major
9/10 Tone minor
15/16 Hemitone
8/9 Tone major
9/10 Tone minor
8/9 Tone major
15/16 Hemitone
An Octave with a greater Third.
C 17/18 C 16/17 D 19/20 D 18/19 E 15/16 F 17/18 F 16/17 g 19/20 g 18/19 a 17/18 a 16/17 b 15/16
A 17/18 A 16/17 B 15/16 C 17/18 C 16/17 D 19/20 D 18/19 E 15/16 F 17/18 F 16/17 g 19/20 g 18/19 a.
An Octave with a leſſer Third

Explanation of the firſt Figure. Between the 2 loweſt Lines, you have the Series of all the 12 half Notes in an Octave, from Are to Alamire, which added together make an Octave or exact Duple Proportion: The ſeveral parts alſo added together make all thoſe inervals of which it is conſtituted. As for example, the two half Notes from A to A ✻ $\frac{17}{18}$, and from A ✻ to B $\frac{16}{17}$ make a Major Tone to which if an Hemitone from B to C $\frac{15}{16}$ be added, you have a leſſer Third

In

Part. I. Plate 4 . page 428.

Fig. 3.

Fig. 6.

Fig. 1.

Fig. 2.
Apr. 21. 10. 1718.
*1
*2
*3
*4

Fig. 3.
Apr. 22. 11. 1718
*1
*2
*3

Fig. 4.

Fig. 8.
E
D
A
a
b
g
C
G
C
g
d
c
F
H
F

Fig. 7.
Bayero
Via Martis
Novembris
Novembris 1680.
Via Cometa
6to Mane
Virginis
Ecliptica
Leonis
Cor Leonis
Bor.
Novemb. 4to Mane
Cometa Observatus Novemb: 11mo St: V. Mane
Bayero
Bor.
Virginis
Ecliptica
Aust.

Fig. 9.
A
C
B
D
K
L
E
I
G
F
M

Fig: 10.

Fig: 13.
Weal
Upminster

Fig: 21.

Fig: 11.

Fig: 12.

Fig: 22.

Fig: 14.

Fig: 15.

Fig: 23.

Fig: 16.

Fig: 17.

Fig: 18.

Fig: 24.

Fig: 25.

Fig: 26.

Fig: 20.

Fig: 27.

Fig: 19.

Fig: 28.

Fig: 31.

Fig: 30.

Fig: 29.

In like manner between the two next lines, you have the ſeries of all the 12 half Notes, in an Octave from C fa ut to C ſol fa ut: the two firſt Tones added together make a greater Third: and ſo you may add a Tone or Hemitone till you arrive at every interval in the Octave, which is ſo call'd becauſe eight ſounds are required for expreſſing thoſe ſeven gradual ſteps whereby we commonly aſcend to it.

It may be alſo obſerv'd, that the proportions falling upon the ſame Notes in two Keys, one finger-board will be ſufficient for both.

'Tis acknowledg'd by all that are acquainted either with Speculative or Practical Muſick, that every interval is divided into two parts, whereof one is greater than the other: An Eighth $\frac{1}{2}$ into a Fifth $\frac{2}{3}$ and a Fourth $\frac{3}{4}$. Again, a Fifth $\frac{2}{3}$ into a greater Third $\frac{4}{5}$ and a leſſer Third $\frac{5}{6}$. Thus alſo a greater Third $\frac{4}{5}$ muſt be divided into a Tone Major $\frac{8}{9}$ and a Tone Minor $\frac{9}{10}$. The Leſſer Third (to comply with the practice of Muſick) is rather compounded of, than divided into a Tone Major $\frac{8}{9}$ and an Hemitone, which is its complement, $\frac{15}{16}$.

Three Tones Major, two Tones Minor, and two of the foreſaid Hemitones, placed in the order found in the Scheme, exactly conſtitute the practical Octave; which is ſo call'd becauſe it conſiſts of eight ſounds, that contain the ſeven gradual intervals. But it is alſo neceſſary to ſet down the Diviſions of the whole Tones, which are the true Chromatick half Notes becauſe there is great uſe of them in Practical Muſick.

To make all our whole Notes, and all our half Notes of an equal ſize, by falſifying the proportions, and bearing with their imperfections, as the common practice is, may be allowed by ſuch Ears as are vitiated by long cuſtom: But it certainly deprives us of that ſatisfactory pleaſure which ariſes from the exactneſs of ſonorous numbers; which we ſhould enjoy, if all the Notes were truly given according to the Proportions here aſſign'd.

It is very eaſie to ſatisfie our ſelves in the Arithmetical Scheme, by thoſe operations which *Gaſſendus* has ſet down. As for example, his rule for Addition is, That two Proportions being given, if the Greater number of one be multiplied by the Greater number of the other, and the Leſſer by the Leſſer, the two numbers produc'd exhibit the compounded Proportions. Thus take a Practical Fifth $\frac{2}{3}$ and a Practical Fourth $\frac{3}{4}$ for the two Proportions given, multiply 3 by 4 and you have 12, then multiply 2 by 3 and you have 6: which compounded proportion of 12 to 6 makes the Practical Octave $\frac{1}{2}$.

Manuduction to the Theory of Muſick, Tom. V. *pag.* 635.

Thus, according to his Arithmetical operations of Addition, Subſtraction, Multiplication or Continuation, and Diviſion, is our whole Syſtem proved, which for the more eaſie application to Practical Muſick, ſhall be alſo ſet forth Geometrically upon the 6 ſtrings of a Viol. *See Figure the 2d.*

This Mathematical fixing of the Frets enables every Practitioner, who ſtops cloſe to them, to give the Proportions of the Notes in a greater exactneſs, than can be done upon the Baſs-Violin or Violin itſelf: ſince they may be ſet forth more perfectly by a pair of Compaſſes dividing a line, than the niceſt Ear can direct.

Though the Frets for the ſeveral Strings do not ſtand in a ſtrait line,
the places are alſo ſhifted in different Keys, yet the Ear naturally dire
the Fingers to them: inſomuch that thoſe perſons, who have all their li
time been accuſtom'd to ſtop upon Frets that go quite croſs the Finge
boards of their Inſtruments, do with very little practice fall right upo
theſe. Such is the power of a Muſical Genius, as may be undeniably prov
by thoſe that play upon the Violin; who, when they change the Key
fall upon the right Stops, tho' they have no viſible direction where to ſto
nor time to alter, by the Ear, the Note they firſt pitched upon.

By this Standard of Regular Proportions may the Voice be formed
ſing the pureſt Notes; they are all the ſame in Vocal and Inſtrumen
Muſick; if then the Inſtrument which governs the Voice be perfect, th
Ear will of neceſſity bring it to perfection. It is a great pity that a goo
natural Voice ſhould be taught to ſing out of Tune, as it muſt do, if it b
guided by an imperfect Inſtrument; and this may be the reaſon why
few attain to that melody, which is ſo much valued; but ſince we no
know wherein perfection lies, a conſtant practice will come to the attain
ment of it. The dividing Wholes into Chromatick Hemitones is ver
neceſſary, but very difficult for the Voice to be broken to: If it learns fro
an Inſtrument whoſe whole Notes and whoſe half Notes are ſuppoſed
be equal, the ſound muſt needs be very uncertain and unharmonical
whereas the proportions truly fixed, would bring it to a perfection in th
niceſt and moſt charming part of Muſick.

The Chromatick Hemitones are the ſmalleſt Intervals our Modern M
ſick aims at, tho' the Antients had their Enharmonick quarter Notes, which
they eſteem'd their greateſt excellency: Theſe may alſo in time b
recover'd, ſince we know their proportions; for as the Diatonick Tone
divided into Chromatick Hemitones, ſo after the ſame manner may th
Chromatick Hemitones be divided into thoſe leaſt Enharmonick Interva
which were ever made uſe of.

But if we go no further, yet this Experiment demonſtrates the tru
Theory of Muſick, and brings the practice of it to the greateſt perfection.

IV. *An Account of a Book omitted.*

Linear Perſpective: or a New Method of repreſenting juſtly all manne
of Objects. By *Brook Taylor*, LL. D. R. S. Secr. 8*vo*. *London* 1715.

The 2d Figure, wherein the Proportions of Muſick are deſcribed Geometrically.

The Explication.

These ſix Lines repreſent the ſix Strings of the Viol in the common Tuning.

The ſounding part of each String from the Nut to the Bridge is ſuppos'd to be 30 Inches long; the two middle Strings C and E are drawn out to 15 Inches, the half of the whole.

'Tis eaſie to meaſure every Interval with a Pair of Compaſſes. Suppoſe you are to take the 20th part of the String G; 'tis an Inch and a half for the firſt half Note: If you take the whole Note from G to A, 'tis the tenth part, and muſt be 3 Inches.

After theſe are taken away, your String will be but 27 Inches long, ſo that if you advance one Note, or a Major Tone further, you muſt take a 9th part of it, which will be three Inches more, whereby you arrive at a greater Third, being the fifth part of the whole String. Thus the Series of all the Notes may be demonſtrated.

All the Strings are Uniſon at the Stops where the Tuning requires: So that though the Proportions be carried on as far as the Frets allow, yet the String is open the ſame with the Stop of that String to which it is tuned; and accordingly the Series of the Notes proceed, as if they were all upon a Monochord.

D d
G
C
E
a
d
D d 19/20
G 19/20
17/C 18
F 15/16
a 17/18
d 19/20
E e 18/19 9/10
A 18/19 9/10
D 16/17 8/9
F 17/18
b 16/17
e 18/19 9/10
F f 15/16
A 17/18
D 19/20
f 15/16
g 16/17 5/6
c 15/16 5/6
F f 17/18
B 16/17 4/5
E 18/19 4/5
g 19/20
c 17/18
f 17/18
G 16/17 3/4
C 15/16 3/4
F 15/16 3/4
a 18/19 3/4
d 16/17
g 16/17 3/4
G 19/20
C 17/18
F 17/18
a 17/18
d 19/20
g 19/20
A 18/19
D 16/17 2/3
g 16/17 2/3
b 16/17 2/3
e 18/19 2/3
a 18/19
g 19/20
c 15/16 15/24
a 18/19 3/5
c 17/18
a 17/18
d 16/17 5/9
b 16/17 8/15
d 19/20
c 15/16 1/2
e 18/19 1/2

This Calculation ſerves but for two Keys A and C, which are called Natural, becauſe they have no eſſential Flats or Sharps.

But becauſe the Compoſer begins upon any Key, and the Series of Notes muſt take its *terminus à quo* from thence; the Inſtrument-maker can provide ſuch moveable Finger-boards as will ſerve exactly for every Key. They are taken out and put in upon the Neck of the Viol, with as much Eaſe as you pull out and thruſt in the Drawer of a Table.

Three, or at moſt five of them will be ſufficient to accommodate all the Keys that are made uſe of.

THE Philoſophical Tranſactions

From the Year 1700. to 1720.

Abridg'd and Methodically Digeſted.

PART II.

The *Anatomical* and *Medical* Papers.

CHAP. I.

A Courſe of Anatomy:

A Courſe of Anatomy at Padua, by Mr. Ray, com. by Mr. Dale, n. 307. p. 2283.

EOdem modo quo alii *Anatomici D. Antonio Marchetti* diſſectionem exorſus eſt ; *Abdominis* nimirum cutem in crucis formam ſecando, umbilico tamen intacto. *Cuticulam* à cute ſeparabat candelam accenſam ſub cute tenendo, quæ cuticulam in veſicam attollebat, unde facile eam ſcalpello ſeparabat. Cuticula à cute niſi vel actuali vel potentiali cauterio, *i. e.* veſicante ſeparari nequit. Sub *Cute* copioſa *Pinguedo* ſemidigitum craſſa abdomen totum inveſtiebat. Erat autem pinguedo crebris fibrillis veluti fulcris ne diffueret ſtipata. 1

Sub *Pinguedine* panniculus ſeu *membrana carnoſa*, quæ tamen hac in parte carnoſa non apparebat; erat etiam & ſub membrana carnoſa aliqua pinguedo ſed parcior. Membranam hanc ille in Brutis duplicem eſſe aſſeruit, qiua Bruta cutim totam movere poſſunt & corrugare, in homine duplex eſt in fronte, unde & frontem contrahere & corrugare poteſt, in nonnullis etiam duplex eſt in Occipite, unde & totum Capillitium commovere poſſunt. *Verùm alii diverſam rationem aſſignant horum motuum, nimirum quia in Fronte & Occipite membrana carnoſa cuti arcte cohæret, & in Muſculum degenerat, quod probabilius eſt.*

Tum *muſculos abdominis* aggrediebatur, & primùm *Oblique deſcendentes*, qui à medio circiter coſtarum notharum exorti ſerratim cum muſculo Thoracis *ſerrato* coaptantur (ut in ſuturis Oſſium). In parte poſteriore muſculus dorſi latiſſimus huic incumbit, atq; idcircò aliquouſque primùm elevandus eſt.

Muſculi recti eminentiis, ſeu proceſſibus oſſium pubis lato tendine adnectuntur, ſuperiùs à cartilagine primæ coſtæ nothæ prope cartilaginem enſiformem utrinq; oriuntur tendinibus nervoſis.

Oblique aſcendentes, à ſuprema margine oſſis *Ilei* radioſis fibris exorti, venam muſculam recipiunt à ramis *iliacis*. Secundò *Obliquè aſcendentes*, in quibus nihil ſingulare, ortum habent à ſummitate oſſis Ilii: Tendo ejus duplex muſculos rectos veluti amplectitur: Una ſc. ejus pars muſculo recto incumbit, quæ antequàm dimidium latitudinis muſculi ſuperavit, cum tendine oblique deſcendentis arctiſſimè cohæret, vel potiùs in unum coaleſcit, ut nulla arte poſſit ſeparari: Altera ſub muſculo recto eodem modo cum tendine muſculorum tranſverſorum coaleſcit. Venam accipit à muſculâ dictâ quæ à ramo Iliaco oritur.

Muſculi recti duas tantum habuêre inſcriptiones nervoſas, cùm in aliis 3, in aliis 4 aut 5 habeant, ut obſervat Veſlingius. In his Muſculis obſervavimus anaſtomoſes venæ Mammariæ internæ & venæ epigaſtricæ.

Muſculi tranſverſi à proceſſibus vertebrarum lumborum oriuntur; non autem Muſculi oblique aſcendentes illis vertebris annectuntur ut ille obſervat.

Muſculos etiam *pyramidales* in hoc cadavere obſervavimus, qui rectis oblique incumbunt.

Obſ. 1. Pinguedo in dorſo Fœminarum liquidior & mollior eſt quàm in maribus.

2. Cutis in iis quæ pepererunt circa Ilia corrugatur, in Virginibus non item.

3. Vena, Arteria & Nervus ſemper ſe mutuò comitantur, arteria ad dextram, vena in medio, nervus ad ſiniſtram.

4. Sub muſculis ſupra peritonæum prope lumbos copioſam obſervavimus pinguedinem, unde in hac parte facilè à peritonæo ſeparantur muſculi, verùm prope lineam albam cum tendinibus muſculorum arctiſſimè cohæret, ut nulla arte ſeparari poſſit.

5. Muſculos etiam in originibus ſeu capitibus ſuis tendines habere aſſeruit.

6. Incepit diſſecare muſculos à capitibus ſeu originibus, quia ita motus ſeu uſus muſculi in motibus faciliùs diſcernitur.

7. Cavendum eſt Chirurgis ne muſculos tranſverſim ad fibras ſecent, quia ita periculum eſt ni nervis (qui ſemper cum fibris parallelis decurrunt) diſſectis, convulſiones oriantur.

8. Si quis velit rectè diſſecare & ſeparare muſculos, debet accuratè obſervare fibras, earumq; ductum ſequi.

9. Muſculi tranſverſi tum in initio, tum in fine latam habent tendinem membranoſam.

Oſtendit tum nobis *vertebras lumbares*, numero quinque; ſingula proceſſus 7. obtinent, ſpinatum unum, tranſverſos duos, obliquè aſcendentes duos, & obliquè deſcendentes duos: Obliquè aſcendentes inferioris cum obliquè deſcendentibus ſuperioris per Ginglymon articulantur: Vertebræ verò ipſæ per Harmoniam: cavitas ſci. ſuperioris gibbum ſeu protuberantiam inferioris recipit.

Os Sacrum ex 6 componitur oſſibus nonnunquam, communiter 5; cum è 6 conſtat, *os coccygis* tria tantùm habet oſſa, cum è quinque quatuor. Os coccygis interiùs curvatur ad commodiorem ſeſſionem.

In difficili partu, *Chirurgus* immittendo digitum in *Inteſtinum rectum*, & retrahendo ſeu reflectendo *Os Coccygis* partum facilitare poteſt: Quod & *Marchettus* ſe feciſſe aſſerit.

Os Sacrum magna habet foramina ad egreſſum nervorum.

Aſſerit ille Oſſa hæc quæ pelvim conſtituunt in fœminis ampliora non eſſe quàm in viris pro ratione corporis, ut alii aſſerunt.

Os Ilium, *Os Pubis*, & *Os Coxæ*, ſeu *Iſchion* in adultis in unum velut os coaleſcunt, in Infantibus diſtincta ſunt, & cartilagine juncta. Omnia hæc tria oſſa femoris acetabulo coeunt, & ſingula partem aliquam cavitatis efformant. In oſſe ilio diſtinxit marginem, coſtam, dorſum, ſinus duos, ſuperiorem unum, ſuper quod nervus è ſummo foramine oſſis ſacri egreſſa tranſiens ad crura deſcendit; inferiorem alium inter eminentias duas, ad commodiorem ſeſſionem.

Nervi egrediuntur ad latera vertebrarum è foraminibus inter duas vertebras formatis.

Oſtendit *Viſcera* & *Inteſtina* in ſitu, *rectum* ſcil. *colon* quod omnia inteſtina circumcinxit; *cæcum* ad dextrum latus digiti parvi magnitudine, quod ille in fœtu & Infantibus nec majorem, nec excrementis repletum unquam inveniri ex ſua obſervatione aſſerebat. Supra cœcum immediate incipit *Ilium* quod & majus eſt & excrementis repletum: deinde *Jejunum*, quod & carnoſius, & vaſis plenius, & magis rubidum, & excrementis vacuum; *duodenum* ad flexuram terminatur.

Pars illa *Meſenterii* cui *colon* annectitur *Meſocolon* dicitur; reliqua pars, cui tenuia inteſtina, κατ' ἐξοχὴν *Meſenteriam* appellatur. *Arteria Meſenterica* inferior per totum colon, atq; etiam rectum ramos ſpargit, unde *Arteria hæmorrhoidalis*, meſenterica ſuperior, ad reliqua omnia fere inteſtina.

Splen in hoc cadavere prægrandis ultra naturalem molem: Hoc ille ebrioſitati aſſignabat.

Colon in hoc cadavere peritonæo adhærebat.

Muſculi Pyramidales à proceſſibus ſeu eminentiis oſſis pubis utrinq; orti & oblique aſcendentes tendinibus ſuis vicinis in *linea alba* terminantur: inſerviunt hi urinæ expellendæ comprimendo *Veſicam* autore *Fallopio*; ubi hi deſunt (ut in nonnullis fit) extremitates muſculorum rectorum latiores ſunt.

Oſtendit inſuper *venam umbilicalem*, quæ in fiſſuram *hepatis* inſeritur, & *ligamentum* degenerat: *Arterias umbilicales*, quæ peritonæo adnexæ decurrunt ad ramos uſq; iliacos arteriæ magnæ: *Urachum*, qui pariter adnexus peritonæo ad fundum veſicæ deſcendit, eamq; ſuſtentat, quinetiam in homine ligamenti duntaxat uſum præſtare, nec omnino perforatum eſſe.

Oſtendit præterea *uteri tubos*, *uteri ligamenta rotunda*, *teſtes muliebres*, & *vaſa Spermatica*, nec non *ligamenta lata*.

Hepatis ligamentum latum. Hepar in viventibus & ſanis non incumbere ventriculo, ac proinde unguenta, fomenta, & epithemata ventriculi regioni exteriùs applicari.

Ligamenta uteri rotunda perforant peritonæum & omnes muſculos, & deinde diviſa unum mittunt ramum ad *Clitoridem*, alterum ad genu uſque. *q.*

Venas & *arterias gaſtricas* & *gaſtroepiploicas* oſtendit & deſecuit accuratè; verùm in his omnino conſentit cum *Veſlingio*, quem adi.

Obſ. 1. *Bubones* nonnunquam oriuntur etiam in his quæ caſtæ ſunt, verùm in iis ſine ſuppuratione poſſunt diſcuti: *Venerei bubones*, niſi *gonorrhæa* ſuccedat, ſemper ſuppurantur.

2. *Valvulam* in coli initio obſervavimus; iliacam paſſionem *Volvulum* dictum oriri ab inflammatione iſtius valvulæ quæ impedit ne excrementa deſcendant, ex propria obſervatione aſſeruit; adeo ſc. anguſtatum vidit foramen ut ne cuſpidem aciculæ potuerit admittere.

Venæ ventriculi ſunt vel propriæ vel communes: Propriæ ſunt 1. *Gaſtricæ ſiniſtræ minores* 3. aut 4. (quarum prima & breviſſima vas breve dicitur) a ramo ſplenico venæ portæ prope lienem ortæ. 2. *Gaſtrica ſiniſtra major* ſeu *coronaria*, quia in ſummitate ventriculi ſparſa coronæ in modum. 3. *Gaſtrica dextra* ſeu *pylorica*. Communes ſunt, 1. *Gaſtroepiploica ſiniſtra*, quæ a ramo ſplenico prope lienem exorta fundum perreptat ventriculi, hinc in ventriculum inde in omentum ramos ſubinde ſpargens, inde in omentum vero unam inſignem epiploicam ſiniſtram dictam. 2. *Gaſtroepiploicam dextram* in hoc cadavere à ramo meſenterico ortam prope pylorum, quæ pariter fundum ventriculi perreptat, hinc in ventriculum, inde in omentum obiter ramos ſpargens, maximo ſuo ramo ſeu trunco *gaſtroepiploicæ dextræ* per anaſtomoſin conjuncta. Eſt hæc vena inſignis, & unum præ cæteris memorabilem emittit ramum *epiploica vena dextra* dictum.

In fundo *veſiculæ felleæ* nulla ſunt conſpicua vaſa quæ *bilem* eo deducant, ſed *poroſitates* quædam quæ bilem tranſmittunt, & proinde cum ſeparatur veſicula ab hepate humor bilioſus manifeſtè exudat. Quinetiam capillares quædam venulæ ab ipſo hepatis parenchymate in membranas veſiculæ ſparguntur adeo, ut ſine effuſione ſanguinis ab *Hepate* dividi nequeat. Aſſerit veſiculam felleam quâ parte hepati conjungitur ſimplici tantum membranâ conſtare, alibi duplici.

Meatus cyſticus ubi in ductum communem terminatur valvulum non habet, ſed *oſtiolum* tantum, quod refluxum bilis impediat.

Hepar 3 habet ſinus, unum in quo jacet cyſtis fellea, alterum in quem intrat vena umbilicalis, tertium ubi tranſit venæ cavæ truncus.

Cyſtis fellea arteriam habuit grandiſſimam, venas parvulas. Obſervaſſe ſe dixit ubi arteria magna eſt, ibi venam eſſe parvam quæ ei reſpondet & vice verſa: *Non credo.*

Opiniones Marchetti *mihi parum probabiles.*

1. Aſſerit præterea, ubi meatus cyſticus obſtruitur, icterum flavum oriri, ubi porus cholidochus, icterum nigrum.

2. Vaſa venæ portæ & cavæ in hepate non conjunguntur per oſcula, ſed per harmoniam aut incumbentiam mutuam vaſorum.

3. Vena portæ intra hepatis parenchyma non induit membranam novam.

4. Se vidiſſe venas lacteas in ipſum venæ portæ truncum inſertas.

5. Se nunquam potuiſſe invenire neque credere dari ullum commune receptaculum chyli: *Experientia mea contrarium evincit.*

6. Se vidiſſe ramum inſignem ductus chyliferi in pancreas deſinentem.

7. Se putare uſum lienis eſſe ſeparare bilem atram à ſanguine, eamque una cum ſanguine ad hepar tranſmittere per ramum Splenicum, ubi per meatum cholidochum expurgatur in inteſtina.

8. Se putare venas lacteas chylum exugere ex inteſtinis, eúmque ad pancreas differre, cujus uſus eſt eum ulterius perficere & exaltare, partemq; excrementitiam in inteſtina per novum vas *Virſungianum* ablegare.

9. Se vidiſſe venas lacteas in meſocolo ad inteſtina ſparſas; quod proculdubio verum eſt. *Veræ nobis viſæ.*

10. Venas hæmorrhoidales externas à vena cava non oriri, ſed ramos eſſe venæ portæ: Ejus autem ſurculos extremos cutim etiam ipſam perforare, & in tubercula ſub cuticulâ deſinere; & his applicantur hirudines. Oritur Hæmorrhoidalis vena aliquando à ramo ſplenico, aliquando à meſenterica, ſæpiſſimè in ipſa divaricatione venæ portæ. Vena hæc ramos ſuos ſpargit per totum meſocolon.

Meſocolon à meſenterio tenuitate ſuâ differt.

Arteriæ ſatis amplæ à cœliaco ortæ tres aut quatuor rami lienem ingrediuntur.

Venæ Splenicæ plures rami per totum lienis parenchyma ſparguntur contra *Sylvium*, qui aſſerit *eas oſculis ſuis duntaxat in lienem biare, ſubſtantiam verò ejus non penetrare.*

Obſ. Cum quis ex morbo diuturniore moritur, lien nigricat; ſi violenter moritur, rubicundior eſt.

Novum vas pancreatis & porus cholidochus eodem in loco *duodenum* perforant; aliquando diverſis foraminibus in inteſtinum exeunt, ut in canibus fit.

Oſtendit nobis in meſenterio nervorum plexum; *cui uſui qui?*

Porus cholidochus in hoc cadavere mihi videbatur eſſe ampliſſimus.

Ren ſiniſter in hoc & aliis omnibus major eſt dextro & ſuperiùs ſitus, & à trunco venæ cavæ remotiùs, unde & emulgens longior eſt. *Huic rei* *rationem ſane probabilem dedit, quia* hepar *in dextro latere incumbens tum illum deprimit, tum augmentum ejus prohibet.*

In dextro latere duas habet hoc cadaver arterias emulgentes, unam conſueto loco in ſinum renum ingredientem, aliam in ſuperiore extremitate.

Uretres in hac fœmina ampliſſimi, quod ille omnibus fœminis commune eſſe aſſerit, quia humidiores ſunt, & plus mingunt.

Glandula renalis dextra ab ipſo venæ cavæ trunco venam accipit; ſiniſtra vero ab emulgente. Glandulæ interius cavitatem habent. Dextra ipſi reni incubuit.

Arteria ſpermatica utraque ab ipſo aortæ trunco infra emulgentes ſibi mutuo proxime oriuntur. Venarum altera ab emulgente, altera nimirum dextra duplici trunco, uno ab emulgente, altera trunco venæ cavæ, qui paulo poſt in unum conveniunt, exoritur.

Affirmavit ſe vidiſſe venas lacteas in prægnantibus ad uterum ſparſas; per quas probabili conjectura aquoſum illum Serum in quo infans natat ad uterum differri putat. *Venas haſce lacteas in Ove prægnante, facillime inveni.*

Meſenterium à tribus ſuperioribus lumborum vertebris oritur.

Monſtravit vaſa ſeminalia, quæ ad teſtes quidem deſcendunt, eos vero non intrant, ſed ſupra ligamenta lata ad convexum teſticuli latus decurrentes, partim in tubos uteri ſparguntur, partim in uterum ipſum.

Tubi uteri ad utrumque fundi angulum ſiti cornibus uteri in animalibus reſpondent, & ſunt omnino cavi, adeo ut ab utero ad extremitates eorum uſque poſſit ſtylus immitti: Tunica eorum interior albicat, inque iis ſæpius reperitur humor ſeroſus albidus, qui ſemen muliebre eſſe creditur.

Teſticuli muliebres epididymidibus carent; una extremitate nervoſis ligamentis utero annectuntur; ſubſtantiam habent molliorem laxioremque teſticulis maſculorum. Unus horum exulceratus cavitatem habuit.

Quòd teſtes tàm in maribus quàm in fœminis ad generationem nihil conducant, memorabili imprimis experimento probavit: Canis nimirum maſculi teſtes execuit epididymidibus integris relictis, deinde canem fœminam in cubiculo concluſit per tres annos, nec ullum admiſit ad eam canem cum ſalaxaſſet præter caſtratum hunc, qui canem iniit & cum eâ ſæpius implicatus eſt. Triennio hæc ter peperit, una vice 7 catulos, altera 9, tertia 5. Re ſatis exploratâ fœminam dimiſit. Alias duas vel tres hiſtorias huic parallelas nobis narravit; unam de Equo caſtrato, relicta tamen epididymide una, qui equas ſæpius imprægnavit, fuitq; in venerem admodum proclivis: Alteram de cane quem ipſius ſervus execuit: Tertiam de homine quodam ruſtico, qui ob bubones venereos utrumq; teſtem amiſiſſet, epididymide unica duntaxat manente, qui tamen uxorem duxit, & tres maſculos filios genuit. Credit ergo ille teſtes non alii uſui inſervire quàm quem *Ariſtoteles* adfert, nimirum ut ſint pondera impedientia ne Spermatica vaſa implicentur; & revera vaſa ſeminalia in eos non terminantur, nec tranſeunt, ſed epididymides ſolùm.

Uteri cavitas peranguſta eſt & minima, verùm tunica uteri ſpiſſior denſiorq; quàm ego credidiſſem.

Ligamenta uteri rotunda non ſunt in uterum perforata, verùm vaſis deferentibus in maſculis quoquomodo reſpondent.

Orificium internum uteri in gravidis lentâ & viſcoſâ materiâ obſeratur, *ut nos ſæpiùs in bove obſervavimus*, adeò ut nihil omninò in uterum penetrare poſſit: Unde nihil ſeminis in uterum projici poſſit, adeoque nec ſuperfœtatio fieri. Narravit tamen nobis ſe audiviſſe de muliere quadam ruſtica in montibus vicinis degente, qui tribus menſibus poſtquam unam peperiſſet, prolem aliam denuò peperit.

Os ipſum uteri Tineæ piſcis ori perſimile, corpus uteri cucurbitæ tonſoris.

Vagina uteri ampla eſt atq; intus rugoſa, in meretricibus verò longo veneris & aſſiduo uſu rugæ iſtæ abolentur, & omnino levis evadit.

P.N. Ad hanc uteri vaginam vaſa quamplurima (venæ ſc. & arteriæ) tendunt, à ramis iliacis interiùs ſive hypogaſtricis orta, miris plexibus & anaſtomoſibus juncta, quæ in ſuperficiem vaginæ ſparguntur, & probabile eſt oſcillis ſuis ſive capillaribus extremitatibus in cavitatem ipſius hiant, in eamq; effundunt Sanguinem menſtruum, quanquam ſe nunquam horum vaſorum orificia potuiſſe invenire aſſerit *Marchettus*; nec mirum. Nonnulli ex his ramis in cervicem etiam uteri ſparguntur.

In pudendo demonſtravit nobis Labia, Clitoridem in ſupremo rimæ angulo, Alas ſeu Nymphas in ſuperiore etiam parte, Urethram ſeu meatum urinarium, & Circulum membranaceum qui pudendum à vagina uteri diſtinguit, quiq; in virginibus membrana *hymene* dicta occupatur totus, excepto foramine in medio per quem menſtrua defluunt.

In defloratis etiam apparet hic circulus qui pudendum hoc loco coarctat, ponè quem vagina laxior & amplior eſt.

Vagina hæc ultra interiùs uteri orificium inferius percurrit, unde ſi membrum virile longius quàm par eſt fuerit, ultra orificium interius uteri in hunc ſinum ſperma projicit, unde uxorem imprægnare nequit.

Ait ſe obſervaſſe etiam in fœminis utero gerentibus uterum fuiſſe duos ferè tranſverſos digitos craſſum.

Obſ. 1. Ratio cur virgines coitu liberantur à morbo illo noſtratibus (*The Green Sickneſs*) eſt quia membrum virile diſtendit nonnihil vaginam uteri & frixione ſua reſerat orificia venarum, adeòq; menſes affluere facit.

2. Locus ubi fœminæ calculo laborantes à Chirurgo debent ſecari eſt in ſuperiore vulvæ parte prope labia, Stylum in urethram immittendo, & ſuper eam ſecando in carnoſo veſicæ collo.

3. Urachus in homine (ne in fœtu quidem dum adhuc in utero matris latitat) perforatus non eſt ex obſervatione *Marchetti*, ſed uſum tantum ligamenti ad ſuſtentandam veſicam præſtat.

4. Se nunquam in medio ureteris hærentem inveniſſe calculum, ſed ſemper vel prope infundibulum, vel prope veſicam.

1610 Decembris. Oſtendit Muſculos pectoris, & primò pectorales dictos, qui inſerviunt adducendo brachio ad pectus, horum & inſertionem q. in *Veſling.* deinde Muſculos ſerratos anticos minores, qui inſerviunt humero antrorſum adducendo, & ſub pectoralibus ſiti ſunt in proceſſus coracoidos inſerti.

Tum muſculos ſerratos anticos majores, qui inſerviunt ſcapulæ antrorſum deorſumque ducendæ; in baſin ὠμοπλάτου inſeruntur.

Pòſt, intercoſtales externos, qui ab inferiore latere coſtæ ſuperioris orti in ſuperiorem marginem coſtæ inferioris inſeruntur; & tandem intercoſtales internos qui à ſuperiore margine coſtæ inferioris orti in inferiorem marginem coſtæ ſuperioris terminantur. Horum muſculorum fibræ ſe invicem obliquè interſecant in crucis Andreanæ formam: Neutrorum ſcil. Fibræ ad coſtas perpendiculares ſunt ſed obliquæ.

Notavimus venas & arterias mammarias; externas, quæ ab axiliaribus oriuntur; & internas, quæ à Subclaviis ortæ, & intra cavitatem Thoracis aliquouſq; progreſſæ in duos dividuntur ramos; unus muſculos Thoracis perforat & in mammas diſtribuitur, alter deorſum tendens uſq; ad medium recti muſculi ibidem cum vena epigaſtrica extremitatibus ſuis per anaſtomoſin conjungitur.

Dixit ſe obſervaſſe ſingulos venæ hujus capillares ramulos in ſingulos mammarum tubulos deſinere, & proinde ſe putare lac non à chylo ſed à ſanguine generari.

Muſculi ſubclavii à claviculis, ubi acromio junguntur, orti, in coſtam primam, ubi cartilagini ſui committuntur, deſinunt:

Obſervavimus Sphincterem & levatores ani dictos muſculos, qui ab infimo oſſis ſacri, ubi coccygi committitur, oriuntur omnes.

In ulceribus ani & fiſtulis cavendum eſt Chirurgis, nè fibras Sphincteris tranſverſim ſecent, quoniam ita amittitur facultas retinendi excrementa.

Vena cephalica diſſepit & diſtinguit muſculos pectorales & deltoides.

In collo primùm obſervavimus platyſma myodes carnoſum, ſcil. panniculus hoc in loco in muſculum degenerat, qui mento affixus caput deorſum trahit.

Deinde Muſculos Maſtoideos dictos: Tum muſculos digaſtricos, qui medio ſuo tendine Styloceratohyoides muſculos perforant.

Oſtendit muſculos oſſis hyoidis, quorum ſex ſunt paria: 1. Sternohyoides. 2. Coracohyoides. 3. Styloceratohyoides. 4. Thyreohyoides. 5. & 6. Geniiohyoides internum & externum.

Præparavit inſuper columellam cum muſculis ei famulantibus quorum duo ſunt paria, nimirum pterygoſtaphylinum internum & externum.

Oſtendit muſculos Cartilaginis Scutiformis, quorum 3 ſunt paria. 1. Sternothyreoides. 2. Cricothyreoides. 3. Hyothyreoides.

Muſculos Cartilaginis arytænoidis, quorum 4 ſunt paria: 1. Thyreoarytænoides, 2. Arytænoides ſeu Sphinſter, 3. Cricoarytænoides laterale, 4. Cricoarytænoides poſticum.

Muſculos pharyngis, quorum 3 paria: 1. Stylopharyngæus, 2. Sphenopharingæus. 3. Cephalopharyngæus, qui potius carnoſum æſophagi initium ſunt, quam muſculi; etiam Muſculus æſophagæus dictus qui gulam conſtringit.

1. In Angina Spuria inflammantur tonſillæ, in legitima Muſculi Laryngis, ſed præcipuè Arytænoides.

2. In Angina Legitima ipſius Parens fecit inciſionem in laryngem inter duos annulos ſuperiores, fiſtulâ argenteâ in vulnus admiſſâ, per quam patiens inſpirabat exſpirabatque, adeoq; eam curavit. Oportet autem ut Chirurgus inciſione facta dividat paulum & diducat curioſè Muſculos Sternohyoides & Sternothyreoides.

3. Dixit ſe obſervaſſe ramulum ductûs Thoracici ſive chyliferi ad pericardium tendentem, per quem immiſſo tubulo inflavit pericardium, unde non abſque ratione conjectabatur lympham in pericardium derivatam eſſe.

4. Pulmo humanus ungulæ bovinæ ſpecie externa ſimilis.

5. Aſperæ arteriæ rami ſeu bronchiæ intra pulmones cartilaginibus angularibus carent.

6. Valvulæ venæ cavæ tricuſpides dictæ ſunt, arteriæ venoſæ mitrales, quia ambæ junctim acceptæ mitram epiſcopalem quoquo modo repreſentant; valvulæ venæ arterioſæ ſigmoides dictæ ſunt, aortæ ſemilunares.

Pericardium in hoc cadavere præter naturam diaphragmati in ſua cuſpide erat adnexum.

Obſervavimus glandulam geminam infra laryngem ſub muſculis Sternothyreoides ad utrumque aſperæ arteriæ latus, quæ in Bronchocele (cui obnoxii ſunt Alpium & montium altiſſimorum incolæ) mirum in modum intumeſcunt.

Obſervavit inſuper ductum Thoracicum unum ramulum mittere ad glandulam parotidem.

18vo *Decembris.* Præparavit Muſculos dorſi nimirum 1. Muſculos trapezios ſeu cuculares dictos à figurâ, *de quibus abunde Veſling.* 2. Muſculos rhomboides in baſin Scapulæ deſinunt. 3. Levatores Scapulæ, patientiæ & pauperum muſculi dicti, quia pauperes cùm eleemoſyna eis negatur, Scapulas levant dicentes, *Oportet patientes eſſe.* 4. Muſc. latiſſimum dorſi, in ſummitatem cubiti inſeritur, & ab officio *Aniſcalptor* dicitur. 5. Serratos poſticos minores, qui ſuperiores. 6. Serratos poſticos majores, qui inferiores. 7. Muſc. longiſſimum dorſi, qui diſcurrit per dorſi longitudinem, initio muſculis ſacrolumbis unitus, ſingulis coſtis duas anſulas ſeu tendines nervoſos largitur, qui ſe mutuò decuſſant in crucis formam, anſulæ ſci. exteriores ſurſum tendunt, interiores deorſum. 8. Muſc. Sacrolumbos qui præcedentibus initio juncti interiùs, & proceſſibus vertebrarum ſpinatis ad collum uſq; protenduntur, ſingulis coſtis anſulas pariter donando, verùm exteriores hujus anſulæ carnoſæ ſunt, & non tendinoſæ, quemadmodum præcedentis. 9. Muſc. Semiſpinatus.

Præparavit inſuper Muſculos capitis & colli; & primò Splenicos dictos, quia Splenem bubulum repræſentant: in occiput inſeruntur, atque etiam (quod non habetur apud *Veſling* alioſve) tendinem ſatis validum à reliquâ ſui parte diviſum ad proceſſum tranſverſalem 2dæ cervicis vertebræ mittunt. 2. Complexos, adeo dictos, quòd quaſi à diverſis muſculis compoſiti videntur. 3. Rectos majores ſive externos. 4. Rectos minores ſive internos, a primæ vertebræ tuberculo exortos. 5. Obliquos ſuperiores. 6. Obliquos inferiores. 7. Muſculos maſtoideos. 8. Longos. 9. Scalenos. 10. Tranſverſales. 11. Spinatos, de quibus conſulantur auctores. Tandem præparavit Muſculos Sacros & Muſculos quadratos.

Monſtravit muſculos faciei. In fronte membrana carnoſa in muſculum degenerat, ibi incipiens duplicari ubi deſinunt capilli.

Obſ. 1. Muſculi labia obliquè moventes ſeu dividentes, Sardonii dicti, in morbo illo riſu Sardeo nimirum debent ſecari.

2. Si caput inungatur pinguedine ſupra Cranium humanum naſcente capillos abunde producit.

3. Qui è Febri maligna moriuntur, iis inteſtina poſt mortem livida aut viridi-cœrulea apparent.

4. Non eſt pericranium à perioſteo diverſum, verùm perioſteum in capite dicitur pericranium, poteſtque in plures, v. g. 7 vel etiam 10 plagulas dividi.

Muſculi temporales, ob tutelam & ut in ſitu contineantur, membranâ propriâ conteguntur, a nonnullis falſò pro pericranio habitâ. Cavendum eſt ne hæc membrana lædatur: Siquidem vulnus ei inflictum, non rarò convulſiones excitat, unde & hujuſmodi vulnera lethalia habentur.

19no Decembris. Præparavit muſculos faciei: Naſi duos, triangulares ſci. & obliquos: Oculi, ſphincterem palpebrarum: Labiorum, elevatores ſci. labii ſuperioris, quorum duo paria; unum ab angulo interiore oculi ortum labiis & naribus commune. Muſculi ab oſſe jugali nati, ideoque Zygomatici dicti, in riſu Sardeo diſſecandi ſunt. Obſervavit in nonnullis hos Muſculos deeſſe. Conſtrictor ſive Sphincter labiorum Muſculus nonnullis Baſiatorius dictus. Depreſſores Labii inferioris ab imo mento exorti, admodum ſpongioſi ubi pili creſcunt. Aliud par, quod Labium inferius & ſuperius ſimul deorſum abducit, in angulos muſculi Sphincteris ſive Oris inſertum.

Muſculos maxillæ inferioris, ſcil. Temporalem, Maſſetarium, pterygoidem internum & pterygoidem externum, qui maxillam ſurſum trahunt omnes; diagaſtricos deinde, qui deprimunt.

Not. 1. In cranio perforando trepano, cavendum eſt a ſuturis; nam ſi Dura Mater (quæ per ſuturas cum pericranio committitur) lædatur, periculum ingens eſt nè ægrotus convulſus moritur.

2. Cerebrum humanum ingens eſt, corporis magnitudinis reſpectu habito.

3. In Cerebri ventriculis obſervavi duo corpora Hippocampi & Bombyces Arantio dicta ob ſimilitudinem.

4. Cerebrum non pulſat per ſe, ſed Arteriarum reſpectu: Nam ſi animalis vivi cranium aperias Cerebrumq; denudes, & ex una parte Menyngem piam amoveas cum vaſis eidem intertextis, videbis alteram partem pulſare,

alteram verò nudam membranâ non item. Aſſerit ſeipſum hoc expertum eſſe, cerebrumque poſt amotionem Cranii plùs horæ quadrante pulſaſſe.

5. Obſervavimus nervorum par 4. ſeu Fallopianum, qui a poſteriore Cerebri parte exorti, ad latera baſis Cerebri reptantes, juxta tertium par exeunt.

6. Obſervavimus plures nervos è 7. conjugationibus non ſimplices eſſe, quamvis ex uno foramine exeunt, ſed revera diviſos & multiplices, nimirum tertium & quintum par ex 4. utrumque nervis conſtat, ſextum ex 8. vel 10. verùm omnes illi ſimul ſumpti non adeo ampli ſunt *ac ego putaveram.*

7. Glandula pituitaria major eſt in homine & ſolidior, quàm ego in animalibus obſervare ſolitus ſum.

8. Sub infundibuli membranâ duo nobis oſtendit corpuſcula alba viciæ magnitudine, teſticuli figurâ, quæ fratrem ſuum primùm inveniſſe dixit, *verum habentur picta apud Veſlingium.*

9. In foraminibus Narium maximis obſervavi 4. corpora aut etiam plura, oblonga ſpongioſa, membranâ tecta, quæ (ut probabile eſt) Mucum ne defluat impediunt.

10. Pia Mater compoſita videtur ex tunicis venarum & arteriarum, quæ eam crebræ perreptant.

Poſt aggreſſus eſt oculi diſſectionem, in quo Muſculos 6. notavit. 1. Elevatorem, Superbum & Hiſpanum dictum. 2. Depreſſores, humiles & Capucinos. 3. Adducentes, bibitorios & Germanos. 4. Abducentes, meretricios. 5. Obliquantes, & 6. Trochleatores, amatorios dictos.

Anterior tantùm tunica humoris cryſtallini aranea dicitur, poſterior pro parte hyaloidis habetur.

Obſervavit Muſculos in ſingulos ingredientes nervos; de Muſculis oculi loquor.

Veſiculæ ſeminales ad vaſa deferentia, immediate ſupra glandulas proſtatas, veluti alæ utrinque adjacent, originem ſeu radicem ſuam juxta glandulas habentes.

Capſulæ Seminales nihil aliud ſunt quàm vaſa deferentia dilatata immediatè ſupra glandulas proſtatas.

Meretrices in coitu habent artem Vulvas coarctandi, os coccygis protrudendo introrſum, adeòque coitum jucundiorem reddendi.

21mo Decembris. Obſervavimus in uno cadavere Arteriam Spermaticam inferiùs à trunco Arteriæ magnæ ortam aſcendere & emulgentem ſupergredi.

Vidimus manifeſtè Capſulam Spermaticam perforatam in Urethram, atque etiam in veſiculas ſeminales, adeò ut in utramque facilè ſtylum admittat. Foramen illud, quòd in urethram exit per tuberculum in ipſo veſicæ collo, ſeu caranculam in urethræ initio, valvulam habet quæ impedit nè Sperma involuntarie exeat, aut in capſulas regrediatur.

Aſſerit ille ſe nunquam inveniſſe ſemen in glandulis proſtatis, neq; agnoſcere ulla foramina per quæ Semen in urethram exeat. *Ego aliter ſentio, & puto Sperma in hiſce glandulis contineri etiam in homine.* Putat ille glandulas ideo tantùm factas eſſe, ut veſicæ collum comprimant, adeoque co ducant ad Semen cum impetu ejaculandum.

In Urethra ſumma, in glande ſci. penis propè extremitatem, cana ſe dilatat & foveolam efficit, in qua ſi materia aliqua acrior aut putri ſtagnet, ſive Sperma ſit, ſive urina, acerrimos dolores creat, & puſtul cauſat.

Gonorrhæa flava vehementiſſimos excitat dolores.

Muſculos manûs diſſecuit. Ii autem ſunt. 1. *Deltoides.* 2. *Coracoi* qui humero attollendo inſerviunt. 3. *Rotundus major.* 4. *Rotundus min* qui humerum deprimit. 5. *Spinatus* inferior. 6. *Spinatus* ſuperior. 7. *In fraſcapularis* ſive *demerſus*; qui humerum circumrotare creduntur. 8. *Bi ceps*, præcipuè notabilis ob duplex initium, quorum alterum in ſinu vel foveolâ oſſis humeri capiti inſculptâ, velut nervus arcûs in fibulâ, tendin ſuo immittitur. 9. *Brachiæus*; qui cubitum flectunt. 10. *Muſculus longu* 11. *Brevis*; qui ſimul juncti. 12. *Anconæus*; ſimul cubitum extendun 13. *Quadratus.* 14. *Teres*; qui pronatores dicuntur. 15. *Supinator longu* 16. *Supinator brevior.* 17. *Muſculus palmaris*, qui per totam manûs volam ex- panditur. 18. *Flexor Carpi externus.* 19. *Flexor Carpi internus.* 20. *Extenſ Carpi externus.* 21. *Extenſor Carpi internus.* 22. *Flexores primi* internodii, *Lum- bricales* dicti, à tendinibus flexorum 2di internodii, orti & carnoſi: 23. *Flexores ſecundi* internodii, *Perforati* dicti. 24. *Flexores tertii* internodii, *Per- forantes* nominati. 25. *Abductor* minimi digiti. 26. *Annularis.* 27. *Me dii.* 28. *Indicis* indicator dictus, præcipuè notabiles. 29. *Adductor Indicis.* 30. *Medii.* 31. *Annularis.* 32. *Auriculares*, muſculi in formam crucis intra digitos collocati, atq; *Interoſſei* dicti. 33. *Flexor* primi internodii pollicis. 34. *Flexor* 2di internodii, qui in 4 partes dividi poteſt. 35. *Adductor.* 36. *Abductor* pollicis. 37. *Flexor* 3tii internodii. 38. *Extenſor* primus pollicis. 39. *Extenſor* 2dus. 40. *Extenſores* digitorum, qui articulos ſingulos velu inveſtiunt.

25to Decembris We ſaw the Operation of cutting a Child out of the Womb, performed in a Carcaſs by *Marchetti* the younger: This is called *Partus Cæſareus*

He told us that himſelf had taken a Child out of the Mother's Womb after ſhe was dead, which lived two or three Days.

Inciſio facienda eſt in uno latere, cavendum à *linea alba* & locis ei vicinis propter muſculorum tendines, qui ibi omnes conveniunt, & ſi ſecentur difficiliùs coaleſcunt: In illo etiam latere, ſeu ibi, ubi infantis ca- put exiſtere, ſeu jacere deprehenditur.

In inciſione facienda magnam adhibere oportet cautionem, ſenſim & leniter ſecando, nè inteſtina vulnerentur: Poſtquam Chirurgus Muſculo omnes & peritonæum perforaverit, de reliquo debet immittere duos digito adeoque omnes muſculos & peritonæum attollere, atque digitos ſurſum verſus ſcalpellum dirigendo, ſecare. Ita cum aperit *uterum* ſummopere caven- dum eſt pariter ne fœtum vulnerat.

Poſtqu

Fig. 9.

B
B
A

Fig. 1.

Fig. 2.

Fig. 8.

B
C
A
D

Fig. 7.

E
E
E
E
E
E
D
A
A
D
D
D

Vena Pulmonalis

Fig. 4.

Fig. 11.

a
c
b

Fig. 3.

Fig. 5.

Fig. 13

Fig. 12.

Fig. 10.

Fig. 14.

Fig. 6.

E
E
E
D
A
A
D
D
C
C

Fig. 15.

G
I
B
A
FH

Poſtquam fœtus eductus eſt, vulnus conſuendum eſt, acu per omnes muſculos & membranas adducto; & fila connectenda ſunt in ſingulis puncturis. Uterus ipſe nullo modo conſuendus eſt.

Hoc facto in uterum injici debet decoctum Saniculæ, Conſolidæ, & aliorum vulnerariorum; item Vinum maximè auſterum. Vulneri autem externo abdominis primò applicari debent lintea albumine ovorum madefacta, deinde Emplaſtra, ut Diapalma, &c. quòd ſi vulnus ad ſuppurationem veniat, immittenda eſt in inferiorem partem turunda.

Dixit ſe nunquam potuiſſe obſervare oſſa pubis in partu ſeparari, nam & ipſe in partu difficillimo ei parti manum impoſuit, nec potuit ſentire ullam disjunctionem aut oblongationem. Habet etiam argumentum ex Hippocrate contra hanc ſententiam deſumptum à callo, Qui inter oſſa fracta aut luxata exiſtere ſolet, & in futurum oſſium ſeparationem impedit.

CHAP. II.

The HEAD.

An unuſual Bone in the Head, and a Diſtemper'd Optick Nerve, by Mr. Cheſelden, n. 387. p. 287.

FIG. 1. Shews a Bone taken from the Falx, or firſt Proceſs of the *Dura Mater*, of a Man who died of violent Headachs.

Fig. 2. Shews the Optick Nerves, the right Nerve being waſted and diſcoloured; the Eyes both appear to be very good. I had not an Opportunity of enquiring into the Caſe of this Perſon; but ſuppoſe it muſt have been a *Gutta Serena.* I open'd another Eye of a Man who died of that Diſtemper; in which I found that part of the Nerves which is within the *Cranium* cruſh'd flat by the interior Lobes of the Brain, their Ventricles being full of Lymph.

A remarkable Fracture in the Skull, by Mr. Amyand. n. 317. p. 173.

II. We had a very remarkable Fracture of the Skull, laſt Campaign at our Hoſpital at *Ghent*; it was in the interior part of the *Squamoſe* Bone, and occaſioned by a Splinter of a Fellow Soldier's Piece burſting, that ſtruck him there. Some time had paſs'd, before the Accidents made us ſuſpect a Fracture, and obliged us to make a Triangular Inciſion upon the Temporal Muſcle; a Fiſſure was diſcover'd, which indicated the Neceſſity of the Trepan. It was apply'd twice, the firſt not making room ſufficient to extract a large piece of the internal Table very much depreſſed. After this all the Accidents diſappear'd; but twelve Days after the Operation, Rigors, cold Sweats, an intermitting Pulſe, and ſome other Signs of an approaching Death, did make us deſpair of the Recovery of our Patient. He died the 15th from the Operation, and about the 20th from his Wound. His Skull was open'd, and in it three very remarkable Fiſſures obſerved. The firſt had, notwithſtanding the Sagital Suture, croſs'd from one Parietal Bone to the other, as far as the Coronal Suture on that ſide oppoſite to the Wound; another had gone croſs the Coronal Bone; and the third was on the Parietal Bone on the ſide of the Wound, pretty near the *Sutura Squamoſa*; but what is moſt ſingular, is, that none of theſe Fiſſures did reach that, upon which the Trepan had been applied. An *Empyema* was found in the *Thorax*, and a conſiderable Impoſthume in the Liver.

A Bullet lying Thirty Years in the Head. By Dr. Fielding. *n.* 320. *p.* 317.

III. At the firſt *Newberry* Fight, in the Time of the late Civil Wars, Dr. *Rob. Fielding* was ſhot by the Right Eye on the *Os Petroſum*, by the Orbit of the Eye to the Skull, which was likewiſe broke, with great Effuſion of Blood from the Wound, Mouth and Noſtrils.

The Surgeon carefully probing the Wound for the diſcovery of the Bullet, but failing of his Intention, on the third day after the Shot, plac'd him Horizontal to the Sun; by which means depreſſing the broken Skull with the Probe, he could ſee the Palpitation of the Brain, but could not diſcover the Bullet.

When the Doctor began to grow old, his Mouth cloſed up, and ſo continued for the ſpace of half a Year, till many Fractures of Bones were come out of the Wound, Mouth, Noſtrils; and afterwards whenſoever a Scale of Bone was to come out, his Mouth would cloſe, inſomuch that ſeveral Years after he prognoſticated to ſome Friends, that a Bone was then coming out, which continued ſo for ſix or ſeven Weeks; at which time finding an itching in the Orifice of the Wound, with his Finger he felt a Bone, upon which he made known to ſome Friends then preſent, that they ſhould ſee him open his Mouth, and taking out a Bone no bigger than a Pins Head, he immediately open'd his Mouth.

At the ſecond *Newberry* Fight it heal'd up, no Art could keep it open. After this, for the ſpace of Ten Years, or more, a Flux of Sanious Matter iſſued out of the Right Noſtril, and then ceaſing there, it flow'd from the Left Noſtril for ſome Years. At length, for the ſpace of two Years or thereabouts, upon riding, the Doctor would ſometimes find a pain on the Left Side, about the Almonds of the Ear, which he attributed to Cold; but more eſpecially after riding in a cold dark Night, which occaſion'd a kind of Deafneſs too; and having ſtop'd his Ear with Wool to recover his Hearing, one Day, either Writing or Reading, ſuddenly an Huffe came in the Ear, which made him ſtart, and the manner not to be expreſs'd, unleſs you can imagine a *Vacuum*; this happen'd about *March* or *April*, 1670. Upon this, all that ſide of the Cheek hung looſe as tho' Paralytick, and under the Ear might be felt a hard Knob.

After this, Tumour upon Tumour appear'd upon that ſide under the Jaw-Bone, which occaſioned his conſulting ſome Phyſicians, two at one time, one of which ſuſpected the Bullet, which, conſidering the Shot, they thought not credible. At length the Tumours coming to the Throat, if he held up his Head a little, it ſeem'd as if one with a Hook did pull down the Jaw-bone; and if any thing touch'd the Throat, 'twas as painful as if prick'd with a handful of Needles. Being at laſt perſwaded to make ſome Applications, a ſmall hole appeared, after that another, and a

third near the *Pomum Adami*; by theſe the Bullet was diſcover'd, and cut out in *Auguſt*, 1672.

An Experiment to diſcover the true Cauſe of the Motion of the Dura Mater. *By Mr.* Ridly. *n.* 287. *p.* 1480.

IV. Canis ſuperiori parte bregmatis terebrata, Membranam duram non ſine aliqua Sanguinis jactura denudavi. Statim prius linteis carptis cohibita hæmorrhagia, ablutaque Sanguinis aſpergine, apparebat Membranæ iſtius, ejuſque Sinus longitudinalis qui per Marginem foraminis decurrebat, motus Syſtalticus, cordis vibrationis quæ citatior ordinario erat æmulus, eique rithmo ad amuſſim reſpondens. Hanc mecum aſtantes vibrationem per ſpatium unius horæ quadrantis conſpexerunt, quo elapſo, eam hamulo acuto prehendere tentabam, ut ita, ea ſuper ipſum ſuſpenſa & cultro diviſa, cerebri ſubtus motum ſatius expiſcarer & ediſcerem: factâ autem ejus perforatione. (quam acute perſentiſcere canis videbatur, cujuſvis autem Convulſionis expers) inſequuta eſt hæmorrhagia, ut prius, brevique ſimiliter reſtricta. Eloto Sanguine, particulari iſto foramine à punctura hamuli in Membrana facto, comparuit Cerebri motus ſyſtalticus, parum id Sanguinis qui in eo hærebat, extus propellens.

Poſtea cuſpide forficis obtuſo caute in aperturam Membranæ vulneratæ immiſſo, eam tranſverſim parte ejus à ſinus longitudinalis tranſitu maxime remota diſſecui; quo facto ſe protruſit cerebrum Pia Matre textum per iſtam aperturam, adhuc manente motu ejus ad tactum ſatis forti, licet membrana ipſa, ob vim reſtitutivam ſuarum fibrarum à plaga accepta diminutam, ad oculos obſcurius vibrari videbatur. Toto hoc tempore vegetus erat canis, levibus tantum tremoribus cum quodam horrore per univerſum corpus affectus.

Poſt horas aliquot hiſce obſervandis tranſactas, quo tempore variis modis excruciatum fuiſſet animal, multumque ſanguinis effudiſſet, tandem ut omnem vim pulſificam quæ vel huic membranæ ineſſe, vel ei aliunde donari ſupponi poteſt ratione fibrarum ſuarum, quarum beneficio motum primo à cerebro ſibi impreſſum validius abſolvit, adimerem, guttulas aliquot oli Vitrioli, unde nigro tingebatur colore, in eam leniter illinebam; Hinc nulla, ſaltem exigua & obſcura ejus percepta eſt vibratio; non obſtante autem hac ejus ad impulſum cerebri obſequendum ineptitudine, admoto digito ſatis diſtinctus cerebri pulſus ſentiebatur.

Peractis his, adhuc alacri manente cane, cum cultri mucronem ad unius unciæ craſſitiem in cerebri molem intrudiſſem, vehementer ſe agitabat canis, & in horribiles corporis jactationes, & pedum anteriorum & poſteriorum convulſiones incidebat; quo tempore immiſſo ut prius in aperturam præfatam digito, fortius ſe movebat cerebrum quam antea; Poſtea ſpecillum profundius adigebam, unde ſenſus dolorifici indicia maxima dedit; tandem cultro integro uſque ad latus cranii oppoſitum trajecto, horrendi exoriebantur ſpaſmi; & jam quo omnis de cerebri motu dubitatio amoveretur, digitis noſtris tum ego, tum adſtantes pari, modo quo aliàs, indentes, Syſtolen Diaſtolenq; ejus cum maximo renixu factam perſenſimus. Quid amplius ad veram genuinamque motus hujus partis cauſam proximam eru-

endam detegendamq; deſiderari poteſt haud video, modo ad phænomena in hoc experimento occurrentia, ſedulo attendatur.

An extraordinary Caſe of an ApoplectickPerſon. By Dr. Arch. Adams. *n.* 313. *p.* 40.

V. I open'd the Head of a Woman who died of an Apoplexy, and in the Left Ventricle of the Heart, I found four or five Ounces of clotted Blood; in the Right Ventricle no Blood at all, but every thing as uſual; and all the Nerves which commanded the Right Side of the Body were as ſtrong as any I ever obſerv'd in a ſound Animal, eſpecially in the Origin, and as far as I could trace them in their Courſe. It was my Opinion, that whichever Ventricle the Obſtruction was in, the Nerves and Muſcles correſponding to that ſide were affected, but here the contrary appear'd plainly; for altho the Obſtruction was in the Left Ventricle, the Senſe and Motion of the Right Side were entirely loſt, and the ſmall Remains of either were obſervable in the Left Side.

An unuſual Cancer. By Mr. Jon. Kay. *n.* 277. *p.* 1069.

VI. My Father had a Cancer, which took its Riſe from the *Os jugale*, and in proceſs of time ſpread itſelf over the whole Cheek; and notwithſtanding the Endeavours of the moſt eminent Surgeons, ulcerated his Eye round, which I ſaw him take out with his own Hand; and afterwards extended itſelf to his Ear, and through his Cheek into his Mouth, and acroſs the upper part of his Noſe, and perforated the Bone there. It likewiſe overran the ſide of his Forehead, fouling the *Os frontis* which came away in pieces, leaving the *Dura Mater* bare as broad as Half a Crown, which running thro the Perforation of the *Cranium*, in a few days putrified and expos'd the Brain itſelf, and ſeveral Portions of it came away freſh and untainted; and what is moſt ſtrange, he perfectly retained his Senſes, and roſe every day to dreſs the Ulcer himſelf, till a conſiderable quantity of the Brain was come away. And when he was confined to his Bed, his Speech firſt fail'd, and he died about four Days after, his Brain being totally conſumed, and nothing remaining in the *Cranium*, but a ſmall quantity of black putrid Matter. He had neither Spaſmus nor Convulſion of any part all the time of his Illneſs.

An unuſual Blackneſs of the Face. By Mr. Yonge. *n.* 323. *p.* 425.

VII. A Girl 16 Years old, had a few hot Pimples riſen on her Cheeks, which Bleeding and a Purge or two cured. She continued very well till about a Month afterward; when her Face, ſo far as is uſually covered with a Vizard Mask, ſuddenly turned black like that of a Negro. This ſurprizing Accident much amazed and frighted the Girl, and the Paſſion and Terror of Mind encreaſed to a great degree, even to Diſtraction, and then they demanded my Aſſiſtance.

By ſome compos'd Anti-Hyſterical Remedies, the Violence of her Fits was much pacified. I alſo directed a Lotion for her Face, which took off the Diſcoloration; but it returned frequently, with no Regularities, ſometimes twice or thrice in Twenty four Hours, ſometimes five or ſix times. It appears inſenſibly to the Girl, without Pain, Sickneſs, or any Symptoms of its approach, except a little warm Fluſh juſt before it appears. It eaſily comes away, and leaves the Skin clear and white, but ſmuts the Cloth that wipes it from the Face; it feels Unctuous, and ſeems like Greaſe, or Soot and Blacking mixt. It hath no taſte at all, which is to me very ſtrange, that a fuliginous Exudation ſhould be inſipid.

She never had the *Menſes*, is thin, but healthful; the Blackneſs appears no where but in the prominent part of her Face. The Blackneſs is ſince divided into a few dark cloudy Specks, which appear but ſeldom, and nothing ſo livid as formerly.

ib. p. 432.

An extraordinary Wen cut off the Cheek. com. by Dr. Bower. *n.* 354. *p.* 713.

VIII. *Alex. Palmer*, of the Pariſh of *Keith*, in the County of *Bamff*, in the North of *Scotland*, about Fifty Four Years of Age, obſerved, when about Twenty Seven, a little hard Swelling in the Muſcle of the lower Jaw on the Left Side, without any Hurt or manifeſt Occaſion; which at firſt went on ſlowly, but afterwards it proceeded more quickly, and the older it grew, it ſtill came on the faſter, until it increaſed to a prodigious Bulk and Weight. From the firſt Appearance of this Tumour to the total Exciſion of it, there were about Twenty Seven Years. He had exceſſive Pains and Uneaſineſs in it, and at laſt it mightily extenuated and emaciated him, who was otherwiſe a ſtrong and robuſt Man.

This Excreſcence was of the natural Colour of the Skin, and ſeem'd to be an *Atheroma*, being a glandulous Subſtance, with ſeveral big Blood-Veſſels in it, and had Hair growing on it, as on the other Parts of the Body, as may yet be ſeen. It was almoſt round, and very hard, and was as ſenſible as any other Part of the Body; for, when the poor Man was working in the Fields, ſome ſix or ſeven Years ago, he accidentally made a great Gaſh or Wound in it with a ſharp Iron, which was very painful; but was cured by a Surgeon, after the manner of an ordinary Wound; the Cicatrice is ſtill to be ſeen in it.

This Excreſcence having grown ſo big, was attach'd to the Muſcle under the Left Eye, call'd *Obliquus minor* or *inferior*, to the Ear and its Muſcles, and to the Muſcle of the lower Jaw, named *Deprimens*. By reaſon of its great Bulk and Weight, it could not hang down freely without ſome Support, therefore it reſted on the top of the Shoulder, which made a conſiderable Dimple in it, that is yet very obſervable; beſides, it was holden up by the Man's Hand in the Day time, and laid on a Pillow in the Night ſeaſon.

Some three or four Days before the total Exciſion was made, the Patient obſerved this Tumour begin to mortify at the lower end, which made him ſo uneaſy, that he took a Knife and cut off a good part of it. This occaſion'd a great Hæmorrhage, ſo that he reckon'd there was loſt a *Scots* Pint, or four Pounds of Blood, before it could be ſtopt. The Patient at laſt applied himſelf to Mr. *Gordon*, Surgeon of the Place, who made a total Extirpation of it, on the 19th of *January*, 1717.

He made a cloſe Ligature, taking in the Baſis of the Excreſcence, and all the looſe Skin, and contracting it as much as poſſible, he cut it entirely off with a ſharp Raſour. There guſh'd out of the Excreſcence, after it was cut off, and was lying on the Ground, as near as could be gueſs'd, two Pounds of Blood; for it was nouriſh'd by ſeveral large Blood-Veſſels, perhaps by ſome Branches of the *Carotide* Artery much inlarged, and other Blood-Veſſels coming from the Ear, and the Muſcles of the Eye and lower Jaw above mentioned. When Mr. *Gordon* brought it to us, which was full three Months after it was cut off, we cut off with a Knife about a quarter of an Inch broad of the Baſis of it; and in that ſmall Space we obſerved four big Blood-Veſſels. The Baſis, as it now appears, is five Inches Diameter, which ſeems too large for the whole ſide of the Face: So that after the Exſection, I think the looſe Skin has turn'd backwards, which may make the Baſis now appear ſo big.

After all this Blood was loſt, the Excreſcence was weighed, and was full Nineteen Pound Weight; ſo that before his own Inciſion, and this Operation, it behoved to be ſeveral Pounds heavier; which is a moſt prodigious Weight to be depending on ſuch a Place. This Tumour was of a Spheroidical Figure, and when meaſured, was Thirty four Inches about by the longeſt way, and Twenty eight by the broadeſt.

The Hæmorrhage, which was great, was ſtopped by the Vitriolic Powders and other Aſtringents, and the ordinary Dreſſing was uſed. So this great Cure was completed in ſix Weeks time, and the Patient entirely recover'd, and goes about his Buſineſs, to the great Admiration and Aſtoniſhment of every Body. The Lid of his Left Eye continues ſtill downwards a little, as does that ſame ſide of the Mouth, which was occaſion'd by the great Weight depending on that ſide of the Face, but it may be expected they may come again to their right Poſture: for the Head, at firſt after cutting, inclined much to the Right ſide, by reaſon of the great Weight on the Left Cheek having been removed; but it now begins to ſtand upright ſince he is perfectly recovered. Tho' the Skin, and even a deal of the muſculous part of the Cheek and lower Jaw was cut away, yet it is grown up again, and is of the ordinary Colour of the Skin, and like the other ſide of the Face, ſo that there grows Hair on that ſide of the Face as well as on the other, which he ordinarily ſhaves; and this is as ſurprizing as any thing in the whole Affair.

Obſervations on Inciſions of the Cornea. *By* Dr. Gandolph. *n.* 322. *p.* 387.

IX. In a blow upon the Eye, in the Month of *April*, 1709. there was a light Contuſion on the outſide of the Part, with very little Alteration to appearance; but a Veſſel being broken within, pour'd forth a conſiderable quantity of Blood. The Eye alſo loſt its Tranſparency, and almoſt its Sight; which was ſo very weak, that it could ſcarce perceive the greateſt Light when objected to it. The *Cornea* appear'd all over red, but without any Inflammation or Blood-Veſſels; it receiving its colour from the Blood pour'd in upon the *Aqueous Humour*. The Patient had been let Blood thrice; and the 8th Day I cauſed the *Cornea* to be open'd near the middle; my Deſign being to make a large Orifice, I determined not to make it at the bottom of the *Cornea*. The Orifice being made, there came forth ſome drops of the *Aqueous Humour* mixt with Blood. The *Cornea* ſtill appear'd as red as before, and was not ſo even as we could have wiſh'd. This Circumſtance made me reſolve to make a ſecond Orifice immediately, as large as the former, but lower. There run out ſome drops of the *Humour*; and the Eye appear'd not ſo red and convex as before. The *Humour* continued coming out of the Orifice for ſome time. We applied nothing to the Eye but a Compreſs (or Stupe) dipt in a Mixture of four Ounces of *Plantain* Water, and two Ounces of a Vulnerary Water.

The Day after the Operation, the upper part of the *Cornea* was tranſparent, the lower part not ſo red, and the whole Membrane appear'd to have recover'd its natural Convexity. It ſeems that all the extravaſated Blood had quite run out, had the lower part of the *Cornea* been open'd, and remain'd ſo for ſome time. I obſerved the Alterations of the Eye for three Days together; in which time the extravaſated Blood ſeem'd ſome times to ſpread over the whole Cavity of the *Cornea*. We judged that the motion of the Patient himſelf, had open'd anew ſome Blood-Veſſel, or had mixed the extravaſated Blood with the *Aqueous Humour*; for we did not perceive all that time that there was any freſh Effuſion of more Blood.

The 5th Day after the two firſt Inciſions, I cauſed a third to be made at the bottom of the *Cornea*; there run out ſome drops of the *Humour*, and continued ſo to do for ſome time; and two Days after, the Eye recover'd its natural Tranſparency. The *Pupil* was now very much dilated; but by little and little it contracted again, but not to its uſual ſmalneſs. The *Iris* all this while kept its motion; ſo that we cannot ſuſpect that the Lancett, in making the Inciſion on the *Cornea*, any ways touch'd upon the *Iris*, becauſe the *Pupil* continued exactly round: And a ſtroke that is able to divide the continuity of the Parts of the Eye, and cauſe a ſuffuſion of Blood, is but too capable of depriving the *Iris* of its natural Power of Contracting.

The *Pupil*, which before the Blow was one Line in Diameter, when the *Iris* was contracted, is at leaſt two Lines in Diameter at preſent. The tranſparency of the *Humours*, and Convexity of the *Cornea* are the ſame as before. The Sight is now reſtored, and there remains no other alteration than what neceſſarily follows from the like Dilatation of the *Pupil*.

From hence we may draw the following Remarks:

1. Inciſions are made on this Part without any Pain.
2. The Orifices unite again without any Scar; which has been before obſerved, but is known to very few.
3. We find that Plants of a diſcuſſive Quality have an ill Effect, the Patient finding himſelf much worſe after uſing a Cataplaſme made of *Cervile* and *Parſley*. Theſe Plants, which are excellent in reſolving Extravaſated Blood in the Muſcular Parts, have an ill effect when applied to the Eye, by cauſing Pain, and rendring the Sight more diſturbed. We had twice Experience of this, and the Patient aſſured us both times, that he found himſelf much better from the uſe of the firſt Medicine.

When there is a conſiderable Effuſion of Blood in the Eye in couching of a Cataract, and no Orifice is made in the *Cornea* to let it out, it may ſo alter the Tranſparency of the *Vitreous Humour*, as to cauſe a loſs of Sight; which ſometimes follows from this Operation.

I made the Inciſion higher on the *Cornea* than it ought to be, becauſe the Perſon that perform'd the Operation, having never before made the like, and deſiring to make an Orifice large enough to diſcharge eaſily the *Aqueous Humour*, I thought it proper to make it near the middle of the *Cornea*, that the Point of the Lancett might not touch upon the *Iris*; which would have been of much worſe Conſequence than a Scar. The Effuſion of Blood, that ſometimes happens in Couching of Cataracts, is diſcuſſed again, either by external Applications, or the help of Nature; but when the Effuſion is very conſiderable, this Operation may be neceſſary to prevent worſe Conſequences.

As for the Scar, that ſometimes follows from an Inciſion of the *Cornea*; I remember I have read in an ancient Phyſical Author, that we need not fear it. But if we practice Inciſions on Eyes affected with Inflammations, Ulcers, or Defluxions, which very much dilate the *Retina* and Veſſels, an Eſchar forms itſelf much more eaſily in theſe Caſes, and conſequently we ought to uſe the greater caution; which was not ſo neceſſary in my Patient, who had no kind of Inflammation on the Eye or *Cornea*.

Obſervables in a human Ear. By Dr. Arch. Adams. *n.* 311. *p.* 2415.

X. The boney Cavity of the Ear is covered at each end by a Membrane; the former is called the Membrane of the Drum, and the other is directly oppoſite to it; the outer is ſtronger than the inner, ſo I call them with ſubmiſſion. They are joined together by the handle of the *Malleus* adhering to the outer, and the upper part of the Stirrup to the inner; which by the intervention of the *Incus* and the Orbicular Bone, make a Chain, and they ſeem to be acted and re-acted by theſe ſmall Bones reciprocally.

Whether Artiſts had any reſpect to this Original, when they firſt deviſed Drums, I cannot ſay; but nothing can more nearly repreſent the Natural, than the Artificial does; the Skins of this anſwering to the Membranes of that, the Wooden Cylinder to the Boney Cavity; the ſound of the Drum would be flat without a Hole in the ſide, and Nature has given a paſſage from the Palate to the Ear. The Skins of the Drum would leſſen the Sound, if they were not kept on the ſtretch; ſo would thoſe of the other flag, if the handle of the Hammer and the Stirrup kept them not on the Tenſe.

This inner Membrane is cloſely ſtretch'd before the Labyrinth, the *Foramen rotundum*, and the paſſage into the *Cochlea*, (I omit the *Foramen Ovale*, becauſe the Foot of the Stirrup exactly ſhuts it) that ſo the ſound may be the bigger upon its approach to the Nerves. The Stirrup is generally broke in diſſecting the Ear, particularly that Cover which goes over the Bone on each ſide; but if it be carefully open'd, the Stirrup is entirely cover'd with a Membrane, which forms a Cavity flatly Oval, and the inſide is Excavated.

XI. *Accounts of* Books *omitted.*

1. CONſilium Ætiologicum de Caſu quodam Epileptico: Quo reſpondetur Epiſtolæ Doctiſſimi viri *Thomæ Hobart*, M. D. Annexa diſquiſitione de Perſpirationis Inſenſibilis materia, & peragendæ ratione. Auctore *Gulielmo Cole*, M. D. *Coll. Med. Lond.* Socio. *n.* 287. *p.* 1485.

2. De Aure humana Tractatus: In quo integra Auris fabrica, multis novis inventis & Iconiſmis illuſtrata deſcribitur; omniumq; ejus partium Uſus indagantur. Quibus interpoſita eſt Muſculorum Uvulæ, atque Pharyngis nova Deſcriptio & Delineatio. Auctore *Antonio Maria Valſalva*, *Imolenſi*, Philoſophiæ & Medicinæ Doctore, in *Bononienſi* Univerſitate ad Inciſionem & Oſtenſionem Anatomicam Profeſſore conducto, necnon Noſocomii Incurabilium Chirurgo. in 4*to*. *Bononiæ*, 1704. *n.* 299. *p.* 1978.

3. Diſſertatio Epiſtolaris de Glandulis conglobatis duræ Meningis humanæ, indeq; ortis Lymphaticis ad Piam Meningem productis. Auctore *Antonio Pacchiono*. 8*vo*. *Romæ* 1705. *n.* 328. *p.* 208.

CHAP. III.

The Neck. The Thorax.

1. *A very large Tumour in the fore part of the Neck. by Dr.* Douglas. *n.* 305. *p.* 2214.

I Lately had the Opportunity of opening a Woman about 50 Years old, who had a very large Tumour, or hard Swelling, in the fore part of her Neck, poſſeſſing all the Space between the whole Extent of the lower Jaw and the upper part of the *Sternum*, with a conſiderable riſing in its middle; laterally its point inclining to the Left ſide, tho' the biggeſt part of the Tumour was on the Right. The Skin on the *Apex* of this protuberating part was thin and ſhrivell'd, of a Colour different from the reſt, and lookt as if the Swelling would have broke in that Place.

The Skin was exceeding thin, having no Fat under it, only in a Cavity between two Lobes, to be afterwards deſcribed, on its Right-ſide, there was a ſmall appearance of ſome; for the Skin being leſs ſtretcht there, the Cells of the *Membrana adipoſa* were not quite emptied. The fleſhy Fibres of the *Latiſſimus colli* were ſcarcely viſible. The *Maſtoidæus* and *Coraco-hyoidæus* were extremely thin, and in their Aſcent they adhered very firm to the ſubjacent Tumour. The *Sterno-hyoidæus* and the *Sterno-thyreoidæus*, that run up the fore-part of this Swelling, were diſtended ſo thin, that it was difficult to ſeparate them from it, eſpecially the laſt named. The Right *Carotid* Artery, in its Aſcent to the Head, ran along the outer edge, which increaſing, did much obſtruct the Current of the Blood that way.

The internal *Jugular*, the *Par Vagum*, and the Intercoſtal Pair went alſo over ſome part of this Swelling in their Deſcent to the *Thorax.* Two of the Lymphatick Glands of the *Jugular* Vein were ſwelled to the Bigneſs of little Eggs, being placed at ſome diſtance one from another, with a Hollow between, where ſome Fat was found; theſe two Lobes made the Tumour very uneven alſo on its Right ſide.

Theſe Muſcles, the *Jugular* with the two Glands adhering to it, and the reſt of the forenamed Veſſels being removed on both ſides, I could eaſily obſerve the Bigneſs, the Figure and the Circumſcription or Limits of this preternatural Tumour, with all its Adhæſions to the adjacent Parts. In Magnitude it ſeemed to exceed that of two Fiſts joined together. Its Figure was almoſt triangular, with a broad Baſis under the Chin, ſloping a little on each ſide, as it deſcended to the upper part of the *Sternum*, where its point was pretty narrow; its Surface was made uneven, by three Riſings, of which the largeſt was turned to the Left Side; the other two being placed on the Right, as above remarked. It adhered by membranous Filaments to the *Maxillar* Glands, to the *Digaſtrick* Muſcle, and to the *Stylohyoidæus*; under which, on the Right ſide, a ſmall portion of it, in the Form of a Nipple, did intrude it ſelf as it were under the Tongue; in the upper and fore part it alſo adhered to the *Os hyoides.*

Laterally it was connected to the *Levator Scapulæ*, and lower down to that part of the *Cucullaris* that terminates into the *Clavicle*; backwards to all the fore part of the *Aſpera Arteria*, between its third or fourth Cartilaginous Ring and the *Os pectoris*, as alſo to that Muſcle of the Head called *Rectus internus major*, and to ſome part of the *Scaleni*; its lower part was engaged under the *Jugulum*, or lunated part of the Breaſt-bone, to which it adhered. It was eaſily freed from its Connexions to all theſe different parts, but not ſo from the *Glandulæ Thyreoidææ*, to which it adhered after a far different manner; for where the *Thyreoidal* Glands are joined to one another, a little below the *Cartilago Cricoides*, on the fore part of the rough Artery, there was no ſeparating of it without cutting its Subſtance; whence it plainly appears, that the Union of theſe Glands was the Root or Beginning of this Tumour: And yet, which is very remarkable, the Glands themſelves kept their uſual Figure, and were no larger than ordinary.

This Tumour was hard and very firm, being exactly of the Conſiſtence of a Cow's Udder when boyled, yet in a few places it was ſoftiſh, containing a liquid and thick Juice. Its Colour was chiefly of a whitiſh Yellow, only in ſome Places it was exceeding red, from its having a greater ſtore of Blood Veſſels, and in others it was very white. I pared off all the ſoft part, and the hard Subſtance that remained I boiled, and then cleared it very well, having left ſticking to it at one Corner a ſoft Cartilaginous Body, which poſſibly, had the Patient lived longer, would have acquired the ſame degree of Induration. It very much reſembles a piece of white unpoliſhed Rock Coral; but whether it may be reckoned oſſeous, or if it be rather the Viſcid Humour of the Glands hardened and concreted into this irregular Chalky or Gravelly Subſtance, or whatever elſe it may be, I leave to better Judgment. Fig. 3. I remember about two Years ago I found in the *Proſtates* of a very old Man a great many Bodies, like white Peas, being of a Subſtance exactly like this, only ſmoother on the outſide; ſome of theſe were in the Body of theſe Glands, others adhered by ſmall Roots to the Muſcular Membrane that inveſts them. Fig. 4.

The firſt Appearance of this large Swelling was about twenty Years ago, cauſed by the breaking of a Vein, as the good Woman uſed to expreſs it, in a hard and very difficult Labour. It increaſed very ſlowly, not arriving to any conſiderable Bulk till a few Years before ſhe dyed; it was never very painful, being a true Schirrus: Many things by ſeveral Perſons had been uſed and applied unſucceſsfully. Its Bigneſs at Length became very troubleſome, in impeding her Swallowing and free Breathing, and at laſt it quite choaked her, by compreſſing the Wind pipe, upon which it lay.

I obſerv'd that the *Uterus* was entirely Schirrous, and diſtended to that degree that it filled up the whole Capacity of the *Pelvis*. Part of the *Colon* and *Ileon* adhered ſo firmly to it, that there could be no Separation without tearing. Both the *Ovaria* and the *Tubæ* grew cloſe to it; and indeed the Confuſion and Mixture of all theſe Parts was ſo great, that if the *Ovaries* had not been ſwelled here and there with Hydatical Tumours, I could not have diſtinguiſhed them. The Neck of the Womb was preſſed down ſo low, that upon a very gentle dilation of the *Labia* it offer'd itſelf to view, being extreamly hard, but yet ſmooth and even, and ſo cloſely ſhut, that I could paſs nothing without cutting. It had ſqueezed the *Veſica Urinaria* ſo cloſe againſt the *Os Pubis*, that it could contain but little or no Urine; which obliged her to make it often, and with pain. The preſſure of this part backwards was ſo great upon the *Inteſtinum Rectum*, that the evacuation of *Fæces* had been obſtructed for the ſpace of five Weeks before ſhe died.

Indeed there was obſerved to come away *per Anum* for ſome conſiderable time, a great deal of *Pus* and ſlimy Matter, but that proceeded from the *Uterus*; for the Acrimonious Humour, which was wont to be diſcharged *per Vaginam*, having been pent up within its Cavity, by the cloſe Conſtriction of the *Collum Uteri*, had corroded, and eat its way through the Subſtance of the Womb into the *Rectum*, by which it had its vent. Which deplorable Caſe I have more than once obſerved in Diſſection.

The thickneſs of the Womb was near two Inches, and in its bottom there was a great deal of this Humour, white and thick, which upon touching made the ends of my Fingers white and rough, by ſhrivelling the *Cuticula*, as if I had waſhed them with a ſtrong Solution of ſome Acid Lixivial Salt. Thus the Cauſtick Salt lodged in Soap affects the Hands of thoſe Women that waſh Linnen. It was very hard to take the *Uterus* out of the *Pelvis*, by reaſon of its ſo cloſe adheſion to the neighbouring Parts. The *Fæces Alvinæ* contained in the Guts, were but few, by reaſon ſhe could not ſwallow any thing ſolid for a long time, but very hard, and in ſeveral diſtinct Clots.

A Tumour in the Neck full of Hydatides. *By Mr.* Hewnden. *n.* 308. *p.* 2344.

II. A Gentlewoman in *London*, aged Twenty five Years, had a large wenny Tumour, the Baſis taking its Origin from all the lower hinder part of the Skull, ſtretching down the Neck near each Jugular, extending it ſelf almoſt as low as both *Scapulas;* on the upper part was a Phlegmon. The Radix being ſo large, I put on a tranſverſe Cauſtick the length and breadth of the Tumour, intending to ſeparate the *Cutis* from the Membrane of the *Cyſtis;* but it being ſo thin where the Phlegmon was, obliged me to divide the *Cyſtis*, out of which I ſav'd about Sixty *Hydatides*, of the bigneſs of a ſmall Walnut; ſeveral more were broken. Theſe *Hydatides* ſwam in a Liquor of the conſiſtence of Whites of Eggs. In this *Cyſtis* I found a large quantity of Atheromatous and Steatomatous Matter, at the Baſis a large *Sarcoma;* the greateſt part I cut off, but fearing to hurt the Muſcles of the Neck, deferred it to the next dreſſing, intending to take the reſt of the *Sarcoma* and Radix of the *Cyſtis* away by Cauſtical Medicines, which I applied with Succeſs, they coming off without making an Eſchar, the Radix being of a Cartilaginous Subſtance. Searching with my Probe to find an Interſtice, it dropt into one; and touching ſome Membranous or Nervous Body, cauſed the Patient to cry out furiouſly. Into that Interſtice I put a piece of *Roman Vitriol*, which came out the next Day all diſſolv'd, with ſome of the Radix. By continual applying the Vitriol, I extirpated all the Radix, and healed the whole *ſecundum Artem.*

Seven Years before that Operation, this Tumour was ſo big, and ſubſided of itſelf.

When I began with Cauſtical Medicines, the firſt I us'd was *Precip. rub.* with which I covered the whole Radix, which came off, and no Eſchar; but it ſalivated the Patient Five Weeks.

A Schirrous Tumour included in a Cyſtis. *By Mr.* Rich. Ruſſel. *n.* 337. *p.* 276.

III. *Auguſt* 18. 1713. I was ſent for to Mrs. *Smith*, who had been reduc'd very low by a Fever, which from her Cough, ſharp Pain under her Breaſt, and other Symptoms, was judg'd Pleuritick. But upon having a Diſcharge from her painful Breaſt, of a thin Gleet, all Symptoms vaniſhed.

When I ſaw her firſt, the Liquor diſcharg'd by a ſmall Pin-hole near the *Papilla*, was little more than would have wet a Handkerchief four times double. Examining of the Breaſt, I found a large Tumour, that lay deep, yielding to my Fingers, and paſting like Dough. I ſearch'd the Abſceſs with my Probe, and twiſted out with it a Matter like Saw-duſt or Bran, mix'd with Hair. Upon laying open the Breaſt, I ſeparated a Cyſtick Tumour, which weigh'd eight Ounces, and contain'd a ſolid Matter like the above-mention'd, mix'd with a Body like Hair.

Upon inquiring into the manner of its coming, ſhe told me, that Eight and Thirty Years ago, ſhe receiv'd a Bruiſe in that Breaſt by a Fall from a Horſe, which was attended with great Pain and Fluxion, inſomuch that the Veins of her Breaſt appear'd Varicous and Turgid, as in a Cancer. But her Pain ceaſing, they ſunk, and left an Indolent Tumour in her Breaſt, ſuppos'd by her Surgeon to be a true *Schirrus.* Since which time it hath always continued nearly in the ſame State, without pain, increaſing very little in Magnitude, but obſtructed in ſuch a manner, that ſhe could not nurſe her Child with that Breaſt. The *Tunic* was pretty thick, nouriſhed with very ſmall Veſſels, but had form'd a *Schirrus* of the Glands it adher'd to, by keeping up a Diſtention of Parts, till there was a Coheſion of their Membranes and Veſſels.

I make no doubt, but this was a Body of diſeas'd Glands, which had ſuffer'd a Colliquation by ſome extravaſated Fluid, and that the Membrane of the Tumour was their proper *Tunic.* After this manner all our Tunicated Tumours ſeem to be form'd; for when an Obſtruction proceeds to Extravaſation, there is a Liquor pour'd out which conſiſts of ſuch Particles, that by degrees makes a Colliquation of the Glandulous Fleſh, which is not very ſenſible of Pain; and by degrees the *Capſula* becomes diſtended with a Matter of a very different Conſiſtence, which gives the Name to the Tumour, either *Steatoma, Atheroma,* or *Meliceris.*

Thus pour Oyl of Olive on Spirit of Nitre, and your Oyl firſt becomes a little hardned, then of the Colour and Conſiſtence of Marrow, till by degrees it is hardned into a white Fat, reſembling that of Animals. The Poſſibility of this Colliquation and Digeſtion we may the eaſier be induc'd to believe, if we conſider how often we find the Glands of the *Viſcera* petrified, without any degree of Pain, or the Membrane in any Meaſure deſtroy'd.

A Schirrous Tumour in the Breaſt which prov'd Mortal. By Mr. Tho. Greenhill. *n.* 300. *p.* 2009.

IV. Mr. *J. D.* was ſuppoſed to die of a Conſumption; foraſmuch as Fourteen Months before he had been violently ſeized with an Inflammation of his Lungs, accompanied with a ſharp Feaver, difficulty of Breathing, Cough, acute Stitches, and Pleuritick Pains, with a ſpitting Blood, &c. He was bled largely in the beginning, and often repeated it during his Sickneſs. But about *Eaſter,* there appeared a Tumour on the Breaſt Bone, Pap, and Pectoral Muſcle, of the Left Side, with a fulneſs under the *Axilla*: From whence there was conjectured to be a collection of Purulent Matter in the Cavity of the *Thorax,* and that the *Sternon* was foul. The firſt from the aforeſaid Tumours, and his ſpitting a bloody and purulent Matter, and the latter from the riſing and inequality of that Part. So ſoon as I had open'd him, and divided and removed the common Teguments of the *Thorax,* I found, inſtead of a riſing of the Bone with Carioſity, only an oblong Tumour, about four Fingers in length, and two in breadth, and a proportionate thickneſs, weighing about three Ounces; it extended itſelf perpendicularly on the Superficies of that part of the *Sternon* which joins with the *Cartilago Enſiformis.* I ſeparated it with my Knife, eaſily, from the Breaſt bone, and found it to be of that Sort of Wens or enciſted Tumours called *Atheroma,* containing a pappy Subſtance like ſodden Barley. Next appeared a very large Tumour on the Left Side of the *Thorax,*

covering

covering the whole Pap and Pectoral Muſcles forwards, with a fulneſs under the *Axilla* of the ſame Arm. Then opening the *Thorax*, I found the ſame Tumour comprehending the *Intercoſtals, Deltoides, Subclavian*, and *Subſcapulary* Muſcles, and the whole Axillary and Mamillary Glands; which being obſtructed, and its Veſſels replete with a creamy pappy Matter, more thick and white than the former, there was produced ſuch an induration of the aforeſaid Glands and Muſcles, which compoſe the upper part of the Breaſt, that it may more properly be eſteemed a *Schirrus*.

The ſame Tumour on the outſide of the Breaſt was ſomewhat bigger than ones Hand, extending itſelf from the Clavicle to the lower part of the Pap; and laterally from the Baſis of the Muſcle quite under the Arm pit. Internally it poſſeſſed a third part of the Cavity of the Breaſt, crouding the Left Lobe of the Lungs to the Right Side, and in its upper part firmly growing to it; which it likewiſe did every way to the Intercoſtal Muſcles. It was about the bigneſs of a Penny Loaf; and the whole Tumour being conſidered together, might reaſonably be allowed to weigh between three and four Pounds; which being cut into, there ouzed out of it, like an expreſſed Sponge, a great quantity of thick, white and pappy Matter. And what is more particularly remarkable, there was form'd a large Sink or *Pelvis*, in the middle of the Axillary Gland, which contained a thinner and diſcolour'd Matter, and had a free Communication to the Veſſels of the Lungs in the upper part of it, where I told you before it was united; and from hence it was that he generally found eaſe when he had ſomewhat emptied it by large Expectorations, and that he could ſo exactly perceive, when any thin Rheums or Matter flow'd to the Part: And it was here only that the Lungs were black and replete wirh ſtagnated Blood, and ſome Globules of the aforeſaid Matter in its *Veſiculæ*. The reſt of the Lungs were pretty clear from any Ulcers or Matter, but of a Sublivid Colour, and ſtrictly adhered on both ſides to the *Pleura*, but particularly on the Left Side, all about the Schirrous Tumour. The *Veſica Fellis* or Gall Bladder was full of Stones, of the bigneſs of a Runcival Pea, and conſiſted moſt of odd Angles, and were formed of a thick viſcous Sediment of Gall (which we found in it) from an obſtruction of its Veſſels, or Jaundice, which he had ſome Years before: They were in Number Twenty two, ſome Triangular, Quadrangular, Quincuncial, *&c.* There was a *Maraſmus* of the External Parts, the waſhing of the Gall, and emptying of all the *Viſcera* and Blood Veſſels in general.

An Apoſtemation in the Lungs cur'd by Dr. Wright. *To Mr.* Cowper. *n.* 285. *p.* 1372.

V. 1. Mrs *Jane Terry* fell ill of the Small Pox in *May* 1701. She was about Eighteen Years old, of a freſh Complexion, and pretty Fleſhy. Her Relations apprehending ſhe might have the Small Pox, removed her to a Nurſe's Houſe, where ſhe had the diſtinct Sort very kindly; her Caſe proceeded ſo very well (as they conceived) that no Phyſician was called to her, till they began to ſhell; only for ſome Days before ſhe had a little difficulty in her breathing, which gradually increaſed till ſhe began to raiſe Blood, which was about the ſeventh Day from their firſt appearing. This raiſing of Blood had increas'd every Day for three Days before I ſaw her; ſhe cough'd and brought up a viſcous Phlegm, ſuch as our Patients vomit when their Stomachs are very foul; only, as meer Phlegm is white, this was all of it as red as Blood: It was not ſtreak'd with Blood, or had a mixture of white Phlegm with it, but was ſo deeply colour'd, that it ſeem'd to be all Blood, only it would not flow as Blood does whilſt it is hot; nor did it coagulate as Blood does when it is cold, but hung from the Baſons when it was pour'd out, as vomited Phlegm does; and in this it differ'd from all the bloody Expectorations I have ſeen, excepting Mr. *Jones* at *Myn Heer Meyſters*'s in *Kenſington*, who cough'd the ſame bloody colour'd Pituita, but in much leſs quantity; for Mrs. *Terry* rais'd above a Pint in Twenty four Hours, for ſome Days, and though a leſs, yet a conſiderable quantity afterwards. Mrs. *Terry*'s afforded a very ſtrong ſmell, but Mr. *Jones*'s had no odour. After ſome Weeks ſhe recover'd again, regain'd her Fleſh, which was waſted in her illneſs; the *Menſes* return'd, and ſhe continued very well from *July* till near *Chriſtmas*.

In *January* ſhe gave this Account of her ſelf. That about three Weeks before *Chriſtmas* ſhe perceived her ſelf a little ſhort breath'd, which increas'd daily, with a fullneſs and weight in her Left Side; that ſhe lay well on the Left Side; but when ſhe turn'd to lie on her Right, ſhe felt as if a Weight fell from the Left to the Right Side, which gave her a ſhortneſs of Breath, and made her Cough. Thus it continued increaſing till *Chriſtmas*, when ſhe began to raiſe a conſiderable quantity of ſtrong ſtinking Pus; ſhe ſaid ſhe eat her Victuals well enough all this time, and was not Feveriſh. When I ſaw her towards the middle of *January*, 1701. ſhe raiſed a conſiderable quantity and often of ſtinking offenſive Pus, which was as fluid as the Pus of other Parts. Her Fleſh was a little abated, but ſhe was at no time Feveriſh; ſhe eat and ſlept pretty well, and had the Catamenia duly. I preſcribed ſuch Medicines as abated that Purulent Expectoration ſeveral times, and ſhe often gave me hopes of her Recovery, ſhe continuing to have the *Menſes* regularly, and being ſtill free from an Hectick; but upon every little Cold, ſhe again raiſed that fetid Pus in a conſiderable quantity. She generally continued pretty free from Coughing ſeveral Hours together, till ſhe perceived ſomething of a fullneſs in her Breaſt, which would oblige her to Cough; and after ſhe had once begun to raiſe, ſhe could not ceaſe till ſhe had brought up two Spoonfuls or more of that fetid Pus. This ſhe did chiefly in the Morning, Afternoon, and at Night. I did apprehend ſhe had an abſceſs in the Left Lobe of her Lungs, and made her lie upon the Bed, with her Head reaching to the Chamber Floor, leaning upon her Left Arm. In this poſture ſhe could at any time, after a little Cough, ſet the Pus a running out of her Mouth, as you have ſeen ſeveral times, till the whole which was therein contain'd, was diſcharg'd.

Then ſhe would get up, and ſeem to be as well as another Perſon, till it was almoſt fill'd again. In the beginning of *May*, Dr. *Torleſſe* and Dr.

Pitts favour'd me with their Aſſiſtance. They ſaw her lye in the Poſture I have deſcrib'd, and ſaw more than two Spoonfuls of ſtinking Pus or Corruption run out of her Mouth, after a little Coughing. This made it ſo apparent that there was an Ulcer in her Lungs, that they immediately approv'd of what I had propos'd to her and her Relations, of the Neceſſity of making an Apertion in her Side, where we could apprehend the Lungs grew to it; for that ſeemed unqueſtionable, from the Poſture of diſcharging it, and ſome little Pain ſhe felt in her Side. Her Relations readily conſented to what theſe Gentlemen had propos'd, and deſir'd your Aſſiſtance. About a Week before, the Pus had begun again to encreaſe, and the Day before theſe Gentlemen ſaw her, ſhe was taken in the Afternoon with a Chilneſs, after which her Pulſe became a little quicker, and ſhe a little Feveriſh, as I believe, you remember ſhe was, when you applied the Cauſtick; this Feveriſh State encreaſed every Day, and after ſome Days a Raſh appear'd, which laſted about fourteen Days before it was quite got off, and left her in a Hectick, with Redneſs in her Cheeks towards Evening, Night-Sweats, continual Looſeneſs, extreme waſting of her Fleſh, and at length a Swelling in her Legs, though ſhe kept her Bed. We felt ſome little Knots betwixt the ſeventh and eighth Rib, which with other Circumſtances made us conclude the Adheſion was in that Part, and would have laid the Cauſtick there, but that it would certainly have ſpread to the Glands of her Left Breaſt, which made us lay it between the ſixth and ſeventh Rib, (*ſurſum numerando*); As ſoon as you could you took it off, and with your Knife gently paſs'd through into the Cavity of her Breaſt, whence iſſued a bloodiſh Water, but no Pus, by bending your Probe you found the Adheſion reach to the lower Edge of the ſeventh Rib, and before the Eſcar was ſeparated, the Pus began to flow at every Dreſſing, and ſo continued, gradually abating, till the Ulcer was cured; during which, a Part of the Inſide of that Rib, above an Inch long, exfoliated, and after that another leſſer Piece of the outſide of the Rib. Towards the latter end of the Cure, ſhe complained very much of a Pain at the *Cartilago Enſiformis*, ſo great that ſhe ſometimes pluck'd out the hollow Tent, which we conceiv'd was occaſioned by its preſſing upon the Nerve. During the firſt ſeven or eight Days of her Raſh, ſhe rais'd very little, if any, of that Pus, nor did it diſcharge any of itſelf then by the Orifice, nor was there a Collection of it in her Breaſt, which made me apprehend, that the Fever did ſo alter the State of her Blood, as not to permit it, to ſeparate its Impurities into the Abſceſs. For ſix Days before the Fever began, ſhe had the *Catamenia* very orderly; by *Auguſt* ſhe was cured, her Side heal'd up, and would not endure it to be converted into an Iſſue; by *October* ſhe recovered her Fleſh, and the *Catamenia* return'd, which had been wanting ever ſince *May*; and now, as you lately ſaw her, ſhe is plump, fleſhy, clear and freſh Complexioned, has little or no Cough, and no fœtid or tabid Expectoration; and ſeems, and I believe is, perfectly cured, having for many Months taken no Medicine.

There are ſeveral Circumſtances in this Caſe, which I cannot forbear making ſome Remarks on.

That there was an Ulcer in the Lungs, and that it has admitted of a Cure, contrary to the general Opinion of Phyſicians. That this Ulcer did contain at leaſt two Spoonfuls, and muſt have been as large as a Hens Egg. That this Abſceſs aroſe from a Collection, with an indiſcernible (if any) Fever, and ſo continued from *Chriſtmas* to about the 10th of *May*. The tender Membranous or Veſiculary Compoſition of the Lungs ſeem to juſtify this Opinion, that it is almoſt impoſſible for them to heal, when there is a conſiderable Diminution of them; the continual and indiſpenſable Neceſſity of their Motion, very much hindring the Coalition of the *Veſiculæ*.

Several Parts of the Body afford a proper Cement to unite and repair them, when hurt or diminiſh'd. Carious and broken Bones ſend forth a *Callus*; when the Skin is conſum'd by Ulcers or Burns, the Parts afford a Cicatrix, which pretty well ſupplies the Defect of the Skin. The Lungs ſeparate a viſcid Pituita, which will be expanded into Fleaks like a Membrane; Mr. *Stringer*, *Sarah Deeping* and ſome other Patients have brought up great Quantities of them, and a little Boy * at Mr. *Tolley*'s in *Kenſington* cough'd up ſeveral Pipes, form'd exactly like the *Bronchia*, and its Divarications, and at firſt View ſeem'd to be the internal Membrane. This Child, two Years before, had an Ulcer in the Right Side of his Lungs, and they adhered to his Back; when I ſeparated them I found a Cicatrix near three Inches long, but very little (if any) Defect in his Lungs. I am of Opinion this Pituita or Mucus doth ſerve to re-unite the Parts of the Lungs, when there is a Solution by an Ulcer.

* *n.* 263. *p.* 545. *Abr. Vol.* III. *p* 63.

Conſumptive Perſons generally flatter themſelves, that they have no Ulcer in their Lungs, becauſe they do not feel a Soreneſs, as in Ulcers of other Parts. This Opinion keeps them from making a timely Application to Phyſicians, whilſt they might receive a ſpeedy and eaſie Cure. When you touch'd the Sound or Ulcerated Parts of her Lungs with your Probe or Finger, ſhe diſcovered no Senſe of feeling you, which may confirm the Opinion of Phyſicians and Anatomiſts, that the Lungs have little if any Senſation. When you touch'd her Heart with your Finger, tho' I believe for not the twentieth part of a minute, ſhe grew very much diſordered, pale and ready to faint, which ſhews Nature cannot ſuffer the leaſt Alteration in its Pulſation, without great Prejudice and Inconveniency.

It is the Opinion of ſome Phyſicians, that the Fever which attends Conſumptive Patients ariſes from ſome Particles of the *Pus*, which being receiv'd into the Blood, and circulating with it, cauſe that Effervеſcence, which we call an *Hectick*. This Patient had no Fever from *Chriſtmas* to *May*, and then unhappily came a continued Fever with a Raſh, which left a Febricitation every Afternoon, with thoſe Symptoms which attend a Hectick.

I have obſerv'd for many Years, that if I could preſerve my Conſumptive Patients from that Hectick Fever, or relieve them who already labour'd under it, I could cure them, tho' the Expectoration was very plentiful and foul. I doubt not, but if ſome of our Faculty applied themſelves more particularly to the Cure of Conſumptions, ſo many thouſands would not dye of it yearly in this City, as appears by the Bills of Mortality.

I do not doubt but ſome Part of her Lungs do adhere to her Side, and it's probable a little part of them do not receive the Air in Inſpiration, but I believe that Defect is very inconſiderable, becauſe ſhe can run up Stairs, and is no more diſordered in her Breath than moſt other People.

The eaſy Diſcharge of the *Pus*, by her lying down in that Poſture, did undoubtedly very much preſerve her Lungs, and prevented its breaking through the Abſceſs, into the Cavity of her Breaſt, and putrifying her Lungs to a greater degree. *Pus generat Pus*, is a noted Aphoriſm, and the Air Bladders of the Lungs are ſo very tender, that they muſt have yielded to the Preſſure of the *Pus*, had it lain long in the Abſceſs, and been only diſcharg'd by violent Coughing. By lying in a proper Poſture, Sir *Tho. Proby*, *Sarah Deeping*, and other Perſons have prevented a greater Solution in their Lungs, and either prolonged their Lives many Years, or recovered their Healths by proper Medicines.

An Anſwer to the foregoing Account, by Mr Cowper, *ib.* p. 1386.

2. Nothing occurs to me ſo remarkable that you have omitted in relating Mrs. *Terry*'s Caſe, as your Prognoſtick of her Recovery, when you ſo often encouraged me to expect Succeſs in her Caſe. You may remember, that the Matter or *Pus* which firſt flowed from her Side was ſo offenſive in its Scent, as obliged the By-ſtanders to quit the Chamber, inſomuch that the Nurſe uſually at the time of Dreſſing, and afterwards, was wont to burn Roſemary, *&c.* to ſuppreſs the Stench. So putrid was the *Pus* that it tarniſhed that end of the Silver Probe I paſt into the Cavity of the Abſceſs, as it did the Top of the Silver Syringe in making Injections. There ſeems no room to doubt that the *Pus* which then flowed from her Side came from the ſame Cavity the *Pus* did ſhe before coughed up, when the Liquor that was injected at her Side came into her Mouth; which ſhe frequently complained of, and particularly of the bitteriſh Taſte of the Tincture of Myrrhe I ſometimes uſed in the Injections.

About two or three Years ſince, I ſaw a Boy in the ninth or tenth Year of his Age, who (ſome time before) after a continu'd Fever was purſu'd with an intermitting one; a Cough follow'd, in which he brought up (at ſhort Intervals) no ſmall Quantity of thick purulent ſtinking *Pus*, which Diſcharge (I think) continued on him not leſs than fourteen or fifteen Months before I ſaw him: His Phyſicians order'd him Iſſues in his Back, which I made as uſual: He had then an healthy Aſpect, his Cheeks florid, and was very briſk and active: When he juſt came from Play he was bid to take a Baſon in his Hand and cough as he was wont, which he did, wherein I ſaw him diſcharge at his Mouth not leſs than 4 or 5 Ounces of the ſort of *Pus* above-mention'd: This his Mother told me he had been wont to do twice every Day; nor did he appear any ways diſorder'd after, but return'd to play immediately. His Phyſicians ſent him into the Country whence he came, where in about a twelve month I heard he dyed, but was not acquainted with his Circumſtances after: What Succeſs the Operation we practiſed on Mrs. *Terry* would have had on this Boy, I dare not determine; tho' I cannot but think it might have been ſafely done to him and another Patient I was ſince call'd to, but I could not obtain the Conſent of the Phyſician that was conſulted.

Another Inſtance (in which a conſiderable Part of the Lungs was obſtructed, and conſequently became uſeleſs, ſome time before Death) was in a Girl of ſixteen, who had been Scrophulous not leſs than 9 Years; the Glands about her Neck and Throat being very much indurated as well as diſtended, her Lips and Noſe were alſo ſwoln: About a Year and a half before her Death ſhe coughed up ſeven or eight Ounces of fœtid *Pus*, in leſs than 24 Hours. On changing the Air of this Town for that of the Country, together with the uſe of Balſamick Pectorals, ſhe recovered a healthful Appearance in her Face; but continued ſomewhat Aſthmatick. On taking cold (as 'tis call'd) her Appetite as well as Digeſtion fail'd her, ſhe grew Feveriſh, and dyed after a few Days Indiſpoſition.

On opening the *Thorax*, I found the Lungs cleaving to the *Pleura* of the Left ſide, in ſuch manner that they could not be ſeparated, without one of thoſe Parts borrowing from the other. A Portion of one of the Left *Lobes* of the Lungs being cut off, ſunk in Water; from which Part 'twas likely the Matter came which ſhe formerly coughed up, tho' the Ulcer was then cloſed, and no Appearance of Matter was to be ſeen in that or any other Part of her Lungs. The Lymphatick Glands at the Divarecation of the Windpipe had by their Intumeſcence ſo compreſt the Canal of the Left ſide, that it wanted more than two thirds of its proper Paſſage for the Air.

In theſe, and ſome other Inſtances I could produce, it's evident that conſiderable Parts of the Lungs may be obſtructed, and the Perſon ſurvive: but Mrs *Terry*'s Caſe demonſtrates the Poſſibility of their Recovery when Part of their Lungs are totally obſtructed, as muſt happen in ſuch large Abſceſſes. But how the remaining ſound Parts of ſuch diſeas'd Lungs become capable of tranſmitting the whole Maſs of Blood from the Right Ventricle of the Heart to the Left, in equal Time and Quantity with the Blood that circulates in the reſt of the Parts, ſeems not eaſily accounted for, when indeed it exacts our Wonder that it is done in a natural State, when all the Paſſages of the Lungs are open and free. Since I had often found Water, injected by the *Arteria Pulmonalis*, return readily from the Lungs again by the *Vena Pulmonalis*, I was tempted to try if melted Wax, when very hot, would not do the like. Which ſucceeded in two young Cats Lungs: for after injecting the Wax (mixt with Oil of Turpentine, and tinged with Vermilion) by the *Arteria Pulmonalis*, I found it had fill'd the Pulmonick Vein with the Left Auricle, inſomuch that ſome of the Wax had reacht the Left Ventricle of the Heart: I don't remember this Experiment ſucceeded, but that ſome of the Wax was extravaſated, and came into the *Bronchiæ* and Wind pipe at the ſame time.

In preparing a Human Heart, by filling its Ventricles, Auricles and Trunks of its large Blood Veſſels with Wax, I found, on injecting the Pulmonick Arteries and Veins with Wax differently tinged, that the Wax paſs'd from the Veins to the Arteries without coming into the *Bronchiæ*, or being extravas'd, tho' the Wax was not injected with near ſo much Force as might be. I muſt confeſs I was never ſo fortunate to make Wax paſs thro the Arteries to the Veins in Human Bodies or Quadrupeds, unleſs in their Lungs, as above noted, and the *Spleen* and *Penis*; Nor do I remember it has happen'd in thoſe Parts, but when the Wax has been impell'd with great Force, tho' I have conſtantly obſerv'd the Communication of Arteries and Veins, of the *Spleen* and *Penis*, more open than in other Parts except the Lungs. I wiſh Dr. * *Morland* had told us in what Part of the Human Body Dr. *Areskin* had made Wax paſs from the Arteries to the Veins, ſo as to demonſtrate their Continuation to the naked Eye, becauſe I have hitherto found the naked Eye unable to diſcover the Extremities of the Arteries and Veins, when the Blood it ſelf was moving in them, in the tranſparent Parts of the *Omentum* or *Meſentery* of the Quadrupeds, or in the Lungs of Frogs or Lizards when living; or after Death, when the Blood has been retain'd in their Lungs in the following manner. On making Inciſion into the Bodies of theſe Creatures their Lungs will ſtart out, and be diſtended with inſpired Air; on which make what haſte you can to paſs a Ligature (*i. e.* a Waxt Thread) and tye it firmly toward the upper part of the Lobe, as near the Heart as you can.

Whence will appear that the Communications between the Arteries and Veins of the Lungs are more open than thoſe of other Parts, at leaſt in the Feet of Frogs: And till it can be ſhewn that melted Wax can be as eaſily injected from the Arteries to the Veins of other Parts in a Human Body and Quadrupeds; I ſhall be inclin'd to think the Communications between the Pulmonick Arteries and Veins in general are more open than the Arteries and Veins of other Parts, except the *Spleen* and *Penis*.

This patent Communication of the Arteries with the Veins of the Lungs ſhews how thoſe Veſſels tranſmit the Blood in equal Time and Quantity with the Blood that moves in the reſt of the Blood Veſſels of the whole Body in a healthful State.

Hence it is, when any of the Blood Veſſels of the Lungs are ſtreightened or totally compreſt (either or both which Circumſtances muſt happen in Mrs. *Terry*'s Caſe) the remaining unobſtructed Blood Veſſels are forced to diſcharge more than they were wont, and in time thoſe Veſſels become ſufficiently dilated to ſupply the Defect. The like happens in the Communicant Branches of the Arteries of any Part, when ſome conſiderable Branch or Trunk is ty'd up, as in the Operation for curing of an * Aneuriſm.

* *n.* 283. *p.* 1292. *infrà Ch.* VIII. *Tract.* 11.

* *n.* 280. *p.* 1191. *infrà Ch.* V. *Tract.* 3.

VI. I ſend you herewith the Account and Figure of a *Polypus*, which I took out of a Child of about a Year old. It's firſt obſervable Diſorders were a quick Pulſe, and a Difficulty of Breathing. In about four Days the Gums were obſerv'd to be ſwell'd, for which they were cut, and all the Symptoms diſappear'd for five or ſix Hours, after which they return'd. Notwithſtanding Blood-Letting and the Application of other Remedies, the Difficulty of Breathing increas'd, the Pulſe became ſtill lower and quicker, and in four Days more the Child dyed.

A Polypus *taken out of the* Vena Pulmonalis. *by Mr.* Cowper. 270. *p.* 797.

The Body was open'd, and the *Viſcera* of the lower Belly were found well conſtituted.

In the *Thorax* the *Thymus* exceeded the natural ſize even in Children. The fore part of the Lungs appear'd to be well diſpos'd, but the back Parts were very hard, and very much inflam'd. Making the Inciſion on the Diſeas'd Part, purulent Matter followed the Knife in ſuch Quantity from divers Cells, that it fill'd the Wounds as faſt as made, and Pieces cut from it ſunk in Water. But as we approached nearer to the Parts unaffected, the Pieces became gradually more buoyant, till at length we came to the fore part which floated.

The Cavities of the right Auricle and Ventricle of the Heart were fill'd with a *Polypus*, which was continued into the ſuperiour and inferiour Trunk of the *Vena Cava*.

Opening the *Vena Pulmonaris* at the *Baſis* of the Heart I found it there compleatly fill'd with a *Polypus* (or Coagulation of Blood) which was continued into all the large Branches of it in the Lungs, and were eaſily drawn out, and when diſplay'd, appear'd as expreſs'd in Fig. 5. Plate 1.

2. This *Polypus* affords us a better Idea of the Contrivance and Structure of the Pulmonary Vein than any Figures of that Veſſel yet publiſht: For tho' in different Subjects of the ſame Species we meet with frequent Varieties in the Diſtribution of the Blood-Veſſels, eſpecially of the Veins; we no where find a more conſtant Regularity and Uniformity than in the Trunks and large Branches of the Pulmonary Vein; of which I have added two Figures, (Fig. 6. and 7.) drawn after a Preparation of that Vein injected with Wax, and freed from the Lungs of an Adult Humane Body. The Original Preparation is to be ſeen among the Anatomical Collections of the Accurate Dr. *Tyſon*.

The Structure of the Veſſel by the ſame, ibid.

Among many very conſiderable Parts of the Humane Body, not ill expreſt in the Tables publiſhed by *Bidloo*, and overlookt, or not known by him, this Trunk of the Pulmonary Vein is one; unleſs he may be allow'd to call it the Left Ventricle of the Heart, as he has done Tab. 22. Fig. 7. A.

The Left Auricle of the Heart (*vid. Fig.* 6. 7.) in Human Bodies being much leſs than the Right, it was neceſſary that the part of this Vein next the Baſis of the Heart ſhould be very large; (*ib.* A A B) leſt the ſudden ſtrong Motion of the Syſtole ſhould cauſe the Refluent blood to recoil in the Branches of this Vein (*ib.* D D E E) and prevent a ready Supply in the ſucceeding *Diaſtole* of the Heart. But the Weight of ſo much Blood lying in the Trunk of this Veſſel (*ib.* A A. B) does effectually prevent its Retroceſſion in the lateral Branches within the Lungs; (*ib.* DD. EE.) and the more, becauſe the Orifices of thoſe Branches (*ib.* DD.) are not diametrically oppoſite (at A A Fig. *ib.*) to the Mouth of the Veſſel on the Baſis of the Heart (*ib.* B) its lateral Branches making Acute Angles with the Trunk, as repreſented Fig. 6.

Oſſifications or Petrifactions in the Coats of the Aorta, by Mr. W. Cowper, *n* 299. p. 1970.

VII. The Oſſification or Petrification in the great Artery, at its Riſe from the Heart, has been ſo commonly found, that ſome think it is conſtant; how it may be in ſome Animals I cannot be certain; but in Humane Bodies, I am well aſſured, whenever it happens it is a Diſeaſe, and does in ſome meaſure incommode thoſe Parts in the due Execution of their Office, as the following Caſes will evidence: But that this Paper may be of ſome Uſe, I ſhall ſet down the Symptoms before Death, which may help our Conjectures when the like offers again. A ſpare Man about 30, who languiſht with an Ulcer in the Thigh, attended with a *Caries*, or Rottenneſs of that Bone at its Articulation with the *Tibia* and *Patella* call'd the Knee, where all thoſe Bones were affected, at length fell into a true *Phthiſis*, and coughed up no ſmall Quantity of *Pus*; ſome Months before his Death, I frequently ſaw him, when he would often offer me his Wriſt, to feel his unequal Pulſe, which was wont to amuſe him; the Artery there miſſing ſometimes one, ſometimes two Strokes in 6 or 7: At firſt he told me he obſerved it miſt but one in ten, but at length thoſe Stops became more frequent, eſpecially on any Agitation of the Body or Mind. Tho' a *Polypus* in any of the great Veſſels about the Heart may induce that Symptom, yet the Continuance of it ſo long before Death, ſhews it owing to ſome other Cauſe, as appear'd on opening the Heart and Great Artery of this Perſon.

Fig. 1. Plate 2. A A. The Trunk of the Great Artery opened and diſplay'd.

a a a. The three Semilunary Valves of the *Aorta*, which hinder the Blood from returning to the Heart, after it is expell'd thence by its *Syſtole* or Contraction; theſe Valves in this Caſe were ſomewhat thicker, and not ſo pliable as naturally, and did not ſo adequately apply to each other, as is expreſt Fig. 4. a a a. Whence it hapned ſometimes that the Blood in the Great Artery (A A A Fig. 1.) would recoil, and interrupt the Heart in its *Syſtole*. But this ſtubbornneſs of theſe Valves was owing to a Bony or ſtony body, markt b. Fig. 1ſt, which appear'd much plainer when the Valves were dry, a is repreſented in the Figure beneath, markt with an *: a a. the two Valves pinn'd out and dry'd, b the Petrifaction or ſtony Body at their Junction. In this Inſtance I obſerv'd the Left Ventricle of the Heart, expreſt at G G. D D. e e. f f. Fig. 1ſt, to be a little dilated from its natural Size, but was not by two parts in three ſo big as the Left Ventricle of the Heart of one I diſſected in the

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Fig:1.

Fig:3.

Fig:4.

Fig:5.

*

M:Vdr Gucht Sculp:

Preſence of Dr. *Sloane*. The Symptoms, ſome Years before the Death of this Perſon, who was about forty Years of Age, were extraordinary Shortneſs of Breath, eſpecially on any Fatigue, with an Intermiſſion of one Stroke in three of the Pulſe; his Poſture of ſitting up was more eligible than any other; he complain'd of great Faintneſs, and now and then Pain about the Heart; the extreme Parts often cold, which towards his Death increaſed more and more on him; his Legs and Arms being gangreen'd ſome Hours before; inſomuch that the Corps was very offenſive in opening, tho' 'twas done within 24 Hours after he expired, in the Month of *November*.

Upon opening the Cheſt, the Heart, particularly its Left Ventricle, was found larger than that of an ordinary Ox, and fill'd with Coagulated Blood. The Valves of the Great Artery A A. Fig. 1. were Petrify'd, inſomuch that they could not approach each other, as expreſt Fig. 2. and 4. But an Orifice, repreſented at Fig. 5. remain'd always open by the Petrifactions b b, Fig. 3. and a a, Fig. 5, which had clogg'd theſe Valves, and hindered their Application to each other, as in a Natural State is repreſented in Fig. 2. and 4. a a a.

The Explication of the Symptoms in both theſe Caſes is obvious enough; for tho' the Perſon firſt inſtanced did not die of the ſame Diſeaſe with the laſt mentioned, yet the Symptoms in his Illneſs plainly ſhewed what muſt follow, from the Diſorders of theſe *Valves*, as they are rendred more or leſs uſeleſs: For as their Office is to prevent the Return of the Blood into the Heart, in its *Diaſtole*, by exactly ſhutting up the Paſſage of the *Aorta* (as the Flaps in Water Engines) ſo if by any Accident they are hinder'd from doing their Duty, as they were by theſe Petrifactions, the Conſequences muſt be, not only a Regurgitation of Blood into the Heart, but they baulk its impulſive Force, when the Muſcular Fibres (which are in theſe Valves) cannot contract to prepare the Paſſage for the Blood of the Left Ventricle, when to be expelled into the *Aorta*. Hence the Intermiſſions of the Pulſe in the firſt Inſtance may be accounted for. In the latter Inſtance, theſe Valves were wholly uſeleſs, the Circulation became more difficult, as appear'd by the Refrigeration of the extreme Parts, Gangreens, *&c.* In both theſe Caſes the Left Ventricle of the Heart was dilated proportionably to the ill conſtitution of theſe Valves, which clearly ſhews theſe Valves give that aſſiſtance to the Heart in its Office that it cannot be without, and that it gradually ſuffers according to their Indiſpoſition.

In an Elderly Gentleman about 72, who had ſometimes Intermiſſions in his Pulſe ſeveral years before his death, I found divers Petrifications in the Mitral and Semilunary Valves of the Left Ventricle of the Heart.

The Explanation of the Fi-gures.

Fig. 1. Shews the Left Ventricle of the Heart open'd, *&c.*

A A A. The inſide of the *Aorta* ſlit open to the Left Ventricle.

B B. The Bulbous Trunk of the *Vena Pulmonalis* divided through, and pinn'd aſide to ſhew

a a a. The three Semilunary Valves of the *Aorta*, which hinder the Blood from returning to the Heart.

b. A ſmall ſtony Body at the Conjunction of two of the Semilunary Valves, expreſt at the * below this Figure.

a a. Parts of the two Valves dryed.

b. The Petrifaction, as it appears in the dryed Valves.

C. Part of the lower Trunk of the *Vena Cava*, cut off immediately above the Liver.

c c c. The Left Auricle open'd and pinn'd out.

D D The Sides of the Left Ventricle divided and drawn aſide, to ſhew its inſide, d d e e f f G G.

d d. The Mitral Valves of the Left Ventricle of the Heart, or *Arteria Pulmonica* divided and turn'd aſide.

e e. The *Carneæ Columnæ*, whence ſpring the Tendons faſten'd to the Valves, d d, expreſt Fig. 3. d f.

f f A Tranſverſe Cord or Tendon, by which the *Columnæ Carneæ* are drawn nearer to each other in the *Syſtole*, or Contraction of the Heart, when the Blood is expell'd into the *Aorta*; whereby the Tendons (expreſs'd f f Fig. 3 and 5) draw the Mitral Valve laterally; by which Means its Orifice g c. Fig. *ibid.* is not only cloſed to prevent the Return of the Blood by the *Vena Pulmonalis*, but at the ſame time it opens a Paſſage for the Blood of the *Arteria magna*, by withdrawing the Mitral Valve, d Fig. 2. from the Orifice of the *Aorta*, a a a g.

G G. The internal Surface of the Left Ventricle, where it is ſomewhat ſmoother as it leads to the *Aorta*.

g g. The Trunk of the Coronary Vein divided when filled with Wax.

h h. The Coronary Artery in like manner divided.

i. One of the Trunks of the *Vena Pulmonalis*.

k k k. The three Orifices of the Trunks of the *Vena Pulmonalis*, as they open into the Bulbous Trunk expreſs'd at BB.

H. The Cone of the Heart.

Fig. 2. A. Part of the *Aorta* next the Heart.

a a a. The three Semilunary Valves, as they appear next the Heart in a Natural State, when the Heart is in *Diaſtole*, and the Blood hinder'd by theſe Valves from returning to its Left Ventricle.

b b. Part of the Baſis of the Heart cut off.

e e. The two *Columnæ Carneæ* of the Left Ventricle.

d. The Mitral Valve.

f f. The Tendons ſpringing from the *Carneæ Columnæ*, and inſerted into the upper and middle Parts of the Valve, as well as to its lower Margin; which is better expreſt in the following Figure.

g. The Orifice of the *Aorta* compleatly cloſ'd by the Application of theſe three Valves to each other.

Fig. 3. Shews the ſame Parts expreſt in the preceding Figure, as they appear'd when the Valves of the *Aorta* were Petrified: The ſame Letters alſo directing to the Parts already explain'd, except a.

a. Part of one of the Valves which was not cover'd with the Petrifaction.

b b b. The Petrifaction on the reſt of the Valves.

† A ſmall Petrifaction on the Mitral Valve.

h h h. Some of the Tranſverſe Tendons which draw the *Carneæ Columnæ* to each other, when the Heart is in *Syſtole*, for the more effectual cloſing the Orifice of the Mitral Valve, expreſt here at g.

Fig. 4 and 5. Shews the Parts repreſented in the two preceding Figures, as they appear view'd towards the Heart, when dry'd and diſplay'd.

A A The Trunk of the *Aorta*.

a a a. Fig. 4. The Semilunary Valves in a Natural State, when the Blood in the Arteries preſſes them cloſe to each other.

b b b b. The Trunks of the two Coronary Arteries cut off.

a a. Fig. 5. The Semilunary Valves Petrify'd.

c. The Orifice of the Mitral Valve next the *Vena Pulmonalis*.

d d d. The Internal Surface of the Mitral Valve leading into the Left Ventricle.

e e e. The *Columnæ Carneæ*.

f f. The Tendons.

g g. The tranſverſe Tendons which draw the fleſhy Columns to each other, when the Heart is in *Syſtole*.

VIII. Fig. 9. Tab. 1. Shews the Beginning of the *Aorta*, or Great Artery from the Heart of a Woman, who died of a Dropſy. *A*. is the Aorta. *B B*. Two Chalk-ſtones, which poſſeſt the Place of the Semilunar Valves. The Left Ventricle of the Heart was dilated to twice its Natural Magnitude. We ſuppos'd theſe Stones occaſion'd the Dropſy, by obſtructing the Valves, and hindering the regular Diſtribution of the Blood.

The like by Mr. Cheſelden, *n.* 337 *p.* 281.

Fig. 10. Shews a Bone taken from the Ventricles of the Heart of a Man, who died Hydropic and Tabid. In this Body the whole Pericardium adher'd to the Heart.

IX. Before the Body was removed from the Bed whereon it lay ſome Hours after Death, the Bliſter in the Neck had diſcharged not leſs than a Quart or three Pints of Serum, before I began the Diſſection.

Diſſection of one dying of an Aſthma, *n.* 336 p. 534. *by Mr* Cowper.

In the Abdomen was a ſmall Quantity of Water, ſuch as is uſual in thoſe that dye of Chronical Diſeaſes: The Parts in the Lower Venter were in a Natural State, except the Kidneys; of which the Right was very much contracted, even to a third part of its Natural Size, and had two large Hydatides, or Bladders of clear Water on its Surface. The Left Kidney was alſo leſſen'd, but not ſo much as the Right. Its Surface like that was unequal, but had no Hydatides in it. The Ureter of the Left Kidney was very much contorted at its Riſe from the *Pelvis*; where its ſides were petrifi'd, inſomuch that its Canal was almoſt render'd impervious for the Paſſage of the Urine.

Nothing was ſound in the Bladder of Urine, but divers ſtones of unuſual Figures, as if they had been pieces of a large Stone, broken to Bits, in whoſe Center a Nucleus had been lodg'd. The Gall Bladder was fill'd with gall Stones. Nor was the Stomach, which he complain'd of (*i. e.* for want of Appetite) any other ways diſorder'd: but a little redder, having more Blood in its Veſſels than is uſual; its Muſcular Fibres being ſtronger than we generally find them in the Stomachs of healthy Perſons.

The Cavity of the Thorax or Cheſt was fill'd with water on both ſides; inſomuch that the Lungs were not above the third part of the natural Magnitude. The Pleura or Membrane that lines the Cavities of the Thorax was very much thicken'd by the Serum or Water; from whence it deſcended by the Muſcles of the Back into his Legs. The Valves of the left Ventricle of the Heart were petrifi'd in ſeveral Places, eſpecially thoſe call'd *Mitrales.* There were ſome ſtony bodies found on the *Bronchia*, at and near the riſe from the Lungs.

A Child with the Inteſtines, Meſentery, &c. in the Cavity of the Thorax. by Sir Cha. Holt. *n.* 275. *p.* 992.

X. The Child was uneaſy and reſtleſs from its Birth, and conſtantly labour'd under a difficulty of Breathing. Its illneſs was like what hath been ſeen in other Children, neither could they perceive it reliev'd by any thing adminiſtred to it; tho' by advice of a Learned Phyſician, but it lay groaning and pining till it died. When the Child was undreſt, an odd ſort of Working was obſerv'd in its Breaſt, and a Crawling perceiv'd round the Ribs and Breaſt, on both ſides, as if a Knot of ſmall Eels or large Earth Worms had been pen'd up within the Cavity. This Relation ſeemed ſtrange, but upon the diſſection we found ſufficient reaſon to believe the account.

When we had opened the Abdomen, there appeared none of the *Viſcera* belonging to the Belly, except the Liver, the Kidneys, *Veſica Urinaria*, and *Inteſtinum Rectum*. We at firſt imagined that the other Inteſtines might be covered by the Liver, which, tho' it be commonly large in Children, in this exceeded the uſual ſize; but upon turning of it up towards the Diaphragm, we only found, under its concave part, the Stomach, not lying in its natural poſture, for the *Pylorus* was drawn by the *Duodenum* croſs the *Vertebræ* of the Back towards the *Fundus Ventriculi*, and part of the *Duodenum* paſſed through a Foramen in the Diaphragm, placed on the Left ſide of that through which the *Gula* deſcends, which occaſion'd the *Pylorus* to lye almoſt *ſub Fundo Ventriculi.* We then reſolved to trace the *Rectum* from the *Anus* upward, not doubting but that it would lead us to the Meſentery and Inteſtines. The *Rectum*, we found, lay in an oblique line from the *Anus* to this new *Foramen*, and was received into it with a Portion of the *Duodenum.* This *Foramen* ſeemed to be formed by Nature, *a primo Ortu*, for tranſmitting thoſe Guts into the *Thorax*; for had it been made by any Force, its ſides would have appeared wounded, or lacerated; but on the contrary, round this Orifice there was a ſmooth Verge, as is ſeen in *Foramine Venæ Cavæ pervio*, or that, *per quod Gula deſcendit*. A Sketch of the Diaphragm with its Perforations I have attempted.

When we took off the *Sternum*, we ſaw the Meſentery *cum adjunctis Inteſtinis* in the Cavity of the *Thorax*, and lying upon the Heart and Lungs. There was no *Omentum* ſpread over the Inteſtines; that, as I remember, was wholly miſſing, as was likewiſe the *Mediaſtinum*. Moſt part of the *Duodenum* lay in the *Thorax*, and all the reſt of the Guts, except the *Rectum*, which (as is already related) aſcended in an oblique line from the *Anus*, and its upper end was inſerted into this Orifice. We enquir'd how this Child, according to the Common Notions of Nutrition, could be nouriſhed? That it was Nouriſh'd ſeems plain, becauſe it daily receiv'd Food, and regularly voided Excrement, as we are aſſur'd by thoſe who attended upon it. For ſalving this Quære, we propos'd to enquire, what communication there was between that Gland, or Glands, in the middle of the Meſentery (commonly called *Pancreas Aſellii*) and the *Receptaculum Chyli* placed between the Internal Lumbar Muſcles, called *Pſoas*; but upon the moſt accurate Search we were capable of making, there was none to be found; for the whole Meſeraick Membrane, and Inteſtines, lay perfectly looſe upon the Heart and Lungs, abſolutely diſengag'd from any manner of Communication with any other part.

That Vermicular Motion, which ſhewed itſelf on the Ribs and Breaſt, we aſcribed to the Periſtaltick Motion of the Guts; and the *Dyſpnæa*, we thought, might be occaſioned by Preſſure made on the Lungs by the Inteſtines and Meſentery, which likewiſe ſo fill'd the *Thorax*, that there wanted room for the Lobes of the Lungs to move freely in, and by conſequence Inſpiration and Expiration would be performed with Difficulty.

See the Figure; where a ſhews the Foramen *through which the* Vena Cava *paſſes. b the* Foramen *thro which the* Gula *deſcends. c the* Foramen *through which part of the* Rectum *and* Duodenum *went into the* Thorax.

Fig. 11. *Plate.* II.

Of the Glands in the human Spleen, by Dr. Douglas. *n.* 349. *p.* 499.

XI. The Subject I found theſe Glands in, was a Boy of about 4 or 5 Years Old, that died of a general *Atrophy*, or Conſumption of all the Muſcular Fleſhy parts of the Body, occaſioned without all doubt from the numerous Glandulous Swellings ſcattered up and down the whole Meſentery; which by compreſſing the Lymphatick Veſſels, called in this place *Vaſa lactea*, prevented the acceſs and ſupply of the Chyle, ſo neceſſary for the continued nouriſhment and increaſe of the Parts. For without the conſtant Recruit of this whitiſh Balſamick Liquor, the Maſs of Blood will in a ſhort time be unfit to perform any of thoſe good Offices, which a freſh acceſſion of Chyle qualifies it for.

In a piece of this *Spleen* we might ſee, without the aſſiſtance of a Glaſs, ſeveral round whitiſh Bodies of a pretty hard Conſiſtence, and abundance of ſmall white and ſofter ſpecks; but both of the ſame nature. Theſe, to me at leaſt, appear to be ſo many diſtinct Glands become viſible;

which in a Natural State are only to be ſeen by a fine Glaſs, as the curious *Malpighius* firſt obſerved in his Treatiſe *de Liene, Cap.* V. *De quibuſdam corporibus per Lienem diſperſis.* Where is a Caſe which was the very ſame with mine.

Anatomical Obſervations by Mr. Chiſe'den, *n.* 337. *p.* 282.

XII. Fig. 12. Plate 1. Shews three Spleens taken from one Body.

Fig. 13. Two Spleens taken from a Man.

Fig. 14. Two Spleens taken from a Woman.

N. B. That in all theſe three Caſes of the Spleen, each had its proper Veſſels, but the Arteries only are there expreſt; and that the Spleens in each Body taken together were but equal in Magnitude to the one we uſually meet with.

An Ureter double of two Thirds of its Length next the Kidneys, and diſtended by Stones paſs'd thro' it.

The *Tubæ Fallopianæ* impermeable; and without *Alæ Veſpertilionis*; the outer Ends being connected to the *Teſtes.*

A Heart with the *Vena Azygos* inſerted into the Right Auricle; and the deſcending *Cava* coming round the Baſis of the Heart, above the Aorta and Pulmonary Veſſels, to enter the Auricle at the lower Part with the aſcending *Cava.*

The Left Ventricle of the Heart of a prodigious Bigneſs, by Dr. Douglas, *n.* 345. *p.* 326.

XIII. I lately open'd a young Man that died of the Palpitation of the Heart, whoſe violent Beating and prodigious ſubſultory Motion, ſome Months before his Death, was not only eaſily felt, by laying the Hand on the Region of the Heart; but ſeen to riſe and fall by lifting up the Bed-Cloaths that covered it. And, which is almoſt incredible, at ſome times the Tremblings and Throbbings made ſuch a Noiſe in his Breaſt, as plainly could be heard at ſome Diſtance from his Bed-ſide. This was accompanied with frequent *Deliquiums,* ſometimes ſlow, ſometimes ſwift, and often intermitting. In the Diſſection of this morbid Heart, I obſerved the following remarkable Particulars.

1. That the *Pericardium* or *Capſula Cordis* was very thick, and firmly adhered or grew by a fibrous Connexion to all the outer Surface of the Heart.

2. Inſtead of the Water called *Liquor Pericardii,* there was only in ſome Places about the *Baſis* of the Heart, a mucilaginous clear Subſtance like a Gelly.

3. In the right Auricle laid open there was nothing preternatural. The aſcending and deſcending *Cava* opened into the ſame as uſual. The *Veſtigium* or Mark of the *Foramen ovale* with its ſemicircular *limbus* was very plain. And the *Orificium* of the *Vena Cordis Coronaria* was extreamly large, yet its Valve was leſs than uſual.

4. In the right Ventricle laid open, the *Valvulæ* called *tricuſpides* were configurate after the uſual manner. The ſides of this Cavity were thin and full of ſmall fleſhy *Columnæ* as they commonly are, with great variety of Furrows and little Holes. The three *Sigmoide* or *Semilunar Valves* in the

Mouth of the *arteria pulmonalis*, were as they always are in a Natural State.

5. The Left Auricle was not much bigger than ordinary: but its muſcular Appendage, called the *Bulb* of the *Pulmonary Vein* by the late Mr. *Cowper*, was extraordinarily dilated and enlarged beyond any thing that I ever ſaw.

6. The Left Ventricle, whoſe Capacity in a natural State is always leſs than the Right, was here conſiderably larger. And if the Experiment had been made before Diſſection, of filling both with any Liquor, this had certainly contained three times more than the other.

7. The *Valvulæ*, called *Mitrales*, placed at the Orifice of this Ventricle, are much thicker in Subſtance than ordinary; and the two fleſhy Columns, called by *Nicolaus Maſſa*, almoſt 200 Years ago, *duo parvi muſculi*, which ſend out abundance of ſmall Tendons to be inſerted into thoſe Valves, were proportionably augmented in Bigneſs.

8. The *Semilunary Valves* in the Mouth of the *Aorta*, or of that great *Vena pulſatilis* that diſpenſes the Blood to all the ſeveral Parts of the human Body, were very much preternaturally affected; as would eaſily appear upon comparing them with thoſe in the Orifice of the *Pulmonary Artery*, in which they are thin and very broad, ſo as to be able to ſhut the Cavity of that Veſſel, and hinder the Blood from returning back into the Ventricle, and likewiſe tranſparent: but in this they are very thick, contracted as it were, and furled together, and of a whitiſh Colour; and in all Appearance, if the Perſon had lived longer, they had turned boney or undergone a Petrification.

This uncommon Structure of the Heart being thus demonſtrated, let us endeavour to account for the following Phænomena. The firſt is the Palpitation of the Heart, which was the chief Symptom and Complaint of the ſick Perſon. The ſecond is the preternatural Dilatation and Enlargement of the left Ventricle. It is not improbable but the firm Adheſion of the *Capſula Cordis membranoſa* to the Subſtance of the Heart occaſioned that uncommon Trembling and Throbbing thereof; its free and eaſy Motion being hindered by that thick *involucrum*, which ſurrounded it ſo cloſe on each* ſide.

As for the Dilatation of the left Ventricle and muſcular Bag of the *Pulmonary Vein*; that is altogether owing to the ill Configuration of the *Valves* we have now deſcribed: for as the great Artery or *Aorta* ariſes out of this Ventricle, it has three Valves, which ſeparating, give Paſſage to the Blood from the Ventricle into the Veſſel; and in a natural State, they ſhut that Paſſage, and ſo prevent the Blood from recoiling into the ſame, if it ſhould endeavour to return. But in this Caſe, by reaſon of its contracted Narrowneſs and Thickneſs, not being able to cloſe or ſhut the Paſſage, the Blood flow'd back again into the Cavity which it had gradually enlarged and dilated to the Bigneſs we ſee. Beſides, the *Muſcular Valves* not being duly qualify'd to perform their Office, the Blood recoil'd into the Auricle, which it had diſtended in like manner. This conſtant Regurgitation

* *v.* Lower *de Corde.*

gurgitation or Reflux of the Blood, is beſides ſufficient of it ſelf to cauſe this extraordinary Trembling.

Of ſome Influence of Reſpiration on the Motion of the Heart, hitherto unobſerved. By Dr. Drake, *n.* 281 *p.* 1217.

XIV. Tho' divers accurate Treatiſes of the *Heart*, and its Action have been written by Learned Men of ſeveral Nations, eſpecially by two of our own Country, Dr. *Harvey*, and Dr. *Lower*, yet there remain ſeveral Doubts and Difficulties about it (in my Opinion) not ſufficiently accounted for. Dr. *Lower* has ſo well accounted for the *Syſtole*, or Contraction of the Heart from the *Mechanical* Structure of it, that he ſeems almoſt to have exhauſted the Subject, and had he been as happy in diſcovering the true Cauſe of the *Diaſtole*, he had left little room for the Induſtry and Sagacity of others about this *Viſcus*. But having judiciouſly and ſolidly explain'd the *Syſtole*, he contents himſelf to aſcribe the *Diaſtole* to a Motion of *Reſtitution*, which Account gives me no Satisfaction : Becauſe the *Syſtole* being the proper, and (as himſelf confeſſes) the only Motion of the Heart, a State of *Contraction* ſeems to be the natural State, and conſequently without external violence, it ſhould have no *Diaſtole* at all.

This will appear more plain, if we conſider the Circumſtances of it, and its Motions, as a Muſcle, with reſpect to orher Muſcles. That Contraction is the proper Action, and State of all Muſcles, is evident from Experience of Fact, as well as Reaſon. For, if any Muſcle be freed from the Power of its *Antagoniſt*, it is immediately *contracted*, and is not by any Action of the Will, or Spirits to be reduc'd to a State of *Dilatation*. Thus, if the *Muſculi Flexores* of any Joint be divided, the *Extenſores* of that Joint being by that Means freed from the contrary Action of their *Antagoniſts*, that Joint is immediately extended without any Conſent of the Will, and in that State it remains; and ſo *Vice verſâ* if the *Extenſores* be divided. From whence it is plain, that the Muſcles have no reſtitutive Motion, but what they derive from the Action of their *Antagoniſts* by which they are ballanc'd. Thus likewiſe the *Sphincters* of the *Gula*, *Anus*, and *Veſica*, having no proper *Antagoniſts*, are always in a State of Contraction, and ſuffer nothing to paſs 'em, but what is forced thro' 'em by the contrary Action of ſome ſtronger Muſcles, which, tho' not properly to be call'd *Antagoniſts*, yet on all neceſſary Occaſions perform the Office of ſuch.

That the Heart is a Muſcle, furniſh'd and inſtructed for Motion like other Muſcles, is (in my Opinion at leaſt) demonſtrated beyond Contradiction, by Dr. *Lower* and others. And, as it is a *Solitary* Muſcle without any proper *Antagoniſt*, and not directly under the Power of the Will, nor exerciſing *Voluntary* Motion, it approaches neareſt to the *Sphincter* kind, which only has theſe Conditions in common with it. But in conſtant and regular Alternations of Contraction and Dilatation, it differs exceedingly from all the Muſcles of the Body. This *reciprocal Æſtus* of the Heart has given the Learned abundance of Trouble; who, finding nothing peculiar in the Structure, which ſhou'd neceſſarily occaſion it, nor any *Antagoniſt*, whoſe Re action ſhould produce it, have been extreamly perplext to find out the Cauſe of it.

The Learned Dr. *Lower* having by ſound Arguments, drawn from the Structure and Mechaniſm of the Heart, eſtabliſht the certainty of its *Muſcular* Motion, reſts ſatisfied, without taking notice of any Aſſiſtance, that the Heart receives from any other Part, except from the Brain, by the means of the eighth Pair of Nerves.

The Accurate *Borellus* in his *Oeconomia Animalis* computes the *Motive* Power of the *Machine* of the Heart to be equal to, or to ſurmount that *Part 2d.* of a Weight of 3000 *l.* The *Obſtacles* to the Motion of the Blood through *Prop.* 67. the *Arteries* he eſteems equivalent to 180,000 *l.* which is 60 times as much *Prop.* 73. as he rates the Force of the Heart at. Then deducting 45,000 *l.* for the Adventitious Help of the *Muſcular Elaſtick Coat* of the *Arteries*, he leaves the Heart with a Force of 3,000 *l.* to overcome a reſiſtance of 135,000 *l.* that is, with 1 to remove 45. *Prop.* 76. This ſtupendous effect he contents himſelf to aſcribe to the *Energy* of *Percuſſion*. But, had he proceeded in his Calculation to the Veins, which he allows to contain conſtantly a quantity of Blood, quadruple to the Contents of the Arteries, and to which this *Energy* of *Percuſſion* does either not reach at all, or but very languidly, he might probably have ſeen a neceſſity for ſome other Expedient to remove ſo inſuperable a difficulty. But not to inſiſt rigorouſly on the exactneſs of this Calculation, we may allow a much greater Deduction than would be juſtifiable, without leſſening the difficulty. But this account I have taken notice of purely for the ſake of the Calculation, which may be of uſe in the Sequel, the account it ſelf being in other reſpects more defective than Dr. *Lower*'s, to which we will return.

The Dr. appears (to me) to have overlookt ſomething of very great moment and importance in the explication of the Action of the Heart. For, though it ſhou'd be granted, that the *Muſcular Fibres* of the Heart, acted by the Nerves, are the immediate Inſtruments of its *Conſtriction* or *Syſtole*, yet it muſt not be denied, that the *Intercoſtal* Muſcles and *Diaphragm* are of great ſervice to aid and facilitate this Contraction, by opening a Paſſage for the Blood through the Lungs, which denied wou'd be an invincible obſtacle. Neither do they promote it that way only. The manner how they farther aſſiſt the Heart in its Contraction will appear manifeſtly, if we conſider the different Poſture, Scituation and Capacity of the Blood Veſſels of the Lungs in the ſeveral times of *Elevation* and *Depreſſion* of the *Coſtæ*.

The *Pulmonary* Artery riſes from the *right* Ventricle of the Heart, and runs into one Trunk, till it comes to the *Aſpera Arteria*; where it is divided, and ſends a Branch along with each diviſion of the *Aſpera Arteria*, according to all the minuteſt ſubdiviſions, of which it is likewiſe ſubdivided, accompanying all the *Bronchi*, in their whole progreſs through the Lungs. The *Pulmonary* Vein, which empties it ſelf into the *Left* Ventricle of the Heart, ſpreads it ſelf on the *Aſpera Arteria* and *Bronchi*, in the ſame manner that the Artery does. The neceſſary conſequence of this diſpoſition is, that this Artery and Vein being coextended with and faſten'd to the *Bronchi*, muſt needs ſuffer ſuch alteration of *Superficial* Dimenſions, as the *Bronchi* do in the *Elevation* or *Depreſſion* of the *Coſtæ*.

While the Ribs are in a ſtate of *Depreſſion* (whether before Commerce with the external Air or after) the *Annular Cartilages* of the *Bronchi* ſhrink one into another, and by that means their *Dimenſions* are exceedingly contracted. In conformity to this condition of the *Bronchi* the *Pulmonary* Artery and Vein muſt likewiſe, either by means of their *Muſcular* Coats contract themſelves to the ſame *Dimenſions*, or lye in *Folds* or *Corrugations*, which is leſs probable. On the other hand, when the Ribs are elevated, and the *Diaphragm* bears downward, the Air ruſhing into the Lungs, ſhoots out the *Cartilaginous* Rings, and *Divaricates* the Branches of the *Trachea*, and by them extends and divaricates the ſeveral diviſions of the *Pulmonary* Artery and Veins, and thereby lengthens and enlarges their Cavities. This enlargement of their Cavities is very conſiderable, not only upon the ſcore of the addition, which they receive in length thereby, but alſo upon the account of their *Divarication*. For whereas, when the Ribs are depreſt, and the Lungs ſubſide, the Blood Veſſels are not only contracted, (as I have already obſerv'd) but their Branches, which are exceeding numerous, approach one another, and lie in juxta-poſition, by which their Cavities are very much compreſs'd and ſtreighten'd: when the Ribs are elevated, and the Lungs turgid with Air, not only the Fibres, by which their Coats in the oppoſite State were contracted, are extended ; but thoſe innumerable Veſſels, which lying before in lines almoſt parallel upon one another, compreſs'd one another, making an *acute* Angle at their Junctures, are divaricated and ſeparated from each other, and make an *obtuſe*, whereby their Channels are widen'd.

Thus a paſſage is open'd to the Blood, from the *Right* Ventricle of the Heart to the *Left*, through the Lungs, to which it cou'd not otherwiſe paſs; and the oppoſition, which the Blood, contain'd in that Ventricle, muſt otherwiſe neceſſarily have made to its conſtriction, is taken off, and the *Syſtole* thereby facilitated. Nor is that all. For the *Diaſtole* being caus'd (as I ſhall in the ſequel ſhew) by the Force of the Blood ruſhing into the Ventricles, this Ampliation and Extenſion of the *Pulmonary* Artery is a ſort of *Check* or *Counterpoiſe* to it, and prevents an endeavour towards two contrary Actions at once, which muſt neceſſarily fruſtrate both. For the Heart being a *Springy*, *Compreſſible* Body, whoſe proper Action, which is Contraction, depends on the influx of certain Fluids into its Fibres, or Subſtance; and containing beſides a Fluid in its *Ventricles*, or great Cavities, in one of which is the Mouth of this Artery ; the action of this Veſſel muſt in great meaſure reſemble that of a *Syringe*, whoſe extremity is immers'd in Water, the Enlargement or Expanſion of the Channels of the Artery anſwering the drawing of the *Embolum*, as the Conſtrictive Motion of the Muſcle of the Heart does the Preſſure of the *Atmoſphere* upon the *Surface* of the Water, the one making way for the fluid, and the other forcing it to follow, where the reſiſtance is leaſt. In this ſenſe we may allow a ſort of Attraction to the *Pulmonary* Artery, depending wholly upon the Action of the *Intercoſtal* Muſcles and *Diaphragm*, which we muſt therefore confeſs to be very ſerviceable and inſtrumental in promoting the *Syſtole* of the Heart.

But the Learned Author has much leſs ſufficiently accounted for the *Diaſtole*, or Dilatation of it, which he aſcribes to a Motion of *reſtitution* of the over-ſtrain'd Fibres, which yet he confeſſes are made for *Conſtriction* only. 'Tis true, he immediately after joyns the *influx* of the *Blood* as a concurrent cauſe; but from the ſlight notice that he takes of it, 'tis plain, that he did not ſo much as dream of any great ſhare it had in that action. His words are theſe. *Quin & (ut obiter hoc moneam) cum omnis motus contractione perficiatur, & Cordis Fibræ ad conſtrictionem ſolum factæ ſint, apparet quoque Cordis motum* totum *in* Syſtole *poſitum eſſe; cumque Fibræ ultra tonum ſuum in omni conſtrictione ejus tendantur, idcirco ubi nixus iſte abſolvitur, motu quaſi* reſtitutionis *Cor iterum relaxatur, & ſanguine a Venis* influente *rurſus diſtenditur; a* nullo *enim cordis motu, niſi* tenſionem ſuam remittente, *& ab* irruente *ſanguine* Diaſtole *ejus libratis adeo viribus ſuccedit.*

De Corde Pag. 75.

If Contraction be the ſole Action of theſe Fibres (as this Great Man confeſſes it to be) and as indeed it is of all *Muſcular* Fibres, I wonder how ſo judicious a Writer came to ſlip into ſuch an abſurdity, as to call their Diſtenſion (vulgarly but improperly called Relaxation) a Motion of *Reſtitution.* For from the Nature of thoſe Fibres, and their diſpoſition in the Structure of the Heart, the Natural State of the Heart appears manifeſtly to be *Tonical*, and its Dilatation a State of Violence; and conſequently the Conſtriction is the *true* motion of *Reſtitution*, and the State to which it will *ſpontaneouſly* return, when the Force is taken off, which is the work of the *Intercoſtal* Muſcles and *Diaphragm.* Thus we are left ſtill to ſeek for the true cauſe of the *Diaſtole*, which ſeems to me to be the main and moſt difficult *Phænomenon*, relating to the Heart and Circulation of the Blood.

But in Mr. *Cowper*'s ingenious *Introduction* to his *Anatomy of Human Bodies*, I find the Share which Dr. *Lower* hints the Blood to have in that Action, further proſecuted, and improv'd into the main Inſtrument of the Dilatation of the Heart, wherein I agree entirely with him. But as to the manner, and Reaſons of its being ſo very Inſtrumental, I can't be ſo perfectly of his Mind. *The Heart* (ſays he) *of an Animal bears a great Analogy to the Pendulums of thoſe Artificial Automata, Clocks and Watches, whilſt its Motion is performed like that of other Muſcles, the Blood doing the Office of a* Pondus. This Explication, being but a Simile without a diſtinct Application to Particulars, is beſide ſo very ſhort, that I can at beſt but conjecture at the Meaning.

By the Blood's doing the Office of a *Pondus*, I ſuppoſe he means, that the Blood contributes in the ſame manner to the Motion of the Heart as the Weights do to that of the Pendulum of a Clock. If ſo, the Blood, according to him, muſt be the Inſtrument of *Conſtriction*; and *Dilatation* muſt be the Natural State, or Spontaneous Motion, to which it wou'd, when under no Violence, return; the contrary of which, I preſume, will appear e'er I have done. But if he means, that the Blood in its Reflux, by gravitating on the Auricles and Ventricles, dilates and expands them, acting therein as a Counterpoiſe to its Contraction as a Muſcle, I could wiſh his Deſign had not bound him up to ſo narrow a Compaſs, and that he had given us an Explanation at large of ſo abſtruſe and ſo important a Phænomenon. Becauſe the Specifick Gravity of the Blood ſeems to me a Cauſe by no Means alone adequate to the Effect, which it is here ſuppos'd to produce. For, if the Blood acts only as a Weight by meer Gravitation, then that Part of it only which deſcends from the Parts above the Heart can be employ'd in that Action. This at the largeſt Computation can't amount to five Pound Weight, and muſt, according to the Computation of *Borellus*, force a Machine, that is able to overcome a Reſiſtance of 135,000 *l.* I leave every Man to deduct what he ſhall upon Examination find reaſonably to be deducted, and yet ſhall reſt ſecure, that it is not to be effected in the leaſt with ſo ſmall a Weight.

But neither does the Refluent Blood gravitate in any ſuch Proportion, as I have here aſſign'd. For to make a true Eſtimate of its Gravitation, we muſt conſider the Circumſtances of the Liquor ſuppos'd to gravitate; in which it very much reſembles Water inclos'd in a recurve Tube, of which if the Length of the two Legs be equal, it may be ſuſpended in the Air full of Water, with the Extremities downwards, without loſing a Drop; altho' the *Diameter* of thoſe Legs ſhou'd be very unequal. The Caſe of the Arteries and Veins is pretty near a Parallel to a Tube, ſo fill'd and inverted. For, if the Arteries and Veins be continued Tubes, (as by the Microſcope they are made to appear) then, ſuppoſing their Contents to have no other Determination of Motion, than their own Weight wou'd give 'em, the contain'd Fluids muſt be Counterpoiſes to each other. For the Veins and Arteries being join'd at the ſmaller Extremities, and the larger of both terminating in the ſame Parallel Line, it is impoſſible, according to the Laws of *Hydroſtaticks*, that the Contents of either ſhou'd overballance t'other. How far then muſt it fall ſhort of forcing the natural Power and Reſiſtance of ſo ſtrong a Muſcle as the Heart by meer Gravitation?

The Blood indeed has a *Progreſſive* Motion thro' its Veſſels, wherein it differs from Water, in a recurve Tube, in the Experiment above ſtated. But, if the natural Gravitation of the Blood contributes nothing to the Dilatation of the Heart, this Progreſſive Motion will not be found much more ſufficient. For, as this Motion is deriv'd entirely from the Heart's Conſtriction (as all Accounts hitherto derive it) cou'd the Blood be ſuppos'd to re-act upon the Heart, with all the Force firſt impreſs'd upon it by the Heart, it wou'd be inſufficient, unleſs we will ſuppoſe the *Force communicated* to be ſuperiour to the *Power Communicant*, which is abſurd. But when the juſt and neceſſary Deductions, for the Impediments which the

Blood meets with in its Progreſs thro' the Veſſels, ſhall be made, the remaining Force will be found ſo exceeding weak, that to prop the Blood thro' the Veins may be a Task alone too great for ſo ſmall a Power, without charging it with the additional Difficulty of forcing the Muſcle of the Heart.

Alphonſus Borellus caſts up the Force of the Heart, and the *Muſcular* Coat of the Arteries, to be together equal to a Weight of 3,750 *l.* and allots 'em a Reſiſtance equal to 180,000 *l.* to overcome which is 45 to 1. To make up for a Diſproportion, by his own Confeſſion, incredible to thoſe who have not conſidered the Matter as he had done, he flings into the Scale the additional *Force of Percuſſion,* which he leaves *indefinite,* and thinks ſufficient to *force any quieſcent finite Reſiſtance whatſoever.* But as this Account and *Hypotheſis* are a Poſthumous Work, I ſhould ſuſpect, that theſe Papers were left unfiniſht by *Borellus;* or at leaſt, that in many Places the laſt Hand was never put to 'em. For, neither in this Place, nor any other of this Work, does he account for any more than the *Syſtole* of the Heart, and the Reſiſtance which is made to the Progreſſive Motion of the Blood in the Arteries only. This alone he found to exceed the Power of the Heart ſo prodigiouſly, that he ſeems to ſhuffle it off his Hands, with a general and precarious Solution, as a Difficulty that he was deſirous to be rid of. For, having aſcrib'd this *ſtupendious* (as he himſelf calls it) effect to the *Energy of Percuſſion,* he takes no Care to ſatisfie his Reader any farther about it, or to refer him, or give him the Expectation of Satisfaction any where elſe; although he has an expreſs Treatiſe on the *Force of Percuſſion,* which was written preparatory to this, and to which he often refers in other Places of this Work. But what confirms my Suſpicion, that this Part was intended for a farther Reviſe by the Author, is, that he has left the Progreſs of the Blood thro' the Veins, and the *Diaſtole* of the Heart, abſolutely untouch'd, tho' they are Difficulties of a much greater Magnitude, than this which he has attempted to account ſo ſlightly for. For in theſe he is excluded the Benefit of *Percuſſion,* and has yet a greater Reſiſtance to overcome without it. Omiſſions of this kind are ſo unuſual with this Author, where-ever he knows himſelf to go upon ſure Grounds; that it is to me an Argument, that he doubted the Sufficience of his *Percuſſion,* and reſerv'd theſe important *Phænomena* for farther Conſideration, without plunging himſelf into ſuch an Abſurdity as to aſcribe to *Percuſſion* any ſuch *Energy* as to be able (ſo broken as it returns to the Heart) by its Re-action to force that Power, from whence only it was firſt deriv'd. But tho' the *Hypotheſis* of *Borellus* may in this caſe be found precarious or inſufficient (a Misfortune that has befallen him in divers Particulars) his *Theory* holds ſtill good. At leaſt it ought to be allowed in Juſtice to his great Abilities and Exactneſs, till ſome body convicts him of ſome material Error in his Calculations.

Suppoſing then the Force of the Heart, and of the Muſcular Coat of the Arteries, as likewiſe of the Reſiſtance, which they muſt overcome, to be computed with any degree of Accuracy, there remains yet ſuch a prodigious Diſproportion to be accounted for, as requires ſome more powerful Agent, than any yet aſſign'd, to make up the Deficiency.

What Aſſiſtance the Heart receives from the Action of the *Thorax* towards the facilitating its Contraction, without which Aſſiſtance there could have been no *Syſtole*, has been already ſhewn. But neither the *Intercoſtal* Muſcles, or *Diaphragm*, which are ſo inſtrumental in that Part of its Action, can contribute any thing to the *Diaſtole*; becauſe they ſerve only to enlarge the Cavity of the *Thorax*, and thereby to open the Paſſage to the Blood from the Heart; and promote its Conſtriction. Whatever therefore the Force is, that dilates the Heart, and is the Cauſe of the *Diaſtole*, muſt be equal to that of the Heart, the *Intercoſtal* Muſcles and *Diaphragm*; to all which it acts as an Antagoniſt. I take no notice of the *Serratus Major Anticus*, and other Muſcles; which have an obſcure Share in the *Elevation* of the *Coſtæ*, becauſe as much may reaſonably be deducted upon the Account of the *Obliquus externus Abdominis*, and other Muſcles; which, having their Inſertions on ſome of the lower Ribs, are as inſtrumental towards the Depreſſion of 'em, and ſo ballance the Account. But the chief Uſe of theſe is in violent Reſpiration. In ordinary Reſpiration their Share is ſmall.

Such a real Power (which may in the leaſt be ſuſpected of any ſhare in this Action) is hard, perhaps impoſſible, to be found in the Machine of any Animal Body; and yet without ſome ſuch Antagoniſt, it is as impoſſible the Circulation of the Blood ſhou'd be maintain'd. All the Engines yet diſcover'd within the Body conſpire towards the *Conſtriction* of the Heart, which is the *State* of *Quieſcence*, to which it naturally tends. Yet we find it alternately in a *State* of *Violence*, that is, of *Dilatation*; and this upon Neceſſity, becauſe upon this *Alternation* depends all Animal Life. Some ſufficient Cauſe external muſt therefore be found, to produce this great Phænomenon, which Cauſe muſt be either in the Air, or Atmoſphere, becauſe we have no conſtant and immediate Commerce with any other *Mediums*.

Some great Phyſicians obſerving this, and, that depriv'd by whatſoever means of Communication with the external Air, we became inſtantly extinct, have imagin'd, that in the Act of Inſpiration certain purer Parts of the Air mixed with the Blood in the Lungs, and was convey'd with it the Heart, where it nouriſh'd a ſort of *Vital Flame*, which was the Cauſe of this reciprocal *Æſtus* of the Heart. Others not quite ſo groſs, rejecting an *Actual* Flame, have fancied that theſe fine Parts of Air mixing with the Blood in the Ventricles of the Heart, produc'd an *Efferveſcence* which dilated it. But theſe Fancies have been long ſince exploded and condemned upon ample Conviction, and 'tis a Point yet undetermin'd, whether any Air does mix with the Blood at all in the Lungs or not. But ſuppoſing that ſome Air may inſinuate it ſelf into the *Pulmonary* Vein, it can no other Way dilate the Heart than by an Efferveſcence in the Left Ventricle, which wou'd not dilate the Right. But this Opinion is contradicted by *Autopſie*, and too laboriouſly confuted by others, to be brought upon the Stage again here.

There remains therefore only the groſs Body of the Atmoſphere to be conſider'd, which is undoubtedly the true Antagoniſt to all thoſe Muſcles, which ſerve for ordinary Inſpiration and Conſtriction of the Heart. This will appear more evidently, if we conſider not only the Power, but the Neceſſity of its Action upon *Animal* Bodies, as well as the want of other ſufficient Agents.

The Heart is a Solitary Muſcle of very great Strength, and the *Intercoſtal* Muſcles and Diaphragm, which likewiſe have no Antagoniſts, are a vaſt additional Force, which muſt be ballanc'd by the contrary Action of ſome equivalent Power or other. For, tho' the Action of the *Intercoſtal* Muſcles be voluntary, that does not exempt them from the Condition of all other Muſcles, ſerving for voluntary Motion, which wou'd be in a State of perpetual Contraction, notwithſtanding any Influence of the Will, were it not for the Libration of Antagoniſt Muſcles. This Libration between other Muſcles is anſwer'd by the Weight of the incumbent *Atmoſphere,* which preſſes upon the *Thorax* and other Parts of the Body. And, as in all other voluntary Motions the Influence of the Will only gives a Prevalence to one of two Powers before equilibrated, ſo here it ſerves to enable thoſe Muſcles to lift up a Weight too ponderous for their Strength not ſo aſſiſted; and therefore as ſoon as that Aſſiſtance is withdrawn, the *Coſtæ* are again depreſs'd by the meer *Gravitation* of the *Atmoſphere,* which wou'd otherwiſe remain elevated thro' the natural Tendency of thoſe Muſcles to Contraction. This is evidently proved from the *Torricellian* Experiments, and thoſe made upon Animals in Mr. *Boyle*'s Engine; where, as ſoon as the Air is withdrawn, and the *Preſſure* thereby taken off, the Intercoſtal Muſcles and Diaphragm are contracted, and the Ribs elevated in an Inſtant, and can't by any Power of the Will be made to ſubſide, till the Air is again let in to bear 'em forcibly down.

As in the Elevation of the *Coſtæ,* the Blood, by the Paſſage that is open'd for it, is in a manner ſollicited into the Lungs, ſo in the Depreſſion of 'em, by the Subſidence of the Lungs and Contraction of the Blood Veſſels, both which are Conſequent thereof, the Blood is forcibly driven, as it were with an *Embolum,* thro' the Pulmonary Vein into the Left Ventricle of the Heart. And this, together with the *general Compreſſion* of the Body by the Weight of the *Atmoſphere,* which ſurrounds and preſſes upon the whole Surface of it, is that Power which cauſes the Blood to mount in the Veins, after the Force impreſs'd upon it by the Heart is broken and ſpent, and which is ſufficient to force the Heart from its natural State to Dilatation. He that is able to compute the Weight of a Column of Air, equal to the Surface of the whole Body, will readily grant it a Power ſufficient for the Effects, which are here aſcrib'd to it. And, when he conſiders, that the Bodies of Animals are compreſſible Machines, he will find that it muſt of Neceſſity affect them in the manner here laid down. But tho' our Bodies be entirely compos'd of *Tubuli,* or Veſſels fill'd with

Fluids, yet this Preſſure, how great ſoever, being equal, cou'd have no effect upon 'em, if the ſuperficial Dimenſions were not eaſily variable; becauſe being compreſs'd on all Parts with the ſame Degree of Force, the contain'd Fluids cou'd not any where begin to recede, and make way for the reſt to follow, but would remain as fixt and immovable as if they were actually ſolid. But by the Dilatation of the *Thorax*, room is made for the Fluids to move, and by the Coarctation of it freſh Motion is impreſt, which is the main Spring whereby the Circulation is ſet and kept going. This reciprocal Dilatation and Contraction of the Superficial Dimenſions of the Body ſeems ſo neceſſary to animal Life, that there is not any Animal ſo imperfect as to want it, at leaſt none to the inward Structure of which our Anatomical Diſcoveries have yet reach'd. For, tho' moſt kinds of *Fiſh*, and *Inſects*, want both moveable Ribs and Lungs, and conſequently have no dilatable *Thorax*, yet that want is made up to 'em by an *Analagous* Mechaniſm anſwering ſufficiently the Neceſſities of their Life. Thoſe Fiſhes which have no Lungs, have Gills, which do the Office of Lungs, receiving and expelling alternately the Water, whereby the Blood Veſſels ſuffer the ſame Alteration of Dimenſions, that they do in the Lungs of more perfect Animals. The Lungs or Air Veſſels of Inſects are yet exceedingly more different in Structure, Diſtribution and Scituation from thoſe of perfect Animals, than thoſe of Fiſhes are, and yet in their Uſe and Action agree perfectly with both, that is, *receiving* and *expelling* the Air, and *varying* the *Dimenſions* and *Capacities* of the Blood Veſſels. Theſe having no *Thorax*, or ſeparate Cavity for the Heart and Air Veſſels, have the latter diſtributed thro' the whole Trunk of their Bodies, by which they communicate with the External Air thro' ſeveral *Spiracula* or *Vent Holes*, to which are faſten'd ſo many little *Tracheæ*, or Wind-pipes, which thence ſend their Branches to all the Muſcles and *Viſcera*, and ſeem to accompany the Blood Veſſels all over the Body, as they do in the Lungs only of Perfect Animals. By this Diſpoſition in every *Inſpiration*, the whole Body of theſe little Animals is inflated, and in every *Exſpiration*, compreſs'd, and conſequently the Blood Veſſels muſt ſuffer a Viciſſitude of Extenſion and Contraction, and a greater Motion muſt thereby be impreſs'd upon the Fluids contain'd in them, than the Heart, which does not in theſe Creatures appear to be Muſcular, ſeems capable of giving. The only Animal that is exempted from this neceſſary Condition of *Breathing*, or receiving and expelling alternately ſome Fluid into and out of the Body, is a *Fætus*. But this, while included in the Womb, has little more than a *vegetative* Life, and ought ſcarce to be reckon'd among the Number of *Animals*. For, were it not for that ſmall Share of *Muſcular* Motion, which it exerciſes in the Womb, it might without Abſurdity be accounted for as a Graft upon, or Branch of the Mother.

In order to account for the Motion of the Blood thro' the Veſſels, it will be neceſſary to obſerve, that the Pulſation of the Heart in a *Fætus* is ſo very weak and obſcure, and the Motion of the Blood ſo extream ſlow and languid, as to be ſcarce, if at all, perceivable, as has been experienc'd in the Diſſection of Puppies before Reſpiration had. To produce ſuch a feeble Palpitation, and creeping Motion, no greater Force ſeems to be requir'd than may be deriv'd from the Communication between the Veſſels of the Mother and *Fœtus* in the *Placenta*. I am not ignorant, that divers Anatomiſts (whom the Crowd have implicitly follow'd) have abſolutely rejected all Communication between theſe Veſſels. But with Submiſſion to great Authorities, I think they have acted arbitrarily, and without ſufficient Warrant from Reaſon or Experiment. For neither are the Arguments which they bring againſt it concluſive, nor the Office which they aſſign to the Umbilical Veſſels in lieu of it, proper, or natural to thoſe Veſſels, or the Reality of the Fact made out by any ſubſtantial Reaſons. Thoſe that reject this Communication uſually do it in favour of one or both of theſe Opinions, that the Arteries of the *Uterus* do depoſite a Nutritive Juice, or a Juice impregnate with Air in the *Placenta*, which is ſuck'd in by the *Umbilical* Vein, and convey'd to the *Fœtus*, for the neceſſary Uſes of Nutrition and Life. Now thoſe that patronize either of theſe Opinions lead Nature an unneceſſary Dance. For if the *Maternal* Blood does really contain any ſuch *Nutritious*, or any ſuch neceſſary Aerial Particles, why ſhould they be ſeparated and extravaſated, to be with Difficulty receiv'd into the Umbilical Vein, and again mixt with the Blood, when they might more eaſily have been imparted by the plain ſimple way of Transfuſion from the Arteries of the Mother to the Veins of the *Fœtus*. And, that this is the Courſe which Nature takes in this Caſe, I am perſuaded from the Eaſineſs and Simplicity of the Method, which readily performs what might be perhaps in vain expected from t'other, and wou'd over and above find 'em, what they ſeem to grope ſo blindly about for, a firſt Mover of the Blood in a *Fœtus*.

Boyle *of the Elaſticity of Air.*
Pechlinus de Aeris & Alimenti defectu.

Thoſe that contend for the Conveyance of a Nutritious Juice thro' the Umbilical Vein from the *Placenta*, are forc'd upon two Difficulties next to Abſurdities. For firſt, they are oblig'd to make this Vein, which, as all other Veins, ſeems dedicated to the Re-conveyance of Blood only, the proper and immediate Channel, through which a very different Liquor is to be carried; and next to give a Power of Attraction or Suction to it; becauſe the Nutritious Juice, which it is thus deſtin'd to carry, is both Viſcous and Stagnant, and has neither Force to drive, nor Subtilty to penetrate, or inſinuate it ſelf into the Capillary Veins; and therefore muſt be drawn or ſuckt as Milk is from the Breaſt, to which the *Placenta* and its Nutritious Juice are by the Favourers of 'em expreſly compared. But if this were the ſole uſe of the *Placenta*, and Umbilical Veſſels, why were the Umbilical Arteries ſent along with the Vein? Their Buſineſs is not to bring any thing back to the *Fœtus*, nor can they contribute any thing to the Benefit of the Mother; for the Uterine Arteries bring all to the *Placenta*, the Um-

Umbilical Vein carries it to the *Fœtus*, and the Uterine Veins convey back again the Surcharge of the Mothers Blood; the Umbilical *Arteries* only have nothing to do, and are ſuperfluous and impertinent, which is contrary to the conſtant Practice of Nature. Yet if *Autopſie* did in the leaſt countenance this Hypotheſis, ſome Defence might ſtill be made; but we find in the Umbilical *Vein* of a *Fœtus* nothing but *Florid* Blood, ſuch as in all probability it received immediately from the *Arteries* of the Mother without any Mixture. And therefore I can't help concluding, that this Opinion engages its Favourers in ſome Abſurdity, without Neceſſity and without Proof.

They that from the *Placenta* ſupply the Body of the *Fœtus* with *Air*, are as much diſtreſs'd as t'other, for they are forc'd to beg the Queſtion twice, which, even when granted, will not anſwer their Ends. Firſt, they ſuppoſe, that an intimate Mixture or Confuſion of *Air* with the Blood, is neceſſary for the Support of animal Life, a *Poſtulatum*, which perhaps the former Part of this Diſcourſe may have render'd unneceſſary; and next that the *Fœtus* is ſupplied with *Air* from, and its *Blood* mixt with it in the *Placenta*. But here again they fetch a Compaſs without Neceſſity or Proof. For if a Mixture of *Air* were neceſſary to a *Fœtus*, why ſhould it be ſeparated from the Mother's Blood, and not rather both communicated together, ſince it is ſo much more eaſie and commodious. But neither does the *Placenta* ſeem to be inſtructed and provided for the Separation of *Air*, but of a much *groſſer Fluid*, deſtin'd to ſome other Uſe, which *Autopſie* confirms. Yet were both theſe Opinions true, they are however defective, and the Circular Motion of the Blood unprovided for.

By the way of *Transfuſion* this great Phænomenon is naturally accounted for, and the Ends, for which the other two Hypotheſes were devis'd, might both be anſwered with more Eaſe. For the *Hyſterick* Arteries tranſmitting their Blood immediately to the Umbilical Vein, may very eaſily tranſmit ſuch Nutritious Juices or Aerial Particles as are contain'd in the Blood, along with it, without depoſiting 'em by the way. By this means ſo much of the Impulſe of the Mother's Blood is preſerv'd, as ſuffices to maintain that languid Circulation, which a *Fœtus* enjoys. For the Blood being driven thro' the Arteries of the *Uterus* into the Umbilical Vein, is convey'd directly to the *Sinus* of the *Porta*, and thence by a ſhort and direct Paſſage through the *Cava* to the Heart; where paſſing through the *Foramen Ovale* to the Left Ventricle, and through the *Canalis Arterioſus* from the Right and Pulmonary Artery, 'tis all deliver'd without coming at the Lungs, to the *Aorta*, and from thence again by the Umbilical Arteries to the Veins of the *Uterus*, making a ſort of *Epicycle* to the main Circulation in the Mother. As this Opinion is favour'd by the Structure and Diſpoſition of the Blood Veſſels on both Parts, ſo there is nothing in it difficult to be conceiv'd, or repugnant to Experience. Late Diſcoveries have made it appear, that the Arteries and Veins are continued

Tubes, and that the latter contain nothing but what they receive from the former, and no Reaſon appears why we ſhould think this Method to be varied in the *Placenta*. On the other Hand, if the Arteries of the *Uterus* were continued to the Veins of the ſame Pair, and thoſe of the *Fœtus* in like manner, without communicating with each other, their Confluence in the *Placenta* ſeems to be altogether impertinent and of no uſe, and the Umbilical Arteries and Vein fram'd for no other Service or Purpoſe, than to give the Blood room for an idle Sally. Mr. *Cowper* has the Honour to re-eſtabliſh this old, but long exploded Truth For by pouring *Mercury* into a Branch of the *Uterine Artery* of a Cow, that went into one of the *Cotyledones* of the *Uterus*, he fill'd thoſe Branches of the Umbilical Veins, which went from that *Cotyledon* to the Navel of the *Fœtus*; which with a Part of the *Uterus* he keeps prepar'd by him.

It would be a weak Objection to alledge that the Obſervation and Experiment being made on the *Uterus* of a Cow, the Inference would not hold from thence to a Woman, the one being *Glanduliferous*, and the other *Placentiferous*; ſince every one of theſe *Cotyledones*, or *Uterine Glandules*, is in all reſpects a little *Placenta*, and all the Difference between 'em is in Number, Name, and Magnitude. Why *Ruminants* differ in this Particular from other *Viviparous* Animals, is beſide the Subject of our preſent Enquiry. But the great Flux of Blood which follows upon drawing the *Placenta* from Women (which is frequently ſo great as to coſt 'em their Lives) is as plain a Demonſtration to Reaſon of the Continuity of the Veſſels, as Mr. *Cowper*'s Experiment is to the Eye.

I have heard it objected by very learned men, that if there were ſuch a *Continuity* of Veſſels, and ſuch *Transfuſion* of Blood, the *Fœtus* muſt neceſſarily periſh through loſs of Blood, upon the ſeparation of the *Placenta* from the *Uterus*, but that on the contrary no viſible flux of Blood does follow while the *Fœtus* continues wrapt in the Membrane, in which condition it may be kept alive ſome hours. To this it may be anſwer'd, that the Circulation in the *Fœtus* being deriv'd from the Mother, may be ſuppos'd wholly to ceaſe upon the cutting off the communication between them, till it is again renew'd more forcibly by *reſpiration*. But if we allow the Motion already impreſs'd upon the Blood to be ſufficient to keep it going a little while, yet it muſt needs be ſo exceeding languid, that the meer reſiſtance of the external Air muſt be more than enough to hinder any Efflux of Blood from a *Fœtus* before Reſpiration. How long Life may be preſerv'd without an *actual* Circulation of the Blood, is a queſtion not of this place. But we have been convinc'd by many and notorious Obſervations and Experiments, that Life has been recover'd a long time after all tokens of Reſpiration, Circulation, or even Life it ſelf have diſappear'd, ſo that we can't think the firſt ſolution either impoſſible or improbable.

I expect to be told, that in the early days of *Geſtation* in *Viviparous*

Animals there is no *Placenta*, or any Adheſion of the *Umbilical* Veſſels to any part of the *Mother*, and conſequently no ſuch *Transfuſion*; and that in *Oviparous* there is no *continuity*, or *communication* of Veſſels of any kind, during the whole time of *Incubation*. But theſe Objections carry neither the Weight nor Difficulty along with them, that they may be ſuppos'd to do; for in thoſe days there is neither *Blood* nor *Blood Veſſels*, and conſequently there can be no *Circulation* of the Blood; and the *Embryo*, of what Species ſoever, is no more than a *Vegetable* at that time; nor does the *Fœtus* of any *Viviparous* Creature enjoy any *Circulation*, or ſhew any ſigns of Animal Life, till after thoſe Veſſels, as well as others requiſite to the Circulation, are compleated.

It muſt be confeſs'd, that *Oviparous* Animals are denied the Benefit of this Communication: But that want is ſufficiently compenſated by a peculiar Mechaniſm, which directly anſwers the ends of *Reſpiration*, and the *Preſſure* of the *Atmoſphere* upon the *Fœtus:* There is at the *Obtuſe* end of an Egg a ſmall Cavity fill'd with Air, which is the Succedaneous Inſtrument to the *Reſpiratory* Organs. For as ſoon as the Contents begin to be warm'd by the *Incubation* of the Hen, or *any analogous* Heat of *Furnace* or *Dunghil*, the ſeveral Humours of the Egg require a *Fermentative* Motion, and the *Air* contain'd in the *Cavity* or *Veſicle* at the *obtuſe* end of the Egg is rarefied, and the Veſicle extended and enlarg'd, and conſequently the other contents are compreſt; to which the *Fermentative* Motion naturally reſiſts. But both Bodies being as well *compreſſible* as *dilatable*, and both having an *Expanſive* Motion impreſt upon them by *Incubation*, the compreſſion and renitency will be mutual; but varied in degree, according as either, through the variation of Circumſtances, ſhall prevail. By this means, an Alteration of Compreſſion and Dilatation will be produc'd in both anſwering the *Reſpiratory* Motion, by which a Motion will be communicated, which, as ſoon as the Organs by which it ſhou'd be regulated are compleated, will in the Body of the *Pullus* be *regular* and *circulatory*.

Fabritius ab *Aquapendente*, and after him our Great Dr. *Harvey*, have aſſign'd divers uſes to this Cavity or Air Veſicle, the Extravagance of which have perhaps deterr'd others from enquiring ſo much into the Uſe, as the importance of it requir'd. But though I can't agree to that *Perſpiration*, *Refrigeration*, and *Reſpiration*, which they make it the Inſtrument of, yet perhaps the *Air*, that was incloſ'd in that *Cavity*, may through the augmentation of the Body of the *Pullus*, and its own *Rarefaction* (which is at laſt ſo great as to occupy half the Shell) break the *Membrane*, which ſeparated it from the *Pullus*, and thereby give ſo much *Reſpiration* as to form the *chirping Voice*, which is often heard before the breaking of the Shell, and with it give an Addition of Strength to enable it to break the Shell. But how it ſhould reſpire ſooner is to me inconceivable.

The incomparable *Harvey* enquires, *Why a* Fœtus, *taken out of th* Uterus *with the* Membranes *entire, ſhall live in Water ſome hours withou* *communication with the* external Air; *where if it be taken out and ſuffer'd once to breath, it can't afterwards ſurvive a moment without the benefit of* Reſpiration. Granting the Fact to be as he has deliver'd it, which yet is not ſo in all caſes, the main difficulty is grounded on a Miſtake, which from the ſtating of the Queſtion I find this Great Man to have ſlipt into. For he thinks, * that a *Fœtus* is ſooner ſuffocated after having once breath'd, than if it had not breath'd at all, and that by breathing it had contracted ſomething which rendered it more periſhable. The Dr. obſerving a *Fœtus* to live longer without Reſpiration, and to diſpence better with the want of Air while included in the Membranes entire, than it could afterwards; infers thence, that the Air does in the firſt Act of Inſpiration impreſs upon the Lungs ſome quality, which renders it ever after more indiſpenſably neceſſary. But allowing his Obſervation, I muſt yet deny his Inference to be good: For deprive a *Fœtus* of means of reſpiring, and then take it out of the Membranes, and it ſhall be as ſoon ſuffocated, as if it had reſpired before. This proves, that this Neceſſity of intercourſe with the Air by way of the Lungs is not the Offſpring, but the Parent of Reſpiration, and that that Learned Man was drawn into a Fallacy of *Non cauſa pro cauſâ*.

** Harv. de Gen. Anim. Cap. de Partu.*

The reaſon of this Neceſſity is the preſſure of the External Air upon the Surface of the Body, from which it was defended by the interpoſition of the Membranes and the Humours contain'd, which are not ſo compreſſible, as the Body of the *Fœtus* itſelf. So ſoon therefore as the *Fœtus* is excluded, and expos'd to the immediate contact of the ambient Atmoſphere, the Veſſels and all the Cavities of the Body muſt neceſſarily be ſo compreſs'd, that the Fluids can't have room for Motion, and conſequently the *Fœtus* could have no Life, if Nature had not contriv'd by the Motion of the *Thorax* to remove and admit that preſſure alternately, and thereby to impreſs a Motion on the Fluids, which is the Spring of Life. But this Motion of the *Thorax* being any way ſuppreſs'd, the equal preſſure of the Atmoſphere on all parts occaſions a total Ceſſaion of Motion, which is Death.

Of the Force of the Heart, by Dr Jurin. *n.* 358. *p.* 863.

XV. Uti non gravate concedimus doctos Viros & olim exſtitiſſe, & hodie reperiri non paucos, qui nullâ inſtructi diſciplinâ Mathematicâ medendi Artem tamen feliciter & cum laude exerceant; ita viciſſim fateri æquum eſt eam doctrinam in Praxi expediendâ non inutilem, ad naturam vero & cauſas Morborum explorandas plane eſſe neceſſariam. Corpora enim Animalium, cum partim ſolidis canalibus partim fluido conſtent per eoſdem jugiter propulſo, Machinas eſſe patet, ac proinde opus eſſe, ad eorum Fabricam, Vires, Actiones, & agendi Impedimenta ſive Morbos rite perſpiciendos, rei Mechanicæ peritiam.

De quibus tamen multa traduntur etiam à Mathematicis Scriptoribus adeo parum accurata, ſecumque invicem & cum ratione pugnantia, ut nobiliſſimæ ſcientiæ non modo commendationem non addant & dignitatem, ſed etiam contemptui & hominum indoctorum ludi-

briis eandem objiciant. Quis enim, non ipſe doctrinâ Mathematicâ imbutus, cum videat, Exempli gratiâ, Cordis Humani vires jam ponderi 3,000 librarum pares, jam 180,000 pondo ſuperantes, jam vero ad uncias 5 vel 8 deductas; Aerem quoque ex Pulmone inter exſpirandum propulſum modo 100, modo 50,000 librarum vi; Quis inquam, qui iſtas concluſiones legerit diſcrimine tam immani à ſe invicem remotas, & tamen omnes demonſtrationibus ſuis munitas, ſi forte ſe à riſu temperet, non tamen inutilem plane & ineptam pronuntiaverit ad explorandas Corporis facultates ſcientiam Mechanicam? Sed meminerint oportet æqui rerum Judices neutiquam mirandum eſſe, ſi quandoque in difficili Problemate vel ſumma Ingenia allucinentur, neque errores, ſiqui forte inciderint, Arti ipſi ſed Artifici imputandos. Quod ut Exemplo manifeſtius declaretur, libet celeberrimi Problematis de Cordis viribus indagandis ſolutionem novam proponere. Utque facilius mihi temeritatis opinionem detraham, qui ejuſmodi inceptum poſt *Alphonſum Borellum* aggredi auſim, utque viam ſimul Lectori expediam ad æquam certamque ſententiam in tantâ ſcriptorum diſſenſione ferendam, primo loco oſtenſurus ſum, quæ in *Borelli* demonſtratione reprehendi debeant, deinde Virorum Doctiſſimorum, *Morlandi*, & *Keilii* ſolutiones, cum eâdem philoſophandi libertate ad examen revocabo.

1. Nobis, quidem, longe præcipuum videtur *Borellianæ* ſolutionis vitium, quod Cordis Potentiam per pondus iners & quieſcens expoſuerit. Cor enim cum & ipſum inter contrahendum movetur, & corpora oppoſita, Sanguinem nempe & Arteriarum tunicas, in motum impellit, patet ejus Potentiam non aliâ ratione ſciri poſſe quanta ſit, quam ut motus hujus quantitatem cognitam teneamus. Motus autem quilibet cum pondere quieſcente comparari non magis poteſt, quam Linea cum Rectangulo.

2. Quod in ipſo Experimento à Circulatore inſtituto, neutiquam conſtet pondus illud ſuſpenſum fuiſſe à ſolâ Muſculorum vi contractrice; quum etiam vis illa, quâ tum Muſculi adhibiti, tum genæ quoque, & ipſa forſitan ligamenta divulſioni ſui ipſorum & fibrarum ruptioni obſtiterint, quâque Muſculi etiam ex cadavere exſecti pondera ſatis magna ſuſtinent, venire in ſubſidium potuerit.

3. Quod vires Muſculorum pondere æqualium a *Borello* pares ſtatuantur: quod profecto dubium admodum videtur, præſertim ubi Muſculi ſunt figurâ diſſimiles.

4. Quod integram Cordis Potentiam, quanta maxima exeri poteſt cum ſummâ fibrarum contentione & molimine, ad ſingulas Syſtoles adhiberi poſuerit. Quum ipſe Circulator, ſi pondus ſuſpenſum vel continenter, vel alternis vicibus breviſſimâ quiete interpoſitâ, ſublevare contenderet, non ita longo tempore plane ſuccubiturus labori fuiſſet.

5. Quod Sanguinis & Arteriarum reſiſtentiam ſexagecuplam ſtatuerit totius Potentiæ Cordis, loco ejus Potentiæ, quæ ad ſyſtolem peragen-

dam à Corde impenditur, quæque forte totius Potentiæ minima pars eſt.

6. Quod in eâ ratione ſexagecuplâ definiendâ errorem inſignem admiſerit. Nam in *Prop.* 60. loco rationis, quam obtinet Summa Potentiarum *P* & *Q* ad Summam *R* & *S*, adhibuit rationem, quæ eſt inter Rectangulum ex Potentiis *P*, *Q* confectum, & Rectangulum ex *R*, *S*. Quod errati ſi per Propoſitiones ſubſequentes corrigatur, habebitur in *Prop.* 73. reſiſtentia longe major, quam ab ipſo *Borello* definita eſt, nempe pondus librarum 1,076,000, loco librarum 180,000, idque ſecundum poſitiones ab ipſo Viro Clariſſimo uſurpatas.

7. Denique quod pondus illud librarum 180,000, quum à Cordis Potentiâ libris 3,000 æquali ſuperetur, miraculi cujuſdam aut monſtri loco Lectoribus obtrudat; & Vim Percuſſionis, quaſi quendam Θεὸν ἀπὸ μηχανῆς in auxilium advocet. Reipsâ enim nihilo plus hic ineſt prodigii, quam ubi pondus 3,000 librarum pondus aliud 180,000 librarum, ad ſubſexagecuplam diſtantiam à centro Libræ inæqualium radiorum appenſum, in æquilibrio ſuſtinet.

Proximus ſequitur Vir Doctiſſimus *Joſephus Morlandus*, qui in Diſquiſitionibus de Cordis vi ſermone Anglicano editis, Methodum peringenioſam expoſuit Potentiam Cordis ad Experimentum revocandi. Hic autem, præter delictum ſupra in *Borello* reprehenſum, quod Cordis vires cum pondere quieſcente contulerit, nobis videtur eo quoque nomine notandus, quod integram Cordis actionem in tunicas Arteriarum diſtendendas impendi poſuerit. Cor enim non ſolum Arterias tendit, ſed Sanguinem quoque certâ velocitate per totum Arteriarum & Venarum tractum propellit.

Supereſt, ut Viri acutiſſimi *Jacobi Keilii* ſolutionem, in Tentaminibus Medico Phyſicis ad Oeconomiam Animalem pertinentibus, non ita pridem cum publico communicatam, expendamus. Qui primus omnium auſus eſt Potentiam Cordis à *Borello* definitam, ac magno Scriptorum conſenſu exceptam & laudatam, non ſolum rejicere, ſed aliam eidem infinito prope diſcrimine minorem numeris diſertis expreſſam ſubſtituere. Hunc autem cenſemus, præterquam quod primum illud *Borellianæ* ſolutionis vitium imitatus ſit, in ſequentibus etiam a vero aberraſſe.

Quod Corollarium *Newtonianum*, quo utitur ad Cordis vires definiendas, aut male intellexerit, aut certe non ſatis apte uſurpaverit. Pondus enim illud ab *Archimede Britannico* determinatum, quo Motus aquæ ex vaſe effluentis generari poteſt, nequaquam generat Motum aquæ; quippe quæ gravitatis vi cadendo ipſa Motum ſuum acquirat. Sed hoc pondus per datum tempus cadendo, Motum concipit Motui aquæ eodem dato tempore effluentis æqualem. Præterea ponit Vir Clariſſimus velocitatem Sanguinis ex Corde effluentis perpetuo æqualem per totam Syſtoles durationem, quam nos inſigniter inæqualem fieri in ſequentibus oſtendemus.

In Methodo illa ſimpliciore, quam poſtea adhibet Vir Doctiſſimus, præter delicta hactenus reprehenſa alia etiam bina admittit.

Adſumit enim Vires Cordis in diverſis Animalibus eam inter ſe rationem obtinere, quæ eſt inter pondera eorundem; quod infra falſum eſſe demonſtrabimus. Tum ponit velocitatem Sanguinis ex ſectâ Iliacâ Arteriâ profluentis, eandem eſſe quâ ex Corde in Aortam emittitur. Atque cum omnis fere ſanguis ex Corde expulſus per Iliacam alteram reſectam emittitur, patet ejus velocitatem tanto eſſe majorem in Iliacâ quam in Aortâ, quanto ſectio Iliacæ circularis à ſectione Aortæ ſuperatur. Præterquam quod velocitas æquabilis, quâ Sanguis per Aortam fluit, longe diſtet ab eâ velocitate, quâcum exit ex ipſo Corde. Similiter fere redargui poteſt & illa Methodus, quâ uſus eſt Vir Cl. ad rationem definiendam inter velocitates diverſas Sanguinis, reſiſtentiâ nunc oppoſitâ, nunc ſublatâ, per Aortam profluentis. Sed cum iſto Experimento non altera ſolum, ſed utraque velocitas major æquo reperiatur, unde ratio, quæ eſt inter ipſas, non magnopere perturbetur, poterit ſatis tuto proportio ab ipſo expoſita, tanquam veræ propinqua, uſurpari.

Curſu hactenus expedito, erit modo nobis ipſis, ſummâ adhibitâ cautione progrediendum. Et primo quidem loco ad ambiguitatem præcidendam neceſſe eſt, ut id, quod quæritur, quale ſit, accuratius paulo declaretur.

Cordis Virium, ſive Potentiæ, nomine ſignificamus vel ipſum Cordis Motum, dum in contractionem agitur, vel Motum ponderis cujuſlibet, quod Sanguini objectum ex Corde proruenti & velocitate idoneâ delatum in partes contrarias, Sanguinis effluxum, adeoque ipſam Cordis contractionem æquali vi librare valet & ſiſtere. Potentiam iſtam, cum à priori vix ſperandum ſit ut definire poſſimus, quod neque fabricam Cordis interiorem, neque cauſæ contrahentis naturam, aut vires ſatis habeamus exploratas, relinquitur, ut eandem per effecta, ſive à poſteriori, æſtimemus.

Cordis actio in Ventriculorum ſuorum contractione omnis conſiſtit. Ventriculi autem inter contrahendum in ſanguinem impingunt, eique Motus ſui partem communicando, eundem magna vi, qua datur porta, urgent, & expellunt. Sanguis hoc modo in Arterias, Aortam & Pulmonalem, protruſus, impetu in omnes partes facto, partim in tunicas Arteriarum ex Syſtole ſua prægreſſâ collapſas & flaccidas, partim in Sanguinem priorem tardius fluentem impingit. Unde gradatim extrorſum truduntur Arteriarum tunicæ, & Sanguis antecedens curſu celeratur. Quod ſi animo concipiantur Arteriæ ſectionibus tranſverſis minimis diſtinctæ, primâ Sanguinis portiunculâ ex Corde in primam ſectionem irruente, partim diſtenditur iſta ſectio, partim Sanguis eâdem antea contentus in ſectionem proximam detruditur, eamque diſtendit, atque iſta actio per ſuccedentes Arteriarum ſectiones continuatur. Deinde ſecunda, & tertia ſanguinis portiuncula, & cæteræ deinceps, in primam Arteriæ ſectionem incidunt, eamque paulo magis dilatant, & ſanguinem eâdem contentum in proximas ſectiones ſucceſſive propellunt; idque fieri pergit, donec omnis ſanguis ex Ventriculis fuerit ejectus. Cæterum id utique obſervandum eſt Arterias, quo magis contractæ & flaccidæ fuerint, eo minus dilatationi obſiſtere, quanto autem magis fuerint dilatatæ, tanto fortius ulteriori diſtractioni reniti; atque idcirco Vim Sanguinis ex Corde prorumpentis primo magis impendi in diſtentionem Arteriarum, quam in Sanguinis præcedentis protruſionem, ſub finem vero magis propelli Sanguinem antecedentem quam diſtendi Arterias, quippe quæ jam rigidæ factæ majorem dilatationem vix admittant.

Sanguis autem ex Corde proſiliens, cum, uti dictum eſt, Motus ſui partem Arteriarum tunicis, partem Sanguini præcedenti communicat, ipſe neceſſario de priſtinâ celeritate remittit; adeoque dum Ventriculorum contractionem moratur, novum ab iis impulſum excipit, ejuſque partem, eâdem ratione atque antea, tunicis Arteriarum & præcedenti Sanguini impendit, unde iterum retardatur, & alium Ventriculorum ictum ſuſcipit, & ſic deinceps, donec omnis ex Ventriculis fuerit expulſus.

Præter cauſam ſupra expoſitam, ſupereſt alia, quâ Sanguis ex Corde effluens gradatim retardatur, adeoque novos ſucceſſive impetus excipit ex Ventriculis ſeſe contrahentibus. Nam Sanguis in Arteriam Aortam influens, etiamſi nulli omnino reſiſtentiæ occurrere ponatur, adeoque nullam pati Motus ſui imminutionem, tamen, cum ex lato in anguſtum fertur, longitudine perpetim creſcit, donec totus in Aortam pervenerit; cumque ſectio Aortæ non minuatur, neceſſario minuitur Sanguinis velocitas. Motus enim Sanguinis eſt in ratione compoſitâ, ex ratione Sectionis Aortæ, velocitate in eâdem, & longitudine Columnæ Sanguineæ, per Theorema noſtrum III. * *de Motu Aquarum fluentium.* Cum vero ea Sanguinis portio, quæ jam pervenerit in Aortam, gradatim retardetur, retardabitur inde Sanguis iſte qui adhuc Ventriculo continetur, & hinc retardabitur ipſius Ventriculi contractio. Unde Ventriculi perpetuò aliam atque aliam Motus ſui partem Sanguini contiguo, his de cauſis perpetim retardato, communicabunt. Patet vero iſthinc, ut id obiter notemus, alium eſſe Motum Sanguinis ex Corde erumpentis, alium ejuſdem jam ex Corde expulſi, & intra Arterias fluentis. Item ictum ſive impulſum Ventriculorum in Sanguinem impreſſum, qui alioqui unicus eſſet futurus, & puncto temporis tranſigeretur, tamen cauſarum dictarum vi, quibus Sanguis perpetim retardatur, per totam Cordis Syſtolen continuari.

* *v Part* I. *Ch.* V. *Tract.* VII. *ſupra.*

Ventriculum itaque alterutrum Cordis Sanguinem impellentem licebit ſpectare, ut datum corpus cum datâ celeritate impingens in aliud corpus quieſcens, cui Motus ſui parte communicatâ ambo corpora communi velocitate deferuntur. Æquatur autem Potentia ejuſdem, vel Facto ex pondere Ventriculi & velocitate ejus initiali, priuſquam in Sanguinem impingat; vel Summæ Motuum ipſius Ventriculi ac Sanguinis ex eodem profluentis, & Motûs qui tunicis Arteriarum & Sanguini præcedenti communicatus eſt; vel etiam, ſi abeſſe ponatur omnis Arteriarum & Sanguinis præcedentis reſiſtentia, Summæ Motuum ipſius Ventriculi & Sanguinis effluentis.

Theorema I. *Motus, quo Machina cava inæqualiter contractilis in contractionem agitur, æqualis eſt ſummæ Factorum ex ſingulis Machinæ particulis ductis in velocitates reſpectivas.*

Patet ex Mechanicâ.

Corol. 1. Machinæ Motus minor eſt Facto ex pondere Machinæ ducto in velocitatem earum Machinæ partium, quæ omnium celerrime moventur inter contrahendum.

2. Motus Machinæ æquatur Facto ex pondere ejuſdem, ducto in velocitatem aliquam mediam inter velocitates earum Machinæ partium quæ omnium celerrime, & earum qnæ omnium tardiſſime, moventur.

3. Si Machinæ plures ſimiles ſimiliter ſeſe contrahant, velocitate mediâ, vel æquabili vel inæquabili, ſimiliter tamen auctâ vel imminutâ in omnibus Machinis; Motus, quo Machina quæque in contractionem agitur, rationem obtinet compoſitam ex ratione quadruplicatâ Diametri homologæ ipſius Machinæ, & ratione inverſâ temporis, quo Machinæ contractio perficitur; vel rationem compoſitam ex ratione ponderis Machinæ, ratione ejuſdem ponderis ſubtriplicatâ, & ratione temporis inverſâ.

n. 359. *p.* 929. *Theorema* II. *Si ex Machinâ cavâ inæqualiter contractili,* A B C D, *aqua per Machinæ contractionem exprimatur, Motus aquæ ex orificio* A *proſilientis æquatur Summæ Factorum ex Sectionibus quibuſvis tranſverſis omnium aquæ filamentorum* A B, A C, A D; *ſingulis ductis in longitudines & velocitates reſpectivas.*

Plate 1. *Fig.* 8. *Demonſtratio.* Loco filamentorum aquæ, concipiatur Machina tubis minimis, inæqualiter amplis, A B, A C, A D, in orificium A deſinentibus, tota conſtare.

Eſt aquæ Motus in quovis tubo æqualis ſectioni cuivis ipſius tubi, ductæ in velocitatem aquæ per ſectionem iſtam fluentis, & longitudinem tubi, per *Theorem.* 3. *De Motu Aquar. fluent.* Proinde Summa Motuum aquæ tubis in omnibus ſimul ſumptis, ſive Motus aquæ ex Machinæ orificio prorumpentis, æqualis eſt Summæ Factorum ex omnium tuborum ſive filamentorum aquæ ſectionibus, ductis in longitudines, & velocitates, reſpectivas. *Q. E. D.*

Corol. 1. Motus aquæ effluentis minor eſt Facto ex orificio A, velocitate aquæ exeuntis, & longitudine filamenti aquæ omnium longiſſimi. Eſt enim Factum ex orificio & velocitate aquæ effluentis æquale Summæ Factorum ex ſectionibus filamentorum ſingulis ductis in velocitates reſpectivas; & Summa horum Factorum, ducta in longitudinem filamenti omnium longiſſimi, major eſt quam Summa eorundem ductorum cujuſque in ſuam longitudinem.

2. Motus aquæ æquatur Facto ex orificio A & velocitate aquæ exeuntis, ducto in longitudinem aliquam mediam inter longitudines filamentorum longiſſimorum & breviſſimorum: vel æquatur Facto ex

quantitate aquæ dato tempore effluentis, & longitudine mediâ prædictâ, applicato ad tempus illud datum.

3. Si Machinæ plures ſimiles aquâ plenæ ſimiliter contrahantur, ſive æquabili velocitate mediâ, ſive inæquabili, ſimiliter tamen in omnibus Machinis auctâ, vel imminutâ; Motus, quo aqua ex Machinæ cujuſque orificio prorumpit, rationem habet compoſitam ex ratione quadruplicatâ Diametri cujuſvis homologæ ipſius Machinæ, & reciprocâ temporis ratione, quo peragitur Machinæ contractio: vel rationem compoſitam, ex ratione ponderis Machinæ, vel molis aquæ, ſive Machinâ contentæ, ſive ex eâdem expulſæ, ratione ejuſdem ponderis, vel molis, ſubtriplicatâ, & ratione temporis reciprocâ.

Problema. *Invenire Potentiam Cordis.*

Sit p = Pondus Ventriculi ſiniſtri, ſive quantitas Sanguinis eidem ponderi æqualis.
S = Superficies interna ejuſdem.
l = Longitudo media filamentorum Sanguinis ex eodem prodeuntium.
s = Sectio Aortæ.
q = Quantitas Sanguinis Ventriculo ſiniſtro contenti.
t = Tempus, quo Sanguis ex Corde expelleretur, ſublatâ Arteriarum & Sanguinis præcedentis reſiſtentiâ.
v = Velocitas variabilis, quâ Sanguis ex Corde proſiliens per Aortam flueret, ſublatâ reſiſtentiâ.
x = Longitudo variabilis Aortæ à Sanguine ex Corde effluente percurſa.
z = Tempus, quo longitudo x percurritur.

Inde velocitas media variabilis Sanguinis Ventriculo contigui, ſive media velocitas ipſius Ventriculi $= \frac{sv}{S}$.

Motus Ventriculi (per *Theor.* 1. *Cor.* 2.) $= p \times \frac{sv}{S}$.

Motus Sanguinis effluentis (per *Theor.* 2. *Cor* 2.) $= \underline{s v \times \overline{l + x}}$.

Horum Summa, ſive Potentia Ventriculi $= s v \times \frac{p}{S} + l + x$. Eſt autem $v = \frac{\dot{x}}{\dot{z}}$. Unde per Methodum *Newtonianam* inverſam, elicitur Potentia Ventriculi $= \frac{sx}{z} \times \overline{\frac{p}{S} + \frac{x}{2} + l}$. Sed cum $z = t$, erit $sx = q$.

Hinc Potentia Ventriculi $= \frac{q}{t} \times \overline{\frac{p}{S} \frac{q}{2s} + l}$.

Simili ratione invenitur Potentia dextri Ventriculi $= {}_{t} \times \overline{\frac{\pi}{\Sigma} + \frac{q}{2\sigma} + \lambda}$.

Literis autem Græcis eadem ſignificantur in dextro Ventriculo, quæ Latinis in ſiniſtro.

Hinc tota Cordis Potentia

$$= \frac{q}{t} \times \frac{p}{S} + \frac{\pi}{\Sigma} + \frac{q}{2S} + \frac{q}{2\sigma} + l + \lambda. \quad Q.\ E.\ I.$$

Si ponatur

$p = 8$ unc. Avoird $= 13.\ 128$ unc. cub.
$\pi = 4 \qquad = 6.\ 564$
$S = 10$ unc. quadrat.
$\Sigma = 10$
$l = 2$ unc.
$\lambda = 1\frac{1}{2}$.
$q = 2$ unc. Avoird. $= 3.\ 282$ unc. cub.
$s = 0.\ 4185$ unc. quadrat. } Ex *Keillianis* Experimentis.
$\sigma = 0.\ 583$ }
$t = 0.\ 1''$

Erit Potentia Ventriculorum æqualis motui ponderum ſubſcriptorum, nempe,

	lib.	unc.
Ventriculi ſiniſtri	9	1
Ventriculi dextri	6	3
Cordis totius	15	4

Quorum ponderum ea eſt velocitas, quâ percurratur longitudo uncialis ſingulis minutis ſecundis.

Cor. 1. Quoties Pulſus fit celerior; aut minuitur reſiſtentia, aut Potentia Cordis augetur, aut minor ſolito Sanguinis copia ſingulis contractionibus ex Corde expellitur.

2. Si Pulſus ſolito tardior fiat; neceſſe eſt, vel augeatur reſiſtentia, vel Cordis Potentia minuatur, vel major Sanguinis moles ex Corde ejiciatur.

3. Auctâ reſiſtentiâ, neceſſario vel Pulſus retardabitur, vel augebitur Cordis Potentia vel Sanguinis quantitas ſolito minor ex Corde exprimetur.

4. Imminutâ reſiſtentiâ, vel Pulſus acceleratur, vel major Sanguinis copia quâque Syſtole ejicitur, vel Cordis vires minuuntur.

5. Auctis Cordis viribus, neceſſario vel augebitur reſiſtentia, vel Pulſus accelerabitur, vel plus Sanguinis ex corde ejicietur.

6. Viribus Cordis imminutis, vel minuatur neceſſe eſt reſiſtentia, vel Pulſus tardior fiat, vel minus Sanguinis ex Corde exprimatur.

7. Cum minor Sanguinis moles ex Corde projicitur; vel accelerantur Pulſus vel Cordis vires minuuntur, vel augetur reſiſtentia.

8. Cum plus Sanguinis ex Corde exprimitur; vel Pulſus tardior fiet, vel augebitur Cordis Potentia, vel reſiſtentia minuetur.

Schol. 1. Ventriculorum ſuperficies internas, cum factu difficillimum videatur, ut accurate determinentur, aut etiam ratio habeatur imminutionis, quam inter contrahendum patiuntur, contenti fuimus præterpropter

propter æſtimare: cum ſive eaſdem 12, ſive 8 unciis quadratis ſingulas æquales ſtatueris, perparva reperiatur Potentiarum facta mutatio. Quod etiam obſervari poterit de longitudine mediâ filamentorum Sanguinis. Præterea differentias, quâ Arteriæ ambæ, earumque rami proximi à Corde progredientes, ſectione augentur, ut æſtimatu perdifficiles & pene inſenſibiles, negligimus. Alioqui eſſet Cordis Potentia tantillo minor ſtatuenda, quam quæ ſupra definita eſt.

2. Determinavit vir celeberrimus, *Jacobus Keillius*, velocitatem Sanguinis, reſiſtentiâ ſubmotâ, ex Corde effluentis, eam circiter, quâ percurrantur pedes, $6\frac{1}{2}$ ſingulis minutis ſecundis. Ponit vero ille celeritatem Sanguinis per totam Syſtolem æquabilem, quam nos inſigniter inæqualem fieri, & perpetim à Syſtoles initio retardari ſupra oſtendimus. Hanc ſi cui definire libuerit, ſubſtituenda eſt, in quartâ Æquatione ſupra poſitâ, Potentia Ventriculi proxime inventa, & ipſi x valor quivis tribuendus, ut eliciatur v, ſive velocitas eidem reſpondens. Ita, cum initio Syſtoles ſit $x = 0$, ſub finem vero $x = \frac{q}{s}$, determinatur inde ea Sanguinis velocitas initio Syſtoles, quâ pedes $14\frac{1}{4}$; in fine autem quâ $4\frac{1}{4}$, minuti ſecundi ſpatio percurrantur. Pariter in dextro Ventriculo, velocitas Sanguinis initialis pedes circiter $10\frac{5}{6}$, ultima vero 3 pedes eodem temporis ſpatio conficiet.

Adhibuimus hactenus eam Hypotheſin, quâ Muſculi Cordis Ventriculos conſtituentes Motum omnem, quo adiguntur in contractionem, Momento temporis concipiunt. Quod ſi ponamus Motum iis communicari non unico quidem Momento, ſed tantillo tamen temporis ſpatio, quod cum totâ Syſtoles duratione comparatum rationem obtineat admodum exiguam; erit Cordis Potentia paululo major ſtatuenda, quam quæ ſupra determinata eſt. Si vero ſtatuatur iſte Motus, procedente Syſtole, in ratione temporis augeri; erit totus Motus in fine Syſtoles acquiſitus duplo major quam ſupra poſuimus, ubi nulla reſiſtentia Sanguini ex Corde profluenti objicitur: Ubi autem ſolita adeſt reſiſtentia, erit idem quintuplo major; quod inſtituto calculo facile patebit. Pari ratione poterit calculus noſter ad aliam quamlibet Hypotheſin, quâ Ventriculorum Motus in duplicatâ vel ſuperiore quâvis ratione temporis augeatur, accommodari. Potentia vero in fine acquiſita ſuprapoſitâ elicietur longe major, nempe ex ratione duplicatâ Potentia tripla, ex triplicatâ quadrupla, ex quadruplicatâ quintupla, & ſic in infinitum.

Nobis autem videtur ſecunda Hypotheſis, quâ Ventriculi parvo admodum temporis ſpatio Motum omnem concipiunt, cæteris longe veriſimilior. Quum neceſſe ſit, ut aliquid temporis impendatur ad Motum quemlibet generandum; neque videatur adeo tarde increſcere Ventriculorum Motus, ut non celerius augeatur, quam ſecundum temporis rationem. Motus enim Muſculorum impetu ſolo Fluidorum quorumcumque, quæ ex Sanguine proveniunt, perfici nequit; quum Brachio alterutro Motum exercere poſſimus Motu Sanguinis per vaſa Corporis univerſa profluentis longe majorem. Relinquitur ergo, ut Muſculorum fibræ Ventriculos Cordis conſtituentium, rareſcentiâ quâdam liquorum in eaſdem influentium, in Motum impellantur. Hæc autem, quoties vim magnam concipit, plerumque ſubita eſt, & fere inſtantanea. Adde quod Ventriculorum Motus ſecundum hanc Hypotheſin longe minor efficitur, quam in tertiâ. Non ſolet autem ſapientiſſimus Artifex, Rerum Conditor, in operibus ſuis plus virium adhibere, quam quantum ſufficit ad finem propoſitum conſequendum.

Cæterum ſive admittatur iſta Hypotheſis, ſive alia quæcunque ex ſupra dictis verior cenſeatur, poterunt omnia Corollaria noſtra eodem jure ex Problemate deduci. Quæ utrum aliquid adjumenti afferant ad Morborum Hiſtoriam explicandam Medico ſagaci conſiderandum permittimus. Facile autem ex morbi cujuſque Naturâ ſciri poterit, utrum aucta ſit vel imminuta reſiſtentia. Augeri vero credibile eſt vel imminui Cordis vires auctis vel imminutis Muſculorum reliquorum viribus, quamvis aliter ſtatuiſſe video virum celeberrimum, *Laurentium Bellinum.*

Theorema III. *Totus Motus reſiſtentiæ, quæ Sanguini ex Corde erumpenti durante Syſtole objicitur, ſive totus Motus, qui Sanguini præcedenti & Arteriarum tunicis communicatur, toti Cordis Potentiæ quamproxime æqualis eſt.*

Dem. Peractâ Cordis Syſtole, quæ pars Aortæ & Arteriæ Pulmonalis Cordi proxima eſt, perſtat plena Sanguine per totam Syſtolem Arteriarum. Nec enim patitur earum fabrica & nexus, quo Cordi conjunctæ ſunt, ut tunicis in ſeſe penitus collabentibus totæ occludantur, neque poteſt earum cavum Sanguine vacare. Alioqui enim, contrahentibus ſeſe reliquis Arteriarum partibus, Sanguis iiſdem contentus retro in vacuum impelleretur, motu & inutili & motui Sanguinis naturali contrario. Tum etiam Valvulæ Semilunares non tenderentur verſus Ventriculos, adeoque Sanguis ex Auriculis in Ventriculos expreſſus, etiam in Diaſtole Cordis, in Arterias protruderetur.

Hinc patet Sanguinem proxime ex Corde expulſum Syſtole peractâ immotum in Arteriis perſiſtere, adeoque tum omnem Ventriculorum Motum excepiſſe, tum eundem totum partim Sanguini antecedenti, partim tunicis Arteriarum communicaſſe. *Q. E. D.*

Theorema IV. *Motus, qui in Syſtole Cordis communicatur Sanguini præcedenti, eſt ad Motum tunicis Arteriarum communicatum, ut tempus Syſtoles Cordis ad tempus Diaſtoles quam proxime.*

Dem. Quum ſanguis per vaſa Corporis univerſa, ſi partes Arteriarum Cordi propiores exceperis, æquabili curſu deferatur; neceſſe eſt, ut tum Motus affrictu Sanguinis ad vaſorum latera deperditus, tum Motus Sanguini redditus à Syſtole ſive Cordis ſive Arteriarum, æqualibus temporibus æqualis ſit. Qui autem Motus à Syſtole Arteriarum Sanguini communicatur, idem eſt præciſe, qui prius à Cordis Syſtole Arteriarum tunicis fuerat impreſſus, cum Arteriæ eodem impetu quo diſtractæ fuerint etiam reſtituantur. Et Syſtole Arteriarum cum Cordis Diaſtole duratione convenit. Unde patet Propoſitum. *Q. E. D.*

Cor. Si ponamus cum viro doctiſſimo *Jacobo Keillio*, Syſtolen Cordis peragi tertiâ parte temporis inter Pulſus binos intercepti; erit Motus Sanguini præcedenti communicatus totius Potentiæ Cordis pars tertia: Motus vero Arteriis communicatus prioris duplus, ſive duæ partes tertiæ totius Cordis Potentiæ.

Theorema V. *In diverſis Animalibus Potentia Cordis rationem obtinet compoſitam ex ratione quadruplicatâ Diametri cujuſvis homologæ Animalis, & ratione inverſâ temporis, quo Cor contrahitur: vel rationem compoſitam, ex ratione ponderis vel ipſius Cordis vel integri Animalis, ratione ponderis ejuſdem ſubtriplicatâ, & ratione temporis reciprocâ.*

Facile demonſtratur vel ex *Corol.* 3. *Theor.* 1 & 2. vel ex Potentiâ Cordis Problemate præcedente definitâ.

Cor. 1. Si ponatur Cordis Potentiam rationem obtinere ponderis vel ipſius Cordis, vel integri Animalis, vel Sanguinis copiæ in toto Animali; erit Animalis longitudo in ratione temporis, quo Cordis Syſtole perficitur, ſive in ratione inverſâ frequentiæ Pulſuum.

2. Si ratio longitudinis integri Animalis major fuerit ratione inverſâ frequentiæ Pulſuum, neceſſe eſt major ſit ratio Potentiæ Cordis ratione ponderis ejuſdem.

Schol. Quum conſtet Experimentis Puerorum Pulſus non eſſe tanto frequentiores Pulſibus virorum quanto pueri virorum longitudine ſuperantur, concludendum eſt, vi ſecundi Corollarii, Potentiam Cordis Virilis majorem obtinere rationem ad Potentiam Cordis Pueri, quam eſt ratio ponderum. Et par eſt ratio in cæteris Muſculis. Nam ſi Corporis robur rationem ponderis ſequeretur, poſſent Pueri æqualia itinerum ſpatia eodem tempore cum viris conficere.

Simili ratione ac Motum ſanguinis ex Ventriculis Cordis erumpentis ope *Theorem.* 2. determinavimus, poterit quoque Urinæ Motus ex Urethrâ profluentis determinari. Nempe ſi ponatur Urethræ Veſicæ longitudo 12 unciis æqualis, & binæ unciæ Urinæ minuti ſecundi ſpatio emittantur, erit Motus Urinæ effluentis æqualis Motui ponderi libræ 1 $\frac{1}{2}$, quod uncialem longitudinem ſingulis minutis ſecundis percurrat. Quoniam vero Urina non ſolis Veſicæ Urinariæ viribus contractivis, ſed etiam Diaphragmatis & Muſculorum abdominalium ope in ſubſidium vocatâ, expellitur, nequit Veſicæ Potentia ex Motu Urinæ profluentis æſtimari.

Remarks by Dr. James Keill, *n.* 361. *p.* 995.

2. Epiſtolam *D. Jurin*, in Actis Philoſophicis nuper publicatam, legi; in quâ, ea quæ à me traduntur de viribus Cordis, infirmare conatur vir Doctiſſimus. Cum æſtimatio virium, quibus Cor Sanguinem expellit, à *Borellio* facta, fere omnibus valde incredibilis videbatur, non me temerarium, non *Borelli* nomini injurioſum, non orbi literato ingratum facturum exiſtimavi, ſi ad verum propius accedere tentarem. In quo Tentamine, non accuratam virium Cordis definitionem mihi propoſitum erat dare, ſed potius methodum, quâ hæ vires forte inveniri poſſent, indicare; & Geometriæ peritiores ad Problematis valde deſiderati inveſtigationem incitare.

Præcipuum quod *Borellio*, D. *Moreland*, mihi objicit vitium eſt, quod in Potentiâ Cordis æſtimandâ, quam rationem ad pondus iners, vel corporis gravitatem obtineat, determinare ſuſcepimus. “ Sed Cor, inquit, cum & ipſum inter contrahendum movetur, & corpora oppoſita, Sanguinem nempe & arteriarum tunicas, in motum impellit, patet ejus *Potentiam* non aliâ ratione ſciri poſſe quanta ſit, quam ut motus hujus quantitatem cognitam teneamus. *Motus* autem quilibet cum pondere quieſcente comparari non magis poteſt, quàm linea cum rectangulo.’ At à nemine certe noſtrum, quod ſcio, eſt *Motus* Cordis cum pondere quieſcente comparatus. Potentiam autem Cordis, ſeu vim Cordis motricem & Sanguinem impellentem cum pondere conferre, quid prohibet non video. Quamquam enim inter *Pondus* & *Motum* corporis ſolidi nulla ſit relatio, vis tamen motrix, ſi in fluidum agit, ad vim gravitatis quandam certe rationem habet. Et revera vis corporis motrix, certam in fluido motûs quantitatem in dato tempore efficiens, æqualis eſt ponderi, quod vi gravitatis cadendo, in eodem tempore, eandem motûs quantitatem ſibi acquirit. Hinc vis, quâ ex orificio aliquo aqua exprimitur, certo ponderi æqualis eſſe dicitur; quia pondus datum, & vis aquam exprimens æquales motus in temporibus æqualibus generant. Hic genuinus Corollarii *Newtoniani* ſenſus mihi videtur eſſe, nec ab hoc ſenſu diſcrepant, quæ de Cordis viribus explicui. Verba *Newtoni* ſunt, *Vis, qua totus aquæ exilientis motus generari poteſt, æqualis eſt ponderi, &c.* quæ non ſatis attendiſſe videtur *Jurinius*, cum dicit *Pondus autem illud quo motus aquæ ex vaſe effluentis generari poteſt*, &c. Sed ſi hâc re à nobis peccatum eſt, cum ſummis certe hujus ſæculi Geometris *Hugenio* & *Newtono* peccavimus, quorum uterque vim fluidorum per vim gravitatis exponit. Nec in *Corol.* prædicto id ſolummodo facit *Newtonus*, ſed in aliis etiam locis oſtendit Methodum, quâ ratio reſiſtentiæ Medii, id eſt, actionis fluidi in corpus ſolidum, ad vim Gravitatis vel centripetam inveniri poteſt, ut videri licet in *Prop. 4ta* & *5ta* Libri ſecundi, eorumque Corollariis. Alia profecto eſt actio fluidorum in ſolidum, & alia ſolidorum in ſe invicem. Fluidum, datâ velocitate motum, datum pondus ſuſtinere poteſt, cum fluidi partes ſibi mutuo continuò ſuccedentes in pondus impingunt, adeoque vis fluidi eſt revera ponderi æqualis; ſed cum Solidorum non par eſt ratio, eorum Vis cum Gravitate comparari nequit.

Me inſuper reprehendit Vir ingenioſiſſimus, quòd velocitatem Sanguinis è corde detruſi, per totam ſyſtolem æqualem poſui, quam ille valde inæqualem eſſe demonſtravit. Verum à me nuſquam Sanguini æqualis data eſt velocitas, ſed pro ſummâ omnium velocitatum mediam poſui. Sed utrum æqualis vel inæqualis eſt Sanguinis è Corde ejecti celeritas, nondum ſatis mihi conſtat; certe quæ pro æquali velocitate ſtat ratio, ea mihi in præſens firmior videtur.

Quod in alterâ methodo quæ ſubjuncta eſt, illi diſplicet, eſt certe aſſumptio illa quæ à *Borellio*, aliiſque viris doctis ſæpius uſurpata eſt, nempe, quod ſimilium muſculorum vires ſunt in ratione ponderum.

Aliam virium rationem in Theoremate *5to* ſtabilire conatur *Jurinius*: ſed cum ex communi omnium Theorematum Principio oritur demonſtratio, communi etiam eorum fato involvetur: Si enim principium illud fallax eſt, (ut mihi videtur) nec ad caſus ad quos adhibetur, congruit; corruunt certe omnia, quæ hâc baſi innituntur. Supponit Vir Clar. Vaſorum tunicas in Sanguinem intus contentum impetu irruere, & motûs ſui partem Sanguini ictu communicare: & hîc in motu Cordis, vult Ventriculum tanquam ſolidum Corpus, datâ velocitate motum, in Sanguinem impingere, & ictu motûs ſui partem illi impertire: quæ ſuppoſitio nec Sanguinis nec Cordis, nec Aeris è Pulmone expreſſi motui competit, nec, ullâ minimorum ictuum reiteratione, horum motibus ita accommodari poteſt, quin quæ inde deducentur concluſiones pro incertis & omnino falſis haberi debeant.

Cum inter Sanguinem & Cordis intimum nullum intercedit ſpatium, ſed eſt alter alteri contiguus, non ictu hoc in illum, ſed preſſu agit: nec ullam in initio ſuæ contractionis celeritatem ventriculi habent, ſed ſe contrahendo velocitatem tempore acquirunt, tanquam gravia cadendo, vel ut fluida rareſcendo, ex quo forte omnis vis Cordis oritur. Adeoque non æquabilis eſt motus contractionis, ut vult Vir Doctiſſimus, ſed eſt motus inſtar cadentis acceleratus. Idem igitur eſt diſcrimen inter ictum quo Cor Sanguinem ferire vult *Jurinius*, & preſſuram quâ Cor revera in Sanguinem agit, quod eſt inter actionem corporis ſolidi moti & vim gravitatis: ſed ipſo fatente hæc comparari nequeunt, adeoque preſſura ſeu actio cordis in Sanguinem per ictum nec à Viro laudato expoſita eſt, nec unquam exponi poteſt. Hanc ſententiam confirmat ipſa Cordis potentia a Viro Cl. inventa. Si enim pondus datâ velocitate motum cordis potentiæ æquale eſſet, tunc Sanguis omni vi Cordis in pondus illud directe impulſus motum ponderis temporis momento deſtrueret: ſed quocunque magno impetu ponderi occurrat Sanguis, nunquam illi omnem motum in inſtanti eripiet, adeoque eſt hoc pondere potentia Cordis minor, nec recte per motum ponderis vires Cordis exponuntur.

Fluidorum vires in corpora ſolida, ubique eodem prorſus modo quo ſolidorum vires in ſe invicem, *Jurinius* æſtimat & perpendit, cum tamen maxima interſit differentia; & ab hoc capite fluit quicquid eſt in illius Propoſitionibus erroris. Ubi enim corpus ſolidum, cujus partes firmiter inter ſe cohærent, in aliud impingit, unaquæque corporis particula ſimul & ſemel ſuam alteri vim impertit: at res aliter ſe habet in fluidis, in queis nulla eſt partium cohærentia, nulla fluidi pars, niſi in ipſo tactu, in corpus ſibi oppoſitum agit: idcirco cum columna aquæ adverſus corpus ſolidum ſurſum vertitur, partes columnæ à corpore remotiores nullam illi vim imprimunt. Corpus etiam ſolidum unicum ſolummodo ictum alteri communicat: at columna fluidi in corpus ſibi oppoſitum continue agit, & minima columnæ pars minimo temporis momento, ictum infinite parvum illi imprimit, eodem prorſus modo quo gravia cadendo agunt, quibus igitur fluidorum motus recte comparatur. Porro omnis motus corporis ſolidi in alterum directe impingentis in temporis momento deſtrui poteſt: ſed motus ſolidi vim fluido imprimentis, non niſi gradatim imminuitur, & in dato tempore evaneſcit, pari ratione, quâ Gravitas in corpus ſurſum miſſum vim ſuam exerit. Ex quibus ſatis abunde conſtat, inter vim fluidi in motum acti, & vim gravitatis magnam eſſe affinitatem, & unam per alteram recte exponi poſſe; vim autem corporis ſolidi ad vim gravitatis referri non poſſe. Cumque hanc differentiam non ſatis attendiſſe videtur Doctiſſimus *Jurinius*, à vero multum aberraſſe mihi videtur. Si igitur ſepoſitâ ſuâ de Vaſorum ictu hypotheſi, & vi preſſuræ, quâ Natura utitur, pro Principio adhibitâ, alia Theoremata de Cordis & Sanguinis motu & viribus, elegante ſuâ demonſtrationis methodo, conſtruere dignabitur, ſeſe dignum, mihi certe gratum, nec eruditis inutile præſtiterit.

Reply'd to, by Dr. Jurin. *n.* 362. *p.* 1039.

3. Queritur primo Vir Clariſſimus, quod ſeſe una cum Doctiſſimis Viris *Borello*, & *Morlando*, tanquam Cordis Motum cum pondere inerti conferentem, injuſte perſtrinxerim. Ego certe, cum prius notaſſem Motum quendam Sanguinis & Arteriarum ex Cordis Vi oriri, dixi tandem ſciri non poſſe Cordis Potentiam quanta ſit, niſi Motus hujuſce quantitatem cognitam teneamus: Motum vero quemlibet cum pondere quieſcente comparari non magis poſſe, quam Lineam cum Rectangulo. Quibus verbis id ſignificare volui, Doctiſſimos Viros non quidem diſerte Motum Cordis cum pondere quieſcente comparare, ſed ipſos, cum Cordis Potentiam per pondus exponerent, nullam oſtendiſſe rationem, quâ Motûs quantitas ex Cordis Potentiâ oriundi poſſet æſtimari. Ex hâc Objectione, ſi recte aſſequor mentem Viri Clariſſimi, ita ſeſe expedire conatur. Cordis Potentia in preſſione conſiſtit, eamque æquabiliter in Sanguinem impendit, eodem prorſus modo, quo Gravitatis vis deorſum pondus impellit, & actione perpetuâ in motum accelerat. Proinde, cum Cordis Potentia ponderi per Corollarium *Newtonianum* definito æqualis eſt, ea Motum eundem durante Syſtole in Sanguinem imprimet, quem pondus iſtud eodem tempore cadendo per Gravitatis vim comparabit. Ita vero cum mentem ſuam exponit Vir Cl. ſublatam iri penitus Objectionem iſtam noſtram confitemur; ſi nimirum Cordis Potentia prædicto ponderi æqualis ſit, eademq; conſiſtat in æquabili preſſione per totam Syſtolen continuatâ. Atqui ex duabus iſtis Propoſitionibus poſteriorem neutiquam probare conatur vir doctiſſimus, ſed Hypotheſeos loco ponit; quamvis nos rationibus quibuſdam adductis contrariam Sententiam conati ſumus veriſimiliorem reddere; nempe, quod Cordis Potentia nequaquam æquabiliter agat in Sanguinem per totam Syſtolen, ſed cum totas vires exiguâ temporis particulâ collegerit, inde uno impetu in Sanguinem irruat, eumque ex Ventriculis expellat, eo modo quem in Diſſertatione noſtrâ Epiſtolari fuſius expoſuimus. Priorem vero Propoſitionem, etiam conceſſâ Viro Cl. iſtâ Hypotheſi, falſam eſſe mox demonſtrabimus.

Corollarii *Newtoniani* ſenſum quod attinet, nolumus Lectori moleſtiam nimiam faceſſere, cum neque putemus ejus intereſſe uter *Newtoni* mentem rectius acceperit; neque ita perſpicue ſententiam ſuam expoſuerit Doctiſſimus Adverſarius, quin periculum ſit, ne aliquem ei ſenſum affingamus, quem ipſe forſitan, ſi poſſet adhuc ſe defendere, foret repudiaturus. Id vero adnotaſſe operæ pretium erit, quod cum loquatur *Keillius* de Vi quâ ex Orificio aliquo aqua exprimitur, *Newtonus* nullum omnino verbum in iſto Corollario poſuerit, quo Aqua per vim aliquam exprimi ſignificetur; ſed pondus ſolum determinaverit æquale iſti vi, quâ totus aquæ effluentis Motus generari poteſt, ſive quod Gravitatis Vi cadendo Motum comparare poteſt Motui aquæ eodem tempore effluentis æqualem.

Quod autem Corollarium illud, ſi non male intellexerit Vir Cl. certe non ſatis apte uſurparit, facile perſpiciet Lector Eruditus, qui animum adverterit, quid interſit diſcriminis inter effluxum aquæ ex foramine in fundo vaſis ſemper pleni quomodo à *Newtono* conſideratur in eo Corollario, & effluxum Sanguinis ex Corde in Aortam. In caſu enim priori aqua jam totam velocitatem comparavit, & per datum temporis ſpatium æquabiliter effluit ex foramine. At Cordis Vis per Hypotheſin *Keillianam* applicatur Sanguini in Ventriculo quieſcenti, & eum primo temporis momento velocitate infinite parvâ verſus Aortam propellit; continuatâ vero æquabili preſſione tandem ei finitam velocitatem imprimit, eamque perpetim auget, donec omnem Sanguinem ex Ventriculo expulerit.

Rurſum in caſu *Newtoniano* conſideratur Motus non quidem totius aquæ Cataractâ contentæ, quæ omnis in motu conſtituta eſt, & diverſa velocitate verſus exitum tendit, ſed aquæ ſolum in ipſo foramine poſitæ & jam exilientis. Vis autem Cordis toti Sanguinis moli Ventriculo contentæ Motum imprimit, totamque Aortam verſus propellit.

Denique negamus pondus quinque unciarum, à Viro Cl. determinatum, poſſe eam Motus quantitatem durante Cordis Syſtole per Gravitatis Vim comparare, quam Cordis Potentia producit, conceſſâ etiam ei Hypotheſi iſtâ, quod Cordis Potentia in æquabili preſſione conſiſtat. Per hanc enim Hypotheſin erit Motus à Ventriculi ſiniſtri Potentiâ productus, ex Calculo noſtro * [*Tranſ.* Numb. 359. p. 932, 934.] æqualis Motui Ponderis Octodecim librarum circiter, quod ſingulis minutis ſecundis longitudinem uncialem percurrat. Motus autem, quem pondus quinque unciarum durante Cordis Syſtole, ſi tollatur omnis Arteriarum & Sanguinis præcedentis reſiſtentia, ſive decimâ parte minuti ſecundi, per Gravitatis vim comparabit, æquabitur fere Motui Ponderis duodecim librarum, quod ſuprapoſitâ velocitate moveatur. Quod ſi cui libuerit adſumptâ hâc Hypotheſi verum pondus definire, quod Cordis Potentiæ æquale eſt, is poſito Calculo eliciet pondus unciarum circiter ſeptem cum ſemiſſe. Hoc enim durante Syſtole Cordis eundem fere

* *v. ſupr. p.* 61, 62.

fere Motum cadendo comparabit, quem producit ipſa Cordis Potentia.

Sed inquiet forſitan aliquis diſcrimen modo expoſitum inter Motum à *Keilliano* pondere acquiſitum, & Motum ex Potentia Cordis oriundum inde proficiſci potuiſſe, quod forte minus accuratæ fuerint poſitiones illæ, quibus Characteres Algebraicos in Calculo noſtro ad numeros revocavimus. Cui dubio ut occurramus, & oſtendamus ſimul nos longe majus diſcrimen inventuros fuiſſe, niſi contigiſſet ut poſitiones iſtæ *Keillio* faverent; operæ pretium erit caſum aliquem ſimpliciorem adſumere, quo data moles aquæ, per datum orificium, dato tempore, per vim aliquam ſive preſſionem æquabilem exprimatur, quæ ſunt conditiones ab Adverſario poſitæ ad Potentiam Cordis definiendam. In eo autem caſu demonſtrabimus neque Motum aquæ effluentis, neque Motum toti tandem moli aquæ per vim illam impreſſum, Motui aquæ in Corollario *Newtoniano*, neque Vim eam ſive preſſionem, ponderi per iſtud Corollarium definito, æquari. Quod ſi præſtare licuerit, corruat funditus neceſſe eſt tota demonſtratio *Keilliana*.

Adſumemus igitur Cylindrum aquæ datum, tubo Cylindrico infinitæ longitudinis contentum; eritque pro orificio iſta ſectio tubi ad quam pertingit utralibet aquæ ſuperficies, alteri autem ſuperficiei Vis applicabitur ope Emboli eâdem Diametro cum ipſo tubo. Perfluat jam dato tempore data quævis aquæ quantitas per dictam ſectionem tubi; tum alia quantitas æqualis per foramen pari Diametro factum in fundo vaſis, quod more *Newtoniano* uſque plenum conſervatur: & primo loco diſpiciamus, utrum pares futuri ſint in utroque caſu Motus aquæ effluentis.

Exponatur tempus effluxus aquæ per rectam *AC*, velocitas autem æquabilis, qua aqua effluit ex foramine in fundo vaſis per rectam *AB*. Unde moles aquæ effluentis ex foramine, cum ſit in ratione temporis & velocitatis conjunctim, exponetur per Rectangulum *ABCD*; & Motus ejuſdem exponetur per ſolidum Parallelepipedon, ex eodem Rectangulo ducto in altitudinem *AB*, quippe qui ſit in ratione compoſita ex rationibus molis & velocitatis.

In caſu altero, ubi aqua per tubum Cylindricum fluit, tempus, ut prius, exponetur per eandem rectam *AC*; velocitas autem aquæ erit in ratione temporis, quippe cum vis adhibita, ex Hypotheſi, in datam aquæ molem æquabiliter agat, & proinde repræſentabitur per rectam mutabilem *FG*, rectæ *AF*, ſive tempori ab initio effluxus, proportionalem. Molecula autem aquæ, particulâ temporis *FH* prædictam Sectionem præterfluens, exponetur per Rectangulum ex ipſâ *FH* ductâ in exponentem velocitatis *FG*; vel ſi evaneſcere intelligatur rectula *FH* per Trapezium *FGIH*, & moles aquæ toto tempore *AC* præterfluens ſignificabitur per Triangulum Rectangulum *ACE*. Et quoniam ex Hypotheſi moles iſta moli aquæ in caſu priore effluenti æqualis eſt, erit Triangulum *ACE* æquale Rectangulo *ABDC*; unde *CE*, ſive velocitas acquiſita in fine temporis *AC*, dupla erit velocitatis *CD* ſive *AB*, quâ aqua ex foramine

Plate 1. *Fig*. 9.

foramine in fundo vaſis effluebat. Motus autem aquæ particulâ temporis *F H* præterlabentis, cum ſit in ratione molis & velocitatis conjunctim, exponetur per Priſma evaneſcens, quod fit ex Trapezio *FGIH* ducto in velocitatem *FG*: Unde totus Motus aquæ toto tempore *A C* præterfluentis exponetur per Pyramidem, cujus baſis eſt Quadratum rectæ *C E*, cujuſque altitudo perpendicularis eſt ipſa *A C*. Quæ Pyramis cum ſit ad Parallelepipedon caſu priore definitum, ut 4 ad 3, erunt quoque Motus aquæ effluentis in utroque caſu in eâdem ratione, & proinde inæquales, quod primo loco demonſtrandum ſuſceperamus.

Proximum eſt, ut oſtendamus Motum tandem impreſſum toti aquæ tubo contentæ non eſſe æqualem Motui in exemplo primo determinato. Hic autem, cum tota iſta moles aquæ per poſitiones ſupra ſcriptas neutiquam definita ſit, adſumemus eam æqualem moli expoſitæ per Rectangulum *A B C D*, quæ in caſu primo effluit ex foramine, quæque in ſecundo ſectionem dictam præterfluit. Unde cum totus Motus ei tandem impreſſus ſit, in ratione molis & velocitatis in fine acquiſitæ, idem exponetur per Parallelepipedon ex Rectangulo *A B D C* ducto in rectam *C E*. Hoc autem eſt ad Parallelepipedon, primo caſu definitum, ex eodem Rectangulo & rectâ *C D*, ut altitudo *C E* ad altitudinem *C D*, ſive in ratione duplâ. Porro, cum molem aquæ tubo contentæ per quodvis aliud Rectangulum, loco Rectanguli *A B C D*, exponere licuiſſet, patet inde Motum hunc poſſe quamlibet rationem ad Motum primo caſu definitum obtinere & idcirco nequaquam eidem eſſe æqualem. Quod erat ſecundo loco demonſtrandum.

Supereſt ut oſtendamus Vim in hoc caſu adhibitam ponderi per Corollarium *Newtonianum* definito non eſſe æqualem. Hæc autem Vis & vis Gravitatis agens in iſtud pondus, cum ambæ ſint æquabiles, erunt in ratione Motuum ex iiſdem dato tempore productorum. Quos cum inæquales eſſe modo demonſtratum ſit, erint illæ vires itidem inæquales. Quod erat demonſtrandum poſtremo.

Pergit Vir Cl. ad alterum illud vitium, quod ego in ejus ſolutione reprehenderam, nempe quod velocitatem Sanguinis ex Corde effluentis æquabilem poſuerit, quam inſigniter inæqualem fieri à me demonſtratum eſt. Negat autem ſe æquabilem velocitatem Sanguini tribuiſſe, ſed pro ſummâ diverſarum omnium velocitatum velocitatem mediam uſurpaſſe. Præterea nondum ſatis ſibi conſtare dicit, utrum æqualis vel inæqualis ſit Sanguinis ejecti velocitas, ſed quæ pro æquali velocitate ſtat ratio, eam ſibi firmiorem videri. Utrum vero, qui velocitatem Sanguinis inventurus molem Sanguinis expulſi ad orificium Aortæ applicat, nullâ factâ mentione neque diverſarum velocitatum, neque velocitatis mediæ, velocitatem Sanguinis æquabilem ponat, penes æquum Lectorem ſit Judicium. Idem quoque facile æſtimabit, utrum Vis aliqua ſive preſſio fluido in vaſe quieſcenti applicata, quæ eſt Hypotheſis Viri Doctiſſimi, id fluidum primo temporis momento, eâdem velocitate quâ in fine, propulſura ſit.

Poſtquam ita ſatisfactum putat Vir Cl. iis Objectionibus, quas contra priorem ſuam Methodum attuleram, jam ad alteram illam faciliorem vindicandam accedit. In hac Ego animadverteram Virum Cl. adſumere iſtam Propoſitionem, quod Vires Cordis in diverſis Animalibus ſint in ratione ponderum, item ponere velocitatem Sanguinis ex ſectâ Iliacâ Arteriâ profluentis æqualem ei, quâ Sanguis ex Corde in Aortam emittitur; quas ambas poſitiones falſas eſſe nobis demonſtratum eſt. Vitium poſterius non defendit Vir Cl. prius vero tuetur *Borelli* & aliorum Doctorum Virorum auctoritate, qui aſſumptionem iſtam ſæpius uſurparunt. Ita quidem, & nos ejuſmodi aſſumptionem in *Borello* reprehendimus, neque valet cujuſquam auctoritas contra legitimam demonſtrationem. Supereſt ergo Viro Cl. ad examen revocanda noſtra demonſtratio. Hanc autem fallaci quodam Principio inniti putat; quo cum omnia Theoremata noſtra ſuperſtructa ſint, communi ruinâ omnia involvit. Ait enim me ponere, quod Ventriculi Cordis, tanquam ſolidum corpus datâ velocitate motum, in Sanguinem impingunt, eoque ictu Motus ſui partem eidem communicant. Quam Hypotheſin Motui neque Sanguinis, neque Cordis, neque Aeris ex Pulmone expreſſi, competere cenſet Vir Clariſſimus.

Quod Pulmonem attinet, quoniam hoc obiter attingere voluit Vir D. agnoſco me conſideraſſe Pulmonem inter contrahendum tanquam datâ velocitate impingentem in Aerem contentum, idque conſulto feciſſe profiteor. Quum enim tum *Bellinus*, tum alii multi Viri Doctiſſimi, quos inter eminet Cl. Adverſarius, multa protulerint de Vi illâ, quâ Aer inter expirandum in Sanguinem Pulmones præterfluentem agit, ejuſque moleculas diſſolvit; quam ſolutionem ipſo expirationis initio cenſent accidere; mihi propoſitum erat hanc ipſorum ſententiam ad trutinam revocare. Videbam autem, quod, ſi aerem per Vim æquabilem ſive preſſionem expelli ſtatuerem, Motus aeri à Pulmone impreſſus initio exſpirandi, ſive reactio aeris in Pulmonem, adeoque in Sanguinem præterfluentem, pro quantitate infinite parvâ habenda erat, adeoque nihil omnino eorum effectuum, quæ ipſi adſcribebantur, præſtare poterat. Ita vero ſi feciſſem, jure queſturos putabam *Bellini* ſequaces, quod inique ſecum ageretur; quippe cum rejiceretur ipſorum ſententia propter demonſtrationem ex Hypotheſi arbitrariâ & eâdem omnium adverſiſſimâ deductam. Malui igitur ex illâ Hypotheſi demonſtrationem deducere, quæ omnium maxime ipſis faveret, maximamque Motus quantitatem exſpirandi initio aeri tribueret. Hæc autem erat, quâ ponebatur Pulmo initio exſpirationis datâ velocitate in Aerem impingere.

Cæterum in Potentiâ Cordis definiendâ iſtam quidem Hypotheſin, quâ ipſius Ventriculi, omni impetu momento temporis concepto, tanquam ſolidum corpus datâ velocitate præditum, in Sanguinem irruunt, primo loco propono, tanquam omnium ſimpliciſſimam, ex eâque ſolutionem deduco. Atqui deinde conſidero tum eam Hypotheſin, quâ Ventriculi Cordis Motum omnem ſuum particulâ temporis admodum parvâ concipiunt, quæque mihi veriſimillima videtur, tum ipſam Hypotheſin *Keillianam*, atque alias infinitas, iiſque omnibus ſolutionem meam accommodo. Adeo ut, ſive iſtud Principium incertum & fallax, ſive verum & ſtabile reperiatur, nihil exinde ſolutionis noſtræ certitudini detrahatur.

Non tamen videmus aliquid argumenti allatum, quo minus iſtam poſitionem nobis adhibere, pari jure atque Viro Cl. contrariam illam de Vi ſive preſſione uſurpare, licuerit. Nihil ſane ſpatii inter parietes Ventriculorum & Sanguinem intercedere non diffitemur, & tamen quare res ictu peragi nequeat nondum liquet. Certe, ſi Cubo Globum contingenti ictus imprimatur, Cubus partem Motus ſibi impreſſi Globo communicabit pari facilitate, ac ſi ſpatium inter eos interceſſerit.

At hæc ſunt corpora ſolida, & ubi de fluidorum Motu agitur, longe alia res eſt. Diſcrimen ſane inter ictus corporum ſolidorum & actionem ſive ſolidi in fluidum, ſive fluidi in ſolidum, fuſius exponit Vir Cl. quod diſcrimen cum me minus advertiſſe cenſeat, ex eo fonte fluere pronunciat quicquid Erroris in meis Propoſitionibus continetur. Ego vero differentiam iſtam ut recte traditam a Viro Cl. lubens admitto, & aio me communem illam doctrinam neutiquam ignoraſſe, cum nihil frequentius in Mechanicis ſcriptoribus occurrat, ſed caſus quoſdam novos expoſuiſſe, quibus ea doctrina cum adhiberi nequiret, alia erat ineunda ratio atque hactenus fuerat uſurpata. Ea tribus verbis abſolvi poteſt. Nam, ut exemplo facillimo utamur, quieſcere ponatur Cylindrus aquæ datæ longitudinis in dato tubo, & moveatur per iſtum tubum Cylindrus alius ſolidus pari diametro, ac datâ velocitate in Cylindrum aqueum impingat. Quid inde futurum eſt? Nempe totus Cylindrus aquæ eo ictu in motum ciebitur, pari ratione, ac ſi fuiſſet & ipſe ſolidus Cylindrus: alter vero Cylindrus Motus ſui partem momento temporis deperdet, & ambo Cylindri communi velocitate per tubum deferentur. Simili modo res eveniet, ſi Cylindrus aqueus per tubum fluens Cylindro ſolido quieſcenti impegerit. Quod ſi Cylindrus aqueus datâ velocitate per tubum feratur, eique occurrat Cylindrus ſolidus aliâ velocitate, ita ut quantitates Motuum Cylindri aquei & ſolidi utrinque pares ſint, jam momento temporis deſtruetur utriuſque Cylindri Motus, pariter ac ſi duo ſolida corpora æquali Motu prædita ſibi mutuo occurrant *.

* *v. infra, Part* 1. *Ch. V. Tract.* VII.

Quod autem nos amicè admodum hortatur Vir Candidiſſimus, ut ſepoſitâ noſtrâ de Vaſorum ictu Hypotheſi, & Vi preſſuræ, quâ Naturam uti cenſet, pro Principio adhibitâ, Theoremata alia conſtruamus; id profecto, niſi gravi morbo impeditus perfunctorie prorſus evolviſſet noſtram Diſſertationem, dudum à nobis præſtitum animadvertere potuiſſet. Quum enim ponimus Motum Cordis in ratione temporis augeri, eâdem utique Hypotheſi utimur, ac ſi Vim preſſionis adhibeamus. Hoc autem poſito, Motum ex Cordis Potentiâ oriundum determinavimus, duplo ſcilicet majorem quam ubi Ventriculorum ictu res peragitur. Calculum vero ipſum, ut ſatis facilem & priori noſtro ſimilem, Lectori reliquimus inſtituendum. Quæ autem ſequuntur Theoremata & in iis Theorema quintum, quod rejiciendum ſtatuit Vir Cl.

tanquam ex Hypotheſi de Ventriculorum ictu deductum, neutiquam pendent ex iſtâ Hypotheſi, ſed ex ipſa Hypotheſi Doctiſſimi Adverſarii pari facilitate demonſtrantur.

CHAP. IV.

The Abdomen.

1. *A Triple Bladder, by Mr.* Buſſiere, *n.* 268. *p.* 752.

IN his beſt Health, the Patient could not make his Water in a full and continual ſtream, the Urine running out by little and little, and with great Efforts of Inſpiration, chiefly when there was but little Quantity of it in the Bladder, which did fatigue him very much, tho the paſſing of the Urine through the Neck of the Bladder was not painful; except the two or three laſt Years of his Life, becauſe of a thick *Mucus*, which then diſcharged with the Urine. That *Mucus* growing in greater Quantity of late, made him apprehenſive it had been cauſed by a Stone in his Bladder; upon ſuch Thoughts he apply'd himſelf to one, in order to be ſearched by him, who accordingly introducing his Catheter, and meeting with ſome Reſiſtance in the *Urethra*, did force the Catheter through the Membranes, and made ſuch Dilaceration in them, that the Patient loſt immediately a great Quantity of Blood; which bleeding continuing for ten Days, without his ſeeking for any Help, brought him under very great Torments, by reaſon the Blood being grumulous in the *Urethra*, could not be forced out but by very violent Efforts and acute Pain, which cauſed a Mortification in the Part, of which he died.

The next Day after his Death, I did open the Body, in which the natural Urinary Bladder was found lying on the left ſide of the *Pelvis* upon the *Ilium* Bone, then ſearching what ſhould be the Cauſe of ſuch unnatural Situation, we did find one large and round Bag, lying under the *Pubis* upon the *Rectum*, filling up all the Cavity of the *Hypogaſter*; then in order to examine the thing more exactly, I did diſſect the *Penis* and the *Rectum*, and having taken them out of the Body, and laid them down upon a Table, the firſt thing I did was to lay open the *Urethra*, to examine whether there was any Carnoſity, as the Surgeon who firſt introduced the Catheter had ſuſpected; but there was none: and that *Ductus* was as plain and ſound as could be, except the Dilaceration which the Catheter had made in it; then having introduced a Conductor into the Bladder, I divided it quite; and firſt it was obſerved, that the round Bag, which was made up of two Bladders, or rather two Cyſtis's, divided one from the other only by a Membrane, that which was next to the true Bladder was ſomething bigger than the Bladder, the other which was lying on the right ſide being much leſſer;

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each of theſe two Cyſtis's had its Orifice open in the Neck of the natural Bladder, which was longer than it is naturally, as you may ſee in the Figure. None of the *Ureteres* were inſerted into any of theſe Cyſtis's, but they were inſerted into the Neck of the true Bladder, by the Orifices of the two Cyſtis's, inſomuch that the Urine could be equally received by them and the Bladder.

Secondly, It was obſerved that the Glandules of the true Bladder were extraordinarily big and red, that Colour being, very likely, the Effect of the Inflammation cauſed by the Dilaceration of the *Urethra.* I have often obſerved that a thick *Mucus*, which runneth out of the Bladder, and which ſome think to be the Matter of an Impoſthume or Ulcer in the Kidneys, is only produced by thoſe Glandules of the Bladder grown ſcrophulous; when that *Mucus* groweth thick and clammy, it cauſeth the ſame Pain on the Neck of the Bladder as if it were a Stone. The Glandules of the great Cyſtis were very ſenſible, but very ſmall, they were not at all ſenſible in the ſmaller Cyſtis.

Now it is eaſie by the Deſcription of theſe Bladders, to give a Reaſon of the Symptoms that Gentleman did undergo, for by the Situation of the great Cyſtis, it is plain that the Water could not come out but by the Force of Inſpiration, its own Muſcles being not able to force it out, and conſequently could not come out but by little and little; and theſe Efforts of Inſpiration were to be the greater, when there was but little quantity of Urine, becauſe it requir'd greater Force to make it aſcend from the Bottom of the Cyſtis, which could not be done but with great Labour and Fatigue.

The Pain he ſuffer'd in the late Years of his Life, was cauſed by the Thickneſs of the *Mucus* running out of the Glandules of his Bladder, his Kidneys being very good and ſound.

A A. The Body of the true Bladder, 1, 2, 3, 4, 5, 6, its Glandules.

B B. The greater Cyſtis.

Plate 3. *Fig.* 1.

C C. The ſmaller Cyſtis, 1, 2, 3, its Wrinkles.

D. Part of the true Bladder overturn'd.

E. The Neck of the Bladder.

F F F F. The two *Urethra*'s.

G. The Inſertion of the Spermatick Veſſels in the *Urethra.*

H H. The *Proſtates.*

I I. *Veſiculæ Seminales.*

K K. *Vaſa deferentia.* *L.* The *Urethra.* *M.* The *Erectores* Muſcles.

N. The *Penis.*

II. *Feb.* $168\frac{2}{3}$. I injected into the *Jejunum* of a Dog, that had for a Day before but little Meat, about 12 Ounces of a Solution of Indigo, in Fountain Water, and after three Hours opening the Dog a ſecond time, I obſerved ſeveral of the Lacteals of a blewiſh Colour, which on ſtretching the Meſentery did ſeveral times diſappear, but was moſt

Experiments for tranſmitting blue Liquors into the Lacteals, by Dr. Muſgrave, *n.* 275 *p.* 996.

eaſily diſcerned when the Meſentery lay looſe; an Argument that the blewiſh Liquor was not properly of the Veſſels but of the Liquor contained in it. A few Days after this, repeating the Experiment in another Company, with the Solution of *Stone Blew* in Fountain Water, and on a Dog that had been kept faſting 36 Hours; I ſaw ſeveral of the Lacteals become of a *perfect Blue Colour*, within very few Minutes after the Injection: For they appear'd ſo, before I could ſow up the Gut.

About the beginning of *March* following, having kept a Spaniel faſting 36 Hours, and then Syringing a Pint of a deep Decoction of *Stone Blue* with common Water into one of the ſmall Guts; and after 3 Hours, opening the Dog again, I ſaw many of the Lacteals of a *deep Blue Colour*, ſeveral of them were cut, and afforded a *Blue Liquor*, (ſome of the Decoction,) running forth on the Meſentery. After this, I examin'd the *Ductus Thoracicus*, (on which, together with other Veſſels near it, I had, upon my Return, made a Ligature,) and ſaw the *Receptaculum Chyli*, and *that Ductus* of a *Bluiſh Colour*; not ſo Blue indeed, as the Lacteals, from the Solution mixing, in and near the *Receptaculum*, with *Lympha*; but much Bluer than the *Ductus* uſes to be, or than the Lymphatics under the Liver (with which I compared it) were.

The Entrance into the Lacteals (which is much the narroweſt Part of all the way from the Mouth to the Maſs of Blood) being thus beyond Exception proved wide enough to admit ſo groſs a Body as *Stone-blue*, we may here in part explain the Admiſſion of Liquors, (as of Diuretic-waters, *&c.*) into the Veſſels in *prodigious Quantities*, in a very little time.

The ſame Wideneſs of the *Pylæ Lacteæ*, makes them eaſie to receive (together with proper Vehicles) thoſe groſſer Bodies, which afterwards compoſe the grumoſe part of the Blood, the Cartilages and Bones. And this open Entrance being allow'd, it will no longer ſeem impoſſible, that with our Nouriſhment, Eggs, or *Animalcula* themſelves ſhould enter theſe Veſſels, there being no manner of Queſtion, but that of both the one and the other, ſome are much leſs in Bulk, than the greateſt Particles of Indigo in the Decoction above-mentioned ſeen in the Lacteals. Add to this the many Species there are of little Inſects, and their great Fertility; ſo many and ſo great, that of the People of the Animal Kingdom, a very ſmall Proportion (perhaps not a quarter Part) comes within view of the naked Eye; and then, we ſhall be the better able to account for the great variety, as well as Numbers of Inſects, obſerved in the Juices of the Body Animal.

But the chiefeſt Uſe of the * Wideneſs of the Lacteal Orifices, is in deducing from thence the Reception of groſs Matters (ſuch as are the Effects of Indigeſtion, *&c.*) which afterwards in the Blood and *Genus Nervoſum*, many times produce moſt ſevere Diſtempers.

* *vid.* Liſter *de Med. Font. Angl. exercit alt. Lond.* p. 48.

A new Syſtem for the Paſſage of the Drink and Urine by Monſ. Morin, *com. by Mr.* Blondel, *n.* 278. *p.* 1101.

III. Having obſerved the extreme Swiſtneſs with which the Drink paſſes ſometimes, as they find that drink *Medicinal* Waters, he thence conjectured that it did not always go the Way, which *Anatomy* ſhews us it takes ordinarily; and that therefore it ought to have another ſhorter Paſſage which is not yet diſcovered. A ſtrong Proof of this his Conjecture is, that thoſe who purge with an Infuſion of *Caſſia* render in a very ſhort time by Urine a Tincture as black almoſt as the Infuſion they have taken; which would not conſtantly happen, if the Drink took always the ordinary Way. He then took Pains to diſcover this unknown Paſſage for the Urine, and he perſuades himſelf that he has found it. To make this *Syſtem* the better apprehended, he began with the Explication of the uſe of the Drink, which is to help the Digeſtion, and to ſerve for a *Vehicle* to the *Chyle*: He ſaid, that the *Urine* is nothing elſe but the Drink it ſelf, which having ſerved for this purpoſe is afterwards caſt out more or leſs loaded; that thoſe who drink much without eating, as when they take the Waters, render their Urine very ſuddenly, and that without Colour. On the contrary, they that drink little and eat much, render theirs ſlower and high coloured; and laſtly, they who both eat and drink very much, renders theirs at firſt, one Part leſs coloured, and afterwards another Part high coloured later.

From whence, and from what he had before ſaid, he inferr'd that the Drink, beſides the ordinary Paſſage which it has to the Bladder by the emulgent Veins Kidneys and Ureters, has likewiſe another by the Pores of the Stomach, and of the Bladder. He called thoſe the firſt Urines which paſs by this new way, and the ſecond Urines thoſe that paſs the ordinary way. He afterwards proved the Poſſibility of this new Syſtem by Experiments. He ſaid, that having taken the Ventricle and Bladder out of a dead Body, and filled them with Water, it run all out through the Pores; and turning them inſide outwards, the Water that was put in them run through them after the ſame manner; and that laſtly, letting them ſwim in Water, it eaſily ſoaked thro into them. From which he concludes that in a living Body, it ought to paſs with much greater Facility by the Tenſion of the Stomach, for the Aliment like a Spunge ſoaks up the Liquor in which it ſwims, and ſo ſwells up the Stomach; which in its Turn again preſſing the Food, ſqueezes out the Liquor from it, and forces it to filter thro' its Pores. With this Preſſion it is eaſy to conceive, that the Drink muſt paſs eaſier through the Pores of the Stomach than the Water, which was put into the Stomach taken out of a dead Body; and that this Liquor re-entring into the Bladder makes the firſt Urine: It is evident likewiſe, that this Preſſion is never ſtrong enough to preſs out all the Liquor from the Stomach, and ſo there remains enough to carry on the Aliment and Chyle, after which it comes away high loaded and coloured, and makes what he calls the ſecond Urines.

He added, the Paſſage of the Drink into the Capacity of the lower Bell did not cauſe the Dropſy, that Liquor, aided by the Preſſure of th Parts that encompaſs it, finding an eaſie Entrance into the Bladder, an none into the Inteſtines, becauſe of the thick *Mucus* that lines them The Eaſineſs of this Paſſage is the cauſe that Mineral Waters ru away ſo ſuddenly by the firſt ways, and by the ſecond; but much more by the firſt, when there is but little Nouriſhment in the Stomach; for there runs more or leſs Urine by the firſt ways than by the ſecond in proportion to the Aliment taken, and to the Surplus of what is neceſſary for the Digeſtion, Reſpect being had likewiſe to what paſſes inſenſibly by Tranſpiration.

This Syſtem being ſo laid down, he gave the Reaſons of two conſiderable *Phænomena*.

The firſt was, the different Colour of Urine that is made at different times: which proceeded from hence, that thoſe that paſs by the firſt ways, are but little charg'd, whereas the other that paſs by the ſecond ways, having ſerv'd for a Vehicle to the Chyle, and circulated with the Maſs of the Blood, are charg'd with the Volatile and Sulphurous Salts, and other Excrements of the Blood, and conſequently more colour'd.

The ſecond *Phænomenon* was the Red, Greeniſh, and ſometimes Blackiſh Colour of the Urine of thoſe that are purg'd with the Infuſion of *Caſſia*. This, according to him, is becauſe the Tincture paſſes by the firſt ways, as was experimented in the Stomach of a dead Man, where this Liquor paſſes indeed more ſlowly and in a leſs Quantity, but always of a Greeniſh Red. It is the ſame of the Red Tincture of the Urine after eating Beetes; of the Violet Brown, which is obſerv'd after Drinking Mineral Waters; of the Smell of Violets after the taking of Pills of Turpentine, and of the ſtrong Smell after Aſparagus, all which comes from the firſt Urines being charg'd with that Colour and Smell, which is not taken away by any thing that's mixt with it, whereas the ſecond Urine, which carries the Chyle and Aliments, has no other Colour nor Smell than Urine ordinarily has. He did not pretend to give this Syſtem as new, but as explained more exactly, and in ſome meaſure demonſtrated by the Experiments he reported.

Part of a Dog's Gut cut and cur'd, by Mr. John Shipton *n.* 283. *p.* 1299.

IV. Præſente *D. Gul. Pleahill, Societ. Chirug. Londin. Cuſtode*, adjutrices etiam manus commodante *D. J. Dobyns*, Juvene in Anatomicis pariter ac Chirurgicis Operationibus optimè verſato,

Cane ex more ligato, vulnus abdomini ſatis amplum infliximus; proximam inteſtini Ilei partem extraximus; vaſa meſaraica, quæ eam irrigabant, ligavimus; inteſtinum tranſverſim duobus locis forfice diſſecuimus; ejuſque portionem duos circiter digitos longam abſtulimus; tum inteſtini vulnus ſuturâ pellionum abdominis interſciſſâ conjunximus; emplaſtro obteximus; vinculiſque ſolutis, faſciâ obligavimus.

Primo

Plate IV. Part II. P. 78.

Fig. 1.

Fig. 2.

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Primo ingreſſu titubare æger, ceu vertigine correptus, debiliſque admodum eſſe videbatur; eadem nocte bis vomuit: Poſt aliquot dies ſuturas aliquatenùs laxatas, ſubmiſſis feſtucis & circa fila contortis, arctius conſtrinximus. Poſteà medicamentis faſciiſque abjectis, intra tres ſeptimanas vulneri ipſe lambendo cicatricem induxit.

Obſervatione forſan non indigna videbitur inteſtini ſimul ac abſciſſum fuit retractio; quod ex utraque parte reſiliens arcteque ſe occludens Sphincterem eleganter exhibuit.

Canem deinde poſt aliquot ſeptimanas, quibus ſanus admodum vegetuſque degebat, ſuſpenſum aperuimus, partemq; inteſtini, qua conſutum erat, in hypochondrio ſiniſtro (ſatis longe ab externo vulnere, quod hypogaſtrio dextro inflictum erat) peritonæo firmiter annexum & in ſacculum ampliatum (*D D D*) invenimus: Omentum etiam (*G*) ei adhærebat, ut & inteſtina ibidem pluribus in locis (* * *e e*) ſibi invicem adnata erant. Ad cicatricem ab interna parte (*c*) accuratius luſtrandam, inteſtinum juxta longitudinem incidimus, quo ex uno latere vulneris labia adducta, (*D D D*) & peritonæo connexa (*f*); ex altero eadem diviſa, proximiſq; adjacentium inteſtinorum partibus agglutinanata fuiſſe apparebat; ita ut exteriores eorum tunicæ, internum ab iſto latere efformantes parietem, inteſtinalem fiſtulam continuarent, cibiſq; devehendis commode inſervirent, *e e*.

v. Fig. 9.

Quanquàm autem hæc in brutis experimenta bene ſucceſſerint, tàmen ſi quiſquam tam inhumanus foret ut ſcienter, vel tam imperitus aut infælix ut inſciè, inteſtinum tenuius homini præcideret, nullus dubito, quin certum ei æque ac perfoſſo corde lethum inferret. Tenuia enim hominum inteſtina tantâ fibrarum carnearum compage haud gaudent, qualicanes natura providè ſtipavit; qui durioribus cibis veſci ſoliti, majori inteſtinalis fiſtulæ motu atq; calore, ad chylum elaborandum, fæceſque (in his plerumque duriores) eliminandas, indigere videntur.

Figuram inteſtini unâ cum vicinis partibus Clariſſ. D. *Cowper* pro ſingulari ſua humanitate accuratiſſime delineare dignatus eſt.

Plate 4.

A a a. Inteſtini Ilei pars ſuperior ventriculum verſus. *B b b.* Ejuſdem pars inferior. *C.* Cicatrix vulneris inteſtini ab interiori parte. *D D D* Inteſtini diviſi labia. *E.* Orificium inteſtini ſuperius. *F.* Orificium inferius. *e e e.* Partes inteſtinorum adjacentium exteriores, portionis inteſtini hoc loco deficientis vicem ſupplentes. *f.* Peritonæi pars inteſtino adnata. *G.* Omentum inteſtino etiam connexum. * * Inteſtini, qua cum altero coaleſcebat, ſeparati veſtigia. *H H.* Aortæ Truncus. *I.* Arteria Cœliaca. *g.* Gaſtrica dextra. *h.* Gaſtroepiploica dextra. *i.* Hepaticæ. *k.* Pylorica. *l.* Gaſtrica major. *m m.* Arteria Splenica. *K.* Arteria Meſenterica ſuperior. *L* Phrenicæ. *O.* Venæ Portæ Truncus. *P P.* Arteriæ & Venæ per Meſenterium diſſeminatæ.

An unuſual Colick, by Dr. Davis *n.* 275. *p.* 965.

V. A perſon, aged between 50 and 60, had been for 3 or 4 years laſt paſt troubled with Gripes, which generally return'd once a month, or thereabouts; his Body for the moſt part coſtive. Laſt *April* coming from *London*, he heated himſelf very much with his Journey, having walked a good part of the way, and as ſoon as he came home, had a return of his Colic pains, which continued upon him for 18 days, notwithſtanding the methods commonly uſed in that caſe, during which time he had no Stool, beſides what the firſt and ſecond Clyſters brought away; his complaint upon his ſeizure was of a pain in his right ſide in the *Regio Iliaca.* Some time before he died, his Belly ſwelled much, and was as tenſe as any Drum, he vomited for 2 or 3 days at the beginning, which left him, and returned not till juſt before he dyed, which was at the expiration of the 18th day, at which time he brought up 2 or 3 mouthfuls of black Choler; but never during his whole illneſs vomited any excrement. Dr. *Shaw* who lived upon the ſpot, prevail'd to have an haſty Inſpection of his Abdomen. He found ſome black Choler in the Stomach, the *Duodenum* and the reſt of the *Inteſtina tenuia* void of excrement, but incredibly diſtemper'd with Wind, and tracing the Canalis of the Guts as far as the *Cæcum*, found that of a blackiſh colour; and from thence for about a yard in length, the Colon mortified and ſo rotten, that the Excrements had made their way thro' it at ſeveral places, into the Cavity of the Abdomen; about 2 inches of the mortified Gut was faſtened to the *Peritonæum* on the right ſide. This part of the Colon was ſo extended with excrements of a ſoft conſiſtence, that they when taken out filled two Chamber-pots; at the extremity of the mortification, towards the *Rectum*, the obſtruction which occaſioned all theſe misfortunes offered it ſelf to view very plainly; for about ten inches of the Colon was doubled, as if you had taken a piece of Tape, and folded it; the two contiguous ſurfaces of the duplicature adhered ſo firmly together, that you could not ſeparate them without tearing the exterior coat of the Inteſtine. Upon ſeparating this coaleſcence, there fell from that part a whitiſh Mucus, the adheſion was about 3 inches broad; the middle of the Duplicature, which made the acute angle, and where the excrements ſtopped, was ſmaller, and the Membranes thinner than in any other part of the Gut; from thence towards the Rectum the Colon was ſound, and void of excrements, occaſioned by the frequent uſe of Clyſters.

An extraordinary Effect of the Colick by Monſ. St. Andre. *n.* 351. *p.* 580.

VI. A Gentleman that came to Town in good Health, meeting with ſome Friends, drank a great deal of new bottled Oat-Ale, after ſome Pints of Wine. Theſe Liquors fermented ſo violently in his Stomach and Inteſtins; that he was taken with a violent *Cholick* the ſame Night.

In the morning an Apothecary was ſent for, who adminiſtred a Clyſter, and took ſome Ounces of Blood to relieve the Patient, who complain'd of a great Pain in his left Side. The Clyſters being repeated the Night following, as alſo the next Morning; and the Patient growing worſe, the Apothecary, without Order of any Phyſician, gave him a violent Vomit: which operated Eight or Nine Times: This added Fewel to the Fire; and the Patient having from that Time been in a deſperate Condition, two eminent Phyſicians were call'd, who order'd that the Clyſters ſhould be repeated: But they not prevailing, I was ſent for about ſix Hours before the Patient died: I found him complaining of a violent Pain in all the Region of the *Abdomen*; a frequent Inclination to vomit; having a great Difficulty of breathing, together with a very ſlow Pulſe; his Belly being as hard as a Stone, though not ſwell'd.

This laſt Indication made me conclude, that the Diſeaſe was a violent Contraction of the Abdominal Muſcles, which had overcome the *Diaphragm*, and that probably the *Inteſtins* might be forc'd into the *Thorax*. Upon which I order'd a Fomentation of hot Milk, adding to every Quart a Drachm of *Liquid Laudanum*, which in theſe Maladies gives great Relief: But before it cou'd be got ready, the Patient expir'd in a violent Convulſion.

In opening this Body, I found the Abdominal Muſcles ſo much contracted, that it was almoſt impoſſible to penetrate them with a very ſharp Scalpel.

Upon Examination, I found the Stomach empty, and ſome Parts of the *Duodenum*, but the *Jejunum* and *Ilium* ſo much diſtended with the fermented Oat-Ale, that the *Ilium* had four Inches of Diameter, and the *Colon* above eight. The *Ilium* was alſo pretty much inflam'd in its inferior Part; and all the Valves of the *Colon* were obliterated by the great Diſtention of that *Inteſtin*. But the greateſt Diſaſter was, the Dilatation made in the *Diaphragm* as I ſuppos'd; made juſt upon the Chink which remits the intercoſtal Nerve to the *Viſcera* of the *Abdomen*, through which a Portion of the *Colon* was forc'd, and the greateſt Part of the *Omentum* and *Pancreas*.

Theſe tender Parts being choak'd ſoon inflamed, a Mortification of them following; and a Rupture of the *Pancreatick Vein* caus'd an internal *Hæmorrhage*, which fill'd all the left Cavity of the *Thorax*, inſomuch that the whole left Lobe of the Lungs was compreſs'd almoſt under the *Muſculus Scalenus*. The Quantity of extravas'd Blood was very great, and it was not in the leaſt coagulated.

VII. Acceſſit curandus nuper Vir Ruſticus, & ex multo tempore extenuatus, laborans torminibus ventris, atque Hypochondriorum Spaſmodicis contractionibus, nil fere dejiciebat per inferiora, & continuo languore exolutus ſpiritu tandem defecit. Aperto cadavere, invenimus inteſtina craſſa, præcipue ubi in vicinia Duodeni flectuntur, mediis protuberantiis calloſis invicem, & cum adjacentibus conſolidata, ut obſcurum eſſet unum ab altero disjungere, & quod pejus erat, eorum

A pain in the Belly caus'd by a concretion of the Inteſtines, by Dr. Meſaporitus, *n.* 303. *p.* 2118.

ſubſtantiam ad inſtar duriuſculæ cartilaginis concretam, & craſſam, ut nulla fere cavitas amplius ſupereſſet.

An extraordinary coſtive Caſe. by Mr. Sherman. *n.* 302. *p.* 2111.

VIII. 1. *Thomas Phillips* of *Eaſthorp*, near *Keldon* in *Eſſex*, was as well in every reſpect as a Child could poſſibly be till he was a year and a quarter old; at which time a very ſtrange and almoſt continual rumbling in his Inteſtines ſeiz'd him, the conſequence was a violent Looſeneſs, for which all the Phyſicians near the place could find no Remedy: But at laſt, when they deſpair'd of the Childs Life, the Looſeneſs terminated in ſuch an unuſual Obſtruction, that he did not go to Stool for a fortnight or three weeks together, and from three weeks it proceeded gradually to the intervals of 17 or 18 weeks, and ſo continued till he came to be about the age of fifteen, when his Body reſumed its natural temper, which laſted 4 or 5 years, but then the Obſtruction returned, and continued, or rather increas'd till he dyed; for it was cuſtomary with him in the laſt years of his Life not to evacuate any manner of Excrement under the interval of nineteen or twenty weeks, and ſometimes (twice at leaſt) he had no diſcharge for one or two and twenty weeks together. He lived to be near 23 years of Age, and walk'd about almoſt to the hour of his Death; for he was ſuddenly ſeiz'd with very ſick Fits (but could not vomit), two or three of which Fits carry'd him off in a few hours; and when he dy'd it was nine weeks after he had any Stool. He died about a year after I wrote to you firſt concerning him. I ask'd him whether he did Vomit often, or had at any time any Excrementous Taſtes in his Mouth, or did Sweat much, or made more Urine than in proportion to his drinking, or whether he found any eaſe when he did either Vomit, Sweat or Urined; all which queſtions were anſwered in the Negative. When he did go to Stool, he evacuated very many times in a day, and ſeveral days together, until he had emptyed himſelf. But his Mother aſſures me, that throughout his whole Life he never diſcharg'd any other than very thin Excrement.

Before his time of Evacuation came about, he was of an extraordinary bigneſs many weeks before his going to Stool, unleſs when he could break Wind, which he often endeavour'd to do, by laying his Body on the edge of a Table or Stool, but could not often by ſo doing produce the deſired effect. He declined the uſe of all Medicines for many years before he dy'd, contenting himſelf with going to Stool once in three or four months, or nineteen or twenty weeks, as above mention'd. But that which amuſes me as much as any thing is, that he generally had a pretty good Stomach, and eat and drank as the reſt of the Family did; nay, till the time that his Body came to be very full, he could do the work of a Man at Plow, or ſuch like Husbandly labour, for Agriculture was his employment. I was very deſirous to have open'd his Body, but could by no means obtain it from his Mother.

2. It's

A Note by Mr. Cowper. *ibid.*

2. It's not improbable, if the *Abdomen* of this Perſon had been opened, but ſome of its Contents would have been found not unlike thoſe I have mentioned, in my Explication of the 34th Table of Prints publiſhed by Dr. *Bidloo*, where I take notice of a young Gentlewoman I diſſected, in whom I obſerved the *Omentum* ſo leſſened, that at firſt it appeared doubtful, if that part had ever been exiſtent in that ſubject; but on ſtrict examination the little Remains of it reſembled a *Congeries* of ſmall Glandules, ſtuffed with a Suet like matter. The whole Canal of the Inteſtines, even from the *Pylorus* to the *Anus*, was diſtended with Excrements, and the Surfaces of all the ſmall Guts adhered ſo ſtrictly to each other, that they could not be parted without tearing their external Membrane, to which the *Omentum* contributed by its adheſion: The whole *Compages* of the Inteſtins very much reſembling that of the external ſurface of the Brain covered with the *Pia mater*, ſo that the Meſentery in that ſubject could not be ſeen till this external incloſure was divided. By this diſorder, 'tis certain, the Periſtaltick motion of the Guts muſt needs be very much leſſened, if not quite hindered. The *Peritonæum* alſo in that caſe was very much thicken'd, and had ſeveral preternatural white bodies ſet at various diſtances on its internal ſurface; the like appeared on the Stomach, which very much reſembled the Miliary Glands on the back part of the *Aſpera Arteria*.

Diſſection of one dying of an Ulcer in the right Kidney. by Dr. Douglas. *n.* 325. *p.* 32.

IX. When the Skin with the other Integuments were taken off, I obſerv'd that part of the Omentum had thruſt itſelf through the Annular Holes of the Abdominal Muſcles on the left ſide and there formed an Epiplocele or Hernia omentalis, as large as a Walnut. The *Omentum* reached as low down as the *Pubis* and Inſide of the *Ilia*, to which it was ty'd, and by Fibrous Connections it adhered to all the *Peritonæum* below the Navel. All the Fat on the *Omentum* and Guts was firm, and hard like Tallow. The Inteſtines and Stomach were quite empty, without either Wind or *Fæces*.

The Left Kidney was much larger than ordinary, being near eight Inches long; its Surface being divided into ſeveral diſtinct Lobes, as in a *Fætus*. The Right Kidney was full of a fœtid purulent Matter: All its inner Subſtance was wholly waſted; and its external or cortical Part was ſtretched ſo very thin, that a ſmall touch of the Finger could eaſily break through it. All the Fat and Glands about the Kidney laſt mentioned were hard, obſtructed, indurate, and big, which made a great compreſſion on the *Muſculus Pſoas* and the *Muſculus Quadratus lumborum*. The Ureter proceeding from this Right Kidney, was covered with a Cruſt or Bed of indurate Glands; and beſides, its capacity was ſtraightned and contracted in ſeveral Places. The Cavity of the *Veſica Urinaria* was very ſmall; its Subſtance ſo very thick and hard, that I could not even by the help of a Blow-pipe diſtend it any wider; Its Inſide ſeem'd excoriated with ſeveral little fleſhy

Caruncles, or red Excreſcences, here and there. There was a remarkable Corroſion in all the Inſide of the *Urethra*. All the Upper and Convex Part of the Liver adhered firmly to the *Peritonæum* that covers the Diaphragm, and to the ſame Membrane where it covers part of the *Muſculus Abdominis tranſverſalis:* Its Subſtance was ſo very tender and ſoft, that it ſeemed to be almoſt rotten. The Gall Bladder was extreamly large and full; the Bilioſe Liquor it contained, being of a whitiſh yellow colour.

Between the *Tunica vaginalis* and *albuginea* of the Left Teſticle, there was a large hydatical or watery Tumor; and upon the laſt named Coat of the ſame Teſticle, there were ſeveral chalky Concretions, about the bigneſs of a Barley Corn each. In the Right Auricle of the Heart there was a large *Polypus*, that fill'd up its Cavity; extending itſelf a great way into the Aſcending and Deſcending Trunks of the *Vena Cava*. All the reſt of the *Viſcera* were as they ſhould be in a Natural State.

About a Year and half ago, the Patient began to decline. His firſt Complaints being a Heat, Sharpneſs or Pain in making Water; conſtant Deſire to Urine, though in great Miſery after. When the Water ſtood a while, there appeared a greaſy Subſtance on its Surface, not unlike the Cream or Ice that is found on the top of *Aqua calcis vivæ*; ſometime after, it depoſited a purulent Matter in great quantity, but without any offenſive ſmell: The Water when made was thick and whitiſh, but when the Corruption ſettled to the bottom of the Pot, it became clear. He ſeldom complained of any great Pain in his Back or Loins; whence they concluded the Ulcer was in the Neck of the Bladder, though the vaſt diſcharge of Matter was an Argument of the contrary: But always was on the Rack when he roſe up after ſitting, and it was a great difficulty to him to get up, which perhaps was occaſioned by the Weight and Preſſure of the Kidney and adjacent indurate Glands, lying on the head of the *Pſoas* Muſcle and *Quadratus lumborum.*

He had often a total Suppreſſion of Urine; but was much relieved by *Sal Succini* and *Corn: Cervi.* He took ſeveral Doſes of *Cantharides* with Camphire, without any ill effect from the Fly, but had little Relief as to his Diſtemper. For Three Weeks paſt he was ſeized with a violent Looſeneſs, which at laſt, in ſpite of all means, carry'd him off.

An Hydropical Caſe, by Mr. Young, *n.* 333. *p.* 426.

X. Mrs. *Dyer* was about 30 Years old, a Mother of ſeveral Children, and very heathful till laſt *January*, when, after frequent watching upon an extraordinary occaſion, ſhe was vexed with a Pain in her Belly, like the Cholick, but proved the Dropſey *Aſcites*; and grew ſo faſt in deſpite of all I could do to help it, that *March* the 9th, being almoſt ſuffocated, I was forced to tap her by an hollow Needle in the uſual Place, and to repeat the Operation ſo often as ſhe filled:

And by that way diſcharged the ſeveral Quantities of Water, at the times here under-mentioned.

		Pints.			Pints.
March	9th I drew	9	*July*	21	16
	14	8		30	16
April	2	12	*Auguſt*	6	14
	16	10		17	14
May	17	14		26	13
	31	14	*Sept*	1, 6, & 22	$11\frac{1}{2}$
June	14	14	*Octob.*	1	3
	24	14		30	15
July	7	17			$102\frac{1}{2}$
		112			112
					$214\frac{1}{2}$

All the while I was pumping that out, I endeavour'd by all the means I could to ſtop the Leak within, but in vain: She dy'd *Novemb.* 4. 1711. and opening her Belly, we found the following remarkable and incredible Things. From the Belly iſſued fourteen Pints of a greeniſh *Serum,* mixt with a very purulent Matter, not a little fetid. The *Inteſtines,* eſpecially the *Colon,* almoſt every where livid, and adher'd in many Places to the *Peritonæum,* altho' they had been ſo long immerſed in Water. The *Omentum* was alſo black, and almoſt conſumed. The *Liver,* which I expected to be indurated, was free of all Faults, only two Superficial Ulcers on the Left Lobe. Both that and the *Peritonæum* (which are uſually full of *Hydatides* in Dropſical Perſons) were wholly free of them; but on the Stomach and Guts were many ſuch.

But we were mightily ſurpriz'd to find a great Bladder diſtended like that of an Ox, to fill up almoſt the whole Region of the Liver and Ventricle, and adhering to the adjacent Parts ſo firmly, that we could not ſeparate them without Difficulty, and get it out whole. Our Surprize, at ſuch a prodigious Appearance, turned into Aſtoniſhment, when we found it the Gall Bladder, and that by its Diſtention it had torn the Liver aſunder; one part of which adhered to the Left ſide of this monſtrous *Cyſtis,* and another Part behind it, towards the Back; and both expanded with it, and faſtened to it, like as the temporal Muſcle to the Skull.

The whole weighed ten pounds and twelve Ounces. It had no Paſſage to let out the Matter it contained, altho' we ſqueez'd it hard to that purpoſe; nor could we find any by Probes: So that we were forced to make way by a Knife, and ſo let out of it ſeven Pints of a black Liquor like Coffee; which having ſtood one Night in a Baſon, near a Quart of thick yellow *Fæces* ſubſided. The Liquor in this Bladder, and what we found in her Belly after her Death, added to what was evacuated before by *Paracenteſis,* amounts to 235 Pints. Beſides the prodigious Quantity of Matter which fill'd this great Bag, we found ſeveral Pieces of Membranes like Gut or Bladder cut into Pieces. During the whole time of her Sickneſs, ſhe ejected by Urine near as much as ſhe drank; and yet by Computation ſhe leaked into the *Abdomen* near a Pint every 24 Hours, from *March* to *November*. When her Belly was near full, her Thighs and Legs us'd to ſwell, and grew diſcolour'd, like an approaching Gangrene, but both went off after Tapping, by the Help of Friction, and a warm Lotion, &c.

Another by Mr. Laſage, *n* 299 *p.* 1977.

XI. I was called, ſome time ago, to open a Maiden Lady 52 Years of Age, who complain'd, about ſix Weeks before, of a Circonſcript hard Swelling on the *Hypogaſtrica regio*, on the Right Side; from that time her Belly grew by degrees to an exorbitant Bigneſs, the great Weight whereof was the moſt conſiderable Symptom, and at laſt ſuffocated the Lady. The Body was mightily emaciated, and the Legs ſwelled few Days before her Death. I expected Water, but there was only a viſcous darkiſh Humour, to the Quantity of 18 Gallons; after the Evacuation of that Matter, I was no leſs ſurprized to perceive a large Heap of Veſicles ariſing from a thick Membrane covering the Guts, it being the *Peritoneum* ſeparated from the Muſcles. I took it out, to examine the better thoſe Veſicular Bodies diſpoſed on the outward Surface of that Membrane, as alſo them that were on its inſide, towards the Guts. The Veſicles were of different Magnitude; ſome of the largeſt had been broken and ſunk, others were broken and almoſt empty, and the others very much diſtended and full; the Matter of all of them was of the ſame nature with the extravaſated Humours. What was contained in the leſſer ones proved to be of different Colour and Conſiſtence, not unlike Gelly, White of Eggs, Gall and Honey; in ſome it was much like the Humour of a true *Meliceris*. There was but little Matter extravaſated in the Cavity of the *Abdomen*; moſt Part was contained betwixt the *Peritoneum* and the Muſcles. The Right Kidney was affected with a particular Dropſie; all the *Viſcera* beſides were in a natural State, two *Polypus*'s were found in the Heart, and two pretty big Stones in the Gall Bladder.

A Diſſection of an emaciated Body dying of an Hydrops, by Mr Vaughan *n.* 281. *p.* 1245.

XII. I diſſected a poor emaciated Creature, that died of an Hydrops, out of whom I took about 20 Gallons of Liquor meaſured.

A Caſe of the Iliaca Paſſio, *by the ſame,* ibid.

XIII. A Youth whom I attended about 30 Hours, was afflicted with the *Iliaca paſſio*, very terrible for the time; he was aged about 14 Years, of a Sanguine Conſtitution. About 3 or 4 Hours before he dyed, I adminiſtred a Terebinthinat Clyſter, which gave, during its ſtay, immediate Eaſe; he ſo continued about an Hour, then this Diſeaſe returned again as ſevere as ever: He ſoon after dyed; but before ſome time he voided ſome of his Clyſter by Vomit. The Caſe being not very uſual, I applied my ſelf to his Friends, in order to Diſſect him; which accordingly I did the next Day. Finding the Liver only ſomething larger than ordinary, I immediately made ſearch from the *Ventriculus* quite to the lower end of the *Inteſtinum rectum*: The *Ventricle* was conſiderably extended; a little ſpace from the Gut *Ileon*, in the *Jejunum*, I found the excrements had made a breach, and ſome quantity was evacuated. I then proceeded to the Gut *Ileum*, I found a conſiderable part of it very livid, not in the leaſt extended; the Colon was much like a Contuſed Wound about 3 or 4 days old. I inſpected into the center of it, where I found it ſomething freſher, not ſo livid as the outſide. About the beginning of the *Inteſtinum rectum* I found another large ruption, where there was more excrements voided.

Of the Human Allantois by Dr. Hale. *n.* 271. *p.* 835.

XIV. I ſhall here give a true and exact account of a Humane *Allantois*, as it appeared in two ſubjects, ſtill by me. One of which I obſerved ſeveral years ago, and the other in *March*, 169$\frac{8}{9}$.

Moſt of the Antients indeed allow of one; not from their experience of it, but becauſe they took it for granted, that Men and other Animals were alike, in the *Viſcera*, *Membranes*, *Veſſels*, &c. Hence *Hippocrates* ſays, that Twins lie in *Sinus's*, and that the *Uterus* has *Cornua*. *Galen* deſcribes the *Navel ſtring* to conſiſt of four Veſſels, beſides the *Urachus*, and the *Allantois* to be like a Pudding reaching from one *Cornu* of the *Uterus* to the other. In ſhort, notwithſtanding the Antients might ſometimes diſſect Humane Bodies, and although, [1] *Herophilus* and *Eraſiſtratus* did open live Men and Women, yet it can't be found what great uſe was made of thoſe opportunities: For the Antients Accounts of many parts, particularly of the *Urachus* and *Allantois*, (as to its Name, Figure, Site, *&c.*) agree only to their appearance in *Brutes*. I ſhall ſay nothing of the *Allantois* in Brutes, ſince it is granted by moſt Anatomiſts, to be in theſe Animals, and ſufficiently deſcribed by Dr. *Needham*, who alſo firſt diſcovered part of the *Allantois* in *Humane* ſubjects, but neither he, nor any other, has taken the right method of finding it entire, and 'tis no wonder they could not truly deſcribe what they never ſaw. [2] Dr. *Needham* ſays, that after the *Amnios* is cleared, and left fixed to the *Umbilical Rope*, you may divide by the fingers, or knife, the remaining part of the *Involucra* into two Membranes. The *exterior* he truly calls the *Chorion*, the *interior* he takes to be the *Allantois*. But by theſe ways of ſeparation, you will preſently tear the *Allantois*, and be able to diſcern only ſome ſmall pieces of it. Beſides, the *Allantois* is at firſt ſight ſo like the *Amnios*, that [3] many who ſuppoſe the *Amnios* double, and that its Coats are eaſily ſeparable, have taken theſe pieces of the *Allantois*, for broken parts of one of the Coats of the *Amnios*. Whereas having firſt found the *Hole* whence the Urine came forth (if the *Allantois* is not too much torn) you may blow up the *Allantois* with a Pipe to its full dimenſions, and then ſee its true Shape, the *Fundus*, the *Crevix*, the inſertion there of the *Urachus*, its relation to the other Membranes, *&c.* Be the *Allantois* never ſo

[1] *Celſus. Præf.*

[2] *cap.* 3d 7th.

[3] *Diemer. lib.* 1. *cap* 31. *Horn. parag.* 69.

ſo much torn, yet this way you may eaſily ſeparate many Inches of it, from the *Chorion*, and *Amnios*. Which eaſie ſeparation demonſtrates a diſtinction of Membranes, ſince no double Membrane can be divided by the breath alone.

Indeed *Hoboken* and [1] *Diemerbroeck* make it a very eaſie thing to ſeparate the *Allantois* from the other Membranes, only by the fingers; but 'tis plain from their deſcriptions that they never ſaw one entire. Amongſt other miſtakes, *Diemerbroeck* ſays that the Urine of a *Fœtus* lies between the *Urinary* Membrane and the *Chorion*; as though not contained in a diſtinct Bladder, but in a Cavity made partly from the *Chorion*, partly from the *Urinary* Membrane. I confeſs *De* [2] *Graaf* tells us, that by blowing with a Pipe into a Hole made through the *Chorion*, all the Membranes of the *Secundines* will appear diſtinct. He has alſo delineated an *Allantois*, with the other Membranes, *&c.* as he ſays he found them. Yet this [3] *Figure* muſt have been drawn from his own fancy, and not from any preparation, for theſe reaſons. 1ſt, Becauſe, by this way of ſeparation, you can only part the *Allantois* from the *Chorion*, but never ſee its true dimenſions, nor any appearance of a Bladder; for a Bladder, as the *Allantois* is, can be ſhew'd only by blowing into its Cavity, or by finding it full. Yet in this *Figure* no ſign can be obſerved, where 'twas blown up, and tied, *De Graaf* alſo ſpeaking of making a Hole only in the *Chorion*. Nor can this *Allantois* be ſuppoſed full of Urine, becauſe 'tis not of the Shape of a full *Allantois*, and our Author himſelf calls it only the inflated part of the *Allantois*. However, I can't conceive how the *Allantois* could remain partly filled with Air (any more than it might with Urine) ſo long as till this *Figure* was drawn, unleſs ſome *Hole* was tied up, whence the Urine came forth, and the Air was blown in. 2dly, Becauſe in this *Figure* the *Umbilical Rope* ſeems to run through both *Amnios*, and the *Allantois*, to its inſertion on the *Placenta*. Whereas the *Allantois* is no where perforated by the *Umbilical Rope*, nor does it any where paſs through the *Amnios*, but only runs under it, at the place of its inſertion on the *Placenta*. If the *Navel-ſtring* could be allowed to enter the *Amnios* at [4] I, and to paſs under it to the *Placenta*, why ſhould it not appear (which it does not) under the *Amnios*, as well as the thin ſubſtance of the *Allantois*? Again, according to *De Graaf*'s poſition of the *Secundines*, (which is the reverſe of *Figure* 1ſt, where the *Navel-ſtring* lies under the *Allantois*) nothing could hinder a plain view of the place where the *Navel ſtring* is ſet on to the *Placenta*. This will be eaſily apprehended, by ſuppoſing the part H in my 1ſt *Figure* to lie uppermoſt, the *Fundus* G and *Navel-ſtrings* being turned over; for then the *Strings* will run over the *Allantois*, as in *De Graaf*'s Cut, and its inſertion appear plain on the *Placenta*, which yet can't be diſcovered in his *Figure*. I am ſure the whole is irregular, and I take it to be fictitious. As for the *Urinary* Membrane G it ſeems to be the *Allantois* of a *Colt* (where [5] *Needham* ſays, the *Umbilical Rope* runs through the *Urinary*

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[1] *Lib.* 1. *cap.* 31.

[2] *de Mul. Org. cap* 15.

[3] *cap.* 15. *Tab.* 22*d.*

[4] *Tab.* 22*d De Graaf.*

[5] *vid.* B. *Tab.* 2*d.*

nary Membrane) not leſs abſurdly added to the *Secundines* of a *Humane Fœtus* than the *Secundines* of a *Whelp* are to a like *Fœtus* by *Veſalius*.

Laſtly, 'Tis moſt evident that *De Graaf* knew nothing of the true Shape of this Membrane, and that he had never ſeen one entire, becauſe [1] he conſents to [2] *Needham*'s deſcription of it as true, which yet is falſe in ſeveral particulars. For 1ſt, the *Urinary* Membrane does not cover the whole *Fœtus* (as he affirms) but only that part of it, which reſpects the *Chorion*, and does not lie on the *Placenta*; for the *Allantois* can be extended at fartheſt but to the edges of the *Placenta*, where the *Amnios* and *Chorion* are ſo cloſely joined by *Fibres*, that no Membrane can come between them. Wherefore 2dly, the *Allantois* is not every where faſtned to the *Chorion*. And conſequently 3dly, the *Allantois* can't be of the ſame Shape that the other Membranes are of, nor be like the *Allantois* of a *Colt*, which contains the *Fœtus* in the *Amnios*; all which nevertheleſs [3] *Needham* aſſerts. In ſhort, Dr. *Needham* had ſeen only pieces of the *Urinary* Membrane, but never an entire one, and ſo could only gueſs at the Shape, *&c.* of it, from what he had obſerved in *Mares*, and *Glanduliferous* Animals. He might have made a better gueſs at the figure, ſite, *&c.* of a *Human Allantois* from that of a *Whelp*, which does not every where encompaſs the *Fœtus*, as he obſerves. *Bidloo*, in moſt of his *Figures* of the *Secundines*, letters ſome *Veſtigia* of the *Urinary* Membrane, but in any of theſe *Figures*, you only ſee broken pieces of one, ſo confuſedly placed, that no *Idea* of its Bigneſs, Shape, or Situation can be formed from them. I muſt confeſs that oftentimes the Membranes of the *Secundines* are ſo torn, that no Art can exhibit an entire *Allantois*. However, among the many *Secundines* that have come under the hands of Anatomiſts, ſeveral no doubt muſt have been entire enough for a fuller diſcovery than they have made, had it not been by their ways of proceeding (*viz.* by knife, fingers, or blowing under the *Chorion*) impoſſible to diſcover any thing plain, or ſatisfactory, even in the faireſt ſubjects.

I come now to anſwer the objections of thoſe who ſtill deny an *Urinary* Membrane to a *Humane Fœtus*.

The difficulty of finding this *Membrane* is by no means an argument againſt the exiſtence of it: But a Woman that dies big with Child is ſo fair a ſubject for the diſcovery of *three Membranes*, that I wonder [4] *Parey* having ſuch an opportunity could find but two, if he was ſo careful as he ſays he was. Dr. *Tyſon* obſerved *three Membranes* ſome years ago, in a like ſubject. After the *Chorion* was divided, and laid aſide, he ſaw *two Bladders*, containing Liquors of different colours, which he preſſing one towards the other, did not mix, but remained diſtinct. This obſervation fully ſatisfied that great Anatomiſt, as to the exiſtence of an *Allantois*; and its figure, texture, ſite, *&c.* might alſo have been diſcovered by him, had not the leſs curious Spectators been impatient to paſs on to other parts of the Diſſection.

[1] *De Graaf. cap.* 15.

[2] *cap* 3*d de format. Fœt.*

[3] *cap.* 7*th.*

[4] *Lib.* 3*d. cap.* 34.

Some deny a *Urinary* Membrane to a *Human Fœtus*, becauſe they ſuppoſe the *Urachus* to be impervious, and that therefore there would be no paſſage for the Urine, and conſequently no need of an *Allantois*. [1] *Needham* indeed ſays that he could never find any ſign of a Cavity in the *Urachus*, yet is of opinion, that by blowing from the *Bladder*, the Air might be forced through a *Humane Urachus*, as eaſily as he has often done it through that of a *Whelp*. I don't underſtand why Dr. *Needham*, and [2] others ſhould inſiſt ſo much upon an apparent Cavity in the *Urachus*, or expect that Air ſhould neceſſarily paſs through it upon blowing, and think that otherwiſe it cannot be fit for the aſſigned Office; ſince many Bodies, as Membranes, *&c.* will not admit Air, *&c.* yet let Water paſs freely through them. It will not ſeem ſtrange, that Water ſhould paſs through the ſubſtance of the *Urachus*, if we conſider that the Cavity of the *Urachus* to the *Navel* is open, as appears by Inflation, or Injections, (to ſay nothing of thoſe who are mentioned to have made Water by the [3] *Navel*) and that the reſt of the *Urachus* is pervious, tho' not plainly hollow, (the Urine rather ſoaking gently, than running through its more ſtrait Tubes) may be gathered from hence, 1ſt, That the ſubſtance of the *Urachus*, (as well as the Cavity of the *Allantois*) is always found turgid with a Liquor, that in colour, taſte and ſmell, ſeems Urinous. 2dly, That ſince the *Mucous* Coat of the Inteſtines is demonſtrated to be vaſcular by Mr. *Leeuwenhoeck*, therefore the Mucous ſubſtance of the *Urachus* may alſo be Vaſcular. 3dly, That Urine may as eaſily ouze through theſe Mucous Veſſels, as other fluids run through Vaſcular *Cartilages*, and *Bones*, &c. or the Chyle into *Lacteals*, (whoſe Orifices, as [4] *Leeuwenhoeck* obſerves, will ſcarce admit of Particles ſo big as the 1000000000 part of a grain of ſand) the great Cavity of the Inteſtines being open at the ſame time; or as eaſily as groſſer parts of the *Semen* paſs the Tubes of the *Teſticles*, whoſe Cavities are not more perceptible. I am ſure the Urine is more aſſiſted in its motion by the *Detruſor Urinæ*, &c. than any of theſe Fluids can be by the *Heart*, or other *Muſcles*.

[5] Others will not admit of an *Urinary* Membrane, they thinking it would be uſeleſs, becauſe they imagine, that when the Bladder is full, the Urine muſt be diſcharged at its *Cervix*, and not at its *Fundus* by the *Urachus*. But in anſwer to this, the Urine can never paſs through the *Cervix*, and *Urethra*, unleſs the *Abdominal Muſcles* contract, becauſe we never void Urine naturally, but by the help of theſe Muſcles, nothing leſs being able to force open the *Sphincter Veſicæ*. Now it being more than probable that theſe Muſcles never act before *Reſpiration*, no Urine can paſs through the *Sphincter*, before the Child breathes. No reaſon can be given why the *Abdominal Muſcles* of a *Fœtus* ſhould voluntarily contract, ſince neither the Quantity nor Quality of the [6] Urine can excite to ſuch an action. For when the Bladder is too full of Urine, it will ouze through the lax ſpongy ſubſtance of the *Urachus*, being gently preſſed by the *Detruſor* alone. There would ariſe many inconveniencies

[1] cap. 3d cap. 4th cap. 7th.

[2] Parey Lib. 3d. cap. 34 Mauriceau Lib. 2d cap. 4th.

[3] Fern. Lib. 6. cap. 13. Senn. Lib. Prac. 3d part 8. ſect. 2d cap. 10. Lauren. Lib. 8th Queſt. 7.

[4] Vol. 2d. Epiſt. 68. pa. 235.

[5] Barth. Lib. 1. cap 36. & 37. Riolan. Lib. 6. cap 3. & 4 Parey Lib. 3d cap. 34.

[6] Barth. Lib. 1. cap. 37. Need. cap. 3. pag. 81.

veniencies from the voluntary contraction of the *Abdominal Muſcles* of a *Fœtus*, as voiding *Fæces* as well as *Urine*, into the *Amnios*, which would be more prejudicial than [1] Sweat, *&c.* Yet if we ſhould ſuppoſe the *Abdominal Muſcles* of a *Fœtus* to act, the Urine will however paſs where it can moſt eaſily, *i. e.* through the *Urachus*, which is partly open, and altogether of ſuch a Texture, as in no wiſe can hinder the paſſing of the Urine, much leſs be able to reſiſt a conſiderable force, as the [2] *Sphincter Veſicæ* can. Beſides, the *Urachus* is not only thus qualified for the admiſſion of Urine, but when the Mother lies down, 'tis almoſt upon a Level with the *Urethra*, and what has once paſſed the *Urachus*, cannot return by reaſon of the Length, Situation, and peculiar Structure of it. Laſtly, the [3] *Pudendi Clauſura* ſometimes happening in both Sexes, demonſtrates that then at leaſt, the Urine can't paſs through the *Urethra*.

[4] *Dioni* not finding any *Allantois*, nor an *Urachus* plainly pervious, thinks there is no need of either, on another account. For he ſuppoſes that the Blood, which ſerves for the Nutrition of the *Fœtus*, is depurated from all Excrement. But I cannot apprehend, what ſhould make this portion of the Blood and Chyle freer from Excrement, than the reſt of the *Maſſa Sanguinea*. There is indeed no portion of it, which does not contain parts unfit for *Aſſimilation* and *Nutrition*. Our Author would have been convinced of this error, had he ever opened *Abortions* of five Months old or upwards, their Bladders being always full of *Urine*, and ſome Fœces conſtantly in the Inteſtines. 'Tis difficult to determine when this ſeparation of *Urine* firſt begins, but I am apt to think it much ſooner than is generally ſuppoſed. *Fig.* the 3d is the *Allantois* of a very ſmall *Abortion*, which I have ſtill by me. Since all the parts are perfectly formed before *Impregnation*, not very long after Impregnation they may begin to perform their Offices. No doubt they begin as ſoon as there is occaſion for any ſeparation, and a ſeparation of Urine is neceſſary, when the *Fœtus* is firſt nouriſhed by the *Umbilical Arteries*.

The Exiſtence of an *Allantois* is denied by [5] ſome who grant an *Urachus*, but will have it convey the *Urine* to between the *Amnios* and *Chorion*. [6] *Diemerbroeck*'s opinion is ſomewhat like this, only he would have the Urine lodged between the *Urinary* Membrane and the *Chorion*. Theſe men don't conſider that the Urine in this caſe would get into the *Amnios* as well as the *Succus Nutritius* of the *Chorion*, whether imbibed from the *Uterus* by the *Chorion*, or ſeparated by its *Glands*. Such a *Succus Nutritius* of the *Chorion* is granted by the [7] Maintainers of the fore-cited opinions, as well as by thoſe who deny an *Allantois* altogether, or [8] ſuppoſe it to have a different Figure, *&c.* from what *Diemerbroeck* aſſigns it. The Tranſudation (or Filtration through the Membranes) of this *Succus* ſeems moſt likely in *Mares* and *Sows*; for in a [9] *Mare*, the *Chorion* is not joyned to the *Uterus*, till ſhe is half gone, and in a [10] *Sow* it does not adhere to the *Uterus* till near the end of her going

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[1] *Harv. de Humor. pag.* 547.

[2] *Galen de uſu part. Lib.* 15.

[3] *Senn. Prac. Lib.* 4. *part* 1. *ſect.* 1. *cap.* 3 *Mauric. Lib.* 2. *cap.* 3.

[4] *pag.* 145.

[5] *Aquapend. Lib.* 1. *cap.* 7. *Fallop.*

[6] *Lib.* 1. *cap.* 31.

[7] *Harv. de Hum. & membra. Needh. cap.* 2. 3. *cap.* 7. *Barth. Lib.* 1. *cap.* 36. *Graaf de mul. org. cap.* 15.

[8] *Needh. Graaf.*

[9] *Harv. de Membran.*

[10] *Need. cap.* 2. *cap.* 3 *&* 7. *Graaf de mul. org. cap.* 15.

going with young. But tis moſt evident that the Urine of a Humane *Fœtus* is not contain'd between the *Chorion* and *Amnios*, nor between the *Chorion* and *Allantois*, from the cloſe connexion of theſe Coats to one another; alſo from the obſervation of Midwives, who often find a Bladder of Water (they call it a By-Water) offering itſelf before the Child, whereas the Humor of the *Amnios* is little, and of the *Chorion* much leſs, and of another Colour, *&c.* at the time of Birth. This By-Water is taken notice of, as an argument for an *Allantois* by Mr. *Cowper*, to whoſe aſſiſtance we owe that the Figures belonging to theſe Papers appear correct.

* Harv. de memb. & humor.

The great * *Harvey* will not allow an *Allantois* even to Brutes, and fancies the *Allantois* and the *Chorion* to be the ſame Membrane, that has two Names, the firſt from its Shape, the other from its Office, or Number of Veſſels. Yet it is plain from *Galen* and all the Antients, that they meant two diſtinct Membranes by the *Allantois* and *Chorion*. Dr *Harvey* thinks that a *Fœtus* does not void Urine, but that the Bladder contains it, till the time of Birth. What was offered againſt *Dini*'s Opinion may ſerve for an anſwer to this alſo. Becauſe 'twas impoſſible for this diligent Anatomiſt not ſometimes to obſerve an *Urinary* Bladder, he has thought of ways to explain ſuch *Phænomena* without granting an *Allantois*. In † Sheep and Does, he had ſeen as it were a certain Proceſs between the Umbilical Arteries full of Urine. This Proceſs is no doubt the *Allantois*, though * *Bartholinus* calls it the *Urachus*. Again, he thinks what is called by others an *Allantois* (if it is not the *Chorion*) is ſome Coat accidentally formed from a Reduplication of the Membranes: becauſe (ſince every Membrane is double) Nature may, upon a ſtreight, lodge the Urine between a Duplicature. Yet he does not tell us how his Duplicature is to be filled, he allowing no *Urachus*. But in ſhort, this *Urinary* Bladder can be no Duplicature of the other Membranes, ſince in all Animals it differs from them, as to Figure, Texture, and in having an *Urachus*, which no other Membrane has. And ſince every Animal that has a Bladder, muſt have a like neceſſity for a Receptacle of Urine till born, ſince alſo the *Urachus* is ever alike inſerted in the ſame *Species* of Animals, and the *Urinary* Bladder conſtantly the ſame, as to Shape, Texture, Situation, *&c.* the *Urachus* and *Allantois*, with its † By-Water, can be no accidental or preternatural things.

† Harv. de memb. & humor.

* Lib. 1. cap. 37.

† Harv. de Hum.

Fig. 1. Repreſents the Secundines of Twins, to ſhew the *Allantois*, and its Relation to other Membranes, *&c.* after the Parts were prepared and dried.

Plate 5.

A A A A Part of the *Chorion* expanded.
B B B a Line, expreſſing the Edges of the *Placenta*.
C C C the *Amnios*, which is united to
D the *Allantois*, at
E E E the Line of *Union*.
F the *Cervix* of the *Allantois*.

Fig. 5.

Fig: 2.

I
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I
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Fig. 8.

Fig. 4.

Fig: 1.

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C
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L
L
R
R
R
Q
A Scale of Inches

G A Hole at the *Fundus* of the *Allantois*, whence the Urine came forth, and where the *Allantois* was blown up.

H Part of that half of the *Allantois*, which lies under the Line of Union, and immediately covered the *Fœtus's*, unleſs it is ſuppoſed that the *Amnios* is continued under the *Allantois*.

I I Two Stiles or Probes thruſt under the *Amnios*. They ſupport the *Allantois*, and keep open the *Aperture* * * * * of the *Amnios*, whence the *Twins* came forth.

K Part of the *Placenta*, with ſome Blood-Veſſels injected.

L L L L the Arteries of the *Navel-ſtring* filled with Red-Wax.

M M The Umbilical Veins filled with Green Wax.

N a Communicant Artery, by the means of which all the Arteries of both Navel-ſtrings were filled at once, and the Veins were filled by one Injection in like manner.

O a Pin that keeps out the *Amnios*, where from the Edge of the *Placenta* it runs partly to the Line of Union, or Adhæſion, and partly over the *Placenta*.

P Part of the *Chorion* at the Edge of the *Placenta*, where it runs under the *Amnios* on the *Placenta*.

Q a Pin that by a Thread helps to pull open the Aperture of the *Amnios*.

R R R the *Urachus*, lying between the Arteries.

a a a Fibres or Veſſels which faſten the *Allantois* to the *Chorion*.

Fig. 2. Exhibits a ſide-view of the ſame Preparation, that the Inſertion of the *Urachus*, &c. may be better ſeen.

N. B. That *A* and all the ſame Letters in theſe three Figures denote the ſame Parts in every one.

S ſhews the Courſe of the *Urachus R* at *F* in pricked Lines.

T part of the *Amnios* raiſed from the *Placenta*, to diſcover the *Placenta K* and *V*.

V that Part of the *Allantois* which is below the Line of Union, near its Neck *F*.

Fig. 3. Shews an entire *Allantois* of a very ſmall Abortion.

N. B. This *Allantois* was eaſily ſeparated from the other Membranes, between which it lay; and the *Amnios* remained an entire Bladder or Membrane under the *Allantois*.

Now ſome object, that what is called the Line of Union can be no real thing. As to this I don't know whether the *Allantois* of Twins may not require ſuch a Conjunction to ſuſtain, and keep ſteady a greater Quantity of Urine: nor can I reſolve whether the *Allantois* of Twins (like that of a ſingle *Fœtus* in Fig. 3.) may not be diſtinct, and ſeparable from the *Amnios*, but was not diſcovered by me to be ſo, thro' want of Skill or Care. However, the Reaſons why ſuch a Line was figur'd are theſe. 1. Although I uſed more Force, with equal Care, to ſeparate the *Allantois* in this Place than in any other (where nevertheleſs the Separation was very eaſie) yet I could not divide theſe Membranes farther than that Line. 2. This Line ſeeming ſo regular as to divide the *Allantois* into two equal Parts, I could not take it to be the Effect of Chance, or my Separation. 3. The Part *H* below the Line *E E* was alike in Tranſparency to that Part of the *Allantois D* above it. Whereas had the *Amnios* been joined to the *Allantois* (as the Objectors ſuppoſe) the *Allantois* below this Line muſt have appeared thicker than that part above it, ſince the *Amnios* alone is much thicker than the *Allantois*. 'Tis eaſie indeed to conceive the *Amnios* running an entire Bladder, or Membrane, under the *Allantois*, and perhaps it may be ſo; but I think it diſingenuous to conceal what I did obſerve, or to make out by Fancy, what I cou'd n't diſcover in Fact.

Others have thought this *Allantois* to be an *Amnios* of one of the Twins belonging to theſe Secundines. This Objection, though it may ſeem plauſible, yet it is of no Force. For firſt, this *Allantois* is much finer to the Touch, as alſo much more tranſparent, than the other *Amnios*; which ſtill remains ſtiff, whilſt the much thinner *Allantois* ſinks under the leaſt Blaſt of Air, notwithſtanding the ſtiles *I I* which aſſiſt it. 2dly, This *Allantois* had two viſible *Urachus*'s, and is of an Oviform Figure, ſomewhat like the common Cuts of a Man's Bladder (for a true Cut of a Humane Bladder I never yet ſaw, for it ought to be made much bigger (as it really is) at its *Cervix, &c.*) Alſo this *Allantois* no where touches the *Placenta*, unleſs at the Neck *F*. But on the contrary, the *Amnios* is of the ſame irregular Figure, as the Poſition, Motion, *&c.* of the *Fœtus* require. Likewiſe, it covers the whole internal Surface of the *Placenta*. 3dly, They who make this Objection muſt ſuppoſe ſome Hole in this Bladder, and in the *Amnios*, through which one Umbilical Rope may paſs from the *Placenta* to the *Fœtus*. But ſuch a *Foramen* would be preternatural, becauſe the Navel-ſtring only runs from the *Placenta* to the *Fœtus*, under a Coat taken from the *Amnios*, and lies with the *Fœtus* in the Cavity of the *Amnios*, that is no where perforated. 4thly, The Hole at the *Fundu G* was ſcarcely wide enough to receive the end of a Man's Finger, whereas the Twins did not want ſix Weeks of their full Time. Since therefore a *Fœtus* of near eight Months could not poſſibly paſs this Orifice, this Bladder could not be an *Amnios*.

Nothing in theſe *Secundines* is preternatural, only ſome things were not before obſerved. Hitherto Anatomiſts have not allowed Twins to lye in a common *Amnios*, but ſuppoſed each *Fœtus* to have a diſtinct *Amnios*. The Reaſon of this Opinion might be, that ſome, denying any Urinary Membrane, called every Membrane they found (except the *Chorion*) an *Amnios*, and theſe finding two Membranes in the *Secundines* of Twins, ſuppoſed them to be two *Amnios*'s. That others granting an *Allantois*, but not diſtinctly diſcovering it, but only two Membranes, alſo imagined them to be two *Amnios*'s. Both of theſe taking that for the *Amnios*, which might really be an *Allantois*. But ſince one *Chorion*, and one * *Placenta* (the || *Placenta* and *Chorion* being ever of the ſame Number) generally ſerve Twins (nay, ſometimes three

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* *Mauriceau* Lib. 2. cap. 4. *Need.* cap. 2. *Barth.* Lib. 1. cap. 36. *Diemerb.* Lib. 1. cap. 30.

|| *Diemer.* Lib. 1. cap. 31. *Graaf* cap. 15.

three *Fœtus's*) why ſhould it ſeem ſtrange, that one *Amnios* (at leaſt ſometimes) and one *Allantois* ſhould ſerve the like Number?

I am not ignorant that * *Mauriceau*, and || *Diemerbroeck* think there is an abſolute Neceſſary for every *Fœtus* to lie in a diſtinct *Amnios*, and that otherwiſe Twins in the ſame Membrane would grow together, and make a Monſter. * *Aquapendens* further ſays, that all *Ova Gemellifica* produce ſome other ſort of *Monſter*. Yet 'tis moſt certain that *Ova Gemellifica* do exclude two perfect Chickens, however not both alive. || The Great *Harvey* indeed thinks it poſſible, that ſuch an *Ovum* may produce a Monſtrous Chick, if its *Vitelli* are contained in the ſame Membrane, *&c.* yet does not poſitively ſay it muſt be ſo. For my Part I can't ſee any more Reaſon why Twins in one *Amnios* ſhould grow together, than that the Hands or Heels of the ſame *Ftœus* ſhould grow to its own Body. How can the Humors that lubricate a ſingle *Fœtus*, and help it to move, joyn two together? ſince the Humours are the ſame, and the Parts of the ſame *Fœtus* as tender as thoſe of Twins are, and lye as cloſe to one another, as Twins do. 'Tis very obſervable, that among all the Monſters we read of, there are very few, which ſeem to be made of two entire Bodies joined together, and that moſt of theſe upon * Diſſection were found to have but *one Heart*, *one Liver*, whence 'tis moſt plain, that theſe Monſters (and no doubt all others) were originally Monſters in the *Ova* before Impregnation, and not ſo from want of the *Amnios*. Yet * *Diemerbroeck* does not a little boaſt of having firſt (as he thinks) found the reaſon why Twins muſt lie in diſtinct *Amnios*'s. But ſince the Matter of Fact (ſometimes at leaſt, as in theſe *Secundines* there was only one *Amnios*, and two regular *Fœtus*) is not true, his Argument for a Neceſſity of two *Amnios*'s for two *Fœtus's*, will never prove valid, even where Twins, and two *Amnios*'s are found. Indeed any Part may be made to grow to any Part, as we ſee in the Cure of Hair-Lips, *&c.* but then the Fibres muſt be firſt broke, before there can be any Union. Now I can't conceive what ſhould naturally break the Fibres of the Twins in the *Uterus*. But although 'tis evident from what has been ſaid, that Twins may lie diſtinct in the ſame *Amnios*, yet there muſt be as many *Urachus*'s as *Fœtus*'s. In theſe *Secundines* I ſaw two running over the *Placenta*, to the Neck of the *Allantois*, which I communicated to ſome Phyſicians, before the Parts began to grow dry. The *Urachus* paſſes under the *Amnios* as the other *Umbilical Veſſels* do, and runs from that Part where the *Umbilical Rope* is ſet on to the *Placenta*, ſtreight to the *Cervix F.* S deſcribes the Courſe of that *Urachus* marked *R* at *F* in the ſecond Figure. The other *Urachus* lay about a quarter of an Inch laterally beyond that marked *R* in the ſame Figure. I mean by two *Urachus*'s, two long roundiſh Bodies, of a depreſſed Figure, they ſeemed as big as a Knitting Needle, and were of a darker Subſtance than the *Placenta* on which they lay. They appear'd in every reſpect, like that part of the Navel-ſtring, which is allowed by all Anatomiſts

to

* *Maur. Lib.* 2. *cap.* 3, and 4.

|| *Diem. Lib.* 1. *cap.* 31.

* *pag.* 19.

|| *Exercit.* 24. *de Generat.*

* *Parey, Lib.* 25. *Cap.* 2.

* *Diem. Lib.* 1. *Cap.* 30. & 31.

to be the *Urachus*, and in like manner ſhrunk in two or three Days, from a Mucous Subſtance to a meer Membrane. Theſe two are the only entire Urinary Membranes that I have prepared. Yet in the *Secundines* that have come to my Hands, I have ever found three diſtinct Membranes eaſily ſeparable.

A Boy that never made Water, by Dr. Richardſon *n.* 337. *p.* 167.

XV. *John Worſnape*, a poor Boy at *North Bierley*, *Ebor.* lived till he was ſeventeen Years old, and never made Water, yet was in perfect Health, vigorous and active. He had conſtantly a *Diarrhœa* upon him, but without much Uneaſineſs. The Obſtruction muſt have been in his Kidneys, for he had never any Inclination to make Water. The Serous Part of the Blood, which ſhould have been thrown off by Urine, was diſcharged by the *Cœliack* and *Meſenterick* Arteries, by the Mediation of the Glands into the Guts. He died of a Fever.

A Boy making Water by his Navel, by Mr. Yonge, *n.* 323 *p.* 423.

XVI. I had once a Boy about 6 Years old brought me, that piſs'd off almoſt all his Urine from his Navel.

One ſwallowing Iron, &c. *by Mr.* Amyard, *n.* 317. *p.* 12[illegible].

XVII. 'Tis now, *Jan.* 20. 1708. about 5 Weeks ſince an Idiot from his Infancy died at *Oſtend*, the Place of his Birth, in the 33d Year of his Age; his Death having been preceded with twelve Days continual remitting Fever, and a conſiderable Tumour and Pain about the Region of the Liver. His Brother, in whoſe Houſe he had been a conſtant Dweller, being deſirous to know the Cauſe of it, deſired Mr. *Ricks*, an eminent Surgeon (who at that time had the Cure of me) to open him; but he ſent his Son, likewiſe Maſter Surgeon of the ſame City, with his Servant; who did the Work in the Preſence of the Brother of the Deceaſed and a Nun, brought thither by the deſire of diſcovering the Cauſe of her Spitting or Vomiting of Blood, to which this Fellow had been very liable, (as well as to bloody Stools) for 6 Weeks before his Death. A large Abſceſs or Impoſthume was found in each Lobe of the Liver, whoſe Bulk did far exceed the ordinary Stint. The Stomach being open'd was found extreamly contracted and ponderous; and indeed it was thought no Wonder, when upon the aperture of it, was found a Bundle of the things following cloſely involved and embraced by the Stomach, *viz.* Nine Cart-wheel Nails, and ſix leſſer; a large and long Iron Screw; two pair of Compaſſes, the one having a Circle two Inches in Diameter; a middle ſiz'd Key; a large Iron Pin as big as my Thumb, and four Inches long, with a Ring at the end of it; another of Braſs, but much leſs; the Handle of an Iron Spring Knife (ſwallow'd as 'tis believed intire, but the ſides and two pieces making up the Spring of it, found aſunder; the Pegs of the Knife, tying thoſe ſeveral Pieces together, were not found;) the upper and lowermoſt end of a Braſs Pommel, inſervient to a Sea-coal Grate, weighing nine Ounces; a broad Piece of Lead weighing

ing three Ounces and a half: the whole conſiſting of 28 Pieces, weighing betwixt 2 and 3 Pounds. They were found all in a Bundle with the largeſt Ends one way, and the ſmalleſt the other; the ſmall End of one of the large Nails was ſo bent, that it would have made a perfect Circle, had not the tip of that ſame Nail been bent back again; this End was forked and wonderfully ſharp, as were likewiſe the Ends of the Compaſſes. None of the Pieces were found poliſh'd, neither cou'd I find the Braſs nor the Lead any ways impair'd or endamag'd; but the Iron pieces were extreamly corroded, eſpecially one of the Sides of the Knife, which had lain in the Stomach about 8 Months, was eaten quite through in two or three places, towards the Blade's End; and three or four Nails mightily indamaged did appear as if ſome particular Menſtruum or Diſſolvent had been poured upon them, capable only to diſſolve that Metal, as *Aq. regalis* has the Property to diſſolve Gold, *Sp. Nitri* Silver, Vinegar Lead, leaving thoſe other Metals joyned and alienated with them untouch'd: the Lead had lain in the Stomach about eight Months, and the Braſs Pin above-mentioned above twelve. It was very eaſy to gueſs at the time thoſe different pieces of Iron had been in the Stomach, in conſidering how much one piece had ſuffered more than the other. This Obſervation is like to give a check to the Notion of thoſe who believ'd that Oſtridges did diſſolve Braſs and Iron by Friction only; for if ſo, I ſee little reaſon why the Iron Branches of the Compaſſes ſhould have been found ſo very much worn out, and the Braſs Branches not in the leaſt impair'd. Mr. *Ricks*'s Son, who open'd him, told me, That the Stomach had been no ways wounded or indamag'd; which does not appear to me probable, when the Patient wns known to have vomited and evacuated Blood by Stole for ſix Weeks before he dy'd, as I have already mention'd It's neceſſary to remark, That this Fellow, from his Youth, had accuſtomed himſelf to ſwallow large Morſels, Glutton like, and without Chewing; which, no doubt, made the Paſſage of the *Oeſophagus* wider, and diſpoſed it to give Entrance to all thoſe Extraneous Bodies. This Idiot, and ſometimes Mad Fellow, was never known to ſleep a Wink, tho' he was often compell'd to go to Bed, and had, to incline him to Sleep, been very much harraſs'd and fatigu'd before: he was always known to eat three times as much as the reſt of Mankind, and when furious, to grow quiet upon the approach of Meat.

Hydatides voided by Stool, by Dr. Muſgrave *n. 295. p. 1798.*

XVIII. In *April* 1708. I was deſired, to ſee a Woman of *Tiverton*, named Mrs. *Pear*. She is about 30, of a tender Conſtitution; had an ill habit of Body, and about Candlemas laſt a Fever; which continuing near three weeks, was at length overcome by Teſtaceous Powders, *Alexipharmacs*, but chiefly by the *Cortex*. In this Fever ſhe had ſowr Vomitings, and a pain in her Stomach; which remain'd long time, and, after the Fever, was accompanyed with a copious Salivation; with Wind, and pains in her ſide, to a degree extraordinary; under all which ſhe labour'd to the time of my ſeeing her. About three weeks before my Viſit, ſhe was ſeiz'd with a Jaundice, and while taking Medicines (*Pilulas & Decoctum Ictericum Fulleri*) for that Illneſs, ſhe brought off ſeveral *Bladders, by Stool*; and continued ſo to do, ſometimes every day, at other times once in two or three days, ever ſince the firſt diſcharge of this kind, to the time of my Viſit. Theſe Bladders were of various ſizes; the leaſt that came off, was of the bigneſs of a great Pins head; the largeſt, equal to a Pullets Egg: They were alſo of differing Colours; ſome white, others more yellow, from the Liquor contain'd in them, which was a ſort of Gelly, like that of Hartshorn, ting'd more or leſs with Saffron.

Before the Diſcharge of theſe Bladders, there was (beſides the Symptoms already mention'd) a Coldneſs, and Sickneſs at Stomach, almoſt perpetual; with frequent Inclinations to Vomit, and Hyſteric Suffocations. Since that Diſcharge, theſe Symptoms are vaniſh'd, and ſucceeded by a Soreneſs of the ſame part, as if ſomething had been torn there. The Bladders came off without pain; many of them whole and entire; one of which I ſaw about the bigneſs of a large Gall, or Marble Stone. Others were broken, and appear'd not unlike the empty Skins of Corrants, Gooſeberries and Plumbs. Only one Bladder came away by Vomiting, and that broken; but to all appearance, had been as large almoſt as a Gooſe Egg. The Gelly thrown up with this Bladder, and which in all likelyhood had been contain'd in it, (before it broke in coming up) was thicker, and more fœtid, than was found in any of the other Bladders. The number of thoſe which came off by Stool, made ſeveral ſcores.

During the whole courſe of this illneſs, the Patient was rather looſe than Coſtive; had no manner of Appetite, and ſeldom ſlept without an Opiat. I found her much waſted in Fleſh, with a dead pale Look; ſuch as argued her to be very low. She had Stools of an unuſual Smell, no way natural, and had vomited a great deal of cold Phlegm. She was very willing to think theſe Bladders came from her Stomach, and urg'd the following Reaſons for her opinion; Firſt, for that, had they been originally in the Bowels, in all likelyhood the Purges (of which ſhe took many in the Jaundice) would, as ſhe ſaid, have carried them off much ſooner. Secondly, From the (almoſt) conſtant pain of her Stomach, and frequent inclination to Vomit, ever ſince her Fever, to the time of the Bladders being Diſcharged. Thirdly, From the Rawneſs and Soreneſs of her Stomach, after the Bladders came off. There was no appearance, in any one of theſe Bladders, of ſuch an Order of Parts, or Organs, as ſhew'd them to be Inſects; nor upon Examination was there any Animal diſcern'd by the naked Eye, in the Liquor contain'd in them: The Medicines given, after I was call'd in, were chiefly of Vulnerary and Digeſtive kinds: That which did her moſt ſervice, (but it was after the Bladders were come off) was Tincture of Myrrh and Gentian, in large and frequent Doſes; and with a proper Vehicle:

Under the uſe of this ſlight, but advantageous Medicine, from a very weak condition, ſhe recover'd an Appetite, *&c.* and is now perfectly well.

Hydatides void-ed by Urine. n. 273. *p.* 897. *by* Dr. Davies.

XIX. A Gentlewoman between 40 and 50 years of age, in the Autumn of 97, drank ſome Aluminous Waters for a month or five weeks, and in a months time after the uſe of theſe Waters, found a pain in the Renal region, where ſhe never had been afflicted with any before; this pain returned after the firſt Paroxyſm in about a months time, and afterwards more frequently, till about the *Chriſtmas* following it viſited her every day; about which time ſhe ſent for me; and had when I came to her the Symptoms of a Stone in her Left Kidney, *viz.* a grinding, and ſometimes a very acute pain on that ſide of the *Spina Dorſi*, a Vomiting, her Urine during the Paroxyſm tinged with Blood, and in it Bloody *Ramenta*; but, what moſt ſurprized me, a dozen at leaſt of *Hydatides*, ſome of the biggeſt of them $1\frac{1}{2}$ inch long, their circumference equalled that of an ordinary Gooſe-quill; in ſhape they exactly repreſented the *Veſiculæ Natatoriæ* in Fiſh, growing ſmaller about the middle, as thoſe generally do, and were filled with a Liquor, which my taſte and ſmell made me believe to be Urine; I never diſcovered any Pus in her Urine, nor had ſhe any pain at the Sphincter of the Bladder, nor in the *Meatus Urinarius*, either before, at, or after making Urine. The Paroxyſm generally laſted 3 or 4 hours; as ſoon as theſe *Hydatides* came away, (which they did not all at once making water, but at ſeveral times) the pain in her Back, *&c.* abated very ſenſibly, and ſhe continued eaſie and well the reſt of the day, excepting an external ſoreneſs, which the pain had cauſed. I thought theſe *Veſiculæ* at firſt to be Membranous, ſince their conſiſtence was ſo tough as to bear taking out of the Chamber-pot and gentle handling, but afterwards was convinced that they ow'd their origin to a glutinous ſlimy matter, becauſe upon long ſtanding in Urine or fair Water they quite diſappeared and were diſſolved, making the Water or Urine to look thick and turbid. By the uſe of Medicines all theſe Symptoms diſappeared, and ſhe continued well when I laſt heard from her, which was about two months ago.

One ſwallowing Stones, by Sir Charles Holt, *n.* 275. *p.* 995. * *See Abr. Vol.* III. *p.* 92.

XX. *Gobſill* (whoſe caſe you printed in the Tranſactions for *June*, 1699. Numb. 253. *) came lately to me, and told me the Stones grew very troubleſome to him; that he had of late vomited up two of them, which he ſhewed me, and I cauſed them to be weighed. One weighed two Drams, and the other one Dram two Scruples and a half. He complains, that his Strength is of late much impaired; that he voids great quantities of Blood by Stool, which keeps him very weak. His Stomach is much decayed, and will retain but few things. His Hands are palſyed; always extream cold, and his Fingers contracted; he is not able to open them without Help, or keep them ſo, unleſs by

Force. His Legs are very likely in a ſmall time to be as uſeleſs to him as his Hands, for, he ſays, they begin to fail him, and in the ſame manner, grow cold, and have little ſenſation in them. But the moſt remarkable of all his complaints, was, a new Progreſs the Stones had either found or made. Formerly at night in bed, they us'd to get up (as he expreſs'd it) to his Heart, and upon turning to his Knees, or ſtanding upright on his Feet, they would drop one by one ſo diſtinctly, that they might be counted, and in this ſtate they always aroſe ſtraight up on the right ſide of his Breaſt; but now they riſe obliquely, and get under his right Arm, inclining towards the *Scapula*, and when they are in this place, by giving him a Blow with the Fiſt on his right Shoulder, they will all fall down in a Lump together, and may very plainly be heard to claſh on the other Stones, which lye as they did formerly juſt above the *Os Pubis*. I made the experiment before Dr *Fowke* and Dr *Davies*, and the matter of fact proved true as he related it.

Prune ſtones found in a Body. by Mr. Vaughan. *n.* 281. *p.* 1244.

XXI. A Gentleman about a Month before Chriſtmas eat ſome common Prunes, about two pound, or more, and about a Month or five Weeks ago he eat about a pound more: About a Fortnight before he died I found, according to his Complaint, that he had ſome Symptoms of the Stone. He had a violent Pain in the *Veſica*, and about the *Urethra*, with Obſtructions in his Urine, &c. I ordered him a Terebinthinate Clyſter, which gave him Eaſe; but ſeeing his Pains encreaſed, I adviſed him to a Phyſician: He made uſe of one of his Acquaintance, he likewiſe preſcribed Clyſters, with Diureticks and Narcoticks, to no Purpoſe. Upon his Death, I obtained Leave of his Relations to diſſect him; accordingly I did with the Help of a Surgeon, Dr. *Weſt* being preſent, with ſeveral others of the deceas'd Patient's Friends. I found upon the Diſſection, the Prune-ſtones paſſed into the *Inteſtinum rectum*, and had there made a Perforation, or Rupture into the *Pelvis*. We tyed one Part of the Gut, and cut out a piece, and emptied it. There was taken out 128 Prune-ſtones in number, beſides what we left behind *in ſtercore*, in the other part of the *inteſtinum Rectum*. There was likewiſe a large *Polypus* taken out of the Left Ventricle of the Heart, &c.

A Ball with a Plumb-ſtone in it, extracted by Mr. Yonge. *n.*282.*p.*1279.

XXII. *Sarah Swayn* of a thin habit and middle ſtature, when but ſix years old, was firſt afflicted with a violent pain, together with a large hard ſwelling on the left ſide of her Belly, which laſted twelve hours, and then went off without uſe of any Remedy or ſenſible evacuation; and at the end of three months returned, laſted and went off, as before. Several years it obſerved that Period, and then changed its intermiſſion from three months to 3 weeks, and ſo continued till ſhe was 35 years old, in which time ſhe married, and bore one Child, the pain of which ſhe averr'd to be much leſs, than what theſe Paroxyſms gave her. During her pregnancy, her Pains nor Intermiſſions had no alteration, and in her whole Life ſhe found no Diet diſturb'd her but Milk and Salt

Meats. About 9 months before ſhe was cured, the Pain and Tumour encreaſed to the bigneſs of a mans two Fiſts, ſhe endeavoured by many Remedies to get eaſe, but in vain, till the torment and watching had ſo weakned her, that ſhe could not riſe out of her Bed nor lie down in it.

In this deplorable condition ſhe was adviſed by a Woman to take a doſe of Powder'd Jalop, but it operated violently, and ſuddenly drove the pain from her ſide down to the *Anus*, where it reſembled a *Teneſmus*, *viz.* a conſtant and violent inclination to ſtools without being able to force off any thing, and after ſhe had been thus tortur'd four days her Urine ſtopt, and two days after that the charitable Neighbours, who had all along given her their beſt aſſiſtance, craved mine. I perceived by their report of the matter, that ſomething obſtructed the paſſage of her Excrements, and ſoon found it ſo by a probe; I then anointed the paſſage with *Popu'keum*, and taking hold of the ſubſtance with a pair of large Forceps, made to extract Stones from the Bladder after Lithotomy, I drew it forth. Abundance of Wind and Excrements guſht out, and continued to flow till her Guts were emptied of all the matter, which had been ſo long retained; after which I ordered her an Anodine Clyſter, and a compoſing draught, and ever ſince (being ſeveral years) ſhe continueth well.

The thing extracted was round, ſomewhat oblong, having on it ſome ſuch impreſſions, as mens Fingers make on Wax or Plaiſter. It then weighed 10 Drachms, now ſcarce an Ounce, it was 5 inches in circumference, and altho it felt and otherwiſe appeared a Stone, it ſwam on Water, which made me, ſee the inſide of it, by cutting it in two with a Knife; externally it was black, and ſmooth as if varniſht, and no thicker, which made me before I had conſidered the whole, think it the expanded skin of the Plumb indurated; next to this thin blackneſs was a cruſt of matter like Brick, the thickneſs of an half Crown, within that appeared a ſubſtance reſembling Paſte-board or chewed Paper, and within that lay a Prune or withered Plumb, with the Stone and Kernel cut aſunder by my Knife. See Figure 3.

Plate 5. Fig. 4, 5, 6.

Inſtances of other perſons who were hurt by ſwallowing of Plumb-ſtones, by Sir Hans Sloane. *n.*282. *p.* 1283.

XXIII. A Man in *Lancaſhire*, being for many Years ill of the Cholic, and receiving Relief from no Medicine, deſired he might be diſſected after his Death, to ſee what might be the Cauſe of his Diſeaſe. This was accordingly done, and they took out of one of his Guts a large Ball 6 Inches about, of an Ounce and half Weight made up of a Spungy Matter which ſwims in Water, and viewed by a Microſcope, appear'd to be made up of very ſmall, tranſparent Hairs or Fibres, wrought together, after the manner of the *Tophus Bovinus* taken out of the Maws of Oxen. In the Middle or Center of it was a common Prune or Plumb-ſtone, which had been ſwallowed, and ſticking ſomewhere in the Guts, had gathered that Subſtance about it which reſembled the ſmall Hairs on the Skins of ſeveral Creatures or Fibres of Plants we eat. Dr *Charles Leigh* in his natural Hiſtory of *Lancaſhire*, &c. in his firſt Table has figured this Fig. 4. He ſhewing me the ſame, I was deſirous to ſee what it was made of, and had it cut for that purpoſe, and found it of a hairy or fibrous ſubſtance, layer upon layer, or coat upon coat, over a Plumb ſtone. This Ball with its Plumb-ſtone and ſeveral Coats, my very good Friend Dr *Leigh* was pleaſed to give me, and it is now in my poſſeſſion, and ſeems to be of the ſame ſubſtance with that mentioned by Mr *Yonge*.

The ſecond inſtance I ſaw of theſe Balls was by the means of Dr. *Will. Cole*, who did me the favour to ſhew me a Letter he had from the Country, and ſome ſmaller Balls than the 2 before mentioned, which had in their centers Plumb-ſtones. The perſon he was conſulted for, had, I think, the Colick to a great degree, and had voided ſeveral of them, they were not ſo Sphærical, but of a compreſs'd figure, ſmooth on the outſide and glaz'd as ſome of the *Tophi Bovini* are; and ſeemed within of the ſame ſubſtance with the former *ſtratum ſuper ſtratum* upon a Plumb-ſtone.

A third inſtance like theſe I ſaw, through the favour of the late Dr. *William Stokeham*, who ſhewed me a Ball about the largeneſs of that I had of Dr *Leigh*, which had been been voided by a perſon after great Sickneſs, and preſerved by the Patients Phyſician, who was one famous for practiſing Chymically ſome years ago, *viz.* Dr *George Thompſon*. I had this laſt Ball in my Poſſeſſion ſome time, and in appearance it was of the ſame ſubſtance, but what was contained in it I could not tell, not being permitted to open it, but that Author tells us it had ſeveral Plumb and Cherry-ſtones, in it. Theſe Balls ſeem to be form'd ſomething after the manner of *Bezoars*, which generally have ſome Seed for their Center or *Nucleus*, on which Coats of another ſubſtance are gathered.

I once ſaw as ſtrange a Diſtemper almoſt as obſtinate and long, as I ever met with, proceed from a great quantity of Strawberry ſeeds, which had lodged in the Guts, and after their Diſcharge, the Perſon was eas'd.

Of ſwallowing Bullace and Sloe-ſtones. by Mr. Derham. *n.* 349. *p.* 484.

XXIV. The Man-ſervant of a neighbouring Clergyman, complained of exceſſive Pains in and about his Stomach; that he lay under a Dejection of appetite; and whenever he eat, he could not retain it, but in a little time vomited it up. By which means he was in a ſhort time brought to a very low and languiſhing Condition, inſomuch as they begun to deſpair of his Life. Upon this, he applyed himſelf to ſome Practitioners in Phyſick; one of which ply'd him with ſtrong Vomits eight Days together, with very little Signs of Succeſs. But ſome time after, having Occaſion to ride ſomewhat more than ordinary, he found himſelf very ſore in his Stomach, and Sick; which ending in violent Vomiting and Straining, brought up the firſt Stones he ever perceived to come from him, which were about Twenty in number,

ber. After this he had frequent returns of the Vomiting up of *Bullace* and *Sloe-Stones*, eſpecially upon ſtrong Exerciſes; particularly moving and ſtooping much in Weeding in the Garden; in Riding alſo, although it was only to water his Maſter's Horſe. Upon theſe Occaſions he would be ſeized with acute Pains in his Stomach, and ſoon after Vomit up more of thoſe Stones. He hath counted above One hundred and twenty *Bullace* and *Sloe-Stones* that have been diſcharged; and many others he could not number, by reaſon they came up when he was in Riding or in his Buſineſs. He is not yet free of them, but is in Pain oftentimes, and Vomits them up, eſpecially in Riding; but after he hath diſcharged them, he is much eaſier for a while. He commonly brings up a ſlimy Matter with them, mixed with Blood or ſomething very like Blood.

The Cauſe of all this Diſaſter the Man aſſures himſelf was, that being in his Youth a great lover of Fruit, he uſed greedily to devour all ſorts he could come at, and *Bullace* and *Sloes* being the eaſieſt to be gotten, he uſed to ingurgitate great quantities of them, without evacuating many of the Stones by Stool, as he well remembers, and as he obſerved others did. Theſe Stones he thinks have lain in his Stomach (ſome of them at leaſt) above ten Years; but he felt no Pains till about four Years ago. And thoſe at firſt were not ſo violent, nor attended with ſuch ſevere Fits of vomiting, and loſs of Appetite, as they by degrees came to be afterwards.

Large Stones voided per Urethram by Dr. Bullen. *n.* 295. *p.* 1804. *com. by Mr.* Lhwyd,

XXV. In agro Ceſtrienſi, loco non longè a vico Malbano diſſito, quidam Thomas Olton annum jam ſeptuageſimum octavum, & quod excurrit, agens, homo ſane probus, & tàm re quam famâ ſpectabilis per multos retrò annos vixit: qui quidem Vir cum veſicæ calculo graviter cruciatus eſſet, opem a me aliiſq; Medicis circumdegentibus non ſemel petiit: eratque a nobis variis medicamentis pro rei exigentiâ, ſed inutiliter tractatus. Tandem verò in uno paroxyſmorum præ omnibus aliis atrociſſimo, calculos duos, benedicente Numine, per Urinæ ductum excrevit: quorum formam exhibet *Fig.* 7.

Piate 5.

Calculum primò egeſtum dolor ferè intolerabilis comitavit, ſecundum verò vix ullus, qui moleſtiæ ſenſum crearet; nec mirum, ſi Urethra a lapide priùs emanante adeo diſrupta, & lacerata, alteri jam tranſituro facilem & patentem præberet exitum. Duo autem hi calculi unum aliquando in veſicâ conſtituebant; quòd ex fracturæ, ſi committantur, convenientiâ, palam eſt. Iſta via, ex neglectâ Chirurgiâ, jam uſque patet; cui quidem incommodo Cornu peramplo, radici penis aptato, quoties Urinam mittit conſulere coactus eſt; ne ſecùs tam ſua, quam adſtantium veſtimenta inter mingendum, inquinaret.

Another Inſtance. by Mr. Thoresby. *n.* 336. *p.* 539.

XXVI. *Joſhua* the Son of *Thomas Spurrit*, upon the *Quarry-Hill* near *Leeds*, having been for a long time ſadly afflicted with the Stone, was the laſt Year in an extraordinary manner tormented. I have three Stones that he voided, which are of a great bigneſs to paſs the *Penis*, and five more that he could not get rid of without the kind aſſiſtance of Mr. *S. Pollard*, a Surgeon, who by an Inciſion made way for them, as they came ſeverally near the *Glans*. When ever one of theſe great Stones broke out, there was a Crack within his Body, as if the Sphincter Muſcle, or Bladder itſelf, was rent. The Youth being diſſected, there were found in the top of his Bladder (which was contracted like a Purſe) two prodigious large Stones; one eſpecially which I meaſured, and it was rather more than 5 Inches and a half one way, and 4 the other; it weighed two Ounces, wanting 3 Drams: The other ſeems lighter, and weighs but one Dram above an Ounce. There were two very odd Stones taken out of the Right Kidney; the Left was wholly degenerated into a kind of Mucilage: And betwixt the Neck of the Bladder and the end of the *Penis* (which was mortified thereby) were lodged no leſs than half a dozen ſuch Stones as this herewith ſent you. There was little Moiſture left in the Bladder; the Ureters being broke off, and almoſt wholly conſumed.

A Stone in the Ductus bilarius, *cauſing the Jaundice. by Dr.* Muſgrave. *n.* 306. *p.* 2233.

XXVII. I ſaw at *Cliſton* in *Dorſetſhire*, a Stone voided, ſome years ſince, by Stool; which was repreſented to me by Mr *Harvey*, as having come from the *Ductus communis bilarius*: But the Largeneſs of it is ſuch, as made the latter part of the account ſeem, at firſt hearing, ſomewhat dubious. Its Figure is Oval; the Length almoſt an Inch; the Breadth, (or ſhorteſt Diameter) $\frac{7}{10}$ of an Inch: It weighed 59 Grains, when I ſaw it; but, at its coming off, was (as I am inform'd) above a Dram in weight; Some part of it being, by frequent handling, rubb'd away. The Surface rough, unequal, divided into ſeveral little Riſings, each about the ſize of half a Vetch, or ſomewhat leſs. *Y. Z.* ſhew the Proportions exactly drawn. The many ſtrong annular Fibres, which appear not only at the Orifice, where the *Ductus communis* opens into the *Duodenum*; but alſo all along the oblique paſſage, of that *Ductus*, between the Coats of the Inteſtine, (which paſſage is, according to Dr. *Gliſſon*'s meaſure, about half an inch in length) do, by way of Sphincter, keep this end of the *Ductus communis* very ſtrait and cloſe. And beſides this ſtraitneſs of the *Ductus*, the two Oblique Inſertions, it makes at ſome diſtance from one another, thro' the two outer Coats of the *Duodenum*, render it yet more difficult, for a ſubſtance of any Bulk, to paſs this way. So that, however great Stones may be generated in the Gall-Bladder, *Ductus Cyſticus*, *Hepaticus*, or *Communis*, it is not eaſy to conceive, How a Stone of the Magnitude here deſcrib'd, could poſſibly, through a paſſage of itſelf ſo very narrow, ſtrait, and difficult be conveyed into the *Duodenum*.

Plate 5. *Fig.* 8.

To prove, That this Stone was not form'd in the *Fiſtula alimentaris*, but (large, as now it is) came this way into it, the Gentleman was pleas'd to let me know, That, before the Diſcharge of this Stone, He had the *Jaundice*; which came ſuddenly on him, and continued ſeveral months, in a ſevere, and moſt exceſſive manner. That this *Jaundice*, beſide the diſcolouring of his Urine and Skin, to a very great degree; beſide Loſs of Appetite, Faintneſs, and many other Symptoms, uſual in this Diſtemper; was alſo accompanyed with a Pain (in or) near the Stomach.

That, during this *Jaundice*, his Stools were of a white colour, as having very little, or no Mixture of Choler in them. That, Travelling under theſe circumſtances, more eſpecially with a conſtant Pain, (as before mention'd,) in his Coach from *London* to *Clifton*; and, after a little time, to *Bath*; he found, a little after his Arrival at *Bath*, this Stone come off by Stool; and, together with it, almoſt a Spoonful of *Gravelly Matter*; and a conſiderable quantity of Choler, as appear'd from the yellowneſs of the Stool: All which happen'd ſo ſoon after he came to *Bath*, as evidently to prove, the Diſcharge of both [Choler and Stone] to proceed from the motion of the Coach. That his deliverance, from the *Jaundice*, commenc'd from the Expulſion of this Stone: For, ſoon after that, the Colour of the Skin and Urine, indeed all the ill Symptoms vaniſh'd; and, in a very little time, (Weakneſs only excepted) He recovered.

Theſe Propoſitions, put together, make a conſiderable Argument, That the Orifice of the *Ductus communis* (how ſtrait, and how ſtrong ſoever) was, in this Gentleman, ſo far dilated, as to give way to the Stone, here deſcribed; that is, dilated to a Circle, in Diameter $\frac{7}{10}$ of an Inch, in Circumference one whole Inch and $\frac{1}{4}$.

The *Jaundice* is often obſerv'd to be a moſt ſtubborn Diſtemper, not eaſily yielding to our moſt probable Methods; and many times to none at all. *Riverius* poſitively affirms, That, when it proceeds from a Stone, obſtructing the Current of the Choler, it is incurable: Urging this reaſon for his opinion; **Calculus, cum diſſolvi non poſſit, morbum facit incurabilem.* When the *Jaundice* is thus difficult of Cure eſpecially when there is a probability (whether from a Pain fixt in, or near the Region of the Liver, or from any good Argument whatſoever.) That it ariſes from the Cauſe now mentioned; rather than to beat over the ſame ground to no purpoſe, or other ground equally improbable; it may not be amiſs, to adviſe Exerciſe on Horſeback, in Coach, or any other ſuch way, as ſhall be likely to diſlodge the Stone, and bring it off. But, to make this Exerciſe effectual, it ought to be Violent, as the Patient can well bear it; and in ſuch manner, as may, by much agitation of the Body, be moſt conducing to the Deſign in hand. The Hiſtory, here mentioned, does ſufficiently recommend this *Gymnaſtic Courſe*; as capable of relieving in ſome Caſes of the *Jaundice*, when the beſt methods of Phyſick (for ſuch we ought to ſuppoſe this Gentleman had preſcrib'd himſelf) fail of ſucceſs.

**de Ictero*

Large Stones evacuated by Mr. Thoresby. *n.* 304. *p.* 2164.

XXVIII. Some large Stones were taken from *William Coldell* of *Green*, *May* the 10th, *Anno Dom.* 1693. They weighed 9 Ounces when firſt

O evacua-

evacuated, and were removed by Diet Drink with an Alkaly Powder, and a Magiſtral Stomach Plaiſter; that the Perſon died 7 years after, of one too large to be evacuated; for upon the griping of it, betwixt the Hypocondrias and Share-Bone, it felt to be as large as a Gooſe Egg.

Several ſolid bodies voided by Urine. com. by Mr. Yonge. *n.* 323. *p.* 414.

XXIX. *Nathaniel Mitchell* of *Loo* in *Cornwall*, aged about 50, was in the Summer 1690, ſeized with violent Colical Pains which he mitigated by Clyſters, but could not perfectly free himſelf of them. About *Michaelmas* 1691. his Pains being very violent, he was relieved by the ſame Remedy; and by the perſuaſion of a skilful Woman, he drank the Powder of Nettle-roots in White Wine; After the firſt or ſecond Doſe he diſcharged a great quantity of Urine, with a very feculent Sediment. About the beginning of *November* 1691. being Coſtive, he eat Mallow roots and Corinths boil'd and mixed with Butter, (his uſual Medicine to render him Laxative.) In a little time after eating it, he was much diſordered, and complained of an Oppreſſion by Wind; at length the Wind (as he termed it) ſettled at the bottom of his Belly, and in a very little time with his Urine he emitted ſome of the Herbs, with above 40 Corinths: A few Days after he piſs'd off ſeveral Parſley-Leaves, which he had a little before eaten. I was called to him about the 12th of *November*, when his Urine being ſhewn me, I thought that part of his Excrements had been evacuated that way, and that ſome Latent Ulcer had made a Paſſage through the *Inteſtinum Rectum* into the Bladder, but found it otherwiſe; for there was no Fætor in the Urine, he had no *Teneſmus*, nor bloody, nor purulent Dejections; but to ſatisfy my ſelf further in this Particular, I ordered him a Clyſter tinctured with Indigo, which he retained above half an hour, but his Urine was not at all diſcoloured with it. I preſcribed Pills of——, two of which came off in his Urine *November* 18. in an oblong form, about the bigneſs of the end of the firſt Quill in a Gooſe's Wing. The Pills I have by me, except the half of one, which I rubb'd abroad with my Fingers. Some time after he piſs'd off a piece of a Raiſon. He lived 'till Midſummer 1692. in which time he ejected at divers times parts of Roots, and other things he eat.

His Wife reſiſted all the Importunity that could be made to have his Body diſſected: So that a great Secret was buried with him.

Stones voided by Stool. by Dr. Holbrooke. *n.* 325. *p.* 28.

XXX. 1. One *Crumbleholm* came to me ſometime ago, and complain'd of a great loſs of Appetite, with Scorbutick Itch, and ever and anon ſevere Convulſive Cholicks below his Navel, all along the *Hypogaſtrium*. They laſt not above a quarter of an Hour, but often return, and raiſe Tumours the bigneſs of a large Walnut, which diſappear and remove as the Pain ſhifts. He has been troubled with it ſome Years, and took Phyſick of almoſt every one he met with; but, as far as I can perceive, not in any regular Method, which gave me ſome hopes, that I might relieve him. Accordingly I began with mild Emollient and Carminative

native Clyſters; purged with *Decoct. Sen. Gereon. Syr. de Spin. Cervin.* & *Tinct. Sacr.* In the Intervals of the Purges I gave *Æthiops Mineral*, with bitter Decoctions Alterative, made more Carminative with *Rad. Zedoar.* and *Caſtor.* He was relieved for that purpoſe; his Appetite and Complection mended, but preſently was as Ill as ever. Then he ſhew'd me the Stones voided by Stool, upon a ſlight Mercurial Purge, which he took laſt *Eaſter.* Upon opening one of them, I found he had ſwallowed either ſome Plumb or Apricock Stones, which by their ſtay in the Inteſtines were incloſ'd in the Excrements, as I take it; and, by the Purge being diſlodged from their *Sinus*, ſent forth, as you find. Hoping then that by ſtronger Evacuations, if I could remove any other that might remain, it might tend to his Cure, I order'd ſtronger Medicines. However, I could not get any more from him; and he being out of hopes, and uneaſy to be kept any longer from his Buſineſs, has left off taking any thing. Laſt Week I ſaw him, and found him much in the ſame Condition, tho' ſomewhat weaker, and ſunk more in his Fleſh.

An Anſwer. by Dr. Cole. *ib. p.* 30.

2. I look upon theſe Stones to be not formed of *adhering Excrements*, as you ſeem to ſuppoſe, but to be made thus. When the Plumb-Stones happen to be included in a fit Glandulous Receptacle, I conceive they may come to be thus coated over by the viſcous Liquor ſecreted out of the Secretory Ducts of thoſe Glandules, which by long lying there may come to acquire ſo great Bulk, by the continual appulſe of the ſame Liquor. This Receptacle I gueſs to be the *Inteſtinum cæcum*, which tho' ſmall naturally, may be, as other Membranous and Glandulous Parts are, capable of a conſiderable Extenſion: So that, when by reaſon of the *Periſtaltick* Motion of the *Inteſtines* above, one of the Plumb-Stones may happen to be, by its pointed Extremity, intruded; the whole may, by the ſame repeated, tho' ſlow Motion, dilate the Cavity ſo, that the whole Body of the Stone may by the ſame Method be ſtill farther and farther protruded, till it come to the further Extremity; which being cloſed muſt be preſumed to detain it there, ſince 'tis hard to conceive it can quickly get out again, that *Periſtaltick* Motion being always forward. One of theſe Stones being thus enter'd, 'tis eaſy to conceive more may be admitted, ſince the firſt cannot but dilate the Paſſage for another that follows, and ſo on till the Cavity be full. Whilſt theſe Stones lye there, they muſt be conceived to offend the part, as having extended it beyond its Natural State: So that the Secretory Ducts of the Gland, of which the inner Coat of that, as well as the reſt of the Inteſtines, is conſtituted, muſt be proportionally dilated; whereby an eaſier way is made for the Liquor they ſeparate, to be excreted. This being of a viſcid and concreſcible Nature, muſt, ſince it cannot get forth, be preſum'd to adhere to the *Subſtratum*, the Stones, and ſo by degrees Incruſt them; which Cruſt by the long confinement muſt grow ſo much thicker, for the ſame reaſon as it began, the Ducts being kept conſtantly open, and the Cavity more and more

dilated, the greater the Incruſtation is. So that I conceive the Symptoms are eaſily accountable for, from the offence given to the part, which being ſenſible, as all Membranous and Fibrous parts are, the Pain muſt grow greater, the greater the Extenſion is; and the change of the Poſture of the Tumor may very well be conceived to proceed from the different Poſtures the Inteſtines put on, by the Chyle or Excrements paſſing along them, and ſometimes filling one part, ſometimes another, as they are protruded further and further, their Lubricity on the Surface, Length, and Confinement obviouſly favouring that *Phænomenon*. I am of opinion, the true *Bezoar Stones* are form'd in the Beaſts, that yield them, in the ſame manner; but whether their Stomachs or Inteſtines have other Cavities capable of receiving and retaining them to their full growth, is to be determined by Anatomy. This I think is certain, that all of them have either a Straw, Stick, or other Subſtance different from the Exterior Matter, which we call the Stone, in the middle of them; and thence I conclude the manner of their Formation to be the ſame. From the continuance of his Symptoms, I believe there may be more behind; and cannot think any other Method more likely to extrude them, than by having his *Abdomen* well anointed with ſome Emollient Oyls or Liniments, and very well agitated backward and forward as much and as long as he can bear, and this both Morning and Evening: After a little while, that the Stones may be preſum'd by this agitation to be ſomewhat diſlodged, ſome gentle Purgatives I conceive may be of uſe to be now and then given to carry them downwards, and with all Emollient Clyſters to ſollicit it gently.

A Ball voided by Stool, by Mr. Thoresby. *n.* 291. *p.* 1595.

XXXI. A Girl about 14 years of Age, having been tormented with Colical, and as was ſuppos'd, Nephritick Pains for ſome time; at length voided a roundiſh Ball *per Anum*, as hard to feel upon as a Stone. After a while, the Pains returning with greater violence, ſo as to make her roul upon the ground, ſhe voided another as hard, and much bigger. Upon which, one Mrs. *Ward*, a neighbouring Gentlewoman, who had been much afflicted with Gravel, gave her ſome of thoſe Medicines which ſhe us'd to take herſelf. Whereupon the Girl voided a third Ball, alſo *per Anum*, with leſs pain, yet the greateſt of the three. The firſt of theſe Balls is ſmooth and gloſſy, of the colour of a right Hazel nut, 3 inches about, and ſomewhat compreſſed. The other two rough and gritty, and in like manner a little compreſſed into a kind of obtuſely triangular figure. The 2d, is 4 inches and a half round about; the laſt, 5 inches and a half. Conſidering their bulk, all 3 are very light, eſpecially the 2 latter and greater ones, of which the laſt weighs but 5 Drachms 36 Grains; and both of them ſwim in water. This lightneſs proceeds from the Matter whereof they conſiſt, which, in ſome places is purely Downy or Fuzzy; in others, mixed with a Gritty ſubſtance, yet not confuſedly, but regularly mixed.

The Fuzzy parts poſſeſt the central part of the Ball, with a ſmall particle of blackiſh Glaſs or other Vitrify'd ſubſtance in the very Centre it ſelf. Over which are ſeveral Coats, gritty and fuzzy, alternately ending in the circumference with a grit, much reſembling the Groundwork and Superſtructure of the Oriental Bezoar ſtone. The Powder of one of theſe Balls ſcraped off with a Knife,is no way mov'd or affected with any ſort either of Alcaline or Acid Liquor dropp'd thereupon. Neither being burn'd doth it ſtink, it conſiſteth therefore of no Animal ſubſtance; but the Girl being of the Green ſickneſs age, the gritty parts (with the glaſſy particle in the Centre, as the moſt ponderous and leaſt moveable) ſeems to be broken off of Tobacco-Pipes, and grown'd ſmall between her Teeth; the downy or fuzzy to be lick'd or ſcrap'd off the Lean of Mutton, or the Rind of Peaches, or ſome other Part or Plant. Her Stomach kneading the Matter into a Coat, as her changeable Appetite ſupply'd it alternately with one or the other ſort.

Balls of Hair taken from the Uterus *and* Ovaria *of Two Women; by* Mr. Yonge, *n.* 309 *p.* 2387.

XXXII. 1: In *November* 1705, I was call'd to deliver a Woman 30 Years old, who had 4 Days laboured in vain to bring forth her firſt Child: The Head, being too big for the Paſſage, ſtuck immoveable at the *Os pubis*; ſo that I could neither faſten a Crochet, nor draw it out by a Cupping-Glaſs fixt to the Scalp with an Air Pump. In this Extremity I directed my Son to open the Childs Head, and take out all the Brains, with ſo much of the Scull as he could; and then by a Cord faſtned round the Neck with a Nooze, to pull it out, which was ſoon and eaſily done. The Child was Corrupted and ſtunk much, ſo did the *Lochia*, which flowed three Weeks; ſoon after they ceaſed, the *Menſtrua* appeared, and the Woman went abroad: About ſix Weeks after her Delivery, ſhe was ſeized with violent Convulſions, and Hyſterick Fits, which laſted near three Days; when a painful Tumor aroſe in the left ſide of her Belly, which ended in an Eruption of white thick Matter near a Pint, with ſmall Knobs of a Subſtance like the Yolk of boiled Eggs: All Symptoms immediatly vaniſht, only ſhe complained of the great Hollowneſs where the Tumor had been.

Four Days after this, the like Swelling appear'd on the right ſide of her Belly, which continued with a ſmall Flux of Matter about five or ſix Months, in deſpight of the many Remedies I uſed to cure her. About that time there appeared in the *Pudenda* a Bunch of ſomething like greaſy Wool, which being drawn forth, proved a Ball, or Wad of Hair, the bigneſs of a Turkeys Egg, immerſed in an Unctuous Slime; adhering on one ſide to a Membrane ſo large as the Palm of a Mans Hand: And in the midſt of it a ſmall Pyramidal Bone reſembling a ſplit Tooth. The Tumor ſunk upon this, and the Fluor ceaſed immediately, and her Lunary Flux (which all this while had not appeared) flowed as uſual, and ſhe continueth in perfect Health ever ſince, full nine Months. The Bone and Hair are fine, ſoft, and indifferently ſtrong, of no great length, of a light brown Colour, intangled like a parcel of Combings. *

* *Dr.* Hook's *Phil. Coll. n.* 2. *Abr.Vol.*III. *p.* 15.

2. About ten Years ſince, Sir *Andrew Leak* gave me a ſmall Bunch of Hair, being part of what had been found in the Belly of a young Woman at *Deal*, by Mr. *Joſ. Nichols* a Surgeon there.

A. D. 1696. At 30 Years of age ſhe fell into a Periodical Fever, and afterward a total ſuppreſſion of her *Menſtrua*; which was ſoon followed with a Pain and Tumor in the right ſide of her Belly, which grew and encreaſed, maugre all the Remedies adviſed by the Neighbouring Phyſicians, till it became bigger and harder than that of a Woman in her laſt Month. When it had grown a full Year, it began to ſoften, and then the Cenſorious People who ſuſpected her Honeſty thought her in a Dropſie. At fifteen Months end, the Belly was ſo diſtended, that it ſeemed ready to Burſt; which made the Patient deſire the Phyſicians to adviſe Mr. *Nichols* to make the *Paracenteſis*; but all were ſurprized, when inſtead of Water there ruſhed out a pint and half of ſweet well-digeſted Matter: The next Day he let out as much more, and then perceived Hair four or five Inches long iſſue forth with the Matter, but ſo faſtned in the Inſide, that he could not pull them out, the Woman complaining he would draw out a piece of her Belly.

She lived but four Days after the Operation; and on Diſſection of her Belly there was found ten Quarts of the ſame Matter which flowed through the Tap hole, and in it floating a Lump of Hair ſo big as an Half penny Loaf, wrapt up in a Fatty Matter, from which being cleanſed, it weighed full half an Ounce. On the Right ſide of the Womb he found a Protuberance bigger than a large Walnut, from which the Hair grew eight Inches long; that Tumor, or rather the *Ovary* being ſeparated from the *Matrix*, there was found in it a perfect Dog-Tooth, ſocketed in a Bone of a triangular Figure, in which another Tooth was growing; the Bone had a *Perioſtium* on it ſurrounded with Fleſh, faſtned at the *Calvaria* to the Scull. She was found on a nice and ſtrict Scrutiny, to die a Virgin, and Intact.

A Bunch of Hair voided by Urine, by the ſame, n. 323. *p.* 414.

XXXIII. 1. A Plethorick Woman about Fifty Years old, that uſed often to be afflicted with Nephritick Pains, employ'd me to relieve her *May* the 9th, 1707. I found by the Purulency and Stench of her Urine, that ſhe had not only Stones and Gravel, but an Ulcer in one or both her Kidneys; and therefore gave her a Doſe of *Cantharides* with *Camphire* made into Pills, and followed it with plentiful Draughts of a ſlippery Emulſion. This made her piſs off abundance of blackiſh Gravel, and white thick Matter like Bird-Lime, without any Pain or ill Symptoms, and ſhe continued eaſie for a Week; then her Pains returned, and went off by the ſame Remedy. About eighteen Days afterwards her Pain ſeeming to threaten a return, I repeated the Medicine; but that Night it gave her very great Pain in the ſide of her Belly, and at laſt threw her into Convulſions, which went off upon the Diſcharge of Urine, of a great deal of Matter, and in it a Bunch of ſhort Hair almoſt rotten: For ſome time after ſhe uſed a Nephritick Courſe, which hath hitherto preſerved her from the Return of Pain, Matter, Stones, and Impediment of Urine.

I herewith ſend a third part of that Bunch.

Microſcopical Obſervations on it. by Mr. Lewenhoeck, *ib. p.* 416.

2. I viewed Part of the hairy Subſtance thro' a Microſcope, and judged it to be the Hair or white Wool of a Sheep; which Wool was broken into ſuch ſmall or ſhort Particles, that ſome of 'em were no longer than ſix Diameters of the Breadth of a Hair; which I ſuppoſe could not proceed from the Body of a Man, but that it was rather found in the Heel of ones Stocking. And the oftner I repeated my Obſervations, the more I was confirm'd in my Opinion; for I could not only diſcover the ſhort broken woolly Particles, but I ſaw alſo a great number of the Ends grinded to pieces as it were; inſomuch that not only the Bark (if I may ſo call it) or outſide of the woolly Particles were rubb'd off, but the inward little Hairs, of which the Wooll is compoſed, were ſo divided from one another, that they appeared with their Ends like little Bruſhes. Moreover under the ſaid Stuff or white woolly Parts, there lay very ſmall Particles compoſed of exceeding ſlender Tubes or Pipes, which I look'd upon to be ſmall Bits of Straw, and they were ſo ſmall, that one Grain of Sand could cover 'em; there were likewiſe other ſmall Particles of the ſame Figure, but I did not take them to be Straw, but rather the outmoſt Husk or Skin of a Grain of Wheat or Rye; and under thoſe I ſaw one Particle cover'd all over with ſmall Hairs, ſuch as we ſee at the Top of Wheat or Rye; as likewiſe ſome few little Bits of Wood, ſomewhat thicker than a Hair of ones Head: there was alſo a ſmall Particle of the outmoſt Skin of a Man, for I could ſee the little Scales of which our outmoſt Skin is compoſed very plain; Now theſe Particles that were not Wooll, might be very eaſily brought into the Stocking, in caſe one ſets ones bare Foot upon the Floor before one puts it on.

There lay moreover in the ſaid Matter an unſpeakably great Number of exceeding ſlender long Particles, which I imagine to be thoſe hairy Particles, of which a little Fibre of Wool (ſetting aſide the Bark or Skin of it) is compoſed; as alſo ſeveral earthy Particles, which I took to be Part of the Dirt of the Floor or of the Foot itſelf. There alſo lay a great many particular little Figures, which I could not diſcover what they were; and theſe laſt mention'd Particles were ſo ſtrongly joyned to ſome little Hairs or Wool by the perſpired viſcous Matter from the Foot, as I ſuppoſe, that I could not ſeparate 'em but by the help of ſome Water. I obſerved amongſt 'em one ſmall Particle, not of a ſingle Feather, ſuch as it appears to our naked Eye upon the Body of a Bird, but rather of the fineſt Down; and the more I unravell'd or ſeparated the Particles of Wooll from one another, ſtill the greater reaſon had I to judge, that the Perſon who had worn the Stocking had been uſed to go often bare-footed upon the Floor. Now ſuppoſing that theſe woolly Particles might have fallen into any Spoon-Meat thicker than ordinary, the Perſon might ſwallow it down without being aware of it. Now my reaſons for gueſſing theſe woolly Particles to come out of a Stocking, and to have been occaſion'd by the motion of the Foot, are becauſe I my ſelf always wear heavy white woollen Under-ſtockings, and I lye in the ſame; inſomuch that I can wear 'em three Weeks together, becauſe I am not inclin'd to ſweat in my Feet; Now having ſeveral times view'd the broken woollen Particles which lye in a heap as it were cleaving together under the Heel, and having alſo ſingled out of them ſeveral Fibres or Threads of Wooll, to prove that they are compoſed of little Hairs; and theſe woolly Particles exactly agreeing with thoſe that were ſent to me; I could no longer doubt that the ſaid woolly Particles that were ſo ſent to me, were any ways different from thoſe Particles that were found in the Heel of the Stocking; 'tis true that amongſt the woolly Particles of my Stockings I never met with any Wood or Straw, but the reaſon of that was, that I have not touched the Ground with my naked Feet for ſome Years, being unable to bear any Cold in my Feet; nay, ſo far that in the Nights, even in the Summer time, I put a Tin or Pewter Bottle filled with warm Water to the bottom of my Feet, by which means I preſerve my ſelf, as I fancy, from that Plague call'd the Gout.

Diſſection of a Woman dying in Child Birth. Communicat. by Dr. Silveſtre. *n.* 269. *p.* 787.

XXXIV. M. Ducheſne quadraginta circiter annos nata, plethorica, & quæ ſæpius etiam gravida menſtruum patiebatur fluxum, & cæterum optimè valens, & obeſa jamque vigeſimum prægnans, urgente legitimo partus tempore, circa quartam pomeridianam die Novembris 12. Anno 1697. horrenda Uteri Hæmorragia corripitur. Vocatus poſt horam nonam *Jacobus Arnaudin* Obſtetriciæ non minus Artis quam Chirurgicæ peritus, & graviſſimo illo ſanguinis profluvio certior factus, nullo prorſus remedio urgenti adeo ſanguinis jacturæ occurri poſſe putabat, niſi quamprimum ſarcina fua Uterus levaretur. Sed, ut plerumque fit, adſtantes nihil temere moliendum, & rem totam naturæ permittendam clamabant. Exinde viribus prorſus exhauſtis, circa mediam noctem iterum accerſitur prædictus *Arnaudin*, qui nulla interpoſita mora operi ſe accingens intra dimidium horæ Puellam mortuam extrahit, ſecundis integris & illæſis abſque labore ſubſequentibus. Uteri tamen Hæmorragia uſque perſeveravit; hancque ſumma jactatio, Pulſus debilitas, Lipothymiæ frequentes, ſudor frigidus, motus convulſivi ſubſequebantur. Tandem circa ſextam horam matutinam vitam cum morte commutavit.

Die ſequenti cadaver ſecuimus: Aperto Abdomine, & remotis tum pingui admodum Omento, tum inteſtinis tenuibus, uterus in conſpectum veni, inſtar majoris Cucurbitulæ, craſſus ac ſatis diſtentus. In dextro ligamento lato Ecchymoſis ingens erat, quæ ex latere uteri, illic ubi

Vaſa *Spermatica* & *Hypogaſtrica* per Anaſtomoſin uniuntur, deſcendebat, & in inferiori parte Colli Matricis, ubi ſcilicet inteſtino recto incumbit, maxime conſpicua erat. Ut omnia facilius examini ſubjicerentur, Uterum cum annexis reſectum, & e pelvi extractum tabulæ impoſuimus. *Vagina* admodum laxa erat: Orificium internum adeo apertum, ut dimidium pugnum facile excepiſſet. Utero ſecundum longitudinem ſecto, hæc conſpiciebantur: fundum grumis aliquot ſanguinis ſcatens, cæterum integrum & ſanum: in hoc autem *Placentæ Uterinæ* adhæſio nobis facile conſpicua; quo loco etiam interna Uteri ſubſtantia inæqualis, craſſior, & quaſi carnoſior obſervabatur. In inferiori parte colli uteri cauſa mortis apparuit; Laceratio nempe duorum digitorum capax, quæ etiam extrinſecus, ubi ſcilicet ea Ecchymoſis, conſpici poterat. An a cauſa aliqua externa uterus diſruptus fuerit, an potius a vehementiori Calcitratione Infantis, ut aliquando accidiſſe memorant obſervationum Medicarum ſcriptores, ignoro: Sed exinde aperta fuiſſe majora vaſa, quæ enormem illam ſanguinis copiam effudere, certum eſt. Diductis tunicis, & tubulo inter duplicaturam immiſſo, flatus ex plurimis anguſtis foraminibus, cum ex ea fundi parte cui *Placenta* jungebatur tum præcipue ex lacera Colli parte, undique erumpebat. In dextro *Ovario* Veſtigium ſeu Cicatriculam manifeſte notavi, ex quo *Ovulum* deciduum, a tubæ cavitate exceptum, ejuſdemque Motu Periſtaltico ad uterum delatum fuiſſe mihi dubium non erat. *Ovarium* ſiniſtrum flaccidum & quaſi exuccum: *Tuba* ex utroque latere aperta, ligamenta rotunda, valida; reliqua omnia ſana obſervabantur. Licet Uterus ſubito poſt Puerperium contrahatur & coarctetur, adeo ut, quod modo dixi, in hac fœmina ſex circiter horis a partu mortua, Cucurbitulæ majoris capacitatem vix ſuperaret; attamen illum inſtar marſupii, vel ſacci digitis extendere facile potui. Et revera fibras carneas in graviditate ſenſim adeo elongatas, & tanto intervallo a ſe mutuo diſcretas, non niſi poſt aliquod tempus priſtinum tonum & robur acquirere poſſe, quis non videt?

A Hydrops Ovarii, *in a Puerpera com. by* Dr Douglas *n.*308. *p.*2317

XXXV. I lately opened the Body of a Woman, aged 27, who dyed the third Day after Delivery, on which I made the following Remarks. She meaſured round the Waſte a Yard and three quarters, and from the *Scrobiculus Cordis* to the *Os Pubis* a Yard and a quarter. All the cutaneous Veins of the *Abdomen* were of a very unuſual and extraordinary Bigneſs, and very much diſtended with Blood. From the largeſt of them, being opened, I extracted ſeveral polypous Concretions. The *Cuticula*, from the *Umbilicus* downwards, was rough and ſcaly to the naked Eye. In ſeveral Parts it appeared gangreened, occaſioned probably by the Sharpneſs of the *Serum* that always ouzed out of it, when ſhe ſcratched the little Pimples or Wheals that aroſe on its Surface; theſe for ſome time uſed to go off without any Scar, but as her Strength decayed they became mortified.

Upon all the *Regio Epigaſtrica* the outward Integuments were very thin, little or no fat being viſible: But from the upper part of the *Regio Umbilicalis*, down to the *Os Pubis*, the Skin was almoſt half an Inch thick, of a whitiſh Colour and hard, ſome of it appearing as if it were granulated, cauſed by ſome obſtructions in the Milliary cutaneous Glands. The Fat under this part of the Skin did exceed the thickneſs of an Inch, being diſtinguiſhed into ſeveral Lobules of an irregular Figure, and lodged in ſo many Cells adhering to the *Membrana adipoſa*, which here alſo was much thicker than it uſually is in a natural ſtate. Her Thighs, Legs and Feet were all *Anaſarcous*, being extremely big and ſwelled, eaſily retaining any Impreſſion made by the Fingers: And her Nurſe told me, that ſhe uſed to wet a great deal of Linnen in drying up the Water, that would always iſſue out from theſe parts on the leaſt rubbing, yet all her ſuperiour parts were extreamly lean and emaciated.

The fleſhy part of the *Abdominal* Muſcles was much extenuated by the great diſtenſion, yet their Tendons were as thick as uſual; and being very eaſily ſeparable one from another, I could plainly obſerve that the Tendon of the *Obliquus Internus* adhered firmly to that of the *Tranſverſalis*, along the edge of the *Muſculus rectus*, and was not double, as *Realdus Columbus*, and all Anatomiſts after him, down to *Diemerbroeck*, who was firſt aware of this miſtake, have maintained: However, this ſtreight Muſcle derives the ſame Benefit from this ſituation, being as it were hemm'd in on one ſide by this firm adhæſion, and on the other by what they call the *Linea Alba*, as if it had indeed been incloſed between the two ſuppoſed Tendons of the *Obliquus aſcendens*; that is, 'tis much ſtrengthned thereby in time of acting. I obſerved alſo, that the Tendons of the two oblique Muſcles, and the fleſhy part of the *Tranſverſalis*, between the anteriour Spine of the *Os Ilium* and the *Pubis*, near its commiſſure, did inſeparably join and unite with one another, forming as it were a thick and hard border, from the outſide of which there was continued over the Blood Veſſels, Nerves and Muſcles, on the Fore part of the Thigh, a large *Aponeuroſis*, which braced them down: The two *Laminæ* of the Membrane of the *Abdomen* being expanded on its inſide. Now this border is what Authors call the *Ligamentum Pubis*, and what I have in another place ſuppoſed to be the firm union of the Tendons of theſe three *Abdominal* Muſcles, with the *Peritonæum*.

Vid. Myograph. comparat. Specim. pag. 5.

Having perforated the *Abdomen* in the moſt convenient dependin part, for it would have been endleſs labour, conſidering the grea bulk of the Tumor, to have laid it bare, by freeing it carefully from the Muſcles and *Peritonæum*; there iſſued out with great Impetuoſity in a riſing ſtream a vaſt quantity of ſlimy Viſcid Water, in colour and conſiſtence very much reſembling a brown, thick and ropy Syru This Water meaſured between 16 and 17 Gallons, beſides what was loſ on the Floor, and imbibed in Sponges and Linnen made uſe of in drying

ing it up. When the Water was quite emptyed, I fancyed it had been all contained in a duplicature of the *Peritonæum*, and had made a Dropſy in that Membrane, becauſe none of the *Viſcera* appeared; for in ſuch a Caſe I have more than once obſerved, that the inner *Lamella* of that Membrane of the *Abdomen* being ſeparated from the outer, is forced inward by the weight of the Water upon the Bowels, to which it cloſely adheres, contracting the Guts and Meſentery into a very ſmall volume. But upon a narrower view I perceived that the thick Membrane, including the Water, could be eaſily ſeparated from the *Viſcera*, having freed it from its adheſions by membranous filaments to the *Peritonæum*, and by Blood Veſſels to the *Omentum*. Now this Bag reach'd from the *Pubis* to the Midriff, and from the Left Region of the Loins to the Right; in a word, it filled up the whole cavity of the *Abdomen*, diſtending her Belly ſo far, that a Plate could eaſily lye upon it, when alive. Having gradually freed it from all the neighbouring parts, and rolled it up, I found it adher'd inſeparably to the Left *Tuba fallopiana*, the Spermatick Veſſels being ramified upon it; and obſerving no *Ovarium*, which in the other ſide was naturally diſpoſed, I concluded that the Bag was nothing but the Membrane of the *Ovarium* covering the *Ova*, preternaturally thickned and diſtended by the collection of the above mentioned humor, and that the Diſtemper was a true *Hydrops Ovarii* inaſmuch as all this vaſt quantity of Water was included in one Bag, being all of the ſame colour and conſiſtence.

All the other *Viſcera* in the *Abdomen* were ſound, and in their natural ſtate. In both Cavities of the Breaſt there was contain'd a great quantity of reddiſh Water. The Liquor in the *Pericardium* was very abundant, and of a greeniſh hue. The Right Lobe of the Lungs was ty'd to the *Thorax*, covering the upper part of that Cavity, but the Left was free from any adheſion. In the left Ventricle I found a large Polypus or Serous Concretion, of a round figure, a white colour, and of a pretty hard Conſiſtence, with ſeveral long Roots of a Red colour, which extended thro the Auricle and Bulb of the Pulmonary Vein into its neareſt divarications in the Lungs.

Having carried home this large Bag, with the *Uterus* appendent, cut off below the Orifice of the *Meatus Urinarius*, and viewed it at leiſure: I obſerved, that the Right Spermatick Vein, which opens into the *Cava* a little below the *Emulgent*, was three times larger than the Left; and from a little above the *Ovarium* it was continued, without any diviſion to its termination. The Right *Ovarium* was in a very natural ſtate. The *Cicatrix* or *Caruncula*, whence the fæcundated *Ovulum* had dropt, was yet remaining, and the Blood Veſſels were ramified upon this *Teſtis*, in a very pleaſant and beautiful manner. The *Tuba Fallopiana*, with its *Fimbria*, were all well diſpoſed.

The Diameter of the Left Spermatick Vein, which opens into the *Emulgent* of that ſide, was much leſs than ordinary. And from the extraordinary narrowneſs of the bore of this Veſſel we may draw a not very improbable Reaſon of ſome Cauſe of this Watery Swelling; for the Blood being hereby hindred in its Reflux to the Heart, a great deal of *Serum* or *Lympha*, thro its ſlow return, muſt needs be thrown off upon the *Ovarium*, already indiſpoſed, whence the gradual Increaſe of the Tumor did proceed. The two Spermatick Arteries were contorted, and full of turnings and windings, from their meeting with the Veins to the *Ovaria* and *Tubæ*. A little below the Kidneys each Artery ſent out a Branch, which was loſt on the *Peritonaum*, and fatty Membrane of the Kidney: And from the ſame places the Veins received two conſiderable Branches. One of the Arteries went off by a narrow Orifice from the ſide of the *Aorta*, the other roſe up from its middle, a little below the firſt.

Between the Bag and the *Uterus* all theſe Veſſels were much dilated, making ſeveral Turnings and Circumvolutions upon the *Peritonæum*, called in this place the *Ligamentum Uteri latum*. The Left *Tuba Fallopiana* was only remarkable in its being much longer and larger than uſual. In the Bag, which was nothing but the Membrane called *Dartos*, which covers all the *Veſicular* Glands of which the *Ovarium* is compos'd I obſerv'd ſeveral little Bladders of ſeveral ſizes, diſtinct from one another, which contain'd a limpid or clear ſlimy *Serum*, in Colour and Conſiſtence like a Mucilage of the *Semen Cydoniorum*, theſe were either Hydatidal Tumours only, or the Eggs themſelves diſtended. This Liquor hardned by a ſlow heat into the Conſiſtence and Colour of the White of an Egg.

All the *Fundus Uteri* was about an Inch and a half thick, but near the *Collum minus* it grew ſomething thinner, which did proceed from the diſtention of its Spongy and Veſicular Subſtance, by the Blood in the Veſſels running thro it in variety of turnings and windings; ſo that when it was cut, it very much reſembled the ſubſtance of the Lungs. Upon the inner Membrane of the *Uterus* I obſerved, upon wiping it with a Sponge, ſeveral little Eminencies, which I took to be the Glands mentioned by *Malphighius*, which ſeparate a Humour, to lubricate and moiſten its cavity. On the upper part of the *Fundus Uteri* I took notice of a great number of ſmall Veſſels, like ſlender Filaments or Threads, running off from its Membrane, and terminating into a reddiſh and ſoft Spongy ſort of ſubſtance, not unlike the *Uvula*, bating its colour, which hung down from that ſide of the Womb in form of a Nipple. Theſe perhaps are the Veſſels, which, in the opinion of ſome, do ſeparate and excern the Matter of the *Lochia* and the *Menſes*, they being only viſible at thoſe times.

Near the beginning of the *Tubes*, I perceived two *Tubercles*, or little Bunchings, about the bigneſs of a Nut, to which perhaps the *Placenta* was faſtned, and to theſe adhered Glandules of a Blackiſh Colour, of different ſizes. The *Collum minus* was compoſed, as it were, of two *Labia*, the uppermoſt was moſt protuberating, and upon it I obſerved ſeveral ſmall Glands, out of which, upon compreſſion, iſſued a viſcid clear Liquor, which is ſaid to ſeal and cloſe up this part, in time of Pregnancy. The lower *Labium* was longer and thinner, its Edges being cut or indented in ſeveral places. The *Rugæ* in the lower part of the *Vagina* run as they are repreſented in Books, but thoſe in the upper part had a quite different Courſe, as they are exactly delineated in the annext figure. Near the Orifice of the *Meatus Urinarius* there were obſervable two very large Caruncles, in Shape like a Mulberry.

This is what I obſerved in the opening of this Woman, I come in the next place to relate, as far as I was informed, the Symptoms that accompanyed her big Belly, and the Method made uſe of for her Recovery.

About three years ago, not long after ſhe had lain in of her firſt Child, ſhe had a violent blow upon the Left ſide of her Belly, very painful for the preſent, but in two or three days, upon keeping herſelf quiet in Bed, the pain and anguiſh went off. About two months after this, ſhe began to feel ſome ſmall pain in the Left Hypogaſtrick Region, where ſhe had lately received the Blow; and ſhe obſerved that ſide of her Belly to grow abundantly bigger than the other: Theſe pains increaſed more and more, till they grew very violent, but upon the Conception, which was three months after ſhe was firſt afflicted with them, they went off, and her Belly ſwelled gradually, as is uſual in Pregnancy, having no other Symptoms but what is incident to that ſtate, only ſhe was much bigger than ordinary; and on that account ſhe forbore the uſe of Medicines, which poſſibly might have been effectual in the beginning of her Diſtemper, had ſhe been aware of her Danger. After Delivery, the ſwelling and bulk of her Belly continued much the ſame as before the Birth, only upon a plentiful evacuation of the *Lochia* it decreaſed a little. When her month was up, ſhe uſed Emeticks, ſtrong Cathareticks, Diuretick Dyet-drinks, and all the train of Medicines commonly uſed in a Dropſy, her ſuppoſed caſe. All the effect they had, was to prevent the farther Increaſe of the Swelling while ſhe uſed them; but upon leaving them off, the Tumor increaſed very remarkably.

Thus ſhe continued about one year, and then ſhe conceived again, which ſhe ſuſpected by the ſtoppage of her *Catamenia*, having always been very regular but at ſuch a time. Her Stomach was always good, ſhe never was very thirſty, ſo drank but little, made Water freely and in great quantity, and was attended with none of the Symptoms of an *Aſcites*, except the Swelling of her Belly: Only when ſhe was half gone, her Legs began to ſwell and pit, growing very big all of a ſudden; from theſe, and likewiſe from her Belly, there would often iſſue out a great deal of watery Humour upon rubbing, as I have mentioned already, eſpecially if ſhe ſcratched the little Pimples, that would often ariſe in theſe parts. About this time ſhe began to be afflicted with a difficulty in breathing, with a violent Trembling and Palpitation of her Heart, and to be often ſubject to great and involuntary Sig ings. She was not able to lye down, but was ſtill obliged to ſleep a ſitting poſture, for fear of being choak'd. After ſhe was brough Bed of a live Child ſhe became exceeding weak, being unable to fe her Breath, and complained much of a heavy Load and Oppreſſion o her Breaſt, and the third day ſhe expired.

The Explanation of the Figures. Plate 6.

Fig. 1. repreſenteth the *Glandulæ Renales*, the *Uterus*, with the Par belonging to it, and the large Bag or Membrane of the *Ovarium* p ternaturally diſtended.

a The *Glandula Renalis* on the Right-ſide. *b* An Eminence, or ri in its middle. *c* A Vein that runs from it to the *Cava*. *d* The *Gl* *Renalis* on the Left Side. *e* A Sulcus or Furrow in its middle. *f* A Ve running from it to the Emulgent. *g* A ſmall Vein that comes fr the Diaphragm, and opens into this Vein before it leaves the Gla *h h* Two ſmall Arteries from the *Aorta*. *i i* Two nervous Twigs fr one of the Intercoſtal *Plexus's*. *A A* The Kidneys. *B B* The *U* cut off. *C* The *Cava* cut off. *D* Its Diviſion into the *Rami Iliaci*. The Internal Branches into which the *Hypogaſtricks* open. *F F* T Emulgent Veins. *G* The *Aorta* cut off. *H* Its Diviſion into the acks. *I I* Its internal Branches, which are ſpread upon the *Uterus*. The external Iliacks of both Veſſels. *L L* The Emulgent Arte *M M* The Spermatick Veins. *N N N N* The Spermatick Arteri very much contorted in their Progreſs, that on the Right ſide be cut off. *O* The Union of the Branches of the Spermatick Vein the Right ſide. *P* The Right *Ovarium*, with Blood Veſſels ramifie its outer Membrane. *Q* The Right Tube. *q* Its *Fimbria*. *R* T Tube on the Left ſide, its *Fimbria* adhering to the large Bag. The Membrane of the Left *Ovarium*, diſtended to a vaſt Bigneſs, the Blood Veſſels ramified upon it. *T* Some of the *Ovula* grown *W* Some Hydatidal Tumours on the Inſide of the great Bag. *V* The *Ligamenta lata*. *U* The *Fundus Uteri*. *X X* The *Ligamenta rotunda*; Membrane that covers them being laid open, that the Veſſels of w they are compos'd may be view'd. *Y* The *Vagina* cut off. *Z* The *Veſi* *rinaria*. ** A ſmall Artery and Vein on each ſide, the firſt going off the Spermatick is ſpread upon the *Membrana Adipoſa* and *Perit* under the Kidney; the latter bringing back the Blood from theſe P opens into the Spermatick Vein.

Fig. 2. Sheweth the *Vagina* and *Uterus* cut open.

A A A The *Fundus Uteri* laid open, and its ſides folded back. 1 1 The inner ſpongy Subſtance, with the Orifices of the Hy rick Veſſels. 2 2 The Glands appearing on the inner Membrane o *Uterus*. 3 3 The ſmall Veſſels, by which the *Lochia*, &c. are ſepa 4. A ſoft Subſtance, depending from the upper Part of the *Uterus*, which the foreſaid Veſſels terminate. 5 5 Two Tubercles, ſeated the Beginnings of the *Tubæ*, to which the *Placenta* adher'd. *B B*

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Fig. 2.

Fig: 1.

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Fig. 3.

Fig. 4.

Vagina laid open. 6. 6. The two *Labia* of the *Collum minus*. 7. 7. Several ſmall Glands plac'd on the upper *Labium*. 8. The Courſe of the *Rugæ* on the upper ſide of the *Vagina*. 9. 9. Their Direction on the under ſide of that Part. 10. 10. Two Orbicular Subſtances, near the Orifice of the *Meatus Urinarius*.

A Fœtus voided by the Navel. by Mr. Birbeck, *n.* 275. *p.* 1000.

XXXVI. One *Raper*'s Wife at *Coxwold*, 12 Miles from *York*, falling in Labour of Child Birth, the Midwife uſing her Endeavour, extracted the Secundine, it offering firſt, and could not, by her beſt Skill, perceive any thing elſe remaining: The Woman's Body falling, and for ſome Days being pretty eaſy, and the Womb contracted, the Midwife took the Secundine to be a Mola, or falſe Conception, but in about a Week's time ſhe began to diſcharge plenty of fœtid Matter by the *Vagina*, which continued, and in time ſhe felt a troubleſome Hardneſs of the *Hypogaſtrium*, which encreaſed daily for above ſix Weeks, by which the Woman was brought ſo low, that they deſpaired of her Life. This Lump, Hardneſs and Soreneſs wrought up to the *Regio Umbilicalis*, and I ſuppoſe continued there fixt for about a Month, at length being exceeding painful, the Neighbouring Gentlewomen took it for a great Boil or Apoſtheme, and apply'd what they uſually do in ſuch Caſes, to aſſiſt its Suppuration and Breaking, which had its Effect, and it broke upon (or rather under) the Navel, diſcharging then (and afterwards) a great quantity of a thin Fætid and diſcoloured Liquor. The Part about it mortified, and the Ulcer inlarged to that Bigneſs, that a Man's Hand might be introduced therein; it continued exceeding painful, and emitted ſuch an exceeding Stench (that neither her ſelf (being extreamly weak and faint) nor any Body elſe could endure to look on it; a ſmall time after they found ſome little Bones wrought out of it, which they ſhewed to me, I found them to be the Bones of a Child's Finger, which made me curious of going to ſee her. When I examined it, I perceived the Fœtus in a confuſ'd Heap or mortified Lump, for with my Probe I felt ſeveral Bones, and at that time extracted (after I had ſeparated and dilated the Mortification about it) above half the Ribs, ſome Vertebres of the Back and other Bones, and cut out above a pound of the Child's mortified Subſtance, as black as Ink, with an extream nauſeous Smell. Every ſecond or third Day for a Month, I extracted what I could, being forced to do it very deliberately, by reaſon of the exceeding Weakneſs of the Woman, who had certainly dyed in the operation, had I forcibly extracted it, and not given her time; for we were obliged every Moment to ſupport her with Cordials, and after every Operation ſhe found her ſelf lightſomer, and by degrees ſweeter, which gave me Hopes of her Recovery, which before I had no Thoughts of. For not only the *Linea Alba* and Muſcles of the *Abdomen*, but the *Peritonæum* and *Omentum* was mortified to a great Breadth, and the Inteſtines lay fairly in view, and expoſed to the Air a long time. When I had extracted Part, and had

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a plentiful diſcharge of thin Fætid Matter, the other diſcharge downwards began to leſſen and abate; ſo that I endeavoured to aſſiſt it by bandage and compreſſes, with deterging and drying injections up the *Vagina*, by which means in a little time I had no diſcharge that way, and thoſe parts became ſhortly perfectly well, and in ſome time after the Ulcer ſeparated (with the aſſiſtance of Fomentations, good Digeſtives and Mundificatives,) from its Putrifaction, contracted and united wonderfully, and hath now been quite cicatrized near 3 months ago, all the whole Abdomen being ſoft, eaſie and well condition'd. The Woman laboured all this Seaſon at Hay and Harveſt.

The Bones of an human Fœtus, voided through an Impoſthume in the Groin. by Sir Phil. Skippon. *n.* 302 *p.* 2077.

XXXVII. *Jan.* 21. 1662. I viſited a Woman 66 Years old in *Drury Lane*, who had a Child conſum'd in her *Uterus* about 28 years ago; She bore two Children after this, one lived 11 Years, the other 16. About 8 Years ago, an Impoſthume broke out in the right Inguen; and then ſeveral Bones of a dead Child were expell'd. She hath a great Swelling now in the Groin, where ſhe feels ſomewhat very hard, which ſhe ſuſpects are Bones.

Two extrauterine Fœtus's by Mr. Yonge. *n.* 323. *p.* 426.

XXXVIII. 1. I receiv'd an Account from a very Learned Divine of *Devonſhire* in theſe Words——; ‘A Gentleman's Servant having kill'd an Ewe, which was thought fat, and taken out the Bowels, found a very unuſual and monſtrous lump of Fat, proceeding like a Wen from the middle of the *Omentum*. I was call'd to ſee this Wonder; and having cut it open, found incloſed a Lamb of the ſame Parts, Feature, and Dimenſions with others of that kind. How it came there? And how nouriſhed? are Queſtions I would have reſolved.

I ſoon apprehended what it was that ſeem'd ſo very ſtrange and unaccountable to my Friend, having thirty Years ſince been ſhewn the like, found in a Bitch, by an expert and ingenious Surgeon in *Oxford*; and from that time obſerved, and conſidered all of that Nature which have occured to me in Books, or otherwiſe; and ſo was ready to tell him that that Lamb was not conceived in the Womb, but in one of the *Fallopian Tubes*; wherein growing too big to be contained, it either broke out into the place where it was found, or ſlipt back toward the upper Orifice, and thorough it into the Belly: That afterward aſſiſted by the prone and inclining Poſture of the Sheep's Body, it ſlipt forward to the *Omentum*, and was there nouriſhed the uſual way, *viz.* by the *Placenta*, which was doubtleſs fixed in the *Tube*, and the *Pedunculus* being kept whole, will eaſily extend from thence to the *Fœtus*, where it lay.

2. I am told by a Gentleman Hunter, that he lately found in the Paunch of an Hare, two full grown young Ones among the Bowels, but almoſt rotten: and three immature *Embryos* in the *Uterus*. The former were certainly *Fœtus*'s broke out of the Womb.

A Child crying in the Womb. by Mr Derham. *n.* 324. *p.* 485.

XXXIX. 1. On *Saturday* laſt I gave my ſelf the Satisfaction of viſiting a Woman brought to Bed *Nov.* 6. 1709. of a Boy, that had Cry'd in her Womb, at times, for five Weeks wanting one Day. The Child appears to be Luſty and Strong, and is, ſince its Birth, a very quiet Child. She told me, the firſt time the Child cry'd was in the Night, as ſhe lay in Bed, after a great Pain which forced her out of Bed, and gave her Apprehenſions of her Labour being nearer than her Reckoning. And every time after, whenever the Child cry'd, ſhe had violent Pains like thoſe of Labour. From the very firſt time of its Crying, the Child ſettled it ſelf on the Mother's Left-ſide, and ſhe never perceived it to ſtir in the leaſt, 'till its Birth approached. Scarce a Day in all the five Weeks eſcaped without Crying little or much. But the Women obſerved, that every other Day it cry'd the moſt, and moſt certainly. The Midwife told me, ſhe heard it cry ſeventeen times in half an Hour. Its Crying might be heard into the next Room; and ſometimes it ſeemed to be ſo Hearty, that the Child would ſob again. Both the Mother and Midwife told me, they found no great Difference between her in her Caſe, and other Women in the ſame Condition.

Remarks, by the ſame. ib. p. 487.

2. *Etmuller* having in his Diſſertation *de abſtruſo Reſpirationis humanæ negotio*, Chap. 9. together with the learned *Diemerbroeck*, doubted the Truth of Accounts of that nature, may in my Opinion receive an Anſwer, in ſome meaſure, from this Caſe into which I have made ſome farther Enquiries. Concerning which I muſt needs ſay, that notwithſtanding I ſhould be as much enclined as any Man to doubt of the Fact, being clearly of Opinion that the *Fœtus* doth not live in the Womb by Breathing, yet the Evidence is ſo clear to me in the preſent Caſe, that I am fully ſatisfy'd it was really *Crying of the Fœtus*, and not Groaking of the Guts, or Womb, or the Effect of any Feminine Imagination. For here we have a thing happening not once, or twice only, but a great many times; almoſt every Day, and divers times in the Day; and that for near five Weeks together. Enough to have diſcovered any Miſtake, or to have undeceived even a fanciful Perſon. In the next Place, we have the Child heard to Cry aloud, ſo as to be diſtinctly heard by Perſons in another Room. Conſequently the Hearers could more eaſily and certainly diſtinguiſh whether the Noiſe was *Crying* or *Croaking*. The Deſcription the Mother and others gave me thereof was, “ That
“ the Noiſe the Child made, was as if a Born-Infant had Cry'd eagerly,
“ ſhut up cloſe in a Tub. In the third Place, The Crying ſeemed to be ſo eager and hearty, as to end in Sobbing, like what is obſervable oftentimes in Born Infants. In the fourth Place, It was heard not alone by the Father and Mother, or one or two beſides, but by many, or moſt of the Neighbourhood, both near and farther off, and many of them Perſons long uſed to Children; who do all with the greateſt Aſſurance affirm it to have been as manifeſt Crying, as ever they heard from a Born-Infant, and nothing like any Noiſe of Wind, or the Guts,

as on Enquiry they all particularly told me. And in the laſt Place, The Midwife told me, that laying her Hand on the left ſide of the Woman's Belly, where the Child lay when it Cryed, ſhe could plainly feel a Motion under her Hand, like that of Reſpiration, every Blaſt of the Child's Crying ſenſibly touching upon her Hand.

Ibid

As to the *Peeping of Chickens in the Egg*, about which *Etmuller* hath the ſame doubt, as of the *Vagitus Uterinus*, I have my ſelf divers times heard that, both from Chickens and Ducks. And a Perſon more Converſant in ſuch Matters than my ſelf aſſures me, That a little before the Hatching, ſhe hath often, and can at any time cauſe ſome Chickens, and Ducklings to peep in the Egg. She ſaith, that ſometimes whole Neſts of Eggs will yield a Cry, ſometimes only ſome particular Eggs: But that ſuch Eggs as have once afforded a Peeping, may be made to Peep and Cry at any time, by ſhaking the Egg, and putting the Youngling into a diſorder. And ſometimes where there hath not been any Noiſe before heard, the Bird hath been made to Cry, by ſhaking the Egg in which it was encloſed. The Cauſe of this Peeping in the Shell, I take to be from ſome Uneaſineſs the young Bird may find there. It being arrived to its perfect State in the Egg, is either weary of its Confinement therein, and deſireth more liberty; or elſe it lies uneaſily, or is offended with ſhaking, and therefore Peepeth and Cryeth, as when uneaſy out of the Shell. And after ſome ſuch manner I take it to be with an Humane *Fœtus*; that it is in ſome Diſorder, and uneaſy in the Womb, and therefore Cryeth as well in, as out of it. Thus I am apt to think it befel the *Fœtus* I have ſo often ſpoken of, *viz.* That it lay very uneaſily in the Womb all the while it Cry'd there, the Mother being in great Pain before, and during the time of the Child's Crying, and the Child it ſelf being cloſely confined, and pent up on the Left-ſide the Mother's Belly, *all the time of Crying only, and not all the 5 Weeks*, as by miſtake I told in my former Letter. Perhaps alſo the Child might find ſome Uneaſineſs from a Bone the Midwife told me ſhe found to ſtick out ſomewhat farther than ordinary: Which, upon Examination, I take to be one of the Vertebræ of the Back-bone. And if this Bone cauſed Uneaſineſs to the Child, it might alſo by that means occaſion perhaps the Woman's Pains I ſpake of.

A Woman who had her Menſes *regularly to 70 Years of Age. by Mr.* Yonge. *n.* 337. *p.* 236.

XL. At *Lamerton*, fifteen Miles from *Plymouth*, dy'd lately a Woman of Eighty ſix Years old; who to the Age of ſeventy had her *Menſes* plentiful and regular. At that time they ceaſed, and ſoon after followed the like Efflux from the *Hæmorrhoids*, which continued till ſhe got on the wrong ſide of Fourſcore. She was till then healthful and ſtrong, of a vigorous Aſpect, ſmooth, plump, and florid in Countenance, like one not half ſo Old: Her Appetite was very good; her Intellects clear and ſound; and her ſight ſo perfect, that ſhe could to the laſt thread a Needle, and read ſmall Print without Glaſſes. When that Flux ceaſed, ſhe became Gouty; and about one Year before ſhe dy'd, there aroſe an Apoſtumation on one of her Wriſts, which open'd and diſcharg'd much Chalky matter and ſome Stones. The Day ſhe dy'd, ſhe aroſe out of her Bed; and after performing ſome Chriſtian Devotions ſhe expired. She was never Sick before the Hæmorrhoidal Flux ſtopt, except once at *Exeter*, where ſhe was born, and then lived, ſhe became inſected with what they called the Plague: It ended in a critical Abſceſs in one of the Emunctories; and which is very ſtrange, during all the time of the Sickneſs, ſhe nurs'd a Male Child, who is yet alive, and one of our Faculty.

Diſſection of a big Belly'd Woman, ſuppos'd to have continued many Years with Child. com. by Dr. Hollings. *n.* 348. *p.* 452.

XLI. A married Woman, about 3 Miles from *Shrewsbury*, about 40 Years old had the common Reaſons to believe herſelf with Child. At the time of her account, ſhe had the uſual Signs of Labour, and a good Midwife aſſur'd her, it was ſo, but ſhe could not be deliver'd without bringing away the Child in pieces. She not conſenting, her pains went ſoon off, and ſhe continu'd without any Diſorder 9 Months longer, when ſhe had again the Pains of Labour, and the ſame Midwiſe aſſur'd her as before, and ſhe perſiſting in her former Reſolution, her Pains after a Day or two went off. Soon after her Belly ſwell'd to a ſurprizing Size, by which ſhe got Subſiſtence for her Family by being ſeen in a Shew. I ſaw her firſt above Twenty Years ſince, when her Belly was almoſt even with her Chin, the Weight of it ſo great that ſhe was oblig'd to ſupport it with a Stool. She could not ſtand without the help of a Rope from the Cieling, which aſſiſted her in changing her Poſture of ſitting. She ſlept commonly with her Arms folded on her Belly, and her Head reſted between them. She had no ſwelling in her Legs: every other Part emaciated as uſual in the like Caſes. Thus this Poor Creature liv'd without any other conſiderable Complaint above Thirty Years, the moſt remarkable Circumſtance, I think, in her Caſe. She dyed in *May* 1715, when this appear'd to be an *Aſcites*.

I need not mention the State the common Teguments muſt neceſſarily be in from ſo great a Diſtention, which had diſtorted many of her Ribs, and forc'd the Diaphragm ſo high, that it was ſurprizing to find her breathing could be ſo long continu'd. The Water was all contain'd in the Duplicature of the *Peritonæum*, 13 Gallons beſides a Quart that was ſpilt: It was Saltiſh, with ſome little fat upon it, and towards the latter Running ting'd with Blood as uſual. There was not any Water in the Cavity of the *Abdomen*, except what was contain'd in a kind of Bladder of the Shape I have ſent, which lay a croſs the *Fundus Uteri*. This was divided by a Cartilaginous Subſtance into two Cavities; in one there was about a Pint and a half, in the other three Parts of a Pint of Water. I believe it was this (I know not how) impos'd on the Midwife. The *Uterus* was of the natural Size without Alteration, except that the *Os Tineæ* and *Collum minus* were fill'd with a gritty Subſtance, hard as Stone, which I take to be the Humor ſeparated

Plate 6, *Fig.* 3.

rated there, and coagulated by Time. Mr. *Cowper Tab.* 15. *Fig.* 4. ſays he found the ſame Parts fill'd with a glutinous Matter, which he thinks is uſeful to prevent Abortion: which if vitiated, Impregnation is hinder'd.

The Liver and other parts contain'd in the *Abdomen*, were forc'd into an incredible ſmall Compaſs (and by that Preſſure a little chang'd in Shape) to perform their Office ſo long; to which the Muſcles of the *Abdomen*, diſtended ſo as to be ſcarce diſcernible, could give but little, if any, Aſſiſtance.

Of the maculæ maternæ. *by* Dr. Mather. *n.* 339. *p.* 65.

XLII. A Woman in *New England*, longing for Peas, but refuſing to gratify her Deſire, by reaſon of a ſort of Bug, wherewith at that time moſt of their Peas were infeſted, had a Child born with an Excreſcence on the Forehead, reſembling one of thoſe Peas, with a black Speck, as the buggy Peas had, which after ſome time dryed away, and ſhell'd out, they fancied as the Bugs are obſerved to leave the Husk of the Pea.

XLIII. *A Paper omitted.*

n. 270. *p.* 819.

A Letter from Dr. *Martin Liſter* to Dr. *Tancred Robinſon*, commending the Doctor's Book of Fevers, and hinting at an Experiment made by him and Dr. *Muſgrave*, concerning Powder'd Blues paſſing the Lacteal Veins. *This Experiment is related at large by Dr.* Muſgrave, p. 75.

XLIV. *An Account of a Book*, viz.

ib. p. 829.

Profluvia Ventris: or the Nature and Cauſes of Looſneſſes plainly diſcover'd, their Symptoms and Sorts evidently ſettled, the Maxims for curing them fully demonſtrated, and all illuſtrated with the moſt remarkable Methods and Medicines in all Ages; with ſome Practical Obſervations concluding every ſort. By *William Cockburn*, M. D. late Phyſician of his Majeſty's Fleet, *F. R. S.* and of the College of Phyſicians, 8vo. *Lond.* 1701.

CHAP. V.

Of the Veins, Arteries, and Blood.

1. *Schemes of Veins and Arteries, preſented to the R. S. by* Mr. Evelyn. *n.* 280. *p.* 1177.

BEing ſome Years ſince in *Italy*, and curious of ſeeing the many repeated Diſſections at the Anatomical Theatre at *Padua*, *Cavalier Veſtlingius* being then Profeſſor, and reading on divers Bodies ſeveral Days, during the Lent; Dr. *Johanno Athelſteinus Leoncenæ*, who was then Operator, by Extracting the Veins, and other Veſſels which contain the Blood, Spirits, *&c.* out of Humane Bodies, (which the many Hoſpitals and Infirmaries of that City plentifully afford) began to apply and diſtend them on Tables, according to their natural Proportion and Poſition, as an Improvement which might be of uſe in Anatomy: Some of theſe Tables being finiſh'd, with the Direction and public Approbation of the Profeſſor and ſeveral other Learned Phyſicians and Anatomiſts, preſent at thoſe Lectures and Operations; and underſtanding that *Leoncenæ* was going ſhortly (I think) into *Poland*, and willing to diſpoſe of his Tables, before he took his Journey; I deſir'd the late Dr. *George Rogers*, (Conſul then at *Padua* for the Students of our Nation in that Univerſity) to purchaſe and procure them for me; which he did, for as I remember 150 *Scudi*; with condition, that he ſhould add a Table more, namely, that of the Liver, Gaſtrick Nerves and other Veſſels, to compleat the Fourth: When theſe were perfected, I immediately ſent them to *Venice*, from whence they were ſhipp'd for *England*: But, upon what Accident or Occaſion I know not, the Veſſel was carried into *Holland*, and lay there a Year or two; (without any Tydings what was become of my Concerns, being then my ſelf at *Paris*) till coming at laſt to be unladen, Sir *Richard Ford* (afterwards Lord Mayor) took care to have them all ſafely convey'd to me at *London*, to my no ſmall Charges.

Anatomical Remarks on them, by Mr. Cowper *ib. p.* 1179. *Plate* 7.

2. That the Arteries are the Veſſels which convey Blood from the Heart to all Parts of the Body, is well known; and we ſee by Fig. the 1ſt, that the common Practice of Nature in diſtributing theſe Veſſels, to ſupply the parts with Blood, is from the next adjacent Trunk, till their Aſcending and Deſcending Trunks become Conical, as well as their collateral Branches: Not that all the Trunks and Ramifications of *Arteries* are Uniform, and become Conical in the ſame manner; nor do all of Them paſs directly to the parts to which They convey Blood; nor do all parts receive *Arteries* from their neighbouring Trunks. The Trunks of the *Carotid*, *Vertebral* and *Splenick Arteries* are not only Contorted in their progreſs, in the Adult; but the Diameters of their bores are variouſly dilated in divers parts of them, eſpecially where they are contorted; but as theſe dilatations of their Trunks are caus'd by the reſiſtance the Blood meets with at thoſe Angles of Inflection, ſo thoſe Enlargements of them afterwards contribute to retard the protruſion of the Blood to the Extremities of thoſe *Arteries*: Hence it is, That as the *Arteries* of the *Fœtus* are not Contorted in ſuch acute Angles as in full grown Bodies, ſo their Trunks are more Conical, and not here and there dilated in divers parts of them, as in the Adult. The Trunk of the Splenick *Artery* has a ſtrait Progreſs in the *Fœtus* and in Infants; but in the Adult I have hitherto conſtantly found it very much Contorted, as expreſt in Fig. 1. 23. The peculiar Contrivances of the Spermatick *Arteries* of *Quadrupeds* as well as Men, ſhew a conſtant deſign in Nature of taking off the Velocity with which the Blood

would otherwiſe paſs thro the Glands of the *Teſtes*: It ſeems to be for this end that the *Teſtes* of moſt Animals (eſpecially Men and *Quadrupeds*) hang out of the Cavities of their *Abdomens*, that the Canals of their Blood Veſſels may be lengthened: for the Spermatick *Arteries* (contrary to all others) ariſe from their Great Trunk, at a far greater diſtance from the *Teſtes* than the *Arteries* of any other part of the Body. Nor would the *Teſtes* (which are ſuch neceſſary Organs) have been thus expoſed to external Injuries, if the end of Nature in lengthening their Blood Veſſels had not been very conſiderable. Beſides this lengthening of the Spermatick *Arteries*, we find Nature ſtill contriving other Impediments to check the Current of the Blood in thoſe Parts; it ſeems for this end that the Spermatick *Arteries* are leſſen'd at their Original from the Trunk of *Arteria Magna* in Men, and that the Spermatick *Arteries* of *Quadrupeds* are ſo much Contorted before they reach their *Teſtes*. The principal Inducement of Nature in making uſe of theſe different Contrivances in the Spermatick *Arteries* of Men and *Quadrupeds* ſeems to be, That if the Humane Spermatick *Arteries* were Contorted, as in *Quadrupeds*, before they reach their *Teſtes*, the Apertures in the *Abdominal* Muſcles of Man muſt be much larger than they now are, and would frequently let the *Inteſtines* deſcend into the *Scrotum*; which we know nevertheleſs often happens: ſuch Ruptures (as they are call'd) are not ſo Incident to *Quadrupeds*, tho the Paſſages for their Spermatick Veſſels (through their *Abdominal* Muſcles) are much wider than in Men, becauſe the poſition of the Trunks of their Bodies is Horizontal, and their Inteſtines therefore cannot preſs on the proceſſes of the *Peritonæum*, as in Men, who are Erect. I ſhall, at preſent, purſue the Thread, and deſcribe the Extremities of the *Arteries*, with their Communications with the Veins, and afterwards produce ſome Inſtances of the Art of Nature in conveying the Refluent Blood to the Heart.

After the Circulation of the Blood through the Heart, Lungs, and large Blood Veſſels, was demonſtrated by Dr. *Harvey*, it was only gueſt how the extremities of the *Arteries* tranſmitted the Blood to the Veins, till Mr. *Lewenhoeck*'s Microſcopes had diſcovered the continuation of the Extremities of thoſe Veſſels in Fiſh, Frogs, *&c.* which is now commonly ſhewn by Microſcopes made by other Hands: Yet there are not wanting thoſe who doubt of the like Continuations of the Extremities of *Arteries* and Veins in Human Bodies and *Quadrupeds*; ſince thoſe Animals it has hitherto been ſeen in (to any ſatisfaction, as Mr. *Lewenhoeck* confeſſes) have been either ſuch Fiſh, or of the Amphibious kind, that have but One Ventricle in their Hearts, and their Blood actually cold, except in Bats, in which it appears very obſcurely: Add to this, that the Blood in thoſe Creatures does not Circulate with ſuch Rapidity as in Animals whoſe Hearts have Two Ventricles. For in all Animals, that have Biventrous Hearts, the Veſſels of the reſt of the Body return their Blood to the Heart in equal time and quantity with thoſe of the

Lungs, notwithſtanding the Inæquality of their Courſe. * This Difference in the principal Organs of the Circulation of the Blood in thoſe Creatures, (on which only theſe Experiments have been hitherto made) mov'd me to make ſome on Animals whoſe Organs differ only from the Human in their groſs Figure, and not in their Intimate Structure: For this end I took a young Cat, about ten or twelve Days old, and faſtened it to a Board as in Viveſection; and making an Inciſion through the *Linea Alba*, the *Omentum* and Inteſtines were extruded; then cauſing the Creature to be ſo held (on the Board) under a large Double Microſcope, where a flat Glaſs for receiving of Objects was placed Horizontally, on which I expanded the *Omentum* or Caul, (a Light being placed underneath) I ſaw the Globules of the Blood move very ſwiftly in the ſmall Veſſels, which are only to be ſeen in the moſt Tranſparent parts of the Membranes of its *Omentum*; but the motion of the Blood ſoon abated, and its Globules were withdrawn from the Extremities of its Blood Veſſels; and in a little time became ſtagnant in their larger Branches. This appearance of the continuation of the Extremities of the *Arteries* and Veins, while the Blood was moving in them, in the *Omentum* or Caul, is expreſt by *Fig.* 4. *A A* ſhews the Trunks of the *Arteries*. *B B* the Veins, which were diſtinguiſhable by contrary currents of the Globules of the Blood in each Veſſel. *C C C* ſhews the Branching of the Extremities of the *Arteries* and Veins, that no longer Aſſociate with each other, but are United, as here expreſt. After I had ſeen this, I attempted to ſhew the like to ſeveral Friends, but did not always ſucceed ſo well as when Mr. *Chambers* and Mr. *Buckeridge* favoured me with their Preſence, at a time when I happened to have a young lean Dog, that was not large; in whoſe *Omentum* we ſaw it very well; but by the Aſſiſtance of an Inſtrument I had prepar'd to expand the Meſentery, we all ſaw it there much better; that part having not only larger and clearer Spaces than the *Omentum*, but its Blood Veſſels are diſtributed more regular, as appears by *Fig.* the 5th, where the ſame Letters of reference ſerve as above.

* Mr. Wotton *in his Reflections upon Antient and Modern Learning. Chap. xviii.*

Thoſe who will entertain themſelves in viewing the tranſparent parts of Living Creatures with Microſcopes, will find that the extremities of their *Arteries* and Veins are not all equally leſſen'd, tho united. In the Tail of the *Lacerta Aquatica*, *Tadpoles*, and in moſt Fiſh I have examin'd, I have frequently obſerv'd ſeveral Communications between the *Arteries* and Veins; in which more than two Globules of Blood have paſt abreaſt: And in the ſame *Area* I have ſeen ſome of thoſe Communications ſo ſmall, as that but one Globule could paſs, and that very ſlowly before the other. In young Fiſh, particularly Grigs, I have frequently obſerv'd a Communicant Branch, ſo very ſmall as that one Globule of Blood only has paſt it in two or three Seconds of a minute: at other times I have found conſiderable Intervals in paſſing of one Globule in ſuch a Communicant Branch; even half a minute, a whole minute, and once in two or three minutes I have ſeen one Globule of Blood only paſs in a particular tract. The prompt paſſing of Liquors, injected by the *Splenick Arteries*, to the *Veins*, ſhews the Communications between thoſe Veſſels are more open than the *Arteries* and *Veins* of other parts, of which I have elſewhere ſpoken. Liquors alſo Injected into the *Pulmonick Arteries* paſs to their *Veins*, though not altogether ſo freely as in the *Spleen*. On viewing the Extremities of the *Pulmonick* Blood-Veſſels in a living Frog with my *Microſcope*, I found their *Communications* much larger than *thoſe* that I had before ſeen in the Membrane between the Toes and in the Feet of the ſame Creature. Nor can we reaſonably doubt of the like patent Communications of the *Arteries* and *Veins* of *Humane Lungs* and thoſe of *Quadrupeds*, when we conſider the *Blood* of their Lungs muſt return to the Heart in equal *Time* and *Quantity*, with *that* of all the parts of the Body beſides, as before noted. Hence it appears that the Bronchial Blood Veſſels (firſt taken notice of by the Accurate *Ruyſch*) are abſolutely neceſſary, elſe the parts of the Lungs could not receive nouriſhment; nor could the Glands of the *Bronchiæ* ſeparate their Liquor, if they were ſupplied with *Blood* from the Pulmonick Blood Veſſels *which* is ſo quickly diſpatched through the Lungs.

On viewing the Membrane that is between the Toes of one of the hinder Feet of a living Frog, after I had frequently taken hold of the ſame Leg of that Creature, to apply it to the Microſcope, I found that Membrane very tranſparent, and without any motion of the Globules of the Blood in it, as if the part had been dead; but while I was looking on it, it was, I confeſs, not a little entertaining to ſee the Globules creep into it by degrees, and at length the Blood move in all the Branches of its *Veins* and *Arteries* as before, when no violence had been offered to the part: While the Blood is thus leiſurely creeping through the Veſſels, you may plainly ſee its Globules compreſt into Oval Figures, which are made more or leſs oblong, by the reſiſtance thoſe Globules meet with by the contraction of the ſides of the Veſſels they paſs through; and this I have more than once obſerved in the Tails of the Water *Newts* or *Lizzards*: But on examining the Blood of theſe Creatures with a Microſcope, and comparing it with the Humane Blood, I found the Globules of the *Lizzards* Blood more incline to an oval Figure, and were as big again as the Globules of Humane Blood, and that of a ſmall Fiſh: which I in like manner viewed at the ſame time. It is not unlikely a ſudden retroceſſion of Blood from the extremities of its Veſſels often happens, and its *Circulation* in the ſame Veſſels is afterwards carried on without any impediment; as on ſome Paſſions of the Mind, Deliquiums by the effuſion of Blood, or otherwiſe. But if the Blood is once become ſtagnant in its Veſſels (eſpecially the *Arteries*) the part is in no ſmall danger of a Mortification, unleſs its neighbouring Veſſels, which enjoy the motion of the Blood, drive on the ſtagnant Blood, and it eſcape by the ſides of the Veſſels that retain'd it. Experience aſſures us, that in Bruiſes when the Blood is extravaſated it goes off either by *Tranſcolation* or elſe cauſes an *Abſceſs*; for there's little reaſon (in my opinion) to ſuſpect any of the ſtagnant Globules of the Blood will be fit to re-unite with the *Circulating* Maſs. But that the Blood after ſtagnation in its Veſſels will ſometimes paſs their ſides, appear'd to me from the following Experiment. On viewing the *Meſentery* of a Dog when living, in which I had before ſeen the Blood paſſing the extremities of the *Arteries* and *Veins*, I conſider'd how to preſerve the Blood in its Veſſels, that I might afterwards at any time ſee it in their Extremities when ſtagnant: For this end I caus'd ſeveral parts of the *Meſentery* to be tyed on as many pieces of ſmall round *Pill-Boxes*, cut tranſverſely like little hoops; on which *Portions* of the *Meſentery* were extended like the head of a *Drum*; and on viewing *them* afterwards with my *Microſcope*, I found the Extremities and Branches of the Blood Veſſels charged with Blood, which before appeared in Motion; ſome of which *parts* of the *Meſentery* I ſtill keep by me. On laying one of theſe *Parts* of the *Meſentery* (thus expanded) in Water, the ſtagnant Blood in its Veſſels diſappear'd; but on juſt immerſing another of thoſe *Pieces* in Water, I could with my Naked Eye ſee the ſtagnant Blood diffuſed in the Interſtices of the Blood Veſſels, and between the Membranes of the *Meſentery*: Hence it's evident, the Blood may paſs the ſides of its *Veſſels* after ſtagnation in *them*; but whether its *Globules* are broken, or what figure renders *them* fit to paſs thoſe pores that are in the ſides of the Veſſels, I leave to the Inquiſitive; but we muſt return to our *Tables*, and firſt of that of the *Syſtem* of the *Vena Cava*.

As the *Arteries* are known to export the Blood, ſo the *Veins* carry it back again to the *Heart*; but having already deſcribed their Extremities, we come next to the large Trunks of the Veins; and here, as in the *Arteries*, we find the common practice of *Nature*, in diſpoſing the Branches of *Veins* to diſcharge the Refluent Blood into the next adjacent Trunk, and ſo on to the Heart. As the *Arteries* afford abundance of Inſtances of *Checks* given to the Velocity of the Current of the *Blood* through ſeveral parts, ſo the *Veins* ſupply us with as many *Artifices* to aſſiſt its regular return to the *Heart*, as well as favour thoſe *Contrivances* in the *Arteries*. The Trunks of the *Carotid*, *Vertebral* and *Splenick Arteries* are not only variouſly Contorted, but are alſo here and there *Dilated*; ſo the *Veins* that correſpond to thoſe *Arteries* are alſo variouſly *Dilated*. The beginnings of the Internal *Jugulars* have a *Bulbous* cavity (Fig. 7. H, H,) which are *Diverticuli* to the Refluent Blood in the *Sinus*'s of the *Dura Mater*, leſt it ſhould deſcend too faſt into the Jugulars. The like has been alſo taken notice of by Dr. *Lower* in the *Vertebral Sinus's*. The *Splenick Vein* has divers *Cells* opening into it near its Extremities in Human Bodies; but in Quadrupeds the *Cells* open into the Trunks of their *Splenick* Veins. The *Spermatick* Veins do more than equal the Length of the *Arteries* of the *Teſtes* in *Men*; their various *Diviſions* and ſeveral *Inoſculations* and their Valves, are admirably contriv'd to ſuſpend the Weight of the Blood, in order to diſcharge it into the larger Trunks of the *Veins*; and were it not that the Refluent Blo from the *Teſtes* is a *Pondus* to the Influent Blood from the *Arteries*, and ſtill leſſens its current in the *Teſtes*; theſe Spermatick Veins, like tho of other parts, might have diſcharged their Blood into the next adjacent Trunk. Who can avoid ſurprize at the Art of Nature, in contriving the *Veins* that bring part of the Refluent Blood from the lower par of the Body? when they conſider the neceſſity of placing the Huma Heart, as well as that of moſt Quadrupeds, ſo far from the *Center* o the Body towards its upper part? It is for that end neceſſary the larg Trunks of the *Veins* and *Arteries* ſhould not aſſociate each other; for i all the Blood ſent to the lower parts, by the Deſcending Trunk of th *Aorta*, ſhould return to the Heart again by one ſingle Trunk (as it i ſent out from thence) the *Weight* of ſo much Blood in the Aſcendin Trunk of the *Vena Cava*, (Fig. 6. C, C, A) (for ſo its lower Trunk i call'd) would oppoſe the force the Heart could give it from the Arterie and hinder its aſcent: For this reaſon the *Vena Azygos* (Fig. 6, b) o *ſine pari*, is contriv'd to convey the Blood ſent to the Muſcles of th Back and *Thorax* into the Deſcending Trunk of the *Vena Cava* (ib. B, A.) above the Heart: Hence it's evident, more Blood comes into th Heart by the Deſcending, or upper Trunk of the *Vena Cava*, Fig B, A. than paſſes out by the Aſcending Trunks of the *Aorta*. Nor do this quantity of Blood convey'd to the Heart by the *Superior Trunk* o the *Cava*, ſeem without ſome other deſign in Nature, beſides Tranſp ting it thither to free the *Inferiour Trunk* from its Weight: But pe it was neceſſary ſo much Blood ſhould be ready there to joyn with th Chyle, (Fig. 6. †) for its better Mixture, before it reaches the Rig Auricle of the Heart.

Vid. Lower de Corde, cap. 1. pag. 1.

Plate 7.

Fig. 1. Repreſents the Trunks and large Branches of the *Arteri* diſſected from an *Adult Human Body*, when diſplayed and dryed; a they are now to be ſeen in the *Repoſitory* of the *Royal Society*. 1. Th Trunk of the *Aorta* cut from the *Baſis* of the Heart. 2. That part o it, whence the Coronary Artery of the Heart does ariſe. 3. That par of the *Arteria Magna*, where the *Canalis Arterioſus* of the *Fœtus* term nates; which in an *Adult* becomes a Ligament. Vid. Fig. 2, 3. 4. That part of the *Axillary-Arteries*, by ſome called the *Subclavian Arteri* 5. The Left *Carotid Artery* (in this ſubject it ſeems) ariſing from a Common Trunk with the Right *Carotid* and *Axillary Arteries*, as in ſom Quadrupeds. 6. The Left *Cervical* Artery in this ſubject ariſing fr the Trunk of the *Arteria Magna*, as expreſt in a Figure given by *Berg* in the *Acta Eruditorum An.* 1698. pag. 295. But in all the Human Bodi in which I have hitherto examined theſe *Arteries*, I have conſtan found them as expreſt Fig. 2. 6. 6. 7. The *Arteries* that carry Blood the lower parts of the Face, Tongue, Adjacent Muſcles and Gland 8. The Trunk of the *Temporal Artery*, ſpringing from the Carotid, a parting with branches to the Parotid Gland 9, and Temples 10, parts adjacent. 11. The *Occiputal Arteries*. 12. The *Arteries* that

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Fig: 1.

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Fig: 3.

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Fig: 4.

Fig: 5.

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M. V.d. Gucht Sculp.

vey Blood to the *Fauces Gargareon* and adjacent Muſcles. 13. The Trunk of the *Carotid Artery* cut off, before it is *Contorted* in paſſing the Skull. 14. The Trunk of the *Artery* of the Arm parting with Branches to the adjacent Muſcles and Parts.

*That part of this *Artery* which is ſometimes prickt in Letting Blood, and makes an *Aneuriſma*, in which caſe this Trunk of the Artery muſt be bared and *firmly tyed above* the *Aneuriſma*; and if it afterwards happens (as it has been frequently known) that the flux of Blood to the *Aneuriſma* in the *Artery* is not very much abated, though the *Artery* has been *tyed above*; the *Operator* in that caſe muſt make another Ligature on the Trunk of the *Artery below* its *Aneuriſma*: Theſe Collateral Communications of the Trunk of the Artery at the bending of the Cubit, preſerve the Circulation of the Blood in the Cubit and Hand, though the *Trunk* is totally compreſt both *above* and *below*; and the ſame *Trunk* afterwards divided between thoſe Ligatures. Hence it is, if one Ligature made above the wound in the Artery is not ſufficient, but the Blood ſtill pours out from below, the *Patient* will ſooner recover the Action and Strength of the Muſcles of the Cubit, than Thoſe in whom the upper Ligature proves ſufficient; the reaſon of which is obvious to any who conſider that the *Communicant Branches* muſt be larger where the lower Ligature is required, than when the ſuperior Ligature only is ſufficient: Theſe *Communicant Branches* (as I have ſeen them in ſome ſubjects) are here markt out in prickt Lines, *vid.* the Fig.

While theſe Papers were lying by me, the two following Inſtances happen'd, in which the *Communications* of the large Trunks of the *Arteries* of the Cubit and Arm were remarkable. The firſt was,

A *Boy* of thirteen years, who, about three weeks before I ſaw him, receiv'd a Wound near the middle of the *Cubit* in which the *Trunk* of the *Artery* (markt in the Fig. †.) was divided. The *Surgeon* who was firſt call'd had frequently bound up the Wound, and put a ſtop to the ſeveral diſcharges of Blood (which they told me did not amount to leſs than 6 or 7 quarts at times) but not without a Compreſs on the Trunk of the *Artery* above the Wound. On another impetuous Flux I was called; but ſeeing no ſmall quantity of Blood diſcharged, I was contented to let the Wound be bound up, in the ſame manner as it had been done before; omitting the Compreſs on the Trunk of the *Artery* above, and adding a piece of Deal-board, on which the Hand and Cubit were faſtened, to prevent any Motions of thoſe parts, as well as the Fingers: Three days after, the applications were taken off, and little or no Blood appear'd; but two or three hours were ſcarce elapſed e're I was alarm'd with notice of a freſh Flux. The *By-ſtanders* being inſtructed in that caſe, to compreſs the *Trunk* of the *Artery* above the *Cubit*, *they* had thereby prevented no ſmall effuſion of Blood, which muſt otherwiſe have happen'd: His *Surgeon* being out of the way, I laid the Trunk of the Artery bare above the Wound as expeditiouſly as I could, being forced more than once to let looſe the Compreſs above to diſcover its

Orifice by the Flux of Blood. I paſſed a *Needle* with ſtrong Waxed Thread under the Artery, and make a ligature on its Trunk, (which lay concealed in the Interſtice of the *Muſculus Flexor Digitorum*, and the *Muſculus Ulnaris Flexor Carpi*;) but notwithſtanding this Ligature on the Trunk of the Artery above the Wound, the Blood ſtill flow'd from the Lower Trunk of the divided *Artery*, yet the velocity of its Current was ſo much abated, that it ſeem'd like Blood flowing from a Vein. I left the Wound with a digeſtive, and the part without hard bandage It being now five Weeks ſince, I hear the Wound is almoſt Cicatriz'd.

A *Boy* about eight years of Age, came to Town with an *Aneuriſma* of the Left Arm, upon Bleeding ſix Weeks before. The Tumour was indeed very large in proportion to ſo ſmall an Arm. After laying the *Aneuriſma* or Tumour bare, and making a Ligature on the Superior Trunk of the *Artery* (in the annext Fig.* . . . . .) I found, on looſening the Compreſs on the ſuperior Trunk of the Artery, very little abatement of the Pulſation of the *Aneuriſma*; I then paſſed a Ligature in like manner on the Trunk of the Artery below the Tumour; but notwithſtanding the Pulſation continued, tho much abated. I then diſcovered another Trunk of the *Artery*, ariſing from the lower part of the Tumour, on which I alſo made another Ligature, and the Pulſation was then taken off. However on cutting off the ſurface of the *Cyſtis* or dilated *Artery*, and clearing it of the coagulated Blood, there ſtill poured out ſome freſh Blood, which was ſoon ſtopt with a common aſtringent; I left the part without any other Ligature or hard Bandage. Is is now eighteen Days ſince the Operation, the Ligatures on the *Arteries* are all come off, and the Pulſation of the *Artery* of the Wriſt begins to be very manifeſt, nor does any Symptom appear that threatens
n.285. *p.*1391. ſucceſs. He has ſince recover'd the intire uſe of his Arm, and is in perfect Health.

15. The diviſion of the Trunk of the Artery of the Arm below the Flexure at the Cubit. 16. The external Artery of the Cubit, which makes the Pulſe, that is commonly felt near the *Carpus*. 17. The Arteries of the Hands and Fingers. 18. The Mammary Artery. 19. 19. The deſcending Trunk of the *Arteria Magna*. 20. 20. The Intercoſtal Arteries. 21. The *Arteria Cæliaca*. 22. The *Arteria Hepatica*. 23. The Trunk of the *Arteria Splenica*. 24. The *Arteria Epiploica Siniſtra*. 25. A Branch of an Artery which paſſes to the bottom of the Stomach. 26. The ſuperior Coronary branch of the Stomach. 27. 27. The ſuperior Meſenterick Artery. 28. 28. The emulgent Arteries. 29. The inferior Meſenterick Artery. 30. 30. The Lumbal Arteries. 31. 31. The two Spermatick Arteries, which in this ſubject ſeem to ariſe at a greater diſtance from each other than commonly. 32. The Iliack Artery. 33. The *Arteria Sacra*. 34. The Internal Iliack Branch. 35. The External. 36. The Epigaſtrick Artery. 37. Branches of the External Iliack Artery, paſſing to the Oblique Muſcles of the *Abdomen*. 38. 38. The Arteries tha

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paſs to the Muſcles of the Thigh and *Tibia*. 39. The Crural Artery. 40. The Umbilical Artery, with thoſe of the *Penis*. 41. That part of the Crural Trunk that paſſes the Ham. 42. The three Trunks of the Arteries of the Leg. 43. The Arteries of the Foot and Toes.

Fig. 2. The Trunks and ſome of the Ramifications of the *Arteries* of an adult Human Body fill'd with Wax, to ſhew the Variety in Nature, and ſupply the defects of the former Figure. 1. The *Aorta* cut off at the *Baſis* of the Heart. *A.* The three Semilunary Valves as they appear when the Heart is in Diaſtole, and hinder the Blood coming back from the *Arteries* into the Left Ventricle of the Heart. *B.* A Portion of the Trunk of the *Arteria Pulmonalis*. *b b* its diviſion before it paſſes to the right and left Lobes of the Lungs. *C.* The deſcending Trunk of the *Arteria Magna*. *D D* The Internal Mammary *Arteries*. 2. The Trunk of the Coronary, cut off. 3. The *Ligamentum Arterioſum*, which in the *Fœtus* is the *Canalis Arterioſus*, and conveys Blood from the Pulmonick Artery to the great Artery. 4. The Trunk of the Subclavian Artery. 5. 5. The Carotids. 6. 6. The Vertebrals. 7. 7. The Arteries which paſs to the lower parts of the Face, Tongue, adjacent Muſcles and Glands. 8. 8. The Trunks of the Temporal Arteries ariſing from the Carotids, giving Branches to the Parotid Glands (9. 9.) and the Temples (10. 10.) *&c.* 11. 11. The Occipital Arteries. 12. The Arteries of the *Fauces*, *Gargareon*, &c. 13. 13. The Contortions of the Carotid Arteries, as they paſs the *Baſis* of the Skull: Theſe Trunks of the Carotid Arteries in Dogs (like thoſe I gueſs of moſt Quadrupeds) are very much Contorted before they reach the *Baſis* of the Skull: on filling theſe Veſſels of that Animal with Wax, I found thoſe Branches of them which paſs to the Brain, firſt clipping the hinder parts of the lower Jaw, immediately under its Condiloide Proceſſes; where thoſe Arteries are received in two *Sinus*'s of that Bone, which *Sinus*'s may alſo be ſeen in the Jaw-bones of other Quadrupeds, but not in Human Bodies. 14. 14. Thoſe parts of their Trunks that paſs by each ſide of the *Sella Turcica*, whence divers ſmall Branches ariſe, and help to compoſe the *Rete Mirabile*; which is more conſpicuous in Quadrupeds than in Human Bodies. 15. 15. The Contortions of the Vertebral Arteries, where we find their Trunks conſiderably dilated. 16. The Vertebral Arteries, as they aſcend on the *Medulla Oblongata* towards the annular Protuberance or *Pons Varoli*. 17. 17. The Communicant Branches of the Vertebral and Carotid Arteries. 18. 18. The Arteries of the Brain diſplayed.

Fig. 3. I chooſe to place this Figure on the Copper Plate of one of the Trunks of the Arteries of the *Tibia* (diſſected from the Leg after Amputation) rather than the following Diſtich, which I find written on the Original Table of this Scheme of the *Arteries*.

Pulſificus Sanguis, de Cordis Ventre ſiniſtro,
Funditur, ut Corpus nutriat hiſce viis.

Before I explain the Letters of Reference of this 3d Figure, it will be neceſſary to let you know that Mr *Stringer* was in his ſixty ſeventh Year when this Artery was taken from him, and near twenty Years before loſt the uſe of both his Legs; and in that time he had been ſo perſecuted with Convulſions in them, that neither Leg was free a quarter of an hour together, whether Sleeping or Waking. At length one of his little Toes mortified, which was taken off by Mr *Goldwyer*, an expert Surgeon of *Saliſbury*; not long after more Toes of the ſame Foot followed the like fate: The Convulſions following that Leg ſtronger and quicker: That part of the Foot next the Toes became tumid and inflam'd, the Tumor extending itſelf above the *Maleoli*: A Sinuous Ulcer paſſed by the ſide of one of the Metatarſal Bones; the extremity of which Bone (whence the Toe was taken off) lying bare. In this condition I found the Left Foot and Leg of this Gentleman, when I had the Honour to wait on him by Command of the Right Honourable the preſent Earl of *Shaftsbury*, he living in the Neighbourhood of that Noble Peer in *Wiltſhire*; where I met with Mr *Goldwyer* above-mention'd; and finding the Leg very chilly, the neceſſity of parting with it was too evident; which Mr *Stringer* ſuffer'd with extraordinary Fortitude, he not ſo much as expreſſing the leaſt outcry during the Operation, tho the part did not want the moſt exquiſite ſenſe of feeling: On the abſciſſion (which was about five or ſix Inches below the Knee) it was unexpected, by me, I muſt confeſs, to ſee ſo little Blood ſpouting from the *Arteries*. The Stump being bound up, and committed to the Hands of two or three Servants, a leſs number not being ſufficient to hold it, by reaſon ſuch ſtrong Convulſive motions purſued the part on the Operation. I was very deſirous to examine the *Arteries* of the amputated Leg, having before diſcovered the Cauſe of a Mortification of the Arm of a Young Gentlewoman who dy'd not long after an Amputation of the part, tho the Gangreen did not appear to reach near the place where the Abſciſſion was made, (*i. e.* below the ending of the *Muſculus Deltoides*); In which Caſe, I found the ſides of the Trunk of the *Artery* of the Arm ſo thickn'd, that the Diameter of its Bore was contracted to leſs than a third part, and would ſcarce admit a common Probe to paſs it, *Fig. G. H. I.* When I had found the ends of the *Arteries* in the Leg above-mention'd, I endeavour'd to paſs my Probe into one of them, but meeting with ſome oppoſition, I ſuſpected I had miſtaken the Vein for the Artery, and that the Valves oppoſed the paſſing of the Probe that way; but on further diſſection I clear'd the Trunks of both thoſe Blood Veſſels, and found the Veins in their Natural ſtate; but the ſides of the *Arteries* were grown Bony or Stony; having clear'd two of their Trunks, I left one of them at *Saliſbury*, the other I brought to Town, and is here Figur'd.

A The Upper part of the Artery cut off in Amputation of the Leg; from ***A*** to *B* The Trunk of the Artery diſtended and dry'd to ſhew

Fig. 7.

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Fig. 6.

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T
V
V
C

Venas has et Humani Corporis Arterias Geminis Tabulis adplicitas Patavio a se pridem deductas: Regali demum Societati dedit *Iohannes Evelynus* ejusdem Socius MDCLXVII.

Fig. 8.

A
A
A
b
a
D
B
C
C
E
F
G
H

M. V. d. Gucht Sculp.

its Canal. *C* That part of the Trunk of the Artery which was ſo contracted by the Petrifaction or Oſſification, that a Probe would not paſs its Canal; From *C* to *D* The Trunk of the Artery opened and expanded. *E E* The Petrifactions or Oſſifications in the ſides of the Artery. *F F* Their ſpecks in the lower part of the Artery, not ſo large as in the upper part, and placed at greater diſtances. *a a* &c. The Branches ariſing from the Trunk of the Artery. *G* A Portion of the Trunk of the Artery of the arm above-mentioned. *H* The ſides of the Artery very much thickned, whereby the Diameter of its *Canalis* was ſo diminiſhed that the Probe *I* would not paſs it.

The Oſſifications in the Coats of *Arteries* have been frequently obſerv'd, eſpecially in their large Trunks within the Cavities of the Thorax and *Abdomen*, but I don't remember the like has been taken notice of in the Limbs; or that ſuch impediments in their Canals have been found the cauſe of Mortifications of particular parts, as in the Inſtance above-mentioned; tho I doubt not, but the like has often happen'd in aged People, eſpecially where we find the progreſs of the Gangreen not very ſwift, and its beginning from no external Cauſe; the conſequences of which are commonly found Fatal. When the Arteries of one Leg (or of any other Limb) are ſo affected, we may well ſuſpect the like in thoſe of other parts; which probably happened in the Inſtance I now mention'd; for tho no Gangreen came on the ſtuup, yet the other Foot and Toes began to Mortifie about 6 weeks after the Amputation, as did the parts about the Hips, which were compreſt in Laying or Sitting, before he expir'd.

FIG. 4. Repreſents the Extremities of the Blood Veſſels, as they appear while the Blood is paſſing them in the *Omentum* of a live Dog, view'd with a Microſcope. *A A* The Branches of Arteries, and *B. B.* the Veins which Aſſociate. *C C* Their leſſer Branches where they paſs from each other, and are United at their extremities.

Fig. 5 The like appearing in the *Meſentery* of a Dog when living. *D, D,* The *Area*, that are here viewed with the Microſcope, as they appear to the naked Eye.

Fig. 6. The Trunks of the *Vena Cava*, with their Branches diſſected from an adult humane Body, done from the Original Scheme in the Repoſitory of the *Royal Society*. *A A* The Orifice of the *Vena Cava*, as it appears when cut from the Right Auricle of the Heart. *a* The Orifice of the Coronary Vein of the Heart. *B. A.* The Superior, or Deſcending Trunk of the *Vena Cava*. *C, C, A,* The Inferior or Aſcending Trunk; ſo diſtinguiſhed from the Motion of the Blood in theſe Trunks, which is contrary to their Poſition. *D D* The Subclavian Veins. † That part of the Left Subclavian Vein, where the Thoracick Duct enters it, and diſcharges it ſelf of its *Chyle* and *Lympha*. *b.* The *Vena Azygos* with its Branches going to the Ribs, *e, e.* *c.* The ſuperior Intercoſtal Veins. *d, d* The internal Mammary Veins. *E, E,* The Right and Left Iliack Branches. *F, F,* The Internal Jugular Veins. *G, G,* The External

Plate 8.

ternal Jugulars. *H, H,* The Veins which bring Blood from the lower Jaw, and its Muſcles. *I, I,* The Trunks of the internal Jugulars cut off at the *Baſis* of the Skull. *f,* The Veins of the *Thymus* and *Mediaſtinum.* *g, g,* The Veins of the Thyroid Glands. *h,* The *Vena Sacra.* *i,* The Internal Iliack Branch. *k,* The External ——— *K, K,* The Occiputal Veins. *L,* The Right Axillary Vein. *M,* The Cephalick. *N,* The Baſilick. *O,* The Median Vein. *P,* The Trunk of the Veins of the Liver. *Q,* The Phrenick Vein of the Left ſide. *R,* The Right Phrenick Vein. *r* A large Vein from the left *Glandula Renalis* and parts adjacent. *S,* The Left emulgent Vein. *T,* The Right Emulgent, in this ſubject very much lower than the Left, which is not uſual. *VV,* The two Spermatick Veins. *XX* Two Communicant Branches between the Aſcending Trunk of the *Vena Cava* and *Vena Azygos,* by which the Wind paſſes into the Deſcending Trunk of the *Cava,* when we blow into the Aſcending at *A.P.C.* tho' the Trunk at *A.P.* and *C.* is firmly tyed on the Blow-pipe. * An uncommon Branch between the lower Trunk of the *Vena Cava* and the Left Emulgent Vein. *y* A Vein which brings Blood from the Muſcles of the *Abdomen* into the external Iliack Branch. *z* The Epigaſtrick Vein on the Right ſide. *l* The *Vena Saphena.*

The reſt of the Branches here diſplayed commonly differ ſo much in various ſubjects, that the particular Deſcriptions of them (which none but the Operator who diſſected them could pretend to be Maſter of) would be perhaps as uſeleſs, as tedious to repeat: Wherefore I paſs to thoſe conſiderable various Trunks which are wanting in this Scheme.

Fig. 7. Some of the large Trunks of the Veins and their *Sinus's* within the Skull, with the Beginnings of the Internal Jugular Veins, filled with Wax and dryed together with the *Falx, &c.* *A* The extremity of the *Falx* cut from the *Criſta Galli.* *a* Its lower *Limbus* that touched the *Corpus Calloſum,* as it divides the Right Hæmiſphere of the Brain from the left; where the Fifth *Sinus* paſſes, which is here dryed and diſappears. *B. B.* The ſecond proceſs of the *Dura Mater,* which ſupported the hindmoſt parts of the Lobes of the Brain, and defended the *Cerebellum* from being preſt by thoſe parts of the *Cerebrum.* *C.* A portion of the *Dura Mater* remaining to the Longitudinal *Sinus.* *D. D.* Several Trunks of the Veins of the Brain cut off before they enter the Longitudinal *Sinus.* *E E* The Longitudinal *Sinus*'s. *F F* The two lateral *Sinus*'s. *G* The Fourth *Sinus.* *g* The Veins from the *Plexus Choroides.* *HH* The *Bulbi* or *Diverticuli* at the beginnings of the internal Jugular Veins. *II* The internal Jugular Veins. *KK* The Trunks of Veins which bring Blood from the lower Jaw and parts adjacent.

Fig 8. The Trunks of the *Vena Portæ* diſſected and diſplayed; done from the Original Scheme in the Repoſitory of the Royal Society. *A A* The Branches of the *Vena Portæ* freed from the Liver. *a* The Umbilical Vein. *B* The Splenick Branch. *C C* The Meſenterick

Branches which are continued from the Inteſtines. *b* The Trunk of the *Vena Pancreatica*, which receives Branches alſo from the *Duodenum*. *c c*. The *Vena Gaſtrica dextra Coronaria Superior*. *D* The Superior Coronary Vein of the Stomach of the Left ſide. *E* The inferior Coronary Branch of the Stomach of the Right ſide, and *F* The ſame Coronary Vein of the Left ſide removed from their proper Situations; from theſe two laſt are continued the *Vena Epiploica Superior dextra* 1, and the *Siniſtra* 2, with the *Media* 3. *G* The Vein call'd *Vas breve*. *d* The *Vena Duodeni*. *H* The *Vena Hæmorrhoidalis* ariſing from the *Rectum* and *Anus*, in this ſubject emptying itſelf into the left Meſenterick Branch; but in other Bodies (and particularly in a Preparation of theſe Veins, which I have now by me) I find this Trunk of the *Hæmorrhoide* Veins ending in the *Ramus Splenicus*.

The length of the Trunk of this Hæmorrhoide Vein,and its progreſs under the Inteſtines, renders it liable to be compreſs'd, and its Refluent Blood retarded; whence its Branches in the *Inteſtinum Rectum* and *Anus* become diſtended with Blood, and cauſe the *Hæmorrhoides Cæcæ* and *Apertæ*; which are frequently attended with Apoſthumations in the *Anus* and Parts adjacent; which Diſorders are the more incident, not only becauſe theſe Hæmorrhoide Veins (like the reſt of the Branches of the *Vena Portæ*) are without *Valves*, and the Blood has an aſcending progreſs in them, together that the long Trunk (*H*) is not only expoſed to the Compreſſions made by the Inteſtines in both Sexes: But particularly the *Uterus* in Women in time of Geſtation, eſpecially near the Birth, ſo compreſſes this Trunk, that it's no wonder we find Women more afflicted with the Hæmorrhoides at that time than any other. Nor are the Iliac Veins, and the Lymphaducts that accompany them without being expoſed to the like Incumbrance in Women with Child, whence the Veins of the Legs and Thighs become Varicoſe, and thoſe Limbs are ſo frequently ſwollen; which in a late Inſtance I was acquainted with, when the Intumeſcence proved ſo great, that at length the Abdominal Teguments were vaſtly extended; but the Gentlewoman recovered, beyond the Expectation of ſome, on the happy Delivery of two large Children.

Experiments relating to the Specifick Gravity of Humane Blood. by Dr. Jurin, *n.* 361. *p.* 1000.

II. 'Tis well known from the Obſervations of Mr. *Leewenhoeck* and others,that human Blood conſiſts of red Globular Particles ſwimming in a pellucid Lympha or Serum; which two different Subſtances, tho' of unequal Specifick Gravities,yet ſo long as they continue to circulate in the Veins and Arteries, are prevented from ſeparating by their Motion and Warmth. But when the Blood comes to ſtagnate and cool in a Porringer, the globular Particles uniting together by their attractive Power, and ſinking by their Weight, which is greater than that of the *Serum*, form the *Coagulum* or *Craſſamentum* at the bottom of the Porringer, the *Serum* ſwimming above it. Things always happen in this manner, when the *Craſſamentum* is at liberty to ſubſide; but it often falls out, that either by its Adheſion to the ſides of the Veins, or by the Bubbles of Air, which the Blood gathers upon falling into the Porringer, and which ſtick to its Surface, the *Craſſamentum* is kept from ſinking, and ſeems to float upon the top of the *Serum*. Theſe Accidents ſeem to have given the firſt occaſion to the Opinion of thoſe who have writ upon this, that the globular Part of the Blood is ſpecifically lighter than the *Serum*, in which it ſwims. But that which has ſo fully eſtabliſht this perſuaſion, is the Authority of the late excellent Mr. *Boyle*, who has left the following Experiments upon this Subject.

The Specifick Gravity of *Serum* of Human Blood was found by weighing a piece of Sealing-wax firſt in *Serum*, and afterwards in Water, to be to the ſpecifick Gravity of Water, as 1024 to 1000. In a ſecond Experiment, which for accuracy was made with an Inſtrument contriv'd on purpoſe, the ſpecifick Gravity of *Serum* was found to be to that of Water, as 1194, to 1000. In a third Experiment made by the ſame Inſtrument, and with *Serum* from the Blood of another Perſon, its ſpecifick Gravity appear'd to be 1186. The Medium between theſe two laſt Experiments is 1190, which has ſince been univerſally receiv'd for the ſpecifick Gravity of *Serum* of Human Blood, the firſt Experiment being declar'd by Mr. *Boyle* himſelf to be leſs exactly made than the other. The ſpecifick Gravity of Human Blood was found by Mr. *Boyle*, to be to that of Water, as 1040 to 1000; though on account of Difficulties by him mentioned, he was far from being ſatisfy'd with this Experiment, and recommended the thing to farther tryals.

Theſe Experiments however having hitherto paſt uncontroverted, and it appearing from them, that the ſpecifick Gravity of *Serum* was greater than that of Blood in the proportion of 1190 to 1040, or of 8 to 7 nearly; it was a neceſſary conſequence of this, that the Blood Globules were ſpecifically lighter than the *Serum*, and that in a very great degree, conſidering the ſmall proportion that the bulk of the *Craſſamentum* was found to bear to that of the *Serum*, from other Experiments. From this it was not improbably conjectured, that theſe Globules were thin Veſicles fill'd with an Aereal Subſtance: and this Opinion ſeem'd to receive a great Confirmation, upon its being obſerv'd, in viewing the Circulation by a Microſcope, that a Blood Globule, in paſſing through a very narrow Veſſel, would change its ſhape from a Globular to an Oval Form, and would recover its former Figure, as ſoon as it was got through its narrow paſſage; which appearance ſeem'd to be naturally accounted for from the Elaſticity of the included *Aura*. Upon this Conjecture have been built a great many Solutions of the Phænomena obſervable in the Animal Oeconomy, and the Diſorders of it; particularly a late ingenious account of Muſcular Motion. I hope I ſhall be eaſily pardon'd for enquiring into the ſoundneſs of the Foundation, when the Superſtructure erected thereupon is ſo conſiderable; and the following Experiments, however trivial in themſelves, will not appear unworthy the Conſideration of the *Royal Society*, if it be found, that they may prevent us from running into Errors of the greateſt Conſequence.

Exp. I. I have ſeveral times cut off a ſmall part of the *Craſſamentum*, when by its adheſion to the ſides of the Porringer it has ſeem'd to ſwim upon the Surface of the *Serum*, and have put it into another Veſſel fill'd with *Serum*: upon which it has immediately ſunk to the Bottom.

Exp. II. When the *Coagulum* has been buoy'd up in the *Serum* by the bubbles of Air adhering to its Surface, I have ſeparated a ſmall part of it, where thoſe Bubbles have been thickeſt, and put it into a Glaſs of *Serum*, in which it has ſwom as before. Then ſetting the Glaſs upon the Air-pump, thoſe Bubbles burſt after one another, as the Receiver was exhauſting, and the Air being again let into the Receiver, the Lump of the *Craſſamentum* ſunk to the bottom of the Glaſs.

Exp. III. I have often placed a Drop of *Serum* upon a clean Glaſs before a Microſcope, in which I had diſſolv'd a very ſmall Quantity of Blood, and obſerv'd, that when the Glaſs was held in a perpendicular Poſture, the Blood-Globules ſubſided to the bottom of the Drop; and inverting the Glaſs, the Globules again deſcended through the *Serum* to the bottom. I had the ſame Succeſs with a ſmall Quantity of *Serum* and Blood in a Capillary Tube. And the ſame thing has been long ſince obſerv'd by the famous Mr. *Leewenhoeck*.

Theſe Experiments undeniably demonſtrate, that the *Craſſamentum*, or globular part of the Blood, is ſpecifically heavier than the *Serum*; and conſequently it is by no means probable, that the Blood Globules are Veſicles fill'd with Air, or any other Fluid lighter than *Serum*. And that they are not fill'd with any ſort of Elaſtick Fluid, will appear from the following Experiment.

Exp. IV. In a ſmall Quantity of *Serum* of Human Blood, I diſſolv'd ſo much Blood, as that the Globules might not lye too thick together, to hinder their being ſeen diſtinctly. Then having lodged a ſmall Drop of this Liquor on the inſide of a thin Glaſs Tube, I fitted the Tube on to the Air-Pump, and placed a Microſcope by it, ſo that I could ſee the Blood Globules through the Tube. This being done, I caus'd the Tube to be exhauſted, keeping my Eye upon the Globules all the time, in order to obſerve whether they dilated themſelves, as the Air was withdrawn; but could not perceive the leaſt Alteration, they appearing exactly of the ſame bigneſs in the *Vacuum*, as they had done before. Whereas if they had been fill'd with an Elaſtick Fluid, they would either have burſt, or have been dilated to at leaſt ſeventy or eighty times their former Magnitude. The Stop-cock being afterwards turn'd, and the Air ſuffer'd to re-enter the Tube, the Blood-Globules ſtill retain'd the ſame bigneſs, as in *Vacuo*. I ſhall proceed to demonſtrate the ſame thing by Hydroſtatical Experiments.

Exp. V. *Nov.* 13. 1713. Having ſuffer'd a quantity of my own Blood to ſtand about 24 Hours in the Porringer, and then drawing off the *Serum* carefully with a ſmall Siphon into a convenient Glaſs, I found by the Hydroſtatical Balance its Specifick Gravity to be to that of Water, as 1029,8 to 1000.

Exp. VI. *February* 21. 1716-7. I examin'd the *Serum* from the Blood of another Perſon in the ſame manner, and found its Specifick Gravity to be 1028,6.

Exp. VII. VIII. and IX. *April* 8th 1717. I obtain'd 3 ſeveral quantities of *Serum* from the Blood of different Perſons. The firſt of theſe was of a deep colour, inclining ſomething to red, and a little Turbid. It's Specifick Gravity was 1029,7. The ſecond was likewiſe a little Turbid, and of a pale whitiſh Colour. The Specifick Gravity of this was 1030,2. The third quantity of *Serum* was perfectly clear, and of the colour of Canary. The Specifick Gravity was found to be 1030.

Exp. X. *January* 15th 1718-9. I drew off all the *Serum* from five or ſix ſeveral Porringers, containing the Blood of different Perſons. This I found to be a little ting'd with Blood, which was occaſion'd by my being oblig'd to draw it off pretty near to the bottom of the Porringers, in order to obtain a quantity ſufficient for my purpoſe. For this reaſon I ſuffer'd it to ſtand about two Days, in which time the Globular part of the Blood was entirely precipitated to the Bottom, and the *Serum* was become perfectly fine and tranſparent. I then drew it off with a Siphon into a Glaſs Vial with a narrow Neck, which I fill'd to a certain mark made in the Neck for that purpoſe. This done, I plac'd my Vial in a nice pair of Scales, in which I had a Counterpoiſe for the weight of the Vial, and found that quantity of *Serum* to weigh 2284¼ Grains. Then pouring out the *Serum*, I fill'd the Vial with common Water to the ſame mark, and found the weight of the Water to be 2219 Grains. From which it follows that the Specifick Gravity of this *Serum* was 1029,4.

Exp. XI. *July* 14. 1719. I procur'd a quantity of Blood taken from the temporal Artery, from which I drew off the *Serum* the next Day, and weighing it in the ſame manner found it's Specifick Gravity to be 1028,8.

The little difference between theſe Experiments may very well be attributed to that which is between the *Serum* of different Perſons; or to the variations occaſion'd by Heat and Cold in the ſeveral Seaſons of the Year, in which they were made. So that from them we may ſafely determine the Specifick Gravity of *Serum* of Human Blood at a Medium to be 1029,5, or in a round number 1030. From which the greateſt Variation in any of theſe Experiments is little more than one in 1000; whereas the difference between Mr. *Boyle*'s Experiments and mine amounts to 160 in 1000.

Exp. XII. *April* 6. 1717. In order to find the Specifick Gravity of Human Blood, which by reaſon of it's tenacity, and ſudden alterations upon ſtanding, cannot be determin'd by the Hydroſtatical Balance; I took a narrow neck'd-Vial, and fill'd it to a Mark, with Blood pour'd immediately out of the Porringer, as ſoon as the Perſon was blooded. This I weigh'd, as I had done the *Serum* before, and found it's Specifick Gravity to be 1051.

Exp. XIII. *Auguſt* 5th 1717. Having fill'd the ſame Vial with the Blood of another Perſon, running immediately out of the Vein through a Funnel, it's Specifick Gravity was determin'd at 1053. Suffering this to ſtand till it was cold, I found the Blood was ſunk a ſmall matter below the Mark in the neck of the Vial. This being fill'd up with the Water, which in ſo ſmall a quantity could make no ſenſible difference from Blood, I found the Specifick Gravity of cold Blood to be 1055.

Exp. XIV. *Auguſt* 6th 1718. The laſt Experiment being repeated in the ſame manner as the Year before, the Specifick Gravity of cold Blood was again found to be 1055.

Exp. XV. *July* 14th 1719. The Arterial Blood, from which the *Serum* was afterwards drawn off for the 11th Experiment, being weigh'd in the ſame manner, it's Specifick Gravity was 1052,5.

As this Arterial Blood and it's *Serum*, differ no more in Specifick Gravity from Venal Blood and *Serum*, than the ſeveral Portions of theſe do from one another, it's plain, that the difference in this reſpect between Arterial and Venal Blood is wholly inconſiderable. The Animal Oeconomy indeed teaches us, that the Serous Liquor is perpetually drawing off from the Arterial Blood by the ſeveral Secretions, but as the quantity ſeparated in one Circulation is very ſmall, the Blood muſt arrive in the Veins nearly of the ſame denſity, as when it runs through the Arteries.

In the 13th Experiment we obſerv'd, that the Blood alter'd it's Specifick Gravity upon cooling from 1053 to 1055; from which we may infer, that if the Blood made uſe of in the 12th Experiment had been ſuffer'd to ſtand till it was cold it's Specifick Gravity would have been 1053; wherefore, taking a Medium between the four laſt Experiments, we may allow the Specifick Gravity of cold Human Blood to be 1054. The difference of 14 Parts in a 1000, between this and the Specifick Gravity determined by Mr. *Boyle*, is eaſily accounted for, if we conſider, that that Gentleman did not make uſe of a Veſſel with a narrow Neck, as plainly appears from the Circumſtances mentioned in his Experiment; and conſequently a ſmall error in the height of the Liquor would make a conſiderable alteration in the Specifick Gravity.

Since therefore the Specifick Gravity of Human Blood is 1054, and that of its *Serum* 1030, it is plain, that Blood is heavier than *Serum* by about one part in 43. From which it manifeſtly follows, that the Globular part of the Blood is ſpecifically heavier than the *Serum*, ſince the Globular part being ſeparated from the Blood leaves the remainder, or the *Serum*, ſpecifically lighter than the intire Maſs. But in order to determine the exact Specifick Gravity of the Blood Globules, it is firſt neceſſary to know the Proportion, which the whole quantity of the *Craſſamentum* contained in Blood bears to the *Serum*. To this end Mr *Boyle* has given us two ſeveral Obſervations of the weights of the *Craſſamentum* and *Serum*, after they have ſeparated one from another in the Porringer. But beſides the difficulty of making this Experiment with any tolerable exactneſs, it is to be conſider'd, that there is a great deal of *Serum* contain'd in the interſtices of the Globules, that compoſe the *Craſſamentum*. This difficulty however is in ſome meaſure anſwer'd by two other Experiments, which Mr. *Boyle* made for this purpoſe, after the following manner. He put a quantity of the *Craſſamentum*, already ſeparated from the *Serum*, into an Alembick, and diſtill'd off the remaining *Serum* to dryneſs, but without drawing off the Oil, or Volatile Salt; after which he weigh'd the diſtill'd Liquor, and the dry Maſs left behind. By comparing theſe Experiments with the two former, it will be found that the entire weight of *Serum* contain'd in Blood is nearly $\frac{13}{15}$ of the whole, and conſequently the weight of the dry'd *Craſſamentum* is only two Fifteenths of the Blood.

But for farther ſatisfaction, an Analyſis was made at my deſire with a large quantity of Blood, amounting to Four Pounds Fourteen Ounces, by that ingenious and skilful Chymiſt, Mr. *John Brown*. From this was obtain'd, with a very gentle heat, two Pounds, Fourteen Ounces, and ſix Drachms of a Phlegmatick Liquor, that had ſcarce any thing of the fœtid Scent, which is uſual in the diſtillation of Animal Subſtances; and it's Specifick Gravity was nearly the ſame with that of common Water, being but 1000,8. This being mixt with a ſtrong ſolution of Alum, ſcarce afforded any *Coagulum*; but exhibited a conſiderable one upon mixture with a ſolution of Roman Vitriol. The diſtillation being continued with the ſame Heat, we had ſeven Ounces more of Phlegm conſiderably impregnated with Volatile Salt, as was manifeſt from the Smell. The Specifick Gravity of this was 1007, and having mix'd it with *Tinctura Martis optima*, Solution of Alum, and of Roman Vitriol, a large *Coagulum* was precipitated. In diſtilling theſe there was loſt by Evaporation, two Ounces and two Drachms.

The third portion of Liquor, being rais'd with a ſtronger Fire, amounted to ſeven Ounces ſix Drachms. This was reddiſh, and Turbid, and ſo ſtrongly charg'd with Volatile Salts, that it might very well deſerve the name of Spirit. It's Specifick Gravity was 1080,1. Beſides theſe we had ſeven Drachms of Volatile Salt, an Ounce of Oil, and eight Ounces Four Drachms of *Caput Mortuum*, which ſtill retain'd ſome ſmall remainder of the Oil, as was manifeſt from its taking Fire at the flame of a Candle. In this latter part of the Operation was loſt three Ounces ſeven Drachms.

Upon making due allowance for the difference between the Specifick Gravities of the three firſt Portions of Liquor and that of *Serum*, as likewiſe for what was loſt in the two ſeveral parts of the Operation, which we may reaſonably conclude to have been of a Specifick Gravity nearly the ſame with that of the Liquor drawn off, it will be found, that the quantity of *Serum* contain'd in this Maſs of Blood was about $\frac{15}{17}$ of the whole Weight, and conſequently that the quantity of *Craſſamentum* was $\frac{2}{17}$ of the ſame Weight. If we calculate therefore upon this Suppoſition, that the weight of the Globular part of the Blood is $\frac{2}{17}$ of the whole, we ſhall find the Specifick Gravity of a Blood Globule to be to that of Water as 1277 to 1000. If we follow the proportion of $\frac{2}{15}$ which reſults from Mr *Boyle*'s Experiments, the Specifick Gravity of a Blood Globule will be 1242.

But this computation is in all appearance a great deal too large; for we cannot be aſſur'd, that our whole quantity of aqueous Liquor was rais'd from the *Serum* of the Blood. On the contrary it is more than probable, that a conſiderable part of it was afforded by the Blood Globules themſelves, eſpecially in the latter part of the Operation, when their texture muſt of neceſſity have been broken and diſſolv'd by the ſtrong Fire that was made uſe of. To prove this, we need only conſider the condition of the dry'd *Craſſamentum*, after the Phlegm is drawn off, that being now a hard and brittle Subſtance: whereas the Globules in their natural State are ſoft and yielding For which reaſons it may perhaps be more ſatisfactory, if we attempt to find the quantity of the Globular part of the Blood after another manner. It appears therefore from Mr *Boyle*'s Obſervation, that the quantity of *Serum*, which may be pour'd off from the *Craſſamentum*, is about one half of the whole Maſs. The remaining *Craſſamentum* conſiſts of the Blood Globules, and a quantity of *Serum* filling up the Interſtices between them; which, if the Globules keep their Spherical Form, may eaſily be found by the principles of common Geometry, to be nearly one half of the bulk of the *Craſſamentum*: but if the Globules by their preſſure againſt one another change their Figure, the quantity of *Serum* will be ſomething leſs. If this quantity of *Serum* lying between the Blood Globules be added to that pour'd off, it appears, that the *Serum* contain'd in Blood is about $\frac{3}{4}$ of the whole bulk, and conſequently that the Blood Globules make about $\frac{1}{4}$ of the whole. From which we ſhall find the Specifick Gravity of the Blood Globules to be to that of Water as 1126 to 1000. If we ſuppoſe the Blood Globules to make $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{3}$, or $\frac{1}{2}$ of the whole bulk, their Specifick Gravity will be reſpectively 1174, 1150, 1102, or 1078. So that upon any of theſe Suppoſitions, the Specifick Gravity of the Blood Globules will be conſiderably greater than that of the *Serum*, and conſequently they cannot be ſuppos'd to be Veſicles fill'd with an Aereal Subſtance.

It will therefore perhaps be askt, What do they really conſiſt of?

In order to come to a Solution of this Queſtion, it may be proper to take notice, That Blood is compos'd of Phlegm, Oil, Volatile and Fixt Salts, and Earth. For as to the Spirit, we look upon it with Mr *Boyle*, to conſiſt of the Phlegm and Volatile Salt united together. That the *Serum*, upon a Chymical Analyſis, exhibits a great deal of the firſt of theſe, and the others in a very ſmall quantity. That on the contrary the *Craſſamentum* yields much leſs Phlegm, but the other Principles much more copiouſly than the *Serum*.

From which-*Data*, I think, we may ſafely conclude, that the *Craſſamentum*, or Globular part of the Blood, conſiſts of ſome Phlegm united with the Oil and Salts, and a ſmall quantity of Earth. But what is the exact proportion of theſe ſeveral Principles to one another; what alterations are produced in the Body by a change of this proportion; how, and in what part theſe Globules are form'd; by what means they preſerve their Figure, without diſſolving in the *Serum*, or uniting with one another; what variations are made in their Specifick Gravities by Heat and Cold; and what are the effects of thoſe Variations, are Queſtions not very eaſy to be ſolv'd, and yet of ſo much importance to the Animal Oeconomy, that it were greatly to be wiſht, we had a number of *Data* ſufficient to determine them.

The following Experiments, ſerving to confirm the Method laſt made uſe of, for finding the Specifick Gravity of the Blood Globules, it may not be improper to relate them.

Auguſt 6. 1719. I took a lump of the *Craſſamentum* and waſh'd it gently in fair Water, to free it from the looſe Globules, which precipitating out of the *Serum*, after the *Coagulum* is form'd, do not unite into one Body with it. This done, I laid it on a Spungy brown Paper, in order to drain off the ſuperfluous Moiſture. After which, weighing it firſt in Air, and then in Water, I found its Specifick Gravity to be $1083\frac{1}{2}$. Another lump of the ſame *Craſſamentum* being weigh'd in the ſame manner, its Specifick Gravity was 1082,9.

September 18. 1719. I found the Specifick Gravity of another piece of *Craſſamentum* to be 1082,1. A ſecond piece from the Blood of a different Perſon gave me 1086,1. A third from the ſame Perſon gave 1086,6. From this it follows that the Specifick Gravity of the Blood Globules is at leaſt 1084, which is the Medium between theſe five Experiments. But if we allow one half of the bulk of the *Craſſamentum* to conſiſt of *Serum*, filling up the Spaces between the Blood Globules, we ſhall find their Specifick Gravity to be 1138. From this we muſt make a ſmall abatement, becauſe ſome part of the *Serum* muſt have been ſqueez'd out from between the Globules, by their yielding to one anothers Preſſure, when the lump of *Craſſamentum* lay upon the Paper: and this will reduce their Specifick Gravity ſufficiently near to 1126, as we had before determin'd it.

Of Human Blood, by Dr. Arch Adams. *n.* 325. *p.* 26.

III. Human Blood view'd in a Microſcope, is ſo far from ſhewing any red Globules ſwimming in *Serum*, that immediately after its Emiſſion, it appears to be a Body of infinite Branches, running in no certain Order, variouſly colour'd; where it lies thickeſt on the Glaſs, 'tis of a dull red; where thin, inclining to yellow; but the whole ſo blended, as to repreſent very near the Top of an Yew-tree, in a

Landskip, having its ſuppos'd Branches of a red and yellow confuſedly intermixt. Not ſatisfy'd with this Appearance, though try'd eleven different Ways, I view'd by diluting one third of thick in the *Serum* of Blood, and laying it on my Glaſs, I could ſee the red Branches, as before, and the Tranſparent fill'd with Particles of great Variety of Figures, which I took to be the Salts of the Blood, but feweſt Globular, and they were pellucid.

If the Fluids moving in an evaneſcent Artery appear Globular, I ſuppoſe, 'tis becauſe the Canal is round, which alters the Caſe much.

At the ſame time, I ſaw ſome Pleuritick Blood, and thought that its Branches ſpread in a different method from the ſound, and more ſtrongly perplext with overthwart Branches, which appear'd black like Blood that had ſtood two or three Days. Whether the Attraction of Particles ariſing from this difference of Figure, may not render the Blood incapable of paſſing thro' the Capillary Artery of the *Pleura* in that Caſe I ſubmit; but ſhould think, ſince the propellent Force of the Heart is leaſt at the Capillary Arteries, there the attractive Force of the Blood ſhould be greateſt; and ſince Spherical Bodies are moſt attractive of any, reſpect being had to their Solidities, were the Blood ſo plentifully ſtock'd with Globules as ſome ſay, we ſhould never be free from Obſtructions, the neceſſary Conſequence of this attractive Force. If in any thing I have err'd I am ready to retract.

An unuſual Hæmorrhage in a Child, by Dr. Monginot, *n.* 268. *p.* 756.

IV. Madam *R*——'s Girl fell into violent Convulſion Fits; and while ſhe was in them voided a large quantity of Blood by the Mouth, the Noſe, the Ears, and the Eyes. The Mother ſhew'd me ſome bloody Tears ſhe had gather'd. All theſe Symptoms were over in half an hours time, and the young Girl, that ſome days before had been taken with a violent Head-ach, attended with a Fever, and a mighty oppreſſion, was very well preſently after that *Hæmorrhagia*. The ſame accidents as I hear, have happened ſeveral times.

She was between two and three Years old, when on a ſudden ſhe complain'd of a violent Head-ach: She was alſo obſerv'd to be feaveriſh, and extremely reſtleſs. At the ſame time her Eye-lids did very much ſwell, and they were ſo heavy, that one could hardly open her Eyes but with a great deal of pain. Theſe ſymptoms did continue for three or four Months, though ſometimes more or leſs violent. At laſt, ſhe fell into Convulſive Motions of her Arms, Legs, and other parts, and theſe were very fierce for two days, till ſhe began to bleed by the Noſe, the Mouth, the Ears and the Eyes. The Linnen Cloaths, that were then applied to the Eye-lids, were all wet and bloody. This *Hæmorrhagia* laſted above a day; and what is more remarkable, When it was expected that the Child ſhould be extremely weakned, ſhe found herſelf ſo well freed from her Illneſs, that ſhe came to her Gaiety again, and ask'd for ſome Victuals. Within twelve Months after ſhe had four of theſe *Attacks*, but not ſo great: in particular, the Convulſion-fits were very inconſiderable, in compariſon of the firſt. The ſame Symptoms did not return again above two or three times every year, and the Head-ach did not continue above eight days before the *Bleeding*: and even then it was much more ſupportable. About two months ago I was ſent for to ſee the Girl, who is now ſeven years old. I found her in Bed, complaining extremely of a Head-ach, attended with a Feaver, a great *Catarrh*, and ſuch a ſhortneſs of breath, as if ſhe had had a *Peripneumonia*. She had been three days in that condition. I told the Mother, my opinion was ſhe ſhould be blooded preſently; but it being very late, it was put off till next morning, and then indeed it was needleſs; for very early in the morning, after ſome Convulſion fits, ſhe began to bleed from the Noſe, the Mouth, *&c.* When I came it was almoſt over, ſo that I could only ſee a little bleeding from the Mouth; but all the people in the houſe told me they had ſeen the Blood drop from all theſe parts. The Girl was then pretty well, without any Feaver or *Catarrh*: She could breath freely, was in good humour, and had a good Stomach; and ever ſince has continued in perfect health.

A very extraordinary Periodical Hæmorrhage, *by Dr.* Muſgrave, *n.* 272. *p.* 864.

V. Mr. *H.* had (from his Infancy, as he has been inform'd, but to his certain knowledge) from as far back as he can remember, up to the twenty fourth year of his age, a *Periodical Hæmorrhage* in one of his Thumbs. The *Time* of the Eruption was about the full of the Moon, ſeldom more than a day before or after it. The Orifice (to ſpeak more exactly) on the Right ſide of the Nail of the Left Thumb. He has not known the Blood to be leſs in weight at any one Turn, or Periodical Diſcharge, than four ounces; and is poſitive, that when he came to be ſixteen years of age, the quantity was then encreas'd to half a pound at each eruption; he and his friends having often view'd the quantity, and found it to be thus much. The *Manner* of the Flux was alſo remarkable; for, without any Pain of Head, Streightneſs of Breath, or any ſigns of Fulneſs, or other Symptoms whatever obſerved by him, excepting only a ſtiffneſs, on the utmoſt Joynt of the ſaid (Left) Thumb; the Blood uſed to ſpin out, with a conſiderable force, on a ſudden, in ſeveral little ſtreams, and continue ſo to do, until the greater part of the quantity was diſcharged. Under this diſcharge, however copious, he was ſtrong and vigorous to the age of twenty four, from his moſt early and tender years. At that age (of 24) finding this Evacuation troubleſome, and being uneaſie under it, he ſear'd with a hot Iron the part, which uſed to open and give vent to the flux of Blood. I ſaw that part, it was hard and callous to the diameter of $\frac{1}{2}$ of an inch. The ſearing had ſtopt the Hæmorrhage to the day I took this account, (which was on *Dec.* 11. 1697.) that is, about twenty years.

This ſtoppage was in its effects very dangerous, and of ill conſequence;

quence; for within one quarter of a year after it he fell into a *Sputum Sanguinis*; bringing up from his Lungs vaſt quantities of Blood. This new Complaint, together with a Cough attending it, reduced him very low; ſo that his Phyſician, old Dr. *Dike* of *Somerſetſhire*, (a perſon of great Learning and Experience in our Profeſſion) thought him utterly loſt in a Conſumption; but by frequent Bleeding, *&c.* deliver'd him from this *Hæmoptoe*: but in a very little time the Patient fell into a moſt violent Colic; from no other occaſion (that he could diſcover) than his late illneſs, putting on a new form, and the matter ſettling on the Bowels. This Colic was in good meaſure overcome by Purging Medicines; but a diſpoſition to it ſtill remains; for he has ever ſince been often troubled with it (as alſo with a Spitting of Blood) on the leaſt exceſs of Cold or Motion. He has, ever ſince the ſtoppage of that firſt Hæmorrhage, been weak, ſickly, of a ſallow, faint look; much impair'd as to health, in compariſon to what he enjoy'd during the time of its Periodical Returns; and gives an Argument, *That when Nature has choſen, and for ſome length of time, exerciſed new and extraordinary methods of Oeconomy, ſhe ſeems to be as fond of their Continuance, as at other times, and in her moſt regular ſtate, ſhe is of that which is her moſt uſual and ordinary courſe.*

An Eruption of Blood from ſeveral parts of the Body. by Dr. Meſaporitus. *n.* 303. *p.* 2144.

VI. Honeſtiſſima viſendæque ſpeciei Juvenis, cui natura ad 18 annum, integerrimam veletudinem hilaremque genium dederat, per multiplices nunc ducta morborum modos incidit in fluxum ſanguinis ex inuſitatis partibus, & tandem in deterrimam partialis ſudoris ſpeciem ex ambitu corporis. Quamprimum in eâ ætate concipiendæ ſpei aptiſſimâ gravi ſenſu correpta fuit, poſt diuturnum ſtomachi laborem cœpit ſanguinem tuſſi expuere occaſione ementitæ pleuritidis, quâ die nonâ *Aprilis* febricitabat cum dolore lateris, & difficili reſpiratione: *Quarto* inſultûs *die* thoracis morbo evaneſcente, capitis dolore, quo & antea laboraverat, gravari, & ſine levamine ſanguinem fundere è naribus obſervabatur; quocirca hujus copiam è pede detraxit *Chirurgus Familiaris*, qui bis eductum primis diebus ex brachio gelatinoſum inſpexerat. Verum haud decreſceret morbus, qui conceptum virus in promptu habebat movere ad ulteriora; hinc ſuperaddita eſt afflictæ Juveni Cardialgia, unde vomitus primum viſcidi, & viridis humoris, mox ſanguinis, nec proinde capitis ægritudine allevabatur, quin circulo quodam ordinato recurrentibus purpureus liquor ex utraque parte, & ad multos dies prodibat. Malorum importuna viciſſitudo nimis adhuc increbuit, cum circa initium *Maii* erumperent Menſes, & prout ipſi mos erat in ſufficienti menſurâ. Chirurgus *Mannæ* potionem exhibuit poſt ſeptimum diem, & ab ea licet non parum perturbata in pejus ægra ruere videretur, attamen paulo poſtea febris in totum ceſſavit, quæ antea plus vel minus una cum vigiliis ingraveſcens fuerat continua. Vix per integram hebdomadam habuit otium morbus, cum febris rurſus invaſit comitata dolore artuum, & potiſſimum ventris quibus licet ſeſe citius

expediverit abſumpto nempe *Oleo Amygd. dulcium rect.* ſine igne extracto, nihilominus conſtans vomitionis moleſtia, & ſanguinis jactura ſummopere urgebant. Diro Symptomati ut occurrerent Clariſſimi Medici, conſuluerunt in minori doſi, & per vices ſanguinem ex *Salvatella* detrahere, anodina, adſtringentia, & alia quam plurima adhibere; quod etſi executioni mandatum fuerit, inconſtanti methodo procedens effectus, frequentiores excretionum vias fere omnes cum attigiſſet, cæpit per inuſitatas errare:

Cum igitur circa initium *Junii* theſaurus vitalis ex auribus effluxiſſet, paulo poſt curioſiori ſpectaculo ex acumine digitorum primo manuum, deinde pedum ſe fundere viſus eſt: Mox ex umbilico, & ex angulo oculi; hinc pluries per ſudorem inde tandem ex medio pectore, poſtea ex pede, eoq; loci, ubi tunditur Saphena, tandem ex utraque volâ manuum, & ex oppoſita parte; poſt biduum ex mento, noctu ex acumine linguæ, ita ut quatuordecim dierum ſpatio diſcrimen hoc, veluti fabuloſum maxima ex parte abſolverit. Motus novitate rei, & Amicorum impulſu veni ad Juvenem, quam miratus viribus mediocriter conſtare, & jucundo vultu ſingula mihi enarrare, prout ab aliis audiveram: Adhuc cernere erat ſuperſtitem cicatriculam in volâ ſiniſtræ manûs ad inſtar inæqualis puncturæ leviter impreſſam, unde ſanguis pridie fluxerat, retulitq; id non ſine ſenſatione triſti factum fuiſſe, uti ſemper contigerat, cum ex ſpeciali loco prodiret: ubi vero ex pectore, aut aliis partibus ſudoris ſpecie emanaverat, nullum apertum pori reliquerat veſtigium; id tantum ex induſiis patebat, quorum duo vidi ad latitudinem ſcuti aurei cruore roſeo ſparſim colorata. Die 14 *Junii* apparentibus menſtruis juſtò copioſioribus, inde ſublatum morbum nonnulli crediderunt, potiſſimum quòd viginti dierum inducias feciſſet. Verum inchoato altero ſudoris exordio, & ad multum tempus procedente miſella debilitari ſentiebatur, & ſanguis diſcolor apparere. Hinc recreandi ſpiritus, mutandæq; fortunæ gratiâ in alterum aerem ſeſe tranſtulit. At qualis ſeceſſerat, rediit ad Urbem die quinta *Auguſti*, imò infirmior, quod præter morbi acerbitatem, artis quoq; methodo exhauſtâ ulteriorem jacturam fuiſſet experta. Hinc non diſſimili modo progrediente ægritudine, & paucis tantum habitis intervallis effundendo ſanguinem modò ex unà parte, modò ex alterâ, ſæpius per ſudorem, quod reliquum erat ejuſdem menſis, & medium *Septembris* confecerat. Petii ægræ domum, & accidit, quod forte expetiveram, ex dolore lancinanti in extremo unius digiti manûs admonitam fuiſſe ſanguinem inde extillaturum, quem notavi, veluti ex profundiori puncturâ, mox ex altera parum diſtanti guttatim decidere, & induſium ſupra Pectus recenti maculâ ſignari videram priuſquam recederam. Poſt triduum mirabiliorem ſanguinei ſudoris copiam exhibuit. Vel nullum, aut exigui momenti ſudorem deinceps paſſa eſt. Attamen reticere non debeo ex levi excoriatione ſupra maleolum caſu torturæ pedis factà uberiorem inde penum ſeroſitatis effluxiſſe, ut ſtipticis remediis, & compreſſis faſciis non facile ſiſteretur. Omnium Senior Medicus *Fa-*

bricius Ardiſonus, qui nonageſimum attingit annum, retulit mulierem annorum triginta ſe olim curaſſe, cui ſuppreſſæ erant menſes, & ſanguis ad multum tempus ex oculi angulo fluebat, revocatis nimirum Menſtruis ad conſueta naturæ.

CHAP. VI.

The Bones, Joynts and Muſcles.

I.

John Marſh of the Pariſh of *Denton* in *Kent*, when about ſixteen years old, was troubled with a Tumor on his Arm, in the end of a continual Fever, which ſeem'd to be a Critical Diſcharge of the Humour of the Feaver on his Arm; he was managed by a Surgeon of the Pariſh for two Years together for this Tumor; at length there being no appearance of a Cure, he was ſent to me: At firſt Dreſſing I found two Sinuous Ulcers in his right Arm, one upwards about the Deltoid Muſcle, the other under the upper part of his Arm, within an Inch and half of the Juncture of the *Cubitus*; the *Sinus* above paſſing upwards within an Inch and half of the Juncture, and downwards to the *Cubitus*. The *Sinus* of the lower part paſs'd downwards to the *Cubitus*, and upwards about an Inch and half. When both theſe *Sinus* were laid open, the Bone ſoon ſhewed itſelf carious and looſe, ſo that I eaſily took it out, and was about five Inches long. Three Weeks after came off another Spelt of Bone of the inner ſide, about two Inches long, having the Channel of the Marrow, Fig. 5. Theſe Ulcers with much Care and Diligence, were cur'd very well in nine Months, and the place of the Bone is ſo well ſupply'd with a ſtrong *Callus*, that he is not only very ſtrong, but can lift Fifty pound Weight with that Arm.

A Callus ſupplying the Place of the Os humeri, *by Mr.* Fawler, *n.* 312. *p.* 2466.

Plate 6. *Fig.* 4.

Fg. 5.

II. A Compound Fracture happening in the Thigh of a young Man about ſeventeen, I was obliged to take out the whole Subſtance of the *Os femoris* about two Inches; and yet by keeping a due Extenſion, Nature did in four Months ſupply ſuch a *Callus*, that the Part is not a quarter of an Inch ſhorter than on the other Side; and the Perſon is as ſtrong as ever, and walks without any Lameneſs.

A Callus ſupplying the Loſs of part of the Os femoris, *by Mr.* Sherman. *n.* 323. *p.* 451.

III. 1. The Wife of *Tho. Steven* of *Maidenhead* in the County of *Berks*, (aged about 62 Years) was ſeized with a Fever about the latter end of *November* 1697. Her Phyſician uſed various Remedies to remove her Fever, which in about 14 Days terminated in a Tumor and Numneſs in her left Foot, both which did by degrees creep up her Leg, and half way up her Thigh. A Fomentation was ordered by her Phyſician made

A Woman that loſt her Leg and greateſt part of her Thigh by a Gangrene, by Mr. Calep, *n.* 313. *p.* 41

made of *Centaur. Abſinth. Hyperic. &c.* boyl'd in a ſtrong *Lixivium*; and after fomenting, he order'd them to anoint her Foot and Leg with *Ol. Terebinth.* wherein *Galbanum* was diſſolved. This Method they had uſed daily for a Month before I ſaw her. Coming *Jan.* 3. 1678. from *Henley* upon *Thames* (where I then lived) to ſee ſome Friends and Relations ~~I had at *Maidenhead*, they deſired me to ſee this poor Woman~~; which I did, and found her in the following Condition, (*viz.*) Her Foot and Leg cold, inſenſible, wither'd, hard as if dry'd in a Chimney, and of a dark Tawney Colour. Her Knee was ſwell'd, and had ſeveral large black Spots upon it, which pitted when preſs'd with my Finger. There was ſeveral Diſcolourations in the Skin half way up her Thigh. She complained of great Pains, eſpecially at Nights in her Knee and Thigh, yet could not feel me when I touched thoſe Parts. Her Fever was now encreaſed again, and ſhe was delirious ſometimes. She begg'd heartily of me for Help; but alaſs! what could I propoſe to relieve her? Nothing but the taking off her Thigh, which ſhe would not conſent to. I was not ſorry for her not admitting of that Operation, becauſe I could not expect any Succeſs in the performing it, by reaſon of her Age, Weakneſs, *&c.* So I took leave of her, ſuppoſing I ſhould never ſee her more. I adviſed her Friends to continue the uſe of the Fomentation, which they did almoſt Night and Day. About a Month after, I coming to *Maidenhead*, was ſurprized to find this poor Woman alive. There was now a Diſcharge of a Black Fætid Matter, at a ſmall Orifice about the middle of the Inſide of her Thigh, which Orifice I enlarged to make a better Diſcharge for the Matter. I likewiſe cut into a Tumour that appeared upon her Knee, but found nothing in it but Wind. I then took leave of her, (as before) adviſing to continue fomenting daily. About a Month or 5 Weeks afterwards, I came to *Maidenhead* again, and found her alive, and to my admiration ſaw how Nature had made a perfect Separation of the mortify'd Fleſh from the ſound, quite round the Thigh, the Bone of the Thigh lying wholly bare, above the breadth of four Fingers, and deprived of its *Perioſteum.* The Fleſh above was freſh and florid, and had good white Matter upon it. I now perſwaded her to let me take off her Thigh, which I did about 2 Fingers breadth in the ſound Fleſh, (becauſe the Fleſh ran tapering down to the Bone,) by which I made the Stump pretty even. The Bleeding was little, by reaſon that the Veins and Arteries (which were eaten aſunder by the mortifying Matter) Nature had cloſed again. I dreſt the Stump with *Pul. Reſtring.* mixt with *Album Ovor.* ſpread upon Pledgets, and dipt in *Ol. Terebinth.* hot. The next Dreſſings I uſed Digeſtives, and performed the reſt of the Cure according to the Rules of our Art.

A Remark by Mr. Cowper. *ib. p.* 43.

2. About the beginning of *October* laſt I hapned to be at *Maidenhead*, where I ſaw the Woman whoſe Caſe is here related; ſhe appeared to be very decrepid, and would have ſhewn me the Stump of her Thigh, but the Coldneſs of the Weather, ſhe ſaid, would make it uneaſie to

her. I felt it through her Cloaths, and the end of the Stump ſeem'd to be not above four or five Inches below the Trunk of her Body.

Since I have ſo frequently found the large Trunks of the Arteries of the Thighs and Legs of Aged People petrified, as I have mention'd in the * *Tranſactions, Numb.* 280. and moſt commonly in thoſe who have had Gangreens in the Legs, *&c.* I am apt to ſuſpect the like happen'd in the Crural Artery of this Woman; which, like a Ligature, did at length put a total ſtop to the Influent Blood below that Stricture.

* *v. ſupr. p.* 134.

IV. The Tumour is of a Schirrous nature, ſpringing from the Thigh Bone, ſomewhat tending to that of a Cancerous. It firſt took its riſe about 2 years ago in a Child 10 years old, juſt above the Patella, without any evident cauſe, and hath, notwithſtanding all poſſible Care, expanded itſelf ſo, that it now occupies the whole Thigh, to the very Groin, and has extended it to above a Dutch Yard, in Circumference. As it encreaſes very much daily, it muſt ſoon exhauſt the Patients Strength.

A Schirrous Tumor in the Thigh by Mr. Amyand. *n.* 317. *p.* 173.

V. The *Os femoris* belong'd to an old Woman turn'd of Fourſcore, who only fell from her Chair whereon ſhe was ſitting, and thereby ſuffered this breach of continuity in the Subſtance of the Bone. She lived three Weeks after it; and tho' it never was reduc'd, yet ſhe complain'd of very little or no pain, which may ſeem very extraordinary. It is obſervable that the Fracture is not only Oblique, near the Neck of the Bone; but that each Trochanter, *i. e.* the two proceſſes near its Cervix, are likewiſe broke ſhort off; and that they were both drawn up almoſt as high as the Head of the Bone it ſelf, by the ſtrong contraction of the *Glutei* and other Muſcles.

A Fracture in the Thigh Bone, by Dr. Douglas. *n.* 349. *p.* 501.

VI. *A Paper omitted.*

An Account of ſeveral Obſervations concerning the Frame and Texture of the Muſcles, by Mr. *Muys* of *Francquer.* From the *Journal Litteraire* for *Jan.* and *Feb.* 1714. *p.* 238.

n. 339. *p.* 59.

VII. *Accounts of Books omitted.*

1. De Arthritide ſymptomatica Diſſertatio. Auctore Gulielmo Muſgrave, M. D. Coll. Med. Lond. & *R. S. S.* 8vo. Lond. 1704.

n. 291. *p.* 1597.

2. De Arthritide anomala ſive Interna Diſſertatio. Auctore eodem. Lond. 1707.

n. 310. *p.* 2435.

CHAP. VII.

Monſters, Longævity.

I. *A Monſtrous Birth. by Mr.* Robert Taylor. *n.* 308 *p.* 2345.

A Poor Woman in a Village near *Hitchin*, being in ſtrong Labour, the Midwife finding the Birth coming very awkwardly, and more Legs than uſual, after a tedious time delivered the poor Woman of Twins (deſign'd by nature doubtleſs,) but joyned together; there being but one Trunk of a Body with 2 Necks, on each a Head, 4 Arms 2 backwards, and 2 forwards, thoſe backwards croſſing each others Shoulder, like two perſons ſide to ſide. There is but one Navel, two Matrix's, two Fundaments, two pair of Hips, 4 Legs: They had gone the full Time, having Hair on their Heads, and Nails on their Fingers and Toes. The Midwife tells me they were alive within half an hour before they were delivered. They look very clear and well. The Children are near * * Inches long, and by reaſon of their being joyned, are about 7 Inches over.

Monſtrous Births, by Mr. Neve. *com. by Mr.* Derham, *n.* 320. *p.* 309.

II. *Dec.* 6. 1706. A Woman at *London Derry* had a Monſtrous human Birth, *viz.* with 2 Heads, 4 Arms, and but one Body at the Navel. It was of both Sexes, Female on the Right-ſide and Male on the Left. The Right hand of the Male was behind the Females Back, and the Left hand of the Female behind the Male's Back, holding each other as in a loving manner. It liv'd but a little while.

Death and Diſſection of John Bayles, *at* 130 *years old. by Dr.* James Keill. *n.* 306. *p.* 2247.

III. *John Bayles*, the old Button-maker of *Northampton*, is commonly reputed to have been 130 years of age when he dyed. There is no Regiſter ſo old in the Pariſh where he was Chriſtened; but the oldeſt people, of which ſome are 100, others 90, and others above 80 years, remember him to have been old when they were young. Their accounts indeed differ much from one another, but all agree that he was at leaſt 120 years. He himſelf did always affirm that he was at *Tilbury* Camp, and told ſeveral particulars about it; and if we allow him to have been but 12 years old then, he muſt have been 130 when he dyed. He uſed conſtantly to walk to the Neighbouring Markets with his Buttons within theſe 12 years, but of late he has been decrepid, and carryed abroad. His Dyet was any thing he could get. I never heard he was more fond of one ſort of Food than another, unleſs it was that about half a year before he dyed he longed for ſome Veniſon Paſty, but had it not. He dyed the 4th of *April* 1706. He lived in 3 Centuries, and in 7 Reigns.

His Body was extremely emaciated, and his Fleſh feeling hard, the ſhape of all the External Muſcles was plainly to be ſeen through the

Skin. The *Abdomen* being laid open, the whole *Viſcera* appeared in good order, but more pale than they are commonly. The *Omentum* was very ſmall. The Stomach was very much diſtended with Wind, and the Bottom of it wore extreamly thin in that part which is next the Spleen, being hardly thicker than thin Writing Paper. In the inner Membrane there were no *Plicæ*. The Liver was pale, but upon cutting was found perfectly ſound. The Gall Bladder was of a larger ſize. The Spleen was not ſo big as one of his Kidneys. His Kidneys were firm and ſound, as were all the Urinary Paſſages. In the Right Kidney were a few ſmall yellow grains of Gravel. The Inteſtines were all ſound; the Meſentery was covered with Fat. The *Cartilages* of the *Sternum* were not harder than uſually they are. The Ribs were brittle, for by leaning gently upon one of them it broke. The Lungs were attacked even to the *Pleura*: They were ſpongious, whitiſh, with many ſmall black ſpots of Blood. The Cavity of the *Thorax* was large and clear. The Heart was large, thick and fat; and though he was always a little Man, yet the Diameter of the *Aorta*, before the *Carotidales* go off, was above two inches, which is conſiderably bigger than ever I remember to have ſeen. The *Aorta* in the *Abdomen*, and *Iliacks*, was for the greateſt part Cartilaginous. The Bones of the Skull were ſound and good. On the inſide of the *Dura Mater*, by the *Falx*, was a ſmall oſſification. The Brain was more firm and ſolid than uſual, and in cutting, hardly moiſtened the ſides of the Knife. The *Ventricles* were full of *Serum*. He had loſt the uſe of his Eyes for ſome years; but his Hearing was good till he dyed. His Genitals, both Teſticles and *Penis*, were of a large ſize.

There is no doubt but that the weakneſs of his Stomach, and the hardneſs of the *Aorta*, were the Cauſes of his Death. The Coats of the Stomach were ſo thin, that they had not ſtrength enough to keep out the Air, and conſequently his Digeſtion muſt have been ſpoiled. He had not eat Meat for ſome years, and of late he lived only on Small Beer, Bread and Butter, and Sugars. And it was impoſſible that his Blood could circulate duly, whilſt the great Artery, having loſt its Elaſticity, by being become Cartilaginous, could give no motion to the Blood. It is very probable that this was the Cauſe of his irregular and intermitting Pulſe, which I have felt ſome years before he dyed. It is obſervable, that the greateſt part of his Blood (which was in greater quantity than I expected) was contained in the Arteries, whereas generally in all dead Bodies the Veins are full, and the Arteries almoſt empty; for the Arteries being diſtended by the Blood, which they receive upon the laſt Syſtole of the Heart, by their natural Elaſticity contract again, and empty themſelves into the Veins, from whence it returns no more; but in this Man, the Great Artery having loſt the power of contracting itſelf, it retained the Blood it received by the laſt Syſtole of the Heart. This account agrees with that given of old *Parre* by the famous *Harvey* in moſt particulars, except in the Cauſes of their Deaths. But in both nothing ſeems more remarkably the effects of old age than the ſmallneſs of their Spleens, which undoubtedly was owing to the contraction of their Fibres in ſuch a lax and ſpongious Bowel. The whiteneſs of Bowels in both muſt be likewiſe either from the ſame contraction or cloſeneſs of the Coats of the Blood Veſſels, or for want of Blood. *Harvey* ſays nothing of the quantity of Blood he found in old *Parre*; but if we may gueſs from his Body being fleſhy, from the goodneſs of his Stomach and Appetite, and from the Diſeaſe he dyed of, there could be no want of Blood in him. In this Man there ſeemed to be more Blood than in ſeveral others I have ſeen, whoſe Bowels appeared more Red. And it can hardly be conceived, that the *Aorta* could be ſo large, without a large quantity of Blood, unleſs there had been ſome Stricture upon ſome other parts of it, which I did not perceive: And therefore it ſeems not improbable, that this whiteneſs of the Bowels was owing to the cloſeneſs of the Blood Veſſels in both. It is no ſmall confirmation of this opinion, that the Fleſh and Skin felt hard, and the Brain firm and ſolid. I might add that it is highly probable, that the ſame diſpoſition might give a cloſeneſs or hardneſs to the Veſſels every where elſe. It is true, this was a Diſtemper, but then it is as true that it is a Diſeaſe of old Age, and may juſtly be reckoned one of the effects of it. And for a farther Proof of what I have ſaid, I cannot but take notice, that in preparing a piece of the ſmall Gut for an Injection, the *Tunica Villoſa* felt more like a fine File than the ſofteſt Velvet; and that I could uſe more violence in injecting the Veſſels than theſe parts will uſually bear. Whoever conſiders how ſoft a Subſtance an Animal Body is at its firſt beginning, and how from time to time it acquires a firmneſs and ſolidity, will eaſily be induced to believe, that Old Age brings a more than ordinary hardneſs to all the Fibres and Veſſels.

The neceſſary conſequence of this hardneſs, and contraction of the Fibres and Veſſels of old people, is a diminution of their Secretions, which *cæteris paribus* are always proportional to the Orifices of the Glands. Hence it is that we find the Skin of old people always dry, their perſpiration being very little. They are likewiſe generally bound, old *Bayles* went to Stool but once in ten or twelve days for ſome years; and old people are always complaining of a want of moiſture, not that the Radical Moiſture is dryed up, but becauſe the natural Secretions, by reaſon of the contraction of the Glands, are diminiſhed. I have already obſerved, that we found in this old Man more Blood than could have been expected in ſuch an emaciated Body, and without doubt it had been larger, if his Stomach and Appetite had been as good as old *Parre*'s. The fullneſs of the Veſſels, and the frequent Rheums and Catarrhs of old People, evince this neceſſary conſequence of the cloſeneſs of the Coats of the Veſſels. From this retention of the excrementitious parts of the Blood, we may expect all the ill conſequences of a vitiated *Plethora*, and languid motion of the Blood; for the Fibres of the Arteries being now become hard, inſtead of aſſiſting, they obſtruct the Heart in circulating the Blood; and the quantity of Animal Spirits ſeparated in the Gland of the Brain muſt likewiſe be leſs, not only becauſe of the retention of the Excrementitious Humours, but alſo becauſe of the cloſeneſs and firmneſs of the Brain itſelf, ſo that the contractions of the Heart and all the Muſcles muſt be weak, and conſequently the motion of the Blood languid. A due conformation of all the Vital parts is moſt certainly neceſſary to bring a Man to a full old Age; but above all the reſt, there are two which to me ſeem to have had the greateſt ſhare in procuring a Longævity to old *Parre* and *Bayles*, by retarding the ill effects juſt now mentioned. The firſt is the Heart, which in both was ſtrong and fibrous; for that being left alone to labour the circulation of a large quantity of ſluggiſh Blood, a great ſtrength is abſolutely requiſite to propel the Blood through unactive Veſſels to the extremities of the Body, and back again. No doubt this is more eaſily done in Men of a low ſtature (as old *Bayles* was) which I apt to think was a qualification of old Age. The ſecond was the largeneſs of their Cheſts, and goodneſs of their Lungs, by which the Air had its full effort upon every Particle of the Blood, in rendring it florid and attenuating it, that it might eaſily move through the contracted Channels of an old Body.

IV. *Martha Waterhouſe* and *Heſter Jager* Siſters in *North Bierley*. *Martha* died in 1711. in the 104th Year of her age: *Heſter* in 1713. in her 107th Year. They had both relief from the Pariſh nigh fifty Years. There are other Inſtances here of Longævity; but joyntly I don't remember any that have come up to them.

Two very aged Perſons. by Dr. Richardſon. *n.* 337. *p.* 167.

V. Dr. *Mather* ſays, 'tis no rare thing with them to have a Gentlewoman ſee many more than 100 of her offſpring. He mentions one Woman that had 23 Children, of which 19 liv'd to Man's Eſtate. Another that had 27; another 26, of which 21 were Sons, one whereof was Sir *William Phipps*; another 39 Children. Here he gives ſeveral Inſtances of Perſons living, with them, to above 100 Years of Age. One *Clement Weaver* lived to 110, his Wife being upwards of 100. This Man, to the laſt Year, could carry a Buſhel of Wheat to the Mill, above 2 Miles. He relates the Caſe of an old Man, above 100, that loſt the memory of ſeveral of the latter Years of his Life, but very well retain'd the Remembrance of what paſt in his younger days.

Longævity and Fertility in New England. *by Dr.* Mather. *n.* 339. *p.* 71.

VI. *A Paper omitted.*

De Monſtris & quaſi Monſtris & Monſtroſis Philippenſibus, by *F. Camelli*, com. by Mr. *Petiver*. Thoſe Particulars in this account which

n. 307. *p.* 2266.

which were moſt conſiderable are deſcribed by Mr. *Petiver* in his *Gazophylacium Naturæ & Artis.*

CHAP. VIII.

General Affections of the Body, and remarkable Caſes and Obſervations not reducible to the foregoing Heads.

1. *Of the Worms in the Itch. by Dr.* Bononio. *n.*283. *p.*1296.

I Found an Itchy Perſon, and asking him where he felt the greateſt and moſt acute Itching, he pointed to a great many Puſtules not yet ſcabbed over, of which picking out one with a very fine Needle, and ſqueezing from it a thin Water, I took out a very ſmall white Globule, ſcarcely diſcernible. Obſerving that with a Microſcope, I found it to be a very minute Living Creature, in ſhape reſembling a Tortoiſe, of whitiſh Color, a little dark upon the Back, with thin and long Hairs, of nimble motion, with 6 Feet, a ſharp Head, with 2 little Horns at the end of the Snout. *Plate* 4. *Fig.* 2. *and* 4.

Not ſatisfied with the firſt Diſcovery, I repeated the Search in ſeveral Itchy Perſons, of different Age, Complexion and Sex, and at differing Seaſons of the Year, and found in all the ſame Animals; and that in moſt of the watry Puſtules, for now and then in ſome few, I could not ſee any.

And tho by reaſon of their minuteneſs, and Colour the ſame with the Skin, 'tis hard to diſcern theſe Creatures upon the Surface of the Body, nevertheleſs I have ſometimes ſeen them upon the Joynts of the Fingers in the little Furrows of the *Cuticula*, where with their ſharp Head they firſt begin to enter, and by this Gnawing and Working in with their Body, they cauſe a troubleſome Itching, till they are got quite under the *Cuticula*; and then 'tis eaſy to ſee how they make ways from place to place by their biting and eating, one ſingle one happening ſometimes to make ſeveral Puſtules, of which I have often found two or three together, and for the moſt part very near to one another.

I examin'd whether or no theſe Animalcules laid Eggs, and at laſt from the hinder part I ſaw drop a very ſmall and ſcarcely viſible white Egg, almoſt Tranſparent, and oblong, like to the Seed of a Pineapple. I oftentimes found theſe Eggs afterwards, from which no doubt theſe Creatures are generated. *Fig.* 3. *and* 5.

From this Diſcovery it may be no difficult matter to give a more Rational account of the Itch, than Authors have hitherto delivered. It being very probable that this contagious Diſeaſe is no other than the continual biting of theſe Animalcules in the Skin, by means of which ſome portion of the *Serum* ouzing out thro the ſmall apertures of the

Cutis, little watery Bladders are made, within which the Inſects continuing to gnaw, the Infected are forced to ſcratch, and by ſcratching increaſe the Miſchief, and thus renew the troubleſome work, breaking not only the little Puſtules, but the Skin too, and ſome little Blood Veſſels, and ſo making Scabs, Cruſty Sores, and ſuch like filthy Symptoms.

From hence we come to underſtand how the Itch proves to be a Diſtemper ſo very catching; ſince theſe Creatures by ſimple contact can eaſily paſs from one Body to another, their motion being wonderfully ſwift, and they as well crawling upon the Surface of the Body as under the *Cuticula*, being very apt to ſtick to every thing that touches 'em, and a very few of them being once lodged, they multiply apace by the Eggs which they lay. Neither is it any wonder if this infection be propagated by the means of Sheets, Towels, Handkerchiefs, Gloves, *&c.* uſed by Itchy Perſons; it being eaſy enough for ſome of theſe Creepers to be lodged in ſuch things as thoſe; and indeed I have obſerved that they will live out of the Body 2 or 3 days.

Nor ſhall we be at a loſs to know the reaſon of the cure of this Malady by Lixivial Waſhes, Baths, and Ointments made up with Salts, Sulphurs, Vitriols, Mercury's Simple, Præcipitate or Sublimate, and ſuch ſort of corroſive and penetrating Medicines. Theſe being infallibly powerful to kill the Vermin lodged in the Cavities of the Skin; which ſcratching will never do, partly by reaſon of their hardneſs, and partly becauſe they are ſo minute as ſcarcely to be found by the Nails. Neither do inward Medicines perform any real ſervice in this Caſe. And if in Practice we oftentimes experience, that this Diſeaſe, when we think it is quite Cured by Unction, does nevertheleſs in a ſhort time return again, this is not ſtrange, ſince tho the Oyntment may have killed all the Living Creatures, yet it may not probably have deſtroyed all their Eggs, laid as it were in the Neſts of the Skin, from which they may afterwards breed again and renew the Diſtemper. And upon this account, 'tis very adviſeable after the Cure is once performed, ſtill to continue the Anointing for a day or two more; which it is the eaſier to do, becauſe theſe Liniments may be be made agreable enough, and of a good ſmell, as particularly is that compounded of the Ointment of Orange Flowers or Roſes, and a ſmall quantity of red Præcipitate.

II. It ſeems to me, that the whole Buſineſs may be reduced to this double enquiry. 1ſt, How a thin Fluid (ſuch as is the Urine) may be ſeparated from the Maſs of Blood, and the remaining parts of the Blood circulate back to the Heart. 2dly, How a thick Fluid (ſuch as is the *Bile* or *Semen* for example) may be ſeparated from the Maſs of Blood and the other Fluids both thinner and thicker than this particular Fluid to be ſeparated, circulate back to the Heart. And

Of Secretions in an Animal Body. by Dr. Morland. *n.* 283. *p.* 1292.

And that I may be the more plain, I ſhall give a general Idea of the Structure of the Glands.

A Gland I conceive to be compoſed, 1ſt, Of the Ramifications of the Blood Veſſels incloſed in a common Membrane, which ſends off ſeveral Fibres, by which theſe minute Veſſels are tied together; and that the Veins are a continuation of the Arteries. Of this Dr. *Areskin* has fully convinced us, by an injection of Wax in an Humane Body, ſo dexterouſly performed, that the Wax being injected by the Arteries filled the Veins at the ſame time; and afterwards by a nice diſſection of the part, where the continuation of the ſmall Ramifications of the Arteries and Veins appeared to the naked Eye. 2dly, I conceive, that when the Branches of the Arteries begin to grow very ſmall, they ſend off ſeveral Ducts, whoſe Orifices are of different Dimenſions. Theſe Ducts are of two ſorts. The Firſt of theſe, which in the ſame Artery are always ſmaller than the ſecond, paſs immediately from the Artery, and open the Veins. The Second which paſs off nearer to the Extremity of the Arteries, unite and carry off a Liquor from the Maſs of Blood for particular ends in ſeveral parts of the Body. It is to be obſerved, that in one caſe the ſecond ſort are only to be found. I imagine, that a thin Fluid may be ſecern'd from a thick one, when the Orifices of the ſecretory Ducts are ſo ſmall, as to admit no other but that thin Fluid, and that at the ſame time the remaining parts of the Blood which are thicker, continue their Courſe in the Veſſel. Again, I imagine, that a thick Fluid may be ſecern'd, when the thinner parts are carried off ſome other way, ſo that the Liquor to be ſecerned will be the thinneſt of the remaining Maſs.

Upon theſe principles I think it will be eaſie to explain the Doctrine of Secretions.

Let us firſt examine how the Secretions are performed: As for Inſtance, the Urine. When the Blood by the Contraction of the Heart is puſh'd into the Arteries, they are dilated, which contracting themſelves puſh it forward into all the parts of the Body, and amongſt the reſt into the Ramifications of the Arteries of which the Glands of the Kidneys are compos'd. By this means the Blood paſſes by the Orifices of the Secretory Ducts; when theſe Arteries contract themſelves they preſs the Blood, and force the thinner parts into the Orifices of thoſe Ducts (which will admit no thicker Fluid) and carry it toward the *Pelvis* and the remaining part of the Blood into the Veins, by them to be carry'd back to the Heart. Thus a thin Liquor may be ſeparated from the Maſs of Blood.

Let us next examine how a thick Liquor may be ſeparated from the Maſs of Blood where thinner Liquors are mixt with it. For Inſtance, let us take the *Gall* or *Semen*. When the Blood is puſht into the *Celiack* or *Meſenterick* Arteries, 'tis forced to paſs into the Glands of the Stomach, *Pancreas*, *Spleen* and *Inteſtines*, &c. where the *Liquor Gaſtricus*, *Succus Pancreaticus*, *Liquor inteſtinalis*, are ſeparated by the abovemention'd

Method. The Blood thus robb'd of various thin Liquors is puſh'd on into the Veins, which anſwer to theſe Arteries, which Veins unite, and form a large Trunk called the *Vena Porta*, which entring into the Subſtance of the Liver, by its ſmall Ramifications chiefly forms the Glands of which the Liver is compos'd. Here again all the Fluids contain'd in the *Vena Porta*, which are thinner than the Bile, are ſeparated from this Maſs of Blood by the firſt ſort of Secretory Ducts (which we ſaid opened into the Veins) and there are diſcharged and mixt with the Blood, which is paſſing toward the Heart. At the ſame time the *Bile*, with the reſt of the Blood which is thicker, continues its Courſe in the Artery: Now all the thin Liquors being ſeparated, the Bile is the thinneſt part of this Maſs of Blood, and ſo may be received by the Excretory Ducts, which are capable to receive it, and no other. The *Semen* being a very thick Liquor is ſeparated much after the ſame manner, *viz.* The Blood being puſht into the Spermatick Arteries, paſſes into the Subſtance of the *Teſticles*, where all the Liquors, that are thinner than that out of which the *Semen* is to be taken, are ſeparated by the firſt ſort of Secretory Ducts, and carried back to the Maſs of Blood. Then this *Liquor Seminalis* being the thinneſt part of the remaining Maſs is ſeparated by Excretory Ducts capable to receive it and no other. After the *Liquor Seminalis* is ſeparated from this Maſs of Blood by the aforeſaid method, it is puſht forward into the Excretory Ducts, where there are other Ducts, which take their origin all along from them, which Ducts are capable to receive the thinneſt parts of the *Liquor Seminalis*, and convey them to the Maſs of Blood; and thus the *Semen* is left behind to paſs into *Vas deferens*. And 'tis worth remarking, that as the *Semen* grows thicker and thicker by continual ſeparation, the Canal in which it is to run, grows larger and larger, as appears by the ſtructure of the Teſticles, *Epididymis* and *Vas Deferens*. Hence we may give a true account, why the Canals of which the Teſticles are compoſed, are ſo long, *viz.* That there might be time enough to ſeparate all the thin Fluids. By this method we ſee, how the thickeſt and thinneſt Fluids may be ſeparated from the Maſs of Blood. And how intermediate Liquors may be ſeparated after the ſame manner by Canals of intermediate Dimenſions. Thus in a word the whole Doctrine of Secretions may be reduc'd to this. To ſeparate a Liquor of any determin'd thickneſs, all the Fluids, which are thinner, muſt be carry'd off by ſmall Canals, and the Liquor to be ſeparated, being the thinneſt of the remaining Maſs, is ſecerned becauſe the Ducts are capable to receive it and no other.

Corollaries. 1. Hence the uſe of the Spleen is evident.

2. Hence appears the origin and uſe of the Lymphaticks.

3. Hence the Texture and uſe of many minute parts of the Body may be diſcovered, which hitherto has been unknown.

Three Caſes of the Hydrophobia. *by Dr.* Mead, *n.* 323. *p.* 433.

III. The firſt was of a Lad of about the Age of nine Years, a ſturdy and bold Boy. A mad Bitch of the Mungrel kind was hunted in the Street, he ſtruck at her with a Stick, and ſhe flying in his Face, bit him in the right Cheek, which was torn with a large Wound to the middle of the Noſe. This was on the 20th of *April.* A Surgeon cured the Wound in about 14 days time, by applying for the firſt three Days, *Theriac. Andromach.* in *Sp. Vin.* and afterwards dreſſing it with *Liniment. Arcæi* and *Balſam. Terebinthin.* No other Care was taken, only a *Bolus* of *Theriac. Andromach.* was given him every Night while under Cure, and quickly after he was bit, he was perſuaded to eat the whole Liver of the Bitch fry'd. He continued very briſk and well to the 22d of *May*; upon that day he ſeem'd dull and ſick, would eat no Dinner, except a little boil'd Spinnage, walk'd out in the Afternoon, and in the Evening complained of his Stomach and Head; his Mother gave him a ſmall Glaſs of Brandy, for he would drink nothing elſe. In the Night he was very bad, ſtartled often, and ſcreamed out as in an Agony, eſpecially when deſired to drink, and complained miſerably whenever he made Urine, ſaying it hurt him. The next Morning he vomited up the Herbs he had eat the day before, unalter'd. I was ſent for that Day in the Afternoon, and found him in a perfect Agony, all in a ſweat, trembling, toſſing himſelf up and down, talking continually, looking very wild; his Pulſe low, and ſometimes quicker, then ſlower: His Urine made the Night before as well Colour'd as ordinary. I deſired him to Drink; he took a little in his Mouth, but as it was going down, he threw it out with Violence, ſaying it hurt him; and praying that he might take no more. We over perſuaded him to hold a little in his Mouth and ſwallow it by degrees and gently; he did ſo with a little more eaſe, but was glad when 'twas over. We bid him ſuck the Drink through a Quill; he try'd, but could not get it down by continual Gulps, but ſtopt as ſoon as a very little was paſs'd, ſtill crying out that it hurt him to ſwallow it. I preſently declared the Caſe to be deſperate. However for the ſatisfaction of the Relations, Bliſtering Plaiſters were apply'd to the Back and on each ſide of the Neck; and a Diuretic *Bolus* of *Sal. Succin. Camphor.* and *Conſerv. Lujul.* was given every ſix Hours; for he ſeemed from the firſt of his Complaint to have a difficulty of Urine.

The next day, the 24th at Noon, I found him much worſe, he had raved all Night; could not bear the ſight of any thing white, and ſaid, that if all the Women in the Room who had white Aprons would go out, he ſhould be well preſently. He ſaid he would drink if we would give him it in a black Cup; but when brought made many Excuſes and could not, though at the ſame time complain'd he was dry, and pleaſed himſelf with talking of full Pots. He eat ſome Bread and Butter heartily, but vomited it up quickly together with a frothy Slime. We dipp'd him in a Tub of warm Water; he ſaid he was not afraid of Water, and was quiet in it for a little while, but ſoon fell into a Convulſion Fit, which obliged us to take him out. I obſerved his Eyes to grow more ſtaring, and the Pupil to be prodigiouſly inlarged. He was thrown continually with ſuch Violence from Place to Place, that it was very hard to keep him in Bed; and quite tired and ſpent, fell into cold Sweats, and dy'd this day at Four in the Afternoon. The next day I obtain'd leave to open the Body. We examin'd the Brain, Throat, Breaſt and Stomach, but met with no extraordinary appearance any where, excepting that there was a great quantity of greeniſh viſcid Bile in the Stomach.

The other Patient was a very luſty vigorous Man of 45 Years. He had ten Weeks before been bit in one of the Fore-fingers near the Nail, by a little Naked Dog of the *Guinea* Breed. On the 8th of *November* in the Morning he complained of a great Sickneſs at Stomach, and vomited green and yellow Choler. The next Morning he took a Doſe of *Rad. Ipecacuanh.* Whilſt he was vomiting, he complained of a difficulty of ſwallowing; and when preſs'd to drink to work off the Medicine, contrived himſelf a way of ſucking the Gruel given him, through a piece of a Tobacco Pipe, but could not get down above one Pint; and tho' he afterwards often try'd this Trick, yet it did not ſucceed. On the 10th he had eight Ounces of Blood taken away at the Arm, and took a Bolus of *Theriac. Andromach.* with *Lap. Contrayerv.* I came to him on the 11th; found him ty'd in his Bed, raving Mad, biting and ſpitting at the By-ſtanders, crying out Murder, making an odd Noiſe as if he cough'd up ſomething from the Throat; this motion I had alſo took notice of in the Boy, and I ſuppoſe this is what ſome Authors have call'd Barking. He ſay'd he would drink if we would unbind him, and give him Water; but as ſoon as it came to his Mouth, he threw away the Cup with the greateſt Fury imaginable, and grew ſo unruly, that he was with much ado ty'd down again. I obſerved that he had a Palſie of his right Arm, for he moved this only by the help of the other; and thoſe who attended him, had taken notice that this Symptom began the Day before, and that at the ſame time he had endeavour'd to Read, but could not, complaining of a Miſt before his Eyes. As he ſeem'd afraid of every Body, ſo he ſhewed the greateſt Enmity to thoſe, for whom at other times he uſed to have the moſt Love and Reſpect. I order'd a Surgeon to take away 20 Ounces of Blood at his Arm: And obſerved it to be very thick and black. He was very tame after this for a few Minutes, but fell again into his outragious Fit, in which he ſoon laid himſelf down quite ſpent and dy'd.

Since theſe Accidents I have had an Account ſent me by a Surgeon from *Stamford* in *Lincolnſhire*, of a young Man of about 18 Years of Age, who dy'd Hydrophobus by the Bite of a Mad Fox, that had been bit by a Mad Dog. The Symptoms diſcovered themſelves three Months after the Wound, which was upon the back of the Hand, and being healed by the Application of *Theriac. Andromach.* had left a ſmall black Scab behind.

Three days before his Death he was ſeized with a Fever, for which he was Blooded, Vomited and Bliſtered; he bit to pieces the Glaſs in which Drink was given him. When Diſſected, the *Fauces* were found very much inflamed; The left Lobe of the Lungs black, with the Veſicles full of black Blood; The ſurface in ſome places, which the blackneſs had not cover'd, appearing Bliſtered, as if raiſed by *Cantharides.* The Liver was hard and of a yellow bilious Colour.

During the whole Violence of the Diſtemper, the *Penis* was obſerved to be continually erected, and as hard as a Bone. This Symptom is particularly taken notice of by *Cælius Aurelianus.*

The Surgeon who opened the Body, with his Knife ſlightly wounded his Fore-finger, and was ſurpriz'd to find that it feſtered, and gave him much more Pain than a greater Cut had at other times done. This I the rather take notice of, becauſe ſomething of the ſame nature happen'd to the Surgeon who Diſſected my Patient. His Hand the following Night was taken with an *Eriſipelas,* attended with great Tenſion and Pain: This was owing to a little Wound made in one of his Fingers a Day or two before, from which, in turning over the Parts, he had rubb'd off the Plaiſter; and it went not off without the continued Application of Cooling and Diſcutient Medicines.

Vid. *Eſſays on Poiſon.*

From all theſe Hiſtories, it may not perhaps be wrong to conclude, that the *Hydrophobia,* (a Name not very proper for the Diſtemper) is the Effect of a particular kind of an Inflamation in the Blood, accompany'd with ſo great a Tenſion and Dryneſs of the Nervous Membranes, and ſuch an Elaſticity and Force of the Fluid with which they are filled, that the moſt common Repreſentations are made to the Mind with too great Effect, and the uſual Impreſſions of Objects upon the Organs cannot be ſuffered: Hence proceed the Timorouſneſs, unaccountable Anxiety and Inquietude, which are always the forerunners of the Dread of Liquids; as alſo did the Pain in making Water, and the ſtrange Averſion obſerved in the Boy at the ſight of any thing White; the *Retina* being really hurt and grieved by the ſtriking of the Rays of Light upon it. Nor is it hard to conceive that when the *Salival* Liquor is hot, and the Throat inflam'd and dry, the ſwallowing of Drink ſhould cauſe ſuch an intolerable Agony; no more than it is that, when things are wrought up to this wretched Condition, the diſmal Tragedy ſhould not laſt above three or four days at moſt, in which the Patient is perfectly fatigued and torn to Death by the Violence of his Actions and Efforts.

Of the Bite of a Mad Dog, by Mr. Dela Pryme. *n.*277. *p.* 1075.

IV. In 1695. my Brother had a pretty Grey-hound Bitch that had Whelps; ſoon after came a Mad Dog and bit this Bitch unknown to the Family, upon which about 3 Weeks after ſhe run Mad, and they were forced to kill her; but ſaving her Whelps, becauſe that no ſign of Madneſs appeared in them, in about 3 Weeks more they all pull'd out one another's Throats except one, which eſcaping, my Brother's men valued and nouriſh'd, made much of it, and ſtroaked it: at length, perceiving it could not lap, nor ſwallow any liquid thing, they put their Fingers in its Mouth, and felt its Tongue and Throat, but finding nothing wrong therein as far as they could diſcover, they let it alone a day or two longer, and then it run mad and dy'd. They being thus dead were ſoon forgot, until that about 3 Weeks after, my Brother's Servant, a moſt ſtrong laborious Man, that had frequently put his Fingers into the Whelps Mouth, begun to be troubled now and then with an exceeding acute Pain in the Head, ſometimes once, ſometimes twice a day, ſo very vehement that he was forc'd to hold his Head with both his hands, to hinder it from riving in two, which fits commonly held him about an hour at a time, in which his Throat would contract, as he ſaid, and his Pulſe tremble, and his Eyes behold every thing of fiery red Colour. Thus he was tormented for a whole Week together, but being of a ſtrong Conſtitution, and returning to his Labour in every Interval, he ſweat and wrought it off, without any Phyſick.

But it went worſe with one of his fellow Servants, a young Apprentice of about 14 Years of age, who had made as much of the Whelp as he, but was not of ſo ſtrong a Conſtitution; he was ſeiz'd alſo with a pain in his Head, was ſomewhat Feveriſh, ſometimes better, and ſometimes worſe, cough'd much, had a good ſtomach, eat heartily, but could drink nothing. I know not what I ail, ſays he, I cannot ſwallow Beer, *&c.* and ſo laugh'd at it. When he went out of doors tho there was but a little North Wind, yet he always ran as if it had been for his life; when they ask'd him why he did ſo, he told them he could not tell——but that the Wind would needs ſtop his breath. A day or two after this he was worſe, and vomited a ſtrange naſty ſort of Matter, like black Blood, which ſtunk like Sallet Oyl, but much ſtronger; which he did ſeveral times; after which he would be pretty well, and walk about, but moſt commonly ran as faſt as ever he could, firſt out of one corner, then into another, then up ſtairs, then down again, as if it was for his life. But upon the third day of his confinement within doors he grew perfectly Mad, would ſtart, and leap, and twiſt his Hands and Arms together, point at people, and laugh and talk any thing that came in his mind. In ſome of his fits he was ſo ſtrong that he was too hard for four young Men to hold him down in the Chair where he ſat: but as ſoon as they were over he was lightſom, and laugh'd, and talk'd, but all his diſcourſe was of fighting, and how if that they would but let him alone, he would leap upon them, and bite, and tear them to pieces: And when one ſaid unto him, that he was ſure that he would not hurt him, he'd been always his Friend; he anſwer'd ſharply, that Friends and Foes were all alike to him, he'd tear them all in pieces, *&c.* About an hour after his fit

came again, which ſoon made him ſpeechleſs, ſeiz'd wholly upon his Brain, and then he dy'd juſt before the Phyſician came.

An extraordinary Spaſmus in two Families. by Dr. J. Freind. *n.* 270 *p.* 799.

V. Æſtate præteritâ magnus apud nos percrebuit rumor puellas quaſdam apud Blackthorn in agro Oxonienſi latratibus crebris more canum correptas eſſe. Duas ſcilicet familias peſtis hæc invaſit: quarum unam fortè adiit, ut de re tam inauditâ ſibi certò conſtaret, Vir omni rerum Medicarum laude præcellens D. Williſius; ille, quâ eſt ſemper in me benevolentiâ, caſum, qualem oculis accepit, mihi impertitus eſt, quem ipſius verbis, hoc eſt optimis, ſic accipe.

Funeſtum hoc & inamænum liberorum ſpectaculum reverſurum ex itinere, ut obiter inviſerem, tum rei novitas, tum amicorum deſideria paulo morabantur. Tunc enim non procul aberam a Villâ dictâ Blackthorn, quæ paſſim hodie celebratur non tam loci antiquitate, quam ſonorâ lue circumquaque nota ac infamis. Ubi primum aures meas infeſtabat terribilis e longinquo ejulantium concentus; domum verò ingreſſum ſtatim ſalutabat vociferantium puellarum horror propinquior, cuilibet harum reſpondente prorſus æquis vicibus violentâ capitis motatione, quaſi ad ejus nutum, pagi nequam tibicinem æmulantes, ingratam pariter modularentur harmoniam Vultu erant ab omnibus ſpaſmis immuni, præter motus oris frequenter oſcillantes; & pulſus iis ſanorum ſimilis, niſi ſub finem paulo debilior. Sonus, ut mihi videbatur, non tam canum latratum referebat, quam eorum erat ejulatibus ſimilis, niſi quod crebrior fieret, ſingultibus illum viciſſim reciprocantibus. Morbus hic novus Cynicus nulli harum quinque quas videre erat, ætati pepercit; cæterum omnes pariter invaſit, ab annum ſextum aut circiter agente uſque decimum quintum. Sæpe inter has vociferationes colloquiis pariter & ſenſibus integris fruebantur: Interdum renovans, more canum venaticorum, cæteras omnes quaſi ad mali ſocietatem vocaret. Tandem deficientibus ſpiritus ſingulæ cedebant paroxyſmo velut Epileptico,quas quidem certo certius lapſuras ſtratus in domo media lectulus expectabat: pauliſper alta quies & qualem ſororum eſſe deceat,concors ac juxta decubitus. Actutum vero novus Spirituum orgaſmus; quæque harum pectus aut alia membra ferire & proximæ cuivis facere moleſtiam. Par puellarum ætate minorum, dum ibi commoratus fui,expergefactæ & ſoporem morboſum & ſorores in lecto deſeruerant: Cæterum iis rediit eadem vociferatio,nec diſſimilis capitis motantis ſpaſmus. Nota & Viſa dico, quæ ſecus mihi fabulæ viderentur; ſed neque liberorum ætas hæc ſuſpectæ eſſe fidei, neque lucri ſtudium procul habitum aut parentum ſinit conditio.

Hiſtoria adeo notabili permotus; 12. Junii 1700. cum Blackthorniam peterem, alteram ibi familiam affectu laborantem inviſi; atque hic quidem filium unum & tres ejuſdem matris filiolas motibus convulſivis ante decem hebdomadas laceſſitos reperi, præeunte nec morbo aliquo nec evidenti cauſâ. Initio quidem puellarum altera tantum correpta eſt; eam prima acceſſio ad duas horas exercuit, reliquæ cum fratre, uti mater retulit, ſororis ægritudine ita turbatæ & perculſæ ſunt, ut intra paucos dies ipſæ etiam terrore victæ ſpaſmodicis inſultibus ſuccumberent. Sub adventum meum omnes Spaſmo immunes præ foribus ludos agebant, quo quidem otio ultra ſemihoram lætæ fruebantur, ſatis longum ſcilicet iis videbatur hoc tantillum intervalli, quod a multis retro hebdomadis fruſtra optârant. Vultus iis colore ſatis probabilis, ſermo alacris, indoles perquam vivida, membra vegeta & ad motum quemvis idonea; nec ulla ab hoc affectu accrevit mali nota, niſi quædam eaque perexigua virium imminutio & languor, pulſus tam impetu quam intervallo rite compoſitus. Tandem puella natu major annum agens decimum quartum, pro more ſpaſmis afficitur; unicum acceſſionis prænuntium ſenſit quaſi Ventriculi intumeſcentiam, quæ gradatim tanquam Globus ad fauces perreptans muſculos Laryngis & capitis monuit, ut ſolitas contractionis tragædias inchoarent; aſcenſus iſte in omnibus notabatur paroxyſmi certus & perpetuus præcurſor, quem ſi ſiſtere aut compeſcere conarentur, vehementiori impetu acceſſit inſultus ſæviitque diutius. Sonus, quem nullâ interpoſitâ morâ ſatis injucunde modulabatur, canum latratum aut ejulatum (aliter ac fama erat) non omnino referebat, videbatur potius inauditum quoddam Cantilenæ genus ex tribus numeris ſive tonis iiſque bis repetitis compoſitum quos ſuſpirium veluti anhelum excepit & tandem ſimplici modulo clauſum, qui reliquis multo vehementior erat ſimulque acutior. Deſunt ſane verba, quibus næniam hanc ad vivum depingam, nullâ forte oratione pingendam; ejus ſcilicet ingenii eſt, quæ fidelibus tantum auribus poteſt percipi, deſcriptioni parum commoda. Quin quomodocumque adumbraram cantilenam continuo fere recinit puella; aliquando enim per intervalla modulos variabat, quandoque cum jam ei ſpiritus deficeret, velocius increbuit tum motus tum vociferatio, donec tandem pene ſuffocata modulum unum aut alterum interponeret concuſſionemque capitis paulum reprimeret; hac arte ſcilicet reſuſcitatis viribus eandem cantiunculam ſtatim de novo integravit Huic ſemper comes adfuit capitis reciproca nutatio, quod jugi agitatione antrorſum retrorſumque ex æquo ferebatur, motu autem ſemicirculari aut obliquo minime jactatum. In collo muſculi valide tenſi & inflati; cætera membra ſpaſmis libera. His tamen cum inſultibus conflictanti haud intercidit vel pedes ſuos pro arbitrio vel prout juſſa erat in quamvis partem flectendi: Toto enim paroxyſmi tempore ſenſibus integris atque imperturbatis fruebatur, & ad libitum imperantis vel ſedebat vel obambulabat, verbo interim alicui efferendo prorſus impar. In vultu nulla coloris immutatio, oculi tanquam in morte immobiles, nulla uſquam niſi in ore diſtortio, quæ ibi ob Muſculos contractos erat omnino Cynica. Durante Paroxyſmo vix digitum feriit pulſus. Hiſce motibus agitata ultra Semihoram perſtitit; in quo quidem ſtatu vere luctuoſo eam relinquere nox tum proquinquans me admonuit. Sorores, cum fratre quanquam una aſtarent, præter morem ſpaſmo intactæ. Noctu ſomnus iis haud malus, ſi nempe horâ decubitus ad eum fuerint proclives: Secus Paroxyſmi ad mane uſque ingruebant, brevibus æque intervallis ac interdiu.

Puellas ex alterâ familiâ cum novo anno morbus hic invaſit; quo quidem tempore etiam fauces iis male affectæ ſunt & tumidæ. Paulo poſtquam eæ, de quibus jam ſermo fuit, ſpaſmis ſuccubuerant, hæ Epilepticis inſultibus cum ſenſuum abolitione correptæ ſunt; interdum pectora furentium more tundebant, interdum velut œſtro percitæ circumcurſitabant, &c. quorum nihil ſub tribus primis menſibus perceperant, iis tantum Symptomatis quæ a me deſcriptæ ſunt vexatæ.

Hunc naturalem eſſe morbum non prohibet ſymptomatum nova facies aut perennis duratio; Siquidem ſpaſmi hujuſce ratio cum communi omnium convulſionum indole congruit, a ſpiritibus ſcilicet animalibus oriunda, qui turbas intra nervos cient, muſculoſque in varias pro orgaſmi genio contractiones impellunt. Ita ut in hoc caſu nihil præter morem agat natura, neque aliquid monſtri præ ſe magis ferat quam in reliquis Spaſmorum generibus; quippe motibus Organicis non minus Mechanice hic uſa eſt, quam in Choreâ S. Viti aut verſatili illâ affectuum Hyſtericorum ſcenâ, ubi nunc cachinnus, nunc ejulatus, nunc violenta pectoris percuſſio ope muſculorum varie atque invitâ voluntate peragitur. Cum igitur muſculorum, tum qui laryngi & Capiti tum qui manibus & pedibus dicantur, eadem omnino ſit vis, eadem ad ſpaſmos aptitudo, quicquid novi & prodigioſi in puellis hiſce reperiri videatur, id non tam a ſymptomatis ingenio quam a parte affectâ derivandum eſt, cujus licet eo modo quo diximus convulſæ exemplum non facile occurrat, eam tamen ita poſſe corripi minus mirabimur, ſiquid apud nos valet Cl. Williſij obſervatio, In pueris ſcilicet qui affectibus cordis & membrorum exteriorum exercitiis nondum aſſueverunt, materiam ſpaſmodicam in nervos proximos h. e. eos 3tii 5ti & 6ti paris ſæpius incurrere; quare illis faciei & oris partes maxime convelluntur.

Ægri in duabus hiſce Familiis erant Conſobrini, utrum in cauſa fuerit cognatus Sanguis, ut altera familia per menſes aliquot intacta morbum tandem affinem ſuſceperit, horoſcopi & ſympathiæ ſtudioſis conjiciendum libens relinquo.

A deaf and dumb Perſon, who recover'd his Speech and Henring after a violent Fever, by Mr. Martin Martin, *n.* 312 *p.* 1469

VI. *Daniel Fraſer* a Native of *Strabarig*, ſome ſix Miles from *Inverneſs*, continued Deaf and Dumb from his Birth, till the ſeventeenth Year of his Age. The Counteſs of *Crawford* kept him in her Family for the ſpace of eight or nine Years: After ſeventeen Years he was taken ill of a violent Fever, but being let Blood his Fever abated, and had not its Natural Courſe:. About five or ſix Months after, he contracted a Fever again, and had no Blood drawn from him, and this went on with its Natural Courſe. Some Weeks after his recovery he perceived a motion in his Brain, which was very uneaſie to him, and afterwards he began to Hear, and in proceſs of time to underſtand Speech; this naturally diſpos'd him to imitate others, and at-

tempt to Speak: The Servants were much amaz'd to hear him, and ſome ran away; he was not underſtood diſtinctly for the ſpace of ſome Weeks; he is underſtood now tolerably well, tho he yet retains the *Highland* Accent, as *Highlanders* do who are advanc'd to his Years before they learn the *Engliſh* Tongue: He can ſpeak no *Iriſh*, for it was in the *Low Lands* of *Scotland* that he firſt heard and ſpoke.

A Cure for Fluxes and Dyſenteries. by the ſame, ib. p. 1470.

VII. A Wound or Scarification croſs the Crown of the Head is us'd in *Scotland* to cure Fluxes and Dyſenteries. The Blood being ſtanched, the Wound is cured as other Wounds commonly are.

A ſleepy Perſon by Dr. Oliver. *n. 304. p. 2177.*

VIII. *May* the 13th, *Anno* 1694, one *Samuel Chilton*, of *Tinsbury* near *Bath*, and Labourer, about 25 years of Age, of a robuſt habit of Body, not fat, but Fleſhy, and a dark brown Hair, happen'd, without any viſible cauſe, or evident ſign, to fall into ſome profound Sleep, out of which no Art uſed by thoſe that were near him, cou'd rouze him, till after a months time; then roſe of himſelf, put on his Cloaths, and went about his buſineſs of Husbandry as uſual; ſlept, cou'd eat and drink as before, but ſpake not one word till about a month after. All the time he ſlept, Victuals ſtood by him; his Mother, fearing he wou'd be ſtarv'd in that ſullen humour, as ſhe thought it, put Bread and Cheeſe and ſmall Beer before him, which was ſpent every day, and ſuppoſed by him, tho no one ever ſaw him eat or drink all that time. From this time he remain'd free of any drowſineſs or ſleepineſs till about the 9th of *April* 1696, and then fell into his Sleeping fit again juſt as he did before. After ſome days they were prevail'd with to try what effect Medicines might have on him, and accordingly one Mr. *Gibbs*, a very able Apothecary of *Bath*, went to him, Bled, Bliſtered, Cupp'd and Scarrified him, and uſed all the external irritating Medicines he could think on, but all to no purpoſe, nothing of all theſe making any manner of impreſſion on him; and after the firſt fortnight he was never obſerv'd to open his Eyes. Victuals ſtood by him as before, which he eat of now and then, but no body ever ſaw him Eat or Evacuate, tho he did both very regularly, as he had occaſion; and ſometimes they have found him faſt aſleep with the Pot in his Hand in Bed, and ſometimes with his Mouth full of Meat. In this manner he lay for about ten Weeks, and then could eat nothing at all, for his Jaws ſeem'd to be ſet, and his Teeth clincht ſo Cloſe, that with all the Art they had with their Inſtruments they cou'd not open his Mouth, to put any thing into it to ſupport him. At laſt, obſerving a hole made in his Teeth, by holding his Pipe in his Mouth, as moſt great Smoakers uſually have, they through a Quill pour'd ſome Tent into his Throat now and then: And this was all he took for ſix Weeks and four Days, and of that not above three Pints or two Quarts, ſome of which was ſpilt too; he had made water but once, and never had a ſtool all that time. *Auguſt* the 7th, which is ſeventeen Weeks from the 9th of *April*, (when he began to ſleep,) he awaked, put on his Cloaths, and walkt about the Room, not knowing he had ſlept above a Night, nor cou'd he be perſwaded he had lain ſo long, till going out into the Fields he found every body buſy in getting in their Harveſt, and he remembred very well, when he fell aſleep they were ſowing of Barley and Oats, which he then ſaw ripe and fit to be cut down.

There was one thing obſervable, That tho his Fleſh was ſomewhat waſted with ſo long lying in Bed, and faſting for above ſix Weeks, yet a worthy Gentleman his Neighbour aſſured me, when he ſaw him which was the firſt Day of his coming abroad, he lookt brisker than ever he ſaw him in his Life before; and asking him whether the Bed had not made him ſore, he aſſured him and every Body, that he neither found that, nor any other Inconveniency at all; and that he had not the leaſt remembrance of any thing that paſt or was done to him all that while. So he fell again to his Husbandry as he uſed to do, and remain'd well from that time till *Auguſt* the 17th, *Anno* 1697, when in the Morning he complaining of a ſhivering and coldneſs in his Back, vomited once or twice, and that ſame day fell into his Sleeping fit again.

Being then at the *Bath*, and hearing of it, I took Horſe on the 23d, to inform my ſelf of a matter of Fact I thought ſo ſtrange. When I came to the Houſe, I was by the Neighbours (for there was no body at home at that time beſides this ſick Man,) brought to his Bed-ſide, where I found him aſleep, as I had been told before, with a Cup of Beer and a piece of Bread and Cheeſe upon a Stool by his Bed within his reach: I took him by the Hand, felt his Pulſe, which was at that time very regular; I put my Hand on his Breaſt, and found his Heart beat very regular too, and his breathing very eaſie and Free; and all the fault I found was, that I thought his Pulſe beat a little too ſtrong: He was in a breathing Sweat, and had an agreeable warmth all over his Body. I then put my Mouth to his Ear, and as loud as I cou'd called him by his Name ſeveral times, pull'd him by the Shoulders, pincht his Noſe, ſtopt his Mouth and Noſe together, as long as I durſt, for fear of Choaking him, but all to no purpoſe, for in all this time he gave me not the leaſt ſignal of his being ſenſible. I lifted up his Eye-lids, and found his Eye-balls drawn up under his Eye-brows, and fixt without any motion at all. Being baffled with all theſe Tryals, I was reſolv'd to ſee what effects Spirit of *Sal Armoniac* would have, which I had brought with me, to diſcover the Cheat, if it had been one; ſo I held my Viol under one Noſtril a conſiderable time, which being drawn from Quick-lime, was a very piercing Spirit, and ſo ſtrong I could not bear it under my own Noſe a moment without making my Eyes water; but he felt it not at all. Then threw it at ſeveral times up that ſame Noſtril, it made his Noſe run and gleet, and his Eye-lids ſhiver and tremble a very little, and this was all the effect I found, tho I pour'd up into one Noſtril about a half ounce Bottle

tle of this fiery Spirit, which was as ſtrong almoſt as Fire itſelf. Finding no ſucceſs with this neither, I cram'd that Noſtril with Powder of White *Hellebore*, which I had by me, in order to make my farther trials, and I can hardly think any Impoſtor cou'd ever be inſenſible of what I did. I tarried ſometime afterwards in the Room to ſee what effects all together might have upon him; but he never gave any token that he felt what I had done, nor diſcover'd any manner of uneaſineſs, by moving or ſtirring any one part of his Body, that I could obſerve. Having made theſe my Experiments I left him, being pretty well ſatisfied he was really aſleep, and no ſullen Counterfeit, as ſome people thought him. Upon my return to *Bath*, and relating what I had obſerv'd, and what proofs this Fellow had given me of his Sleeping, a great many Gentlemen went out to ſee him, as I had done, to ſatisfie their Curioſity in a Rarity of that Nature, who found him in the ſame condition I had left him in the day before; only his Noſe was inflamed and ſwelled very much, and his Lips and the Inſide of his Right Noſtril bliſter'd and Scabby, with my Spirit and *Hellebore*, which I had plentifully dos'd him with the day before: His Mother upon this for ſome time after wou'd ſuffer no body to come near him, for fear of more Experiments upon her Son. About ten days after I had been with him, Mr *Woolmer*, an Experienc'd Apothecary at *Bath*, called at the Houſe, being near *Tinsbury*, went up into the Room, and finding his Pulſe pretty high, as I had done, takes out his Launcet, lets him Blood about Fourteen ounces in the Arm, tyes his Arm up again, no body being in the Houſe, and leaves him as he found him; and he aſſured me he never made the leaſt motion in the world when he prickt him, nor all the while his Arm was bleeding. Several other Experiments were made by thoſe that went to ſee him every day from the *Bath*, but all to no purpoſe, as they told me on their return: I ſaw him my ſelf again the latter end of *September*, and found him juſt in the ſame poſture, lying in his Bed, but remov'd from the Houſe where he was before about a Furlong or more; and they told me, when they remov'd him, by Accident, carrying him down Stairs which were ſomewhat Narrow, they ſtruck his Head againſt a Stone, and gave him a ſevere knock, which broke his Head, but he never mov'd any more at it than a dead man wou'd. I found now his Pulſe was not quite ſo ſtrong, nor had he any Sweats, as when I ſaw him before. I try'd him again the ſecond time, by ſtopping his Noſe and Mouth, but to no purpoſe; and a Gentleman then with me ran a large Pin into his Arm to the very Bone, but he gave us no manner of tokens of his being ſenſible of any thing we did to him. In all this time they aſſured me no body had ſeen him eat or drink, tho they endeavour'd it all they could, but it always ſtood by him, and they obſerv'd ſometimes once a day, ſometimes once in two days all was gone. 'Tis farther obſervable, he never foul'd his Bed, but did his neceſſary occaſions always in the Pot. In this manner he lay till the 19th of *November*, when his Mother hearing him make a noiſe, ran immediately up to him, and found him Eating; ſhe askt him how he did? He ſaid, Very well, thank God: She askt him again, Which he lik'd beſt, Bread and Butter, or Bread and Cheeſe? He anſwer'd, Bread and Cheeſe: Upon this, the poor Woman overjoy'd left him to acquaint his Brother with it, and they came ſtrait up into the Chamber to diſcourſe him, but found him as faſt aſleep as ever, and all the Art they had cou'd not wake him. From this time to the end of *January* or the beginning of *February*, (for I cou'd not learn from any body the very day) he ſlept ſo profoundly as before, for when they call'd him by his Name he ſeemed to hear them, and be ſomewhat ſenſible, tho he could not make them any anſwer. His Eyes were not now ſhut ſo cloſe, and he had frequently great tremblings of his Eyelids, upon which they expected every day when he would wake, which happened not till about the time juſt now mention'd, and then he wak'd perfectly well, not remembring any thing that happened all this while. 'Twas obſerv'd he was very little alter'd in his Fleſh, only complained the Cold pincht him more than uſually, and ſo preſently fell to Husbandry as at other times.

Strange Effects of the Scurvy, at Paris. *by Mr* Poupart. *n.* 318. *p.* 223. *From* Hiſt. Academ. Sc.

IX. The Diſeaſe, which I am now going to treat of, was a true Scurvy; for they who were ſick of it, felt, as common Scorbutick Perſons do, pains in their Thighs, the Calves of their Legs, their Belly, and Stomach, and were deprived of the Motion, or Uſe of their Limbs, tho' they ſtill retained their feeling. They were troubled with Head-achs, Convulſions, and ſuch ſtrange itching in the Gums, that the Children pulled off certain Pieces of them with their Nails. The Blood which came from them, was Watery, Salt and Corroſive; and the Stink, which came from their Mouth, was intolerable. They had hard blew Spots on their Legs and Thighs, frequent Hæmorrhages, or Bleedings at the Noſe and Fundament, and alſo ſo great a Weakneſs in their Knees, that they could not go without reeling or ſtaggering. Theſe were the Symptoms which they had in common with other ſcorbutick Perſons.

When we removed theſe ſick Perſons, we heard a ſmall clattering of their Bones.

I obſerved at the opening of all thoſe Bodies or Cadavers in which we heard the aforeſaid little noiſe, that the *Epiphyſes* were intirely ſeparated from the Bones, which by rubbing againſt each other occaſioned this clattering. We have opened ſeveral young Perſons, in whom we alſo perceived a ſmall low noiſe when they breathed. In all theſe ſort of Bodies we found, that the Griſtles of the *Sternum* were ſeparated from the Bony part of the Ribs; and as the Griſtles are of a ſofter Subſtance than the *Epiphyſes*, the noiſe, which their rubbing produced, was greater than that of thoſe Bones which rubbed againſt the *Epiphyſes*. They, in whom we heard this noiſe at the time when they breathed, are all dead, except one young Man, whoſe Ribs were viſibly reunited to the Griſtles, for after his Cure, we heard no more of this noiſe. All thoſe, in whoſe Breaſts any Matter or Seroſity were found, had their Ribs ſeparated from their Griſtles, and that bony Part of thoſe Ribs, which were over againſt the *Sternum*, was rotted for the length of four Fingers; which is an Evidence, that the *Lympha* of theſe Bodies was extreamly cauſtick. The greateſt part of thoſe Bodies, which were opened, had their Bones black, worm-eaten, and rotten.

Moſt of the Sick went ſtaggering: This is an Accident common or uſual to ſcorbutick Perſons, and very well known to moſt Phyſicians; but the reaſon of it, which you have here, is not ſo well known. It is certain, that the ſupport of the Joints proceedeth from the Force and Spring of the Ligaments, which bind the Bones cloſe to each other; the Ligaments of theſe ſick Perſons, were corroded, looſe, and the Bones were ſeparated from each other; which proceeded from this, that inſtead of finding in their Joynts that ſweet Oily *Lympha* (which commonly aboundeth there in order to make the Joynts ſupple, and give them an eaſie free Motion) there was nothing but a greeniſh Liquor, which by its over cauſtick Quality had corroded the Ligaments, and conſequently deſtroyed the Force of their Spring. All the young Perſons under eighteen, had in ſome degree their *Epiphyſes* ſeparated from the Body of their Bones, and by the leaſt endeavour or force we ſeparated them entirely. The reaſon of it is this, that young Perſons have not yet their *Epiphyſes* ſo ſtrongly faſten'd to the Bones, ſo that when they are never ſo little ſoak'd with that Corroſive *Lympha* which is in the Joynts, that cauſtick Liquor may eaſily ſeparate them entirely from the Bones. All the Bones, which we found entirely ſeparated from their *Epiphyſes*, were more than twice as big as they ſhould be in their Natural State, becauſe theſe *Epiphyſes* were ſeparated in them only, whoſe Bones were well ſoak'd with a Water which had penetrated into their very Subſtance and made it ſwell.

The Bones of thoſe which recovered, or were recovering, remained ſwell'd, without giving them any pain: They might grow leſs in time, as it happens to Children, which are troubled with the Rickets, whoſe Bones grow dry by little and little as they grow up. All they who had any Difficulty of Breathing, or had their Breaſts ſtuff'd or ſtopp'd up, had there good ſtore of *Lympha*, or Matter; and we often found more or leſs of them in their Lungs, according as they were oppreſſed. We have ſeen ſome ſick Perſons, whoſe Breaſts have been ſo oppreſſed, that they died all on a ſudden; in the mean while we found no Seroſity neither in their Breaſts nor in their Lungs: But the *Pericardium* was entirely faſten'd to the Lungs, and the Lungs were glued to the *Pleura* and *Diaphragma*; and all the Parts were ſo mixed and blended together with each other, that they all made up but one Maſs or Lump, ſo confounded, that one could ſcarce diſtinguiſh the one from the other: Now as the Lungs were ſqueez'd together in the midſt of this Maſs, they were deprived of their Motion, and the ſick Perſon was choak'd for want of Breath. The cloſe Adheſion, and Confuſion of theſe parts one with another, proceeded from this, that being ulcered as they were, they muſt needs ſtick to each other. The ordinary or common Scorbutick Perſons have the Glands of their Meſentery much obſtructed and ſwell'd; thoſe we treat of, have theirs partly corrupted, and Impoſthumes on the Subſtance of it. In the Liver of ſome few, the Matter or Corruption was hardned, and as it were petrified; their Spleen was three times bigger than it ſhould be, and fell to pieces as if it had been compoſed of coagulated Blood; and ſometimes the Kidnies and the Breaſt were full of Impoſthumes.

There were ſome Bodies or Cadavers of thoſe of Fifteen, in which, if we ſqueez'd betwixt two Fingers the end of the Ribs, which began to be ſeparated from the Griſtles, there came abundance of corrupted Matter, which was the ſpungy part of the Bone; ſo that after the ſqueezing of it together, there remain'd nothing of the Rib, but two bony Plates. We have ſeen ſome certain Perſons, who had no other Token of the Scurvy, but ſome ſlight Ulcerations in the Gums: They had afterwards ſome ſmall red, hard Tumours on their Hands, their Inſteps, and in ſome other parts of the Body. After that, there appeared large Impoſthumes on their Groin and under their Arm-pits, attended with ſeveral blue Spots all over their Body, which were the certain Fore runners of Death. We found that the Glandules under their Arm-pits were very big, and ſurrounded with Matter or Corruption; as well as the Muſcles of their Arms and Thighs, whoſe Intervals were all filled with them. We obſerved ſome whole Arms, Legs, and Thighs, were of a reddiſh Black, and as it were burnt; which proceeded from that black and coagulated Blood, which we always found under the Skin of thoſe Perſons. We alſo found their Muſcles ſwelled, and as hard as Wood, which proceeded from the Blood which was fix'd in the Body of the Muſcles, which were ſometimes ſo full of it, that their Legs remained bent without being able to extend or ſtretch them out. We obſerved that the blue, red, yellow, and black Spots, which appear in their Bodies who have the common Scurvy, proceed purely from extravaſated Blood under the Skin. As long as the Blood kept its red Colour, the Spot was red; if the Blood is black or coagulated, the Spot is alſo black; when there is ſome Bile mix'd with it, the Spot is of a yellowiſh black; in ſhort, according as the Blood is mixed with the Humours of different Colours, ſo alſo the Spots appear of a different Colour. We ſometimes ſaw on the Bodies of theſe Perſons certain ſmall Tumours which grew bigger every Day: we applied Emollient Ointments to ſoften them, and thoſe Tumours on their breaking, formed a Scorbutick Ulcer, which proceeded from the Blood with which the Tumour was filled;

for as often as we took off the Plaiſter, we ſtill found under it a great deal of coagulated Blood; we put on a freſh Plaiſter, and ſome time after we ſtill found under it coagulated Blood: We continued dreſſing of them after this manner, and by thus taking away the Blood, we dried up the Tumour, and the Perſon was cured. Some old perſons had ſuch large Bleedings at the Noſe and Mouth, that they died of it, it being impoſſible to ſtop it, becauſe the *Lympha* of theſe perſons was ſo ſharp and corroſive (as I ſaid before) that it corroded and eat through the Coats of the Veins. And this kind of Hæmorrhage was ſo much the harder to ſtop, becauſe the Blood of old Perſons is more fluid and watery than that of young Perſons, who are ſeldom ſubject to this Accident.

Old Perſons, as well Women as Men, were troubled with ſuch mighty Fluxes, that the weakeſt of them died under them; but if they had ſtrength enough to withſtand them, they were ſoon cured. There were ſome of theſe Sick Perſons, who were ſo Coſtive in their Body, that they never could go to Stool without taking ſome Gliſters. Several of them had ſuch large Swellings over all their Bodies, their Hands, Arms and Feet, that they ſeemed to have been blown up. We cured ſeveral of them by proper Medicines, Gliſters, and ſweetning Juleps. A Youth of Ten Years old, had his Gums much ſwelled and ulcered; his Teeth were eaten up to the Roots of them, and ſerved no longer; and his Breath was intolerably ſtinking. The Chirurgeon was obliged to pull out all his Teeth, for the better dreſſing of his Mouth, tho' they would have fallen out of themſelves: His Gums were healed, but there aroſe a Tumour on the ſide of his Tongue as big as a Walnut. In the middle of this Tumour there was a bluiſh Hole, which degenerated into an Ulcer, which eat up half the Tumour, the other half remained whole and entire. Some ſmall time after, there appeared another Tumour in the Cheek, which was very hard: It was Blue in the middle, and turn'd to an Ulcer alſo as the firſt. This Youth died all on a ſudden, when it was leaſt expected, and all the inward Parts of his Body were Corrupted.

All they who died ſuddenly, without having any viſible Cauſe of their Death, had the Auricles of their Heart as big as one's Fiſt, and full of coagulated Blood, which, by putting a ſtop to the Circulation of the Blood, brought an inevitable Death on them. There came in the Cheeks of ſeveral a ſmall White Ulcer, which was hard all round; unleſs we took care to ſtop it preſently, and to take it off with the Spirit of Vitriol, it grew preſently livid or blue, black and ſtinking, and eat up part of the Cheek, ſo that one might ſee the Teeth thro' it. We have ſeen ſeveral from the Age of Eighteen to the Age of Thirty, who were without pain caſt down ſtupid and without any Motion. They had their Mouth open, their Eyes ſunk in, their Looks frightful, and appeared rather like Statues than Men. All theſe Perſons had no apparent Sickneſs, only their Gums were Ulcered;

their Skin was ſmooth and Fair, without any Spots or Hardneſs: Yet we found their Muſcles were Gangreen'd, and all wet with a black corrupted Blood, and in handling of them, they fell into Pieces in our Hands. There was a Man who had a Carbuncle on his Inſtep, his Lips and his Noſtrils were chopped, and a ſtinking Water flow'd gently from his Noſtrils. This Man linger'd out a long time in a dying Condition: His *Cadaver* made me afraid, I durſt not open it. A Young Man, who as to all outward appearance ſeem'd not to be very Ill, died ſuddenly. We found his *Pericardium* was ſo eaten up, that there remain'd but a little of it, and his Heart was Ulcer'd all about very deeply.

Scorbutick Perſons are commonly better in the Summer, than they are in the Winter, which may proceed from their great Tranſpiration. On the other ſide, theſe were indifferently well from the Month of *April*, to the beginning of *June*, the Spots, hardneſs and other Accidents of the Scurvy then diſappearing; but on the coming of the great Heats, all thoſe Accidents returned They, who were ſo well as to be in a readineſs to quit the Hoſpital, relapſed again: Their Legs and Thighs grew all Black, and Death often put a Period to there Miſeries. This Diſorder might arrive from this, that there was ſuch a great quantity of corroſive *Lympha* in them, that it was in a manner impoſſible for it to be carried off by Tranſpiration, ſo that by ſtagnating in their Bodies it grew hot, fermented, ſower, and putrified; from thence aroſe thoſe Corroſions, Ulcers, and great Impoſthumes, Corruptions and other Accidents which we ſpoke of before. All theſe Poor People eat very heartily to the laſt Moment of their Life; this proceeded from a ſharp Humour, with which their Stomach always abounded, which created in them a kind of *Fames Canina.* Nothing is ſo apt to corrupt the Blood as long Want; the uſe of ill Food is ſtill worſe; Cold ſtops the Circulation of the Blood, and makes the Blood remain too long in the Parts, where it ſoureth and ſoon corrupteth; Sadneſs and Grief (which theſe poor Creatures are ſubject to) is worſe than all the reſt; and what all theſe may do when they meet altogether *in* one Perſon, we may eaſily judge. They produced there *Lympha*'s of different Colours, with which the Belly, the Breaſt, and ſeveral other Parts of their Bodies were fill'd. Thoſe *Lympha*'s were ſo Cauſtick, that having put our Hands into their *Cadavers*, the Skin of them came off, and our Faces were thereby Ulcered; ſo that we were obliged to riſe in the Night to waſh our Faces with Freſh Water, to take off the Heat and Inflammation of it. But that which was very ſurpriſing in this great Diſeaſe, was that the Brains of theſe poor Creatures were always ſound and entire.

Diſſection of the Body of Mr. Dove. *by Mr.* Cowper. *n.* 335 *p.* 512.

X. The Body in divers parts appear'd of black, blue, livid, and various Colors, before I made any Inciſion into it, particularly the Back, where the Blood was ſettled, had a cadaverous Blackneſs, where the

Cuticula was here and there veſicated, or diſtended with *Serum*. Of this there was no Appearance before Death.

The Muſcles of the *Abdomen* had a Mortified Appearance, being a blackiſh green Colour. The Liver was intirely ſphacelated. The Spleen had large mortified Spots on its Surface: Both theſe Parts were ſpecifically lighter than in the Natural State; inſomuch, that Portions of each of them ſwam on the Surface of the Water, and ſeemed to have more Air in them than we commonly find in the Lungs in their Natural State. The reſt of the *Viſcera* in this lower Cavity were not in ſo low a State; tho' the Guts had here and there blackiſh Spots on them.

The Pectoral Muſcles were in little better State than thoſe of the *Abdomen*; nor were the Intercoſtal Muſcles like thoſe of the Limbs. I am apt to think all the Muſcles imployed in Reſpiration, had more or leſs of this Blackiſh Appearance. The Right Lobes of the Lungs were diſeaſed; and the ſame ſide of the *Thorax* had a ſmall quantity of *Serum* in it. The Lungs on the other ſide were in no ill Condition. The Heart was very flaccid and large: The Right Ventricle and *Vena Cava* had no ſmall *Polypus* in them. The *Vena Pulmonaris* was exceedingly dilated next the *Baſis* of the Heart. The Left Ventricle of the Heart was furniſhed with a ſmall *Polypus*, and a great quantity of Grumous Blood. The Great Artery was very thin, and appeared not a little extended, and had ſome Cartilaginous Bodies interſperſed in its Membranes.

In the Head; the *Dura Mater* was found inſeparable from the *Cranium* in its upper part. A *Polypus* was drawn out of the upper great Vein of the Brain, called *Sinus Falcis ſuperior*.

The Carotide Arteries were very thin, and much larger than they ought to be, before they entered the Subſtance of the Brain. In ſhort, all the Blood-Veſſels which I examin'd were very much dilated, and ſeemed to be charged with as much Wind as Blood.

A Child deliver'd full of the Small-Pox. by Mr. Derham, *n.* 337. *p.* 165.

XI. There was, in the Pariſh next *Upminſter*, a Great-bellied Woman who had the Small-pox, and was pretty well recovered, ſo that ſhe was able to take ſomething to purge her after it: And on *Auguſt* the thirtieth laſt, ſhe took a Purge (I know not what) which did not work; and on *September* the Firſt, another Purge, which gave her only a Stool or two. Upon which, *September* the Third, ſhe took another ſtronger Purge, that work'd ſo violently upwards and downwards, that ſhe fell into Faintings and Convulſions: about which time I conceive her Child died within her, but of which ſhe was not deliver'd till *September* the eighth. The Child was a Female, and in appearance well made, luſty and ſtrong. At its Delivery, the Midwife judged it had been dead five or ſix Days; ſo that the Belly was burſt, and the Bowels came out, and the whole Body was inclining to be rotten. The Child was very full of the Small-pox, ſo full, that the Midwife ſaid, hardly a Pins head could be put between the Bliſters, which were very Plump, and full of Matter, like the Puſtules of an Adult, when the Small Pox is at the Height, only a little depreſſed in the middle. But as full as the Child was, the Mother had as few, and very favourably.

Of procuring the Small Pox by Inoculation. by Dr. Timone. *com. by Dr.* Woodward. *n. 339. p. 72.*

XII. The *Aſiaticks* have introduced this Practice of procuring the Small Pox by a ſort of Inoculation, for about 40 Years at *Conſtantinople*; and tho' at firſt 'twas us'd with Caution, yet the Succeſs it has met with in thouſands of Subjects for 8 Years paſt has put it beyond all Suſpicion and Doubt; ſince none who have had it this way have been found to die of the Small Pox; when at the ſame time 'twas very mortal, when it ſeiz'd the Patient the common way, of which half the affected dy'd.

They that have this Inoculation practis'd on them are ſubject to very ſlight Symptoms; ſome being ſcarce ſenſible they are ill or ſick; and what is valued by the Fair, it never leaves any Scars or Pits in the Face.

The Method of the Operation is thus. Choice being made of a proper Contagion, the Matter of the Puſtules is to be communicated to the Perſon propoſed to take the Infection; whence it has, metaphorically, the name of Inciſion or Inoculation. For this purpoſe they make choice of a Boy, or young Lad, of a ſound healthy Temperament, that is ſeized with the common *Small-Pox* (of the diſtinct, not Flux ſort) on the Twelfth or Thirteenth day from the beginning of his Sickneſs; they with a Needle prick the Tubercles (chiefly thoſe on the Shins and Hams) and preſs out the Matter coming from them into ſome convenient Veſſel of Glaſs, or the like, to receive it; it is convenient to waſh and clean the Veſſel firſt with warm Water: A convenient quantity of this Matter being thus collected, it is to be ſtop'd cloſe, and kept warm in the Boſom of the Perſon that carries it, and as ſoon as may be, brought to the place of the expecting future Patient.

The Patient therefore being in a warm Chamber, the Operator is to make ſeveral little Wounds with a Needle, in one, two or more places of the Skin, till ſome drops of Blood follow, and immediately drop out ſome drops of the Matter in the Glaſs, and mix it well with the Blood iſſuing out; one drop of the Matter is ſufficient for each place prick'd. The Punctures are made indifferently in any of the fleſhy Parts, but ſucceed beſt in the Muſcles of the Arm or *Radius*. The Needle is to be a three edg'd Surgeon's Needle; it may likewiſe be perform'd with a Lancet: The cuſtom is to run the Needle tranſverſe, and rip up the Skin a little, that there may be a convenient dividing of the Part, and mixing of the Matter with the Blood more eaſily perform'd; which is done, either with a blunt Stile, or an Ear-picker: The Wound is cover'd with half a Walnut-ſhell, or the like Concave

Veſſel, and bound over, that the Matter be not rubb'd off by the Garments; which is all removed in a few Hours. The Patient is to take care of his Diet. In this place the Cuſtom is to abſtain wholly from Fleſh and Broth for 20 or 25 days.

This Operation is perform'd, either in the beginning of the Winter, or in the Spring.

Some, for caution, order the Matter to be brought from the Sick by a third Perſon, leaſt any Infection ſhould be convey'd by the Cloaths of the Operator; but this is not material.

As to the Proceſs of the Matter, in reſpect of the *Idioſyncraſy*; the *Small Pox* begins to appear ſooner in ſome than in others, in ſome with greater, in others with leſſer Symptoms; but with happy Succeſs in all. In this Place the Effloreſcence commonly begins at the end of the ſeventh day. Which ſeems to favour the Doctrine of *Criſes*.

It was obſerv'd, in a Year when the common *Small-Pox* was very mortal, that thoſe by Inciſion were alſo attended with greater Symptoms. Of 50 Perſons, who had the Inciſion made upon them almoſt in the ſame day, four were found in whom the Eruption was too ſudden, the Tubercles more, and Symptoms worſe. There was ſome ſuſpicion, that theſe four had caught the common *Small-Pox* before the Inciſion was made. It is enough for our preſent purpoſe, that there was not one but recovered after the Inciſion: In thoſe four the *Small-Pox* came near the confluent ſort. At other times the inoculated are diſtinct, few and ſcatter'd; commonly 10 or 20 break out; here and there one has but 2 or 3, few have 100: There are ſome in whom no Puſtule riſes, but in the Places where the Inciſion was made, which ſwell up into purulent Tubercles; yet theſe have never had the *Small-Pox* afterwards in their whole Lives; tho' they have cohabited with Perſons having it.

It is to be noted, that a no ſmall quantity of Matter runs for ſeveral days, from the place of the Inciſion.

The Pocks ariſing from this Operation are dry'd up in a ſhort time, and fall off, partly in thin Skins, and partly contrary to the common ſort, vaniſh by an inſenſible waſting.

The Matter is hardly a thick *Pus*, as in the common, but a thinner kind of *Sanies*; whence they rarely pit, except at the place of the Inciſion, where the Cicatrices left are not to be worn out by time, and whoſe Matter comes near the nature of *Pus*.

If any Apoſteme breaks out in any (which Infants are moſt ſubject to) yet there is nothing to be fear'd, for it is ſafely heal'd by Suppuration. If any other Symptom happens, 'tis eaſily cur'd by the common Remedies.

Obſerve, they ſcarce make uſe of the Matter of the Inciſious Pox, for a new Inciſion. If this Inoculation be made on Perſons who have before had the *Small-Pox*, they find no alteration, and the places prick'd preſently dry up; except in an ill Habit of Body, where poſſibly a ſlight Inflammation and Exulceration may happen for a few days.

To this time, he ſays, I have known but one Boy, on whom the Operation was perform'd, and yet he had not the *Small-Pox*, but without any miſchief; and ſome Months after catching the common ſort, he did very well. It is to be obſerv'd, that the places of the Inciſion did not ſwell. I ſuſpect this Child prevented the inſertion of the Matter, for he ſtruggled very much under the Operation, and there wanted help to hold him ſtill. The Matter to be inſerted will keep in the Glaſs very well for 12 Hours. He goes on.

I have never obſerv'd any miſchievous Accident from this Inciſion hitherto; and altho' ſuch Reports have been ſometimes ſpread among the Vulgar, yet having gone on purpoſe to the Houſes whence ſuch Rumors have ariſen, I have found the whole to be abſolutely falſe.

It is now eight Years ſince I have been an Eye-witneſs of theſe Operations; and to give a greater Proof of the Sedulity I have uſed in this Diſquiſition, I ſhall relate two Hiſtories.

There was in a certain Family, a Boy of 3 Years old, afflicted with the *Falling-Sickneſs*, the *King's-Evil*, an *Hereditary-Pox* and a long *Maraſmus*. The Parents were deſirous to have the Inciſion made upon him; the *Small Pox* were thrown off with eaſe; about the 40th day he dy'd of his *Maraſme*. In another Family, a Girl of 3 Years old, troubled with the like Fits, ſtrumous, attended with an *Hereditary-Lues*, and labouring under a colliquative Looſeneſs for three Months. The Operation was perform'd on this Child; ſhe came off very well of the *Small-Pox*, which was all over the 15th day; on the 32d ſhe dy'd of her Looſneſs, which had never left her the whole time.

Ætiologia. *by the ſame. ib.* p. 76.

2. Contagium Variolarum per puris infuſionem propagari haud equidem mirabitur qui Æſculapii templum vel à primo limine ſalutavit, & fermentationis doctrinam ſubodoratus eſt: Nec obſcurior eſt inſitionis modus quàm panificium aut ars cereviſiaria, in quibus ex admixto fermento maſſæ fermentandæ turgeſcunt; conciliato nimirum motu inteſtino minimarum particularum principiis active pollentium. Si quis quærit interim cur variolæ periculoſæ alioquin & perſæpe lethales, ex inſitione ſine ullo periculo excludantur. Dico: Variolæ communes vel concurrente pravâ aliquâ ſpeciali aeris diatheſi ſuſcitantur, vel ab effluviis à varioloſo corpore emanantibus per contagium propagantur. Primus caſus in paucis individuis accidit, & concurrente quidem vel inſigni cacochymiâ, vel ſaltem varioloſi ſeminii in talibus individuis latitantis accerrimâ exaltatione: Secundus caſus communiſſimus eſt. In primo caſu miaſma malignum aereum, in ſecundo virulenta contagii corpuſcula indolis (probabiliter) ſalino-ſulphureæ ſed ſpecificam fracedinem ſeu ranciditatem nacta, ſtatim ac per reſpirationem hauriuntur ſpiritus ipſos, & labe quidem teterrima inficiunt; ſubſequenter autem maſſam ſanguineam & lympham vitiari manifeſtum eſt. Spiritus ſta

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tim inſici rationi conſentaneum eſt, tum quia in fontes ſpirituum, cor ſcilicet & cerebrum, ſtatim ingreſſum habent virulentum aporriæ, tum ratione analogiſmi inter miaſmata & effluvia iſta ipſoſque ſpiritus, cum utraque ſpirituoſo-aereæ texturæ ſint. Deducitur etiam cita & prava ſpirituum infectio à tot tantiſque nervoſi ſyſtematis ſymptomatibus, quæ malas plerumque comitantur variolas, & præcipuè à convulſionibus epilepticis quæ infantibus accidunt ipſo momento, quo varioloſo inficiuntur contagio multo antequam febris illos corripiat. Maſſam autem ſanguineam inquinari præter febrem purulenta tuberculorum excluſio teſtatur. Lymphæ verò vitiatæ fidem faciunt glandularum in faucibus tumor, ſcreatus, & enormis multoties ptyaliſmus. Inter hæc circularis etiam ſequitur noxa. Sed præcipuè ſanguinis particulæ ab indebita ſpirituum irradiatione in plures ataxias & anomalias perducuntur. Duobus tamen potiſſimum modis in variolis communibus mortem contingere obſervavi.

Primus eſt quando paucis erumpentibus variolis, & tardè ad maturitatem procedentibus,mala alia oboriuntur ſymptomata; ſecundus quando nimia tuberculorum copia cadaveroſam putredinem inducit. In primo caſu malignæ vulgo dicuntur variolæ: cauſa autem eſt vel nimia fuſio & diſſolutio maſſæ ſanguineæ, vel ejuſdem coagulatio & grumeſcentia. Si enim impetus ſpirituum exploſivus juſto plus augeatur, particulæ maſſæ ſanguineæ nimium ad invicem atteruntur, comminuuntur, & tenuiſſimas nanciſcuntur acrotitas: ſanguis in hoc ſtatu ſolertis naturæ mechaniſmum eludit, cumque nil fœculentioris in glandulis ſecretoriis cribriſque deponat, œconomiæ animalis functionibus requiſitas filtrationes & tranſcolationes celebrari haud patitur: improportionata etenim eſt figura particularum liquidi ad configurationem pororum in tubulis & colatoriis ratione ſubtilitatis nimiæ: filtratione enim defæcarentur particulæ ſanguinis ſi naturalem ſervarent ſchematiſmum & molem: hinc dicitur peſſim fieri per incraſſationem. Præter hoc celeritas ipſa tranſitûs ſanguinis in cauſa eſt ut nihil deponatur in colatoriis. Torrens,ubi nimio impetu & præcipiti curſu fertur,aquas turbidas defæcari haud patitur; quia vis centripeta gravitatem admixti terrei ſequens ſuperatur à fortiorum pulſoria virtute aqueorum globulorum rapide ruentium: virtus enim fortis, verbi gratia, ut unum non poterit lineam perpendicularem deſcribere, ubi virtus fortis ut duo ad lineam horizontalem protrudit: ſic etiam haud pluit vento flante intenſiſſimo; eadem geometrica proportione (probabiliter loquendo) ſanguinis particulæ aucto ab effrænibus ſpiritibus motu, tubulos colatorios præterfluunt nullâ factâ fæcum depoſitione. Hæc probabilia fiunt à ſumma pulſûs celeritate, febre intenſiſſima, ſudore nullo, & urina cruda. E contra quandoque contingit ut ab acutis, & ſcindentibus deleterii fermenti particulis frangatur, corrodatur, vel ſaltem relaxetur elater ſpirituum: elangueſcente igitur ſpirituum motu, torpidiores etiam hebetioreſque fiunt ſanguinis lymphæque particulæ: igitur dum in labyrinthæis tubulorum anfractibus moram indebitam contrahunt,

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alias turmatim invicem complicari, alias autem, congeſtione factâ, ſuper alias incidere, & diverſo ad invicem ſuperficierum ſuarum contactu à naturali configuratione deſciſcere, & novas induere angulorum dimenſiones neceſſe eſt. Sic igitur diverſa ab illa, quam ſuperius narravimus, figurarum ad tubulorum meatus improportione, pari tamen calamitatis eventu, dædaleæ naturæ machinationes irritas fieri contingit. Hæc probabilia fiunt à pulſu tardo & raro, ac febris carentia, quandoque in ſumma malignitate obſervatis, paucis & tardè erumpentibus variolarum puſtulis. Ulterius à trepidatoria, ſeu ſubſultoria ac tumultuoſa furentium ſpirituum irradiatione, inæqualis eodem tempore in diverſis partibus maſſæ ſanguineæ, & arteriarum etiam venarumque contingere poteſt impulſus. Sive igitur fibrillæ aliquæ (ut quidam volunt) reperiantur in ſanguine, ſeu chili nondum bene aſſimilati ſint portiones uſibus peculiaribus dicatæ; probabiliter iſtarum motum turbari contingit: has enim in circulatorio motu ſecundum longitudinem ſuam naturaliter moveri neceſſe eſt: ab inæquali autem preſſione dicta rectilineam figuram perdere, & in ſpiras ac ſemicirculos criſpari coguntur: has igitur ſic contortas tranſverſaliter poſtmodum in circulatione raptari, ad invicem implicatas convolvi, &, ramoſis ſchematibus obortis, racematim adeo conglobari neceſſe eſt, ut in majuſculos tandem grumos coaleſcant; ſive demum fibrillæ illæ non dentur, certè cujuſcumque figuræ ſint maſſæ ſanguineæ particulæ, illas à naturali deſciſcere ſituatione ex hac motûs inæqualitate contingit: Confuſæ igitur particulæ iſtæ & ad invicem implicatæ ſtatim vehiculi ſui, ſeri ſcilicet globulis per expreſſionem à ſuo contubernio exploſis, majorem ratione molis auctæ gravitatem nanciſcuntur, ideoque impulſivæ circulatoriæ facultatis vim ſuperant: Has igitur hîc illic reſtitare ac ſtagnare neceſſe eſt, prout in hoc vel illo loco prima mutua cohæſio forte contigerit: Hinc livida ſtigmata, & ſimul (quod ſæpe obſervavi in variolis cum petechiis erumpentibus) frequens ſequitur mictus, quo limpidiſſimum ſerum in magna copia excluditur. En fuſio, & coagulatio. Hinc mirum non eſt cur moriantur aliqui in variolis cum petechiis, convulſionibus, ſyncope, vigiliis nimiis, hæmorrhagiis, delirio, vomitibus enormibus, dyſenteriis, *&c.* quamvis haud multa puſtularum putrilagine perfundantur: In ſtygium enim veluti characteriſmum variolarum fermentum multoties evehitur, ita ut quamvis haud magnam craſſi puris copiam progignere aptum ſit, ſpiritibus tamen, liquidis & ſolidis, ſuprarecenſita mala modis vel explicatis vel aliis conſimilibus communicare poſſit, ſicque mortem inferre; & hoc ante undecimum plerumque. Veniamus nunc ad ſecundum modum. Diverſa enim aliquando contingit pernicies & longè alterius generis tragœdia: quamvis enim abſint illa ſymptomata, nimia tamen puris, materiæ ſcilicet cadaveriſatæ, copia corpus obruitur. Pus autem generari probabile eſt quando ſulphureis oleoſiſque maſſæ ſanguineæ particulis in fracedine & fuſione conſtitutis acido-ſalinarum particularum coaffuſio contingit. Huic aſſerto facem accendunt innumera chymica

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experimenta quibus manifeſtè edocemur, ſolutionibus pinguium ſulphureorum per alkalia factis acido quolibet coaffuſo ſtatim maſſam albicantis coloris emergere. Multoties igitur miaſma ſeu fermentum variolarum per reſpirationem hauſtum ratione indolis propriæ acerrimæ & fortaſſis ſepticæ tales in maſſam ſanguineam particularum acido-ſalinarum & oleoſo-ſulphurearum producere poteſt combinationes, ut non ſeminia ſolum variolarum, quæ omnibus individuis (mole tamen minima) à nativitate indita ſunt, agitentur, actuentur, & in purulentam abeant putrilaginem, ſed maſſa ipſa ſanguinea tota acorem contrahat, & motu quodam corruptorio putreſcat & cadaveriſetur. Sic igitur, incendio veluti coorto, ulterius furere fermenteſcentes particulas contingit, quam varioloſis ſeminiis per deſpumationem eliminandis opus ſit : hic motus non eſt depuratorius heterogeneis ſecernendis inſerviens, ſed deſtructivus & corruptorius, fermento nempe maſſam totam ſuperante & invertente ; fracidis ſcilicet rebellibuſque particulis victoria potitis, & omnes alias in ſua caſtra migrare cogentibus. Hoc manifeſtè obſervamus in variis potulentis, in quibus, fermentatione aliquando excitatâ, motus ſuccedit corruptivus liquores totaliter vitians : hinc videmus aliquos, quamvis ſuprarecenſitis ſymptomatibus immunes, immenſo tamen, ut ita dicam, putredinis oceano ſuffocatos : Et hoc periculum uſque ad vigeſimum ſecundum protrahitur. Ultimo loco conſiderandum ſolida etiam & nobiliores partes in hiſce caſibus peſſimè affici, & in ſpaſmos inordinatos fieri : variis horum diſtortionibus tubulorum meatus vitiari, at functionum munera depravari neceſſe eſt. Ecce igitur continentia, contenta, & impetum facientia, quorum triumviratu humani corporis reſpublica regitur, una eademque ruina ut plurimum involuta : mirabiturne quiſpiam malorum inde Iliadem in hominis perniciem pullulare ? Obſervandum ulterius multis, qui peſte laboraverint, communibus variolis etiam poſt annum correptis, bubones eoſdem intumuiſſe, qui antea in peſte eruperant : nonne hoc etiam ſummam malignitatem teſtatur ? Inſitionem modo ad rationis trutinam revocemus. At hercule longe aliter in hoc contagionis modo rem procedere quis eſt qui non fateatur ? Primum enim Spiritus nullatenus inficí manifeſtum eſt : deinde non lymphæ, non ſanguini labes illa teterrima inuritur, non ſolidis vitium aliquod communicatur. Hinc ſymptomata omnia levia, nulla peſſima, nulli infantibus epileptici inſultus. Contagionis enim hujuſce fermentum non ſpirituale, non aereum & acutum eſt, ſed humorale, iners ac pigrum : venena autem quo ſubtiliora eo pejora : Ratione igitur improportionis nulla inter fermentum hoc & ſpiritus eſſe poterit lucta. Pus equidem variolarum in ipſa ſubſtantia ſanguini immediatè infuſum ſtatim in largum veluti pelagus exceptum diluitur, involvitur, abſorbetur, obtunditur : ſic illud miteſcit, ſic in manſuetiorem indolem circuratur. Contagioſæ iſtæ particulæ ſanguinem ingreſſæ ſtatim ſibi congeneres varioloſi ſeminii particulas ſanguini à nativitate inditas inveniunt ; iis igitur conſermenteſcunt, ſed invicem combinatæ ac complexæ haud amplius ſui juris ſunt, ut ulteriores excitent turbas, regiam vitæ petant, ſpirituum theſauros diripiant; nam mutuis compedibus conſtrictæ fixantur, præcipitantur, craſſioreſque & hebetiores fiunt, quam antea fuerint. Statim igitur volubilioribus aquearum particularum globulis tamquam aptis vehiculis ſuperincumbentes, ſanguinis motu à centro ad peripheriam tendente, ſecundo veluti amne, ad ambitum corporis protruduntur, eliminantur. Nonne manifeſtè videmus haud pus generari in inſititiis variolis, ſed ſanioſam, dilutiorem videlicet aqueamque magis materiam? Nonne ex hoc phænomeno palam eſt acido-ſalinas fermenti contagioſi particulas haud oleoſas paſſim ſanguinis particulas in cadaveroſam purulentiam pervertere, ſed blandioribus potius lævioribuſque aqueis particulis eaſdem dilutas & ſaturatas foras aſportari? Ex negatione fovearum & cicatricum nonne manifeſtum eſt acres, aculeatas, pungentes & corroſivas ſalini fermenti particulas à balſamicis ſtatim ſanguinis globulis obtundi, ſpiculis ſuis orbari, & hebetiori figura modificatas, vi veluti mochlica, extra propelli? Integra interim ſervatur maſſæ ſanguineæ textura, inviolata conſiſtentia. Nullam hîc vides fuſionem, nullam grumeſcentiam, nullum corruptorium aut deſtructivum motum. In inſitione enim tantum ſolummodo ſanguis fermenteſcit, quantum impuro à puri conſortio ſeparando, ac per deſpumationem extrudendo ſatis eſt. In hoc fermentationis motu ſolum per undulationem quandam leviter aliquando afficiuntur ſpiritus, lympha, & ſolidæ partes; & ſiquæ ad iſta contagii particulæ perveniunt, certè (quod inſitionis adumbrat metaphora) non niſi ſilveſtri acrimonia privatæ, ac veluti dulſificatæ pervenire poſſunt.

On the ſame Subject at Venice, *by Dr.* Pylarinus. *n.* 347. *p.* 393.

3. In Byzantio operatio hæc Medica latuit initio per aliquot annos, raró quoque, & inter humiliores dumtaxat recepta: Immaniter autem graſſante nuper Variolarum epidemia, latiùs innoteſcere cœpit; Nunquam tamen ſublimiores auſa eſt ingredi aulas; donec Nobilis quidam nec obſcurus inter præſtantiores Græcos, & ex antiquo Caryophyllorum ſtipite clarus, mihi verò intimiori amicitiæ titulo notus, anno ſalutis 1701. ſub hyemis finem, ſerio me quidnam de hac inſitione ſentirem, conſuluit; & an ad eandem in quatuor propriis filiis celebrandam præſtarem aſſenſum: Nam tùm temporis lethaliter totam ferè civitatem morbus hic invaſerat; qui, ſummum ei de natorum ſalute metum incutiendo, anxium valdè reddiderat. Verùm quid ipſe ſuper ignota re decernerem, nulla præcedente noviſſimæ hujuſce methodi notitia, penitus ignorare me dixi; ac ſimul operatoris conveniendi copiam petii. Triduo peracto, cum ad amicum denuò acceſſiſſem, & de eadem materia ſermo iteratò inter nos eſſet initus; Ecce paulò poſt mulier Græca ad decentiam ſatis compoſita cubiculum intravit; quæ totam operationis ſeriem, modum, locum, tempus, cæteraſque circumſtantias, ut ego deinceps aperiam, clarè ſatis lateque nobis expoſuit; quamquàm ipſa veram ex inſitione excitationis variolarum cauſam haud intellexerit: His experimenta & caſus innumeros

tutò ſemper & ad ſalutarem eventum perductos annexit; è quibus aliquot (omnes enim quis potuiſſet in urbe ampliſſima exquirere?) veriſſimos eſſe ex aſſertoribus fide dignis deprehendi: Idcircò re bene penſitata, rationi & naturæ haud abſonam omnino comperi: præſertim autem caſibus jam dictis permotus, amico jam fervidius poſt aliquot dies conſilium iterùm expetenti, me haud alienum, ſub levi tamen hæſitantia, præbui; Quâ, tamquam dato aſſenſu, arreptâ, & de ſervando regimine toto ægrotationis tempore ſatis edoctus, inſitionem per mulierem Græcam in quatuor filiis audacter inſtituit: quorum tres natu minores (quinquennes, & vix ſeptennes) leviter ægrotarunt; pauciſque apparentibus puſtulis poſt hebdomadam, febre penitus & periculo evaſerunt; Ætate verò major, octavum ſuper decimum agens annum, graviter decubuit: Nam continenti febre & malignante correptus, ſuperveniente difficilium ſymptomatum ſyndrome, pluſculiſque quamquàm non copioſis exanthematibus obrutus, vix poſt decimum quartum diem morbum eluſit: Quod ego atrabilari ejus temperamento, ſucciſque pravis, ut & neglectæ prius (contra datam admonitionem) corporis expiationi tribuendum velim. Felix operationis eventus mirum quam multas nobiliorum familias ad imitationem traxit; Ut hodie ſine hæſitantia, præter timidiuſculos aliquos, unuſquiſque tranſplantationis emolumentum ſentire velit. Soli Turcæ, utpote Fati decretis addicti minuſque dociles, hanc neglexerunt hucuſque.

Naturalis eſt penitus hæc operatio, nulloque obducta ſuperſtitionis fuco; quamvis ipſum tranſplantationis nomen prima facie hæſitantiam pariat. Differt toto cœlo à curationibus Sympatheticis; eo magis à magnetiſmo illo tranſplantatorio, per quem morbi ex uno ſubjecto (mediante imaginata quadam, gratis efficta, & imperceptibili Mumia) in aliud traduci dicuntur; qua de re Tenzelius, Bartholinus, Maxuellus, Etmullerus, aliique ex recentioribus, docti alias viri, agunt ſedulo; qui vetuſtas antiquorum in medicina quiſquilias expurgandas eſſe cum jactitent; inter quiſquilias ipſi quandoque ſe volutant, noviſſima vanitatis amurca venerandam hactenus puriſſimamque Scientiam deturpantes. Quare, ut verum fatear, quemadmodum operationes hæ Magneticæ, vel Sympatheticæ ſuperſtitioſæ vanitatis ſuſpicione non carent (ut in unguento armario, pulvere ſympathetico & ſimilibus) utpote extra activitatis ſphæram in diſtans agentes; ita variolarum tranſplantatio, vera, mera, pura Phyſica eſt; quia puris mediis Phyſicis, & ad oculum patentibus, nec non ipſo ſenſibili contactu completur, ut è mox dicendis clarius eluceſcet.

Hujuſmodi igitur variolarum exercitatio fit per metaphoricè ſic dictam inſitionem ſive tranſplantationem; quæ nihil aliud eſt, quam fermenti morbifici ſeu puris ex variolis extracti in corpus ſanum, per vulnuſcula ad hoc facta, inſertio.

Phyſicus excitationis modus hoc pacto ſuccedit. Intruſum in vulnuſcula pus veri fermenti ſuſcipit indolem; Hinc beneficio circulationis per vaſa & canales proprios ad ſanguinis maſſam delatum, diſpoſitas ad hoc particulas, & deliteſcente vitio turgidas ſtatim aggreditur, inficit, venenumque communicando latitans in illis ſeminarium fermentativum excitat, agitat, actuat, inque motum ciet; Unde ebullitio univerſalis, ſeu fermentatio oritur; Vi cujus impuriores & heterogeneæ partes ſeparatæ criticè protruduntur ad cutem; natura interim placidè virtute talis operationis totum opus moderante.

Sed ad Tranſplantationem ipſam, & ejus celebrandæ formam properandum; modum quò eadem mulier tranſplantatrix tutiſſimè operabatur fideliter deſcribendo, ordinem, cæteraque omnia; ex quibus regula quædam inſtitui poterit pro operatione hac obeunda. Cæterùm quamvis de omnibus oculatus Teſtis, ut ingenuè fatear, eſſe non poſſim; ex operatricis tamen ore multa, plura ex fideli relatione tranſumpſi; plurima & potiora ipſemet obſervavi. Omitto quæ ad majorem confirmationem univerſalis fama canit. Quibus omnibus præſtandam eſſe fidem candidè ac ſtrenuè aſſevero.

Ergo primo tempus ad celebrandam inſitionem opportunum ſeligendum eſt; Quod ſecundùm operatricem hybernum deſideratur; & non niſi tali tempore ipſa inſitionem inſtituebat: Vernum ego pariter idoneum crediderim, propter clementiorem aeris temperiem.

Secundo ſelectiſſimum adhibet fermentum; Pus ſcilicet inſerendum non ex quolibet ſubjecto recipit; ſed variolis epidemicè graſſantibus è puſtulis jam maturis decumbentis alicujus pueruli alias ἐυσάρκου, iiſque benignis, punctione illud extrahit exprimitque; & in conchulam aliquam vel vitreum vaſculum mundiſſimum, nec nimis actu frigidum reponit reconditque; quod vaſculum bene ſartum tectum in pediſſequi ſinum fovendum intrudit; mox ſine mora ad operationem properat: Pus ex inſititiis rejicit, ut inefficax. Quod tamen ego benignioris indolis, nec minoris interim energiæ fecerim: qua in re experientia conſulenda.

Tertio, temperatiſſimum vult patientis inhabitandum cubiculum quoad aeris modificationes.

Quarto, ad operationem jam celebrandam accedens mulier, frontem in confinio capillorum & quidem medio in loco; mentum & utraſque genas acu ferrea vel aurea pungit; non rectà, ſed obliquè impingendo, cutemque acuta cuſpide à ſubjecta carne aliquantulùm ſeparando, hinc eadem acu pus jam præparatum in vulnuſculum inſtillat è vaſculo, intruditque; ſuperinducta per faſciam ligatura: Manus item ambas in metacarpis, pedes in metatarſis eodem modo ferit, puſque inferit, faſciaſque leniter ſtringit; ſeriò imponendò patienti ne partes illas ſcalpat madefaciatve. Potiùs carnoſiora pungerem loca, quatenus inflammationibus dolorique minus obnoxia, nec tendinibus intertexta.

Præter hanc operandi formam cæteri omnes rejiciuntur modi, utpotè abſoni, inuſitati, malè ſuccedentes, infeliciſque exitus.

Interim lectulo moderatè manendum, neque plus quàm opus fuerit jacendum.

Quinto bonum in ſex rebus nonnaturalibus regimen, præcipue in victu, injungit ; Non ſolum enim vino & carnibus, verum etiam earumdem juſculo rigoroſè patientes ad quadrageſimum diem interdicit : Pluribuſque monitum non curantibus ſæpius malè ſucceſſit ; nam ad oculos, pœna erroris, novas erupiſſe puſtulas, aliaque non parvi diſcriminis ſymptomata ſuperveniſſe, viſum eſt.

Sic ritè peracta tranſplantatione, non omnibus eodem tempore intervallo ſuſcitari ſolent variolarum ſymptomata ; Variè enim fermentum agit ; ſerius vel citius, prout unicuique proprium favet temperamentum, ætas, robur : Quamquàm variolæ ipſæ, in ſeptimo ferè ſemper apparere incidant, qui dies vere criticus eſt. Nec defuere, quod raro contingere ſolet, quibus ſtatim primo die effloruerint.

Symptomata ægrotantibus evenientia variant ſecundùm temperamentorum diverſitatem, ſuccorum in maſſa ſanguinea habitudinem, & particularem in ſingulis naturæ diſpoſitionem : remiſſiora nempè vel intenſiora ingruunt ; At communiter graſſantibus non diſſimilia, quamvis clementiori ut plurimum facie : Plures vix alterationem læſionemve ſentiunt aliquam.

Excitatæ variolæ ferè ſemper ſunt de genere diſtinctarum ; nec numero multæ ; Decem, ut plurimum viginti, triginta, raro ad centum, rariſſime ad ducentas erumpunt.

Notandum primò nonnullos, unico dumtaxat vulnuſculo ad brachium inflicto contentos, excitaſſe variolas ; pauciſque apparentibus puſtulis præſervatos tamen impoſterum fuiſſe à contagio.

Notandum ſecundò accidiſſe interdum, ut ex inſitione nullæ penitus excitatæ fuerint Variolæ, vel ob non præexiſtentem ullam prorſus variolicam in corpore diſpoſitionem, vel ob enervatum infractumque fermenti contagium : At poſtea graſſante populariter morbo, correpta ſunt promiſcuè ejuſmodi corpora jam inſitionem paſſa, communi cæterorum ſorte.

Tertiò, Inſertionis loca ſeu vulnuſcula in puſtulas ſemper evadere ſolent : Quibuſdam verò excreſcunt in purulenta tubercula, nullis interim apparentibus puſtulis ; Nonnullis in apoſtemata quidem majora degenerant magnam puris copiam effundentia : Non ſemel eadem loca, in pedibus præcipuè manibuſque, ſummo cum dolore intumeſcunt; pureque effuſo ſubſident, iterumque in tumorem attolluntur. Quibuſdam, rariſſimè tamen, ad glanduloſas partes & emunctoria, poſt aliquod tempus, abſceſſus emergunt, ac ſuppurantur paulatim : ludente in diverſiformi corporum craſi natura.

Poſtremò, nunquam fere ex tranſplantatione funeſti quid accidiſſe, obſervatum fuit hactenus ; etiamſi in quocumque ſexu, temperamento, ætate celebrata fuerit ; quinimò, ritè recteque tractata, & in corporibus per peritum medicum aptè præparatis, certiſſimam promittit ſalutem. Variolæ enim hoc modo excitatæ benignioris ſunt indolis, quam ſunt illæ quæ populariter graſſantur ; Utpotè ex fermento ſeu contagio, omni malignitate carente permotæ : Ebullitio, per quam

maſſa ſanguinis agitatur ac totum opus perficitur, blandè non violenter moderante natura, conamina ſua molitur: Sed præter hæc, tempus ad operationem atque tempeſtas magis idonea pro tranſplantatione ad libitum eligi, ut & corpus inciſioni ſubjiciendum congruis adminiculis ad recipiendam illam ex arte preparari diſponique poterit; Quod revera maxime ad ſalutarem fauſtumque morbi ſucceſſum momenti cenſeri debet.

A way us'd in Scotland *to prevent the* Small-Pox. *by Mr.* Martin. *n.* 312. *p.* 2470

4. When the Small-pox is Epidemical in the main Land over-againſt *Sky Iſle*, on the S. E. and Eaſt, and likewiſe in *Sky Iſle*; the Natives bath their Children in the Infuſion of Juniper Wood, and they generally eſcape; whereas thoſe who neglect this Precaution, are often obſerv'd to die.

A Plant us'd in drawing a Felon, ibid.

XIII. The Plant Water Lilly being apply'd to the pained part of the Body where the Felon is fixed, it is obſerved that it forces its Paſſage quickly in that place thro' the Skin.

A Woman that lay ſix days covered with Snow. by Mr. Bowditch. *n.* 337. *p.* 265

XIV. *Joanna Crippen* of *Chardſtock* in *Dorſet*, *January* 24. 1708-9. coming homeward, it ſnowing very hard, having no manner of Suſtenance with her only a quarter of Tobacco, a pound of Worſted Yarn, and three Pence in Copper, not ſo much as a Bit of Bread, Bisket, or the like, was conſtrain'd to travel, as well as a poor tired Creature could, towards her own home; but going not far from thence was met by a Man of the Pariſh. He ſeeing her tumbling in the Snow a diſtance off, as he was going to his home, and finding her lying in a Ditch, help'd her up, and bid her obſerve to go in his Track, which he obſerv'd ſhe did indifferently well. But ſhe had not gone a quarter of a Mile, before ſhe was forc'd to lye down under a Hedge, having loſt one of her Shoes; and her Cloaths, which were very mean, were with the Brambles and Thorns torn almoſt quite off her Back: In which place ſhe lay from *Monday* Evening about ſix a Clock, until *Sunday* following about 4 in the Afternoon, and then was diſcovered by ſome Neighbours, who went to ſearch for her; and after ſome time, found her buried in four Foot of Snow or thereabouts, it being more than ſo much higher before the Thaw. One of the Men with his Pole thruſting at her, cry'd out, She was there; then the reſt opening the Snow, found ſhe was Alive. She immediately ſpoke, and begg'd he would not poot her too hard (as ſhe expreſs'd it) for ſhe was almoſt naked; and deſired that ſome of he Women would come to her and take her forth, which accordingly was done; they finding her without Stockings or Shoes, an old Whittle about her Shoulders, with a large Hole in it, which ſhe had eat through; the Snow melting down on her, which ſhe drank to quench her Thirſt. From thence ſhe was brought near my Habitation, where the beſt Care has been taken of her. She had a

Mortification on one of her great Toes, which is now in a good way of Recovery; and ſhe now is very hearty, and in a fair way of a perfect Recovery.

She was very ſenſible at the firſt taking her out, and ſtill continued ſo; ſhe knowing every Body perfectly well. Her Tobacco and Three Pence were in her Pocket. You may aſſure your ſelf, ſhe had no manner of Food with her, as Bread, or any Eatable whatſoever.

A Calenture, by Dr. Oliver *n.*290.*p.*1562.

XV. In *Auguſt* 1693, I was called about 4 in the Morning, to ſee a Sailor on board the *Albemarle* Man of War in the Bay of *Biſcay*, in a violent Calenture. He was between thirty and forty Years of Age, brown Hair'd, pretty tall, but thin, and had not much Fleſh about his Bones. When I ſaw him firſt, I found him in the Hands of three or four of his Comrades, who were hardly able to manage him, becauſe of his ſtrugglings, and conſtant endeavours to get from them. I obſerved, he very often cryed out, he would go into the green Fields; his looks were as furious and wild as thoſe of a Lyon, and every now and then he would heartily curſe thoſe that held him. The firſt thing I did was to examine his Pulſe, I felt, 'tis true, a diſorderly motion of the Blood in the Artery, and a burning fiery heat all over his habit of Body, but could perceive no diſtinction, or vibration of Pulſe at all. The Surgeon of the Ship, a good experienc'd Man in that way, had, before I came, attempted to bleed him, but though the Vein of the Arm was fairly open'd, yet could he not procure an ounce of Blood from thence. Upon that I order'd him to open the frontal Vein, which ſucceeded no better, for that ſoon ſtopt too. This put me upon trying a third time, what effects the opening the Jugular Vein might have. From this Vein, though our Orifice was pretty large, we had about two Ounces of florid thick Blood, and then it quite ſtopt there too. I was, I muſt confeſs, not a little ſurprized at this; and ordered the Surgeon to unbind his Arm, and try whether he could make him bleed again at that Orifice, which I remember he did in a ſmall quantity, and then ſtopt as before. However, having three Orifices open at that time, we drew Blood ſometimes from one, and ſometimes the other, where we ſaw it run moſt freely. After ſeveral Eſſays of this kind, I always obſerved as the Veſſels emptied, he bled more freely, and as faſt as I deſired. Not long after this, for he bled well enough now, I obſerv'd his Strugglings were not ſo ſtrong, his Ravings and crying after green Fields left off, his wild Looks much abated, and not only his Pulſe had recovered its due and regular Vibrations, but his heats were moderated too, and the Fury of his Spirits lay'd to that degree, that he that juſt now was as furious as a Lyon, was grown ſo tame that one Man was able eaſily to manage him as he pleaſed. In this half hour, as near as we could gueſs, we took from him about Fifty Ounces of Blood from the three Orifices mention'd. By this time I thought we had enough, ſo I order'd him to his Hammack, as ſoon as we had ſecured the Orifices from bleeding again, and directed the Surgeon to give him an Ounce of Diacodium in a Draught of Barley Water, as he went into it: Upon this he ſlept till about Noon, when he awaked with no other Complaint, but of weakneſs from his loſs of Blood, and a ſoreneſs all over his Body, occaſion'd, I preſume, from his violent Convulſions and Endeavours to get looſe.

'Tis very probable, that when they are ſeiz'd with this violent Heat and Diſorder, which for the moſt part happens in the Night, they ſteal privately over-board into the Sea, imagining they're going into the green Fields. And this I take to be the reaſon we ſee ſo few; tho I have heard frequently in the *Mediterranean* in Summer time, and very hot Weather, of Seamen loſt in the night, which the Sailers took for granted were gone off upon ſuch like occaſions unobſerved. And I remember very well, this Perſon was actually going over-board, when one of his Brethren, who ſuſpected his deſign, as he told me, caught hold of him juſt as he was going to leap off, call'd for help, and ſecur'd him by this Accident. And laſtly, Calentures happen oftner by Night than by Day, becauſe our Ships are moſt cloſely ſhut up by Night, and are leſs airy than they are in the day time.

Of ſtrange Epileptick Fits, by Dr. Cha. Leigh *n.* 280. *p.* 1174.

XVI. We have this Year had an epidemical Fever, attended with very ſurprizing Symptoms, in the beginning the Patient was frequently attacked with the *Colica Ventriculi*; Convulſions in various parts, ſometimes violent Vomitings and a Dyſentery; the Jaundice, and in many of them a ſuppreſſion of Urine; and what Urine was made, was highly ſaturated with Choler: About the ſtate of the Diſtemper, large Purple Spots appear'd, and on each ſide of 'em two large Bliſters, which continued 3 or 4 days; theſe Bliſters were ſo placed about the Spots, that they might in ſome meaſure be term'd Satellites or Tenders; of theſe, there were in many four different eruptions; but the moſt remarkable inſtance I ſaw in this Fever, was in a poor Boy of *Lymm* in *Cheſhire*, one *John Pownel*, about 13 years of Age, who was afflicted with the following Symptoms; upon the Criſis or turn of the Fever, he was ſeized with an *Aphonia*, and was ſpeechleſs 6 weeks with the following Convulſions; the Diſtemper infeſted the Nerves of both Arms and Legs, which produc'd the *Chorea Sancti Viti*, or *St Vitus*'s Dance; the Legs ſometimes were both ſo contracted, that no Perſon could reduce them to their natural poſition; beſides theſe, he had moſt terrible Symptoms, which begun in the following manner; he cou'd perceive the Fits to come on about the *Os Sacrum* or extremity of the Back-bone, and the Region of the Navel, and then the diſorder, as he imagined, united about the top of his Head; he immediately afterwards fell into violent Convulſions in the *Abdomen* or loweſt Cavity, with that violence, that ſometimes two or three Perſons were forced to lye upon him to keep him in Bed, his Body being frequently rais'd from it; after this, the Nerves of the Lungs were immediately affected, and then he barked in all the uſual notes of a Dog, ſometimes Snarling, Barking, and at the laſt howling like an Hound; after this, the Nerves of the Mandibles were convuls'd, and then the Jaws claſh'd together with that violence, that ſeveral of his Teeth were beaten out, and then at ſeveral times there came a great foam from his Mouth; afterwards he had an extream wild look, ſnatching at any thing near him, and would have tore off his Fleſh, had he not been prevented by the Perſons about him; this made me conjecture he might formerly have been bit by a Mad dog, which had introduced the *Hydrophobia*; but I was convinced to the contrary, for I put a Baſon of Water by him, and he was not in the leaſt afraid of it, nor attempted to lap it. I ſaw him in three of theſe Fits, but at other times in theſe Convulſions, he roar'd like a Bull, made a noiſe like a Hog, and ſometimes like that of a Goſling; all which different ſounds, (I take it) proceed from the different contractions of the Lungs, variouſly forcing out the Air, and conſequently as they were differently convuls'd, form various ſounds; In a weeks time I recover'd the Boy his Speech, his Senſes return'd, his Convulſions vaniſh'd, and the Boy is now very cheerful.

Of the Plague at Copenhagen *in* 1711. *com. by* John Chamberlayne, *Eſq. n.* 337 *p.* 279.

XVII. This Diſeaſe began to ſhow itſelf firſt in this City, about the beginning of *July*, 1711. It increaſed till the beginning of *September*; after which it diminiſh'd by little and little to the End of the Year, at which time it totally ceaſed. It appears, that before this Diſtemper there were about Sixty Thouſand Souls in *Copenhagen*: From whence they infer, that there is Born every Year about Two Thouſand, and that there Dies nearly the ſame Number; which being Multiplied by Thirty makes Sixty Thouſand. In the Six Months which this Diſtemper continued, it is thought it carried off about 25000 Souls. It is true, the Publick Liſts reckon but 22535; but it is agreed by all, that in the laſt Week of *Auguſt*, and the two firſt Weeks of *September*, each of which carried off above 2300 Souls, there died a great many, of which there was no Notice taken. Almoſt the very ſame happened two Years before at *Dantzick*; where before the Plague broke out, there died Weekly from 45 to 50; but the Number of the Dead increaſed by degrees to the beginning of *September*; ſo that in the firſt Week of that Month there died 2205 Souls, in the ſecond Week 2070, and in the third 2075. After which the Mortality decreaſed to the End of the Year.

It is obſervable, 1. That there were ſome Houſes which eſcaped the Infection; but that there were few where it did not carry off more than one or two Perſons; and that there were many in which it did not leave a Soul alive. 2. That generally ſpeaking, this Diſtemper was moſt fatal to the meaner ſort of People; there ſcarce dying any Perſon of Note; but on the contrary a great number of the Poor.

Which may be attributed to ſeveral Cauſes: The firſt, and moſt general of which, is their naſty manner of Living. The ſecond is, that this ſort of People live very cloſe together, and as it were heaped one upon another; ſo that ſometimes there are four Families in one Room. The third is, the fooliſh Curioſity they have of ſeeing the Dead Bodies. And fourthly a great many of them are ſo bigotted to the *Turkiſh* Notion of Predeſtination, that they ſay, if it pleaſes God that I ſhould die of this Diſeaſe, I ſhall not eſcape it; and if it be his pleaſure that I ſhall live I can't die: And upon this Notion they go abroad every where, and ſo catch the Infection. There are ſome of 'em alſo, which make no ſcruple of lying in the ſame Beds, where others have dy'd. The 3 Sorts of Trades of which there died moſt were Coffin makers, (who took meaſure of the dead Bodies) Surgeons, and Shoomakers. The Care that was taken and Medicines us'd did great Service, I was told that *Theriaca* did little Good; and they obſerv'd the ſame alſo at *Dantzick*.

An emaciated Child diſſected, by Dr. Blair. *n.* 353. *p.* 631.

XVIII. This Child was five Months old, and was ſo emaciated, that he appear'd rather to have decreaſed, than to have increaſed in Bulk, from the time of his Birth; his whole Body not weighing above five Pounds. The Skin and Muſcles of the *Abdomen* were very thin, but the Peritoneum was preternaturally thick. The Ventriculus was more like to an Inteſtin than to a Stomach, its length being five Inches, and its breadth but one Inch. The Coats of it were thick and fleſhy, and the Cavity very inconſiderable. The Pylorus, and almoſt half of the Duodenum were Cartilaginous, and ſomething inclin'd to an Oſſification, ſo that no Nouriſhment could have paſſed into the Inteſtins, tho' the Stomach had been capable of containing it, which makes it no Wonder that the Body was ſo emaciated. There were ſcarce any foot-ſteps of the Omentum to be ſeen, even at the Bottom of the Stomach, to which it uſually adheres. The right Lobe of the Lungs adhered firmly to the Ribs and had three Exulcerations, which contain'd purulent Matter. It was ſo very thin and compact, that it ſeem'd as if that Lobe had never been of uſe in Reſpiration. The left Lobe was of a more florid Red; ſpongy, and free from any Adheſion.

Upon enquiring after the Symptoms this Child had been affected with, his Mother told me, he ſeem'd to be healthy till he was about a Month old, when he was ſeized with a violent Vomiting, and a Stoppage of Urine and Stool. Some time after, both theſe became more regular, but the Vomiting ſtill continued. He ſeem'd to have a great Appetite, taking what Suck, Drink, or other Food was offer'd him, with a kind of eagerneſs; but he immediately threw it all up again. He had all along breathed freely, and had no Cough, notwithſtanding the Exulcerations above mention'd. This confirm'd me in the Opinion that he had never breath'd by the right Lobe of the

Lungs. There could be nothing more emaciated than this Child was.

XIX. A Gentlewoman in *New England* ſwoons upon the ſeeing any one cut their Nails with a Knife; which if done with a pair of Sciſſars has no Effect upon her.

Of Antipathies. by Dr. Mather. *n.* 339. *p.* 641.

XX. The Wife of a Perſon vomited, upon ſeeing her Husband take a Vomit; the Perſon that took it being not moved, but forced to take a freſh Emetick.

Of the Force of Imagination, by the ſame. ib.

XXI. *Nathaniel Hulme,* aged about 17, had the Small-Pox about 8 Years of Age: Soon after which he had a great Itch, almoſt to the degree of a Leproſie, with which his Finger-nails and Thumb-nails began to grow thick, and by degrees hardned into Horns; which grew in 7 or 8 months to the length of an Inch, and ſome almoſt 2 Inches, and ſome much longer. It began in the Fore-finger of his Left-Hand, and ſo to all the reſt of that Hand, which had as many Horns as Fingers and Thumb. All which Horns about the end of 12 Months fell off by degrees; that which grew firſt ſalling off firſt, without any pain, unleſs when cut off, as they were at firſt, there appearing great quicks (as they call them) or Roots under the Nails. By degrees they came on the Thumb, and then on the Fingers of the Right Hand; which grew to the ſame length in about a years time, and then fell off, he having ſhed them 5 or 6 ſeveral times. One of the Horns that grew on the Ring Finger of the Left Hand, was a quarter long. They are now at preſent all come off his Left Hand, but growing again: That on his Little Finger is two Inches long. I have now by me one or two of them. This Account I took in 1702.

He had in 1704 all the Fingers of both Hands armed with ſuch like, and ſome as long as theſe I ſend. He has on every Toe alſo, but keeps them cut, that he may be able to wear Shoes. I think he cannot live long, being miſerably overſpread with his Leproſy.

One of the Horns above-mentioned is now in the Repoſitory of the Royal Society.

A Boy with Horn-like Excreſcences growing on the Fingers. n. 297. *p.* 1899.

XXII. Mrs. *Lovelock* had been ſeized ill of a Fever, which affected her ſo, as to make her Light-headed to a great degree, convulſed and reſtleſs; upon ſhe took great quantities of Opiats, in order to compoſe her; but could never procure any thing like Sleep; but ſtill as ſhe took them they ſeem'd to refreſh her, and make her ſenſible, but cauſed nothing of reſt. The quantity ſhe took from *Tueſday* Night twelve a Clock to *Friday* Night twelve a Clock was as follows, *Jan.* 29. four Bolus's with two gr. each of *Laud. Lond.* made up in Venice Treacle. Six Pills, with two grains each of the ſame. A Bolus with eight

A Perſon who took a great quantity of Opium, without cauſing Sleep. by——— *n.* 275. *p.* 999.

eight grains of the ſame in Venice Treacle. *Jan.* 30. Twelve Pills with two grains each, and one Bolus with ten grains of the ſame in Venice Treacle. *Jan.* 31. four Draughts with ten grains of the ſaid *Laud. Lond.* one ounce of *Syr. de Mecon.* in each Draught. So that in all ſhe took in the time above-mentioned 102 grains of *Laud. Lond.* and ʒiij. of Venice Treacle, and ℥iiij. of *Syr. de Meconio.* She died *Feb.* 1.

XXIII. *Accounts of Books omitted.*

n. 270. *p.* 832. 1. Sanctorii Sanctorii de Statica Medicina Aphoriſmorum Sectiones ſeptem, cum Comentario Martini Liſter. 8vo. *Lond.* 1701.

n. 273. *p.* 914. 2. Jac. Gaveti Acad. Monſpel. Alumni Avenionenſis Doct. Medici, & apud Camberienſes practici, nova Febris Idea, ceu Conjecturæ Phyſicæ circa Febris naturam. 8vo. *Genevæ*, 1700.

n. 284. *p.* 1373. 3. Hippocratis Aphoriſmi, cum Commentariolo, Authore Martino Liſter, è Medicis Sereniſſimæ Majeſtati Reginæ Annæ. *Lond.* 1703.

n. 320. *p.* 324 4. An Account of Animal Secretion, the quantity of Blood in the Human Body, and Muſcular Motion. By *James Keill*, M. D. 8vo. *Lond.* 1709.

n. 337 *p.* 101. 5. A Deſcription of the Plague, which happen'd at the Royal City of *Dantzick*, in 1709. Written in *High Dutch*, by Dr. *John Chriſtoph. Gottwald*, and communicated by Dr. *John Philip Breynius*, as the beſt Account of that Diſtemper there publiſh'd.

n. 303 *p.* 2152. 6. Joh. Conradi Beckeri, Phil. & Med. Doct. Tractus Alsfeld. Medici Ordin. Paradoxum Medico Legale de ſubmerſorum morte ſine pota aqua, aliquot cadaverum ſectionibus detectum, & è principiis mechanicis illuſtratum. Cui adjicitur Dodecas obſervationum circumſtantiis curaque rariſſimarum. Gieſſæ Haſſorum, Anno 1714.

n. 343. *p.* 263. 7. Bibliographiæ Anatomicæ Specimen, ſive Catalogus omnium pene Auctorum, qui ab Hippocrate ad Harveium rem Anatomicam ex profeſſo vel obiter ſcriptis illuſtrarunt, *&c.* Cura & Studio Jacobi Douglas, M. D. R. S. S. & in Coll. Chirurg. Lond. Anat. Prælect. 8vo. *Lond.* 1715.

CHAP. IX.

Pharmacy, Chymiſtry.

I, 1. *Of Purging and Vomiting Medicines, by Dr.* Cockburn. *n.*303 *p.*2119.

QUO rectiùs diverſas emeticorum & purgantium Medicamentorum Doſes, pro Temperamentorum & ætatum varietate decernamus. Supponendum eſt, *Primo*, iſtiuſmodi Medicamenta nihil prius poſſe operari quàm in ſanguinis maſſam appulerint, & eidem penitus immiſceantur: Conſtat enim, ni Nauſeam moveant, nullum ab iis effectum ſenſibilem produci, multo poſt tempore quo in eandem devehi poſſint. *Secundo*, generaliorem eorum effectum eſſe temperamenti ſanguinei aliorumque Liquorum gyrantium Alterationem. Ex hiſce duobus Poſtulatis concludimus, quod ubi ſanguinis Craſis eſt eadem, Medicamentorum Doſes, ad certum aliquem eliciendum effectum, ſanguinis quantitati proportionales eſſe: Si namque certa quædam Doſis exigatur ad unius ſanguinis v. gr. Craſin alterandam ad certum aliquem gradum, duplicem oportebit Doſin adhibere ut Libræ duæ ad gradum alterentur, triplam ut tres, *&c.* Et univerſaliter ſi ſanguinis quantitas b exigat Doſin d, quantitas ſanguinis mb Doſin md exiget, & eſt $b : d :: mb : md$.

Coroll. Quum Quantitas ſanguinis & cæterorum humorum in gyrum actorum rite, ex Animalis pondere, poſſit æſtimari (partes enim quas ſolidas nuncupamus ſunt tantum Canales qui liquores iſtos continent) exinde ſequitur doſium quantitates, *cæteris paribus*, eſſe corporis ponderi proportionales; adeoque medicamentorum doſin infanti, recens nato, propinandam eſſe ad eorundem doſin provectioris ætatis hominibus ut eſt infantis pondus ab hominis. Exempli cauſâ, pilularum Rudii gr. xxx unicâ plerumque doſe homini exhibentur, & hominum pondus commune eſt 160 Librarum, & infantum Librarum 12; quapropter, ut ſunt 160 Hominum pondus ad 12 infantum, ita ſunt gr. xxx Hominum doſis ad gr. $2\frac{1}{4}$ quæ ſunt infantum; ac ſemper in eâdem ratione Medicamentorum doſes augendæ veniunt ut creſcit infans; dehinc doſes exhibendæ manent eædem ad annum quinquageſimum, ex quo tempore ſanguinis quantitas & vis minuuntur, quâ quoque ratione minuendæ ſunt Medicamentorum doſes.

Hâc ratiocinandi methodo, homines omnes eodem gaudere Temperamento ſimul & Augmentum Ordinemque ſecretionum ad annorum numerum, eſſe æqualem ſuppoſuimus; Quum vero hominum Temperamenta; ſeu Conſtitutiones, ſint diverſiſſimæ pro varia ſanguinis humorumque gyrantium Craſe, Doſium quantitates non ſemper proportionantur corporis ponderi. Diverſum id ſanguinis temperamentum in certâ quadam partium ejuſdem cohærendi diſpoſitione conſiſtit, quâ ſanguis magis vel minus fluidus evadit; unde provenit quod medicamentorum in ſanguinem Operationes fiant diverſæ pro variis Cohærentiæ gradibus. Ponamus enim homines duos æquali ſanguinis quantitate pollentes, cujus Cohærentiæ gradus ſint diverſi; manifeſtum eſt, medicamenta iſta facilius laxioris texturæ ſanguini admiſceri quam firmius cohærenti, commiſcendique Proclivitas quam habent medicamentorum partes particulis ſanguinis eſt ſemper ut Fluiditas directe vel reciproce ut eſt ſanguinis Tenacitas, & medicamenti in ſanguinem vis erit in eâdem ſimiliter ratione: Adeoque ut medicamentum in iſtis diverſi temperamenti hominibus æqualiter operetur, propinandæ doſes ſanguinis Tenacitati proportionales eſſe debent, ſupponendo itidem ſanguinem in utroque pari Celeritate circuitum agere. Si vero ſanguinis velocitas ſit diverſa, medicamentorum operationes, *h. e.* Secretionum quantitas ab eis producta, erunt ut ſanguinis velocitas: Secretiones enim in qualibet Glandula, in dato tempore, ſunt ſemper ut ſanguinis quantitas quæ in eandem Glandulam eodem tempore devehitur, *h. e.* ut ejus Velocitas. Sanguinis etiam velocitas, *cæteris paribus*, ſemper ſe habet ut ejus Fluiditas, ſeu reciproce ut ejus cohærentiæ gradus: Si igitur ſanguinis velocitas eſſet unice conſideranda, hoc caſu quantitas doſium ad eundem effectum producendum erit directe ut cohærentiæ gradus in partibus Sanguinis.

Prop. I. *In hominibus duobus æqualem ſanguinis quantitatem habentibus ſed Cohærentiæ gradibus differentis, Medicamentorum Emeticorum & purgantium doſes, ad eundem effectum eliciendum neceſſariæ, ſunt in duplicata ratione graduum cohærentiæ ſanguineæ.*

Ubi enim Sanguis eâdem velocitate movetur, doſis quantitas ut cohærentiæ gradus ſit oportet, & ſi gradus cohærentiæ eſſent iidem, doſis quantitas eſt reciproce ut velocitas; adeoque, ubi nec cohærentia nec velocitas ſunt eædem, quantitas doſis eſt in ratione compoſita ex directa ratione graduum cohærentiæ in ſanguine & reciproca velocitatis: Atqui reciproca velocitatis ratio eſt directæ Tenacitatis vel cohærentiæ graduum rationi æqualis. Ideoque Doſis quantitas eſt in ratione compoſita graduum cohærentiæ & graduum cohærentiæ, *h. e.* Doſes propinandæ ſunt in duplicata ratione eorundem. Q. E. D.

Prop. II. *Doſium quantitas hominibus diverſæ quantitatis ſanguinis exhibenda, qui ſimul diverſis pollet Cohærentiæ gradibus, eſt in ratione compoſita ex ratione ponderis hominum & duplicata graduum Cohærentiæ.*

Quum enim cohærentiæ gradus ſunt iidem, doſium quantitas eſt ut hominum pondus; & ubi hominum pondus eſt idem, doſium quantitas eſt in duplicata ratione graduum cohærentiæ, adeoque quum neuter eſt idem doſis, Quantitas eſt in ratione compoſita ex ratione ponderis hominum & duplicata graduum cohærentiæ. Q. E. D.

Coroll. Hinc, Quantitatem & Qualitatem Sanguinis in quolibet homine ſcientes, doſes ad purgandum & vomendum neceſſarias non ita arduum eſt determinare. Hæc qualitas, ſeu Temperamentum, ſan-

guinis a perito Medico ex Pulſibus, Urina, & aliis ſecretionibus facile invenitur; adeoque Medicus, obſervando quæ doſes datæ conſtitutionis homines purgant, minimo negotio doſes cuilibet conſtitutioni vel Temperamento idoneas ſtatuet.

Schol. Quæ hactenus in genere demonſtrata ſunt, ex vulgari Medicorum circa Purgationem & Vomitionem Hypotheſi eodem ſequerentur pacto; quoniam Diſpoſitio, quam habent Ventriculus & Inteſtina ut ſtimulentur, eſt ut memorati quantitatis & Cohærentiæ in Sanguine gradus. Ita, vel ex illa Suppoſitione, Veritas hæc maxime conſtaret, licet poſtulati ſimplicitate deſtituitur, atque ob id non adhibetur.

2. From what has been ſaid it is manifeſt in general, 1. that theſe *n.* 314 *p* 36. Medicines operate either upon the account of their being mixed with the Blood, or by their ſtimulating the Stomach and Guts: 2. That this their Operation is more or leſs according to the Quantity and Thickneſs of Blood, *h. e.* a greater Quantity, and the thickeſt Blood require the greateſt Doſes: And 3. that when the Quantities of Blood are the ſame, the Doſes of Purging and Vomiting Medicines are in a duplicate proportion of the Bloods thickneſs. As alſo, that in every caſe theſe Doſes muſt be in a proportion compounded of the Quantity of Blood and thoſe Squares of its thickneſs. Now ſince the Operations of Purgative and Vomitive Medicines depend ſo much on the Quantity and Viſcidity of the Blood, which have not been duely conſider'd before; it is no wonder that the Practice of Phyſick in theſe Evacuations has been ſo uncertain, and that the moſt expert Phyſicians, from their moſt accurate Obſervations, could never determine the true Doſes of Medicines, which alter ſo much according to the various Subjects they work upon; they not being acquainted with the true Method of determining either the quantity of the Blood, or the degrees of its thickneſs. We will here ſubmit our Solution to common Obſervations, and try whether every thing propoſed in it, does not exactly anſwer Matters of Fact, and the Viſible Operations of Nature.

Firſt then, it plainly follows, thar theſe Medicines always purge beſt and moſt conſtantly in a liquid form; becauſe they are more eaſily convey'd into the Blood, and can ſtimulate more parts, and that upon the account of this their Fluidity; whatever may be the way that Purges and Vomits work, or whatſoever their Nature may be. This explains very eaſily a very common Obſervation, hitherto very difficult to Phyſicians, about the different Operation of the ſame Medicine in different forms: Why, *viz.* the Infuſion of a due quantity of a Purging Medicine produces its effects ſooner and more conſtantly than a like quantity of the ſame Medicine in a Powder, tho' ſtill more conſtantly in a Powder than a Bolus, tho' ſtill ſooner and more conſtantly in a dry Bolus, than if it be given in Pills made into that form with Gums that do not purge; and this difference in Purging ſhall even be notable, according to the diſſolubility of the Gums. From whence it follows, that the Evacuation made by ſuch Medicines, is in proportion to the quantity of thoſe Medicines that happens to be diſſolv'd, and not to the quantity adminiſter'd.

Secondly, That purging by Draughts is the moſt excellent form, and will always have the moſt conſtant effect.

The next Conſideration is, that a certain quantity of any purging Medicine affects us after a different manner, according to the different Quantity and Conſtitution of the Blood, or its thickneſs; and it was ſhewn in the Solution, that if its thickneſs were the ſame, the Doſe ſhould always be as its Quantity, but the Blood differing likewiſe in thickneſs, the Doſes of Purging and Vomiting Medicines muſt be augmented on account of its thickneſs. This is perfectly well confirm'd by daily experience; where we find, that People ſick with a manifeſt thickneſs of Blood, as in Dropſies, the Jaundice, *&c.* take far greater Doſes than they did at any other time when they were not ſick, or in that manner. By a further Diſquiſition into this matter, we find that the Doſes muſt not only be greater where the thickneſs of Blood is greater; but that they muſt be encreas'd in a duplicate proportion of their Viſcidity. This is evident by the Tables in *Caſſia*, *viz.* 9 : 8ʒ :: 4 : 3ʒ, 1℈, 13 $\frac{1}{3}$ *gr.* and therefore *alternando* 9 : 4 :: 8ʒ : 3ʒ, 1℈, 13 $\frac{1}{3}$ *gr.* Therefore the Doſes are as the Squares of the Conſtitutions. So likewiſe 9 : 8ʒ :: 16 : 14ʒ, 13 $\frac{1}{3}$ *gr.* and *alternando* 9 : 16 :: 8ʒ : 14ʒ, 13 $\frac{1}{3}$ *gr. h. e.* the Doſes are as the Squares of the Conſtitutions. The ſame is true in any other Conſtitution beſides the mean: For Example, in the loweſt and higheſt 4 : 16 :: 213 $\frac{1}{3}$ *gr* : 853 $\frac{1}{3}$. So that by this means we are not only led directly to a right uſe of theſe Medicines, and are able to find the true cauſe why the ordinary Doſes produce ſo very different effects in different Conſtitutions; but likewiſe, *The Quantity of Blood in any Perſon being given together with the ordinary and extraordinary effect of a Doſe of a Purging Medicine, the Change of that Perſons Conſtitution, and the Nature of that Change may be determin'd.* It cannot but be a great ſatisfaction to the mind to find a Doctrine founded on a few ſimple Experiences leading us into the cauſe of many more that are very complext, difficult, and obſcure; which is ſufficient to prove its conformity to Nature. But my preſent endeavour being to rectify the common Practice of theſe Medicines by this Doctrine, I ſhall frame, by this Method, Tables of the Purging and Vomiting Medicines in preſent uſe; better adapted to Experience than are hitherto to be found.

The Method of framing ſuch Tables, is by ſetting off the practicable Conſtitutions in the different Ages that I have obſerv'd to take notable Quantities of Purging and Vomiting Medicines; ſo that by comparing theſe Conſtitutions with the Ages, we have the different Doſes in all thoſe caſes, which is all that is requir'd for a better practice; tho' a more proper occaſion may produce a more nice and exact diviſion

of

TABLES,
Shewing the Doſes of purging and vomiting Medicins according to the Solution of Dr. COCKBURN'S Problem.

	Medicins	Ages.	Conſtitutions.	Doſes		
				ʒ.	℈.	Gr.
Medicins whoſe common Doſe is ʒj.	Caſſia. Catholicon Diacarthamum. Elect. lenitivum. Succus radic. Irid. ----Fumariæ. Syrup. de Rhamno. ----de Pomis magis. ----Roſar. cum Helleb. Tamarind. Sal cathartic. amar. ---Mirabile. Mann. ʒij. **Emetica** Vin. emeticum, ſeu Bened. Suc. Aſari. Senecionis.	16	2	3	1	$13\frac{1}{3}$
			3	8	0	0
			4	14	0	$13\frac{1}{3}$
		9	2	2	2	0
			3	6	0	0
			4	10	2	0
		6	2	1	2	$6\frac{2}{3}$
			3	4	0	0
			4	7	0	$6\frac{2}{3}$
		3	2	0	2	$13\frac{1}{3}$
			3	2	0	0
			4	3	1	$13\frac{1}{3}$
Common Doſe ʒß	Confectio Hamech. Elect. Caryocoſtin. Diaphœnicon. E ſucco Roſarum. **Emet.** Syrupus emeticus.	16	2	1	2	$6\frac{2}{3}$
			3	4	0	0
			4	7	0	$6\frac{2}{3}$
		9	2	1	1	0
			3	3	0	0
			4	5	1	0
		6	2	0	2	$13\frac{1}{3}$
			3	2	0	0
			4	3	1	$13\frac{1}{3}$
		3	2	0	1	$6\frac{2}{3}$
			3	1	0	0
			4	1	2	$6\frac{2}{3}$
Common Doſe ʒj	Agaricus. Aloe. Carthamus. Ebuli ſem. ---Cortex. Hermodactyli. Mechoacanna. Pil. Aggregativæ. ---Coch. major. ---Fœtidæ ---ſine Quibus. Pulv. Diaſennæ. Rhabarbarum. Soldanella. Senna. Turbith.	16	2	0	1	$6\frac{2}{3}$
			3	1	0	0
			4	1	2	$6\frac{2}{3}$
		9	2	0	1	0
			3	0	2	5
			4	1	1	0
		6	2	0	0	$13\frac{1}{3}$
			3	0	1	10
			4	0	2	$13\frac{1}{3}$
		3	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$
Common Doſe ʒß	Jalappa. Juli juglandis. Lap. Lazuli. ---Armenius. Opoponax. Pil. Coch. minor. ---de Gut. Gamandræ. ---Rudii. Pul. Warvicenſis. Sagapenum. **Emet.** Rad. Ipecacuana.	16	2	0	0	$13\frac{1}{3}$
			3	0	1	10
			4	0	2	$13\frac{1}{3}$
		9	2	0	0	10
			3	0	1	$2\frac{1}{2}$
			4	0	2	0
		6	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$
		3	2	0	0	$3\frac{1}{3}$
			3	0	0	$7\frac{1}{2}$
			4	0	0	$13\frac{1}{3}$
Common Doſe ℈i	Reſina jalappæ. Extr. rhabarbari. Pil. de Hermodactylis ℈ij. pulv. Cornach. ℈ij. **Emet.** Gilla vitrioli.	16	2	0	0	$8\frac{8}{9}$
			3	0	1	0
			4	0	1	$15\frac{5}{9}$
		9	2	0	0	$6\frac{2}{3}$
			3	0	0	15
			4	0	1	$6\frac{2}{3}$
		6	2	0	0	$4\frac{4}{9}$
			3	0	0	10
			4	0	0	$17\frac{7}{9}$
		3	2	0	0	$2\frac{2}{9}$
			3	0	0	5
			4	0	0	$8\frac{8}{9}$
Common Doſe Gr. **VI.**	Colocynthis. Euphorbium. Eſulæ Cortex. Elaterium. Gum. Guttæ. Gran. Gnid. Ricini ſem. Scammon. Tr. Alhand. **Emet.** Croc. Rulandi. Turbith minerale.	16	2	0	0	$2\frac{2}{3}$
			3	0	0	6
			4	0	0	$10\frac{2}{3}$
		9	2	0	0	2
			3	0	0	$4\frac{1}{2}$
			4	0	0	8
		6	2	0	0	$1\frac{1}{3}$
			3	0	0	3
			4	0	0	$5\frac{1}{3}$
		3	2	0	0	$\frac{2}{3}$
			3	0	0	$1\frac{1}{2}$
			4	0	0	$2\frac{2}{3}$
Common Doſe Gr. **III.**	**Emet.** Hep. Antimonij. vitr. Ant. Merc. vitæ. Tart. emet. Merc. præc. per ſe. ----ſolar. ----ruber. Hercules Bovij.	16	2	0	0	$1\frac{1}{3}$
			3	0	0	3
			4	0	0	$5\frac{1}{3}$
		9	2	0	0	1
			3	0	0	$2\frac{1}{4}$
			4	0	0	4
		6	2	0	0	$\frac{2}{3}$
			3	0	0	$1\frac{1}{2}$
			4	0	0	$2\frac{2}{3}$
		3	2	0	0	$\frac{1}{3}$
			3	0	0	$\frac{3}{4}$
			4	0	0	$1\frac{1}{3}$

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of Conſtitutions, very much to the advantage of the Practice of Phyſick in all Diſeaſes. The Ages wherein theſe different Doſes are taken, I find to be four; when a Man is about 16 or 20 Years of Age, and weighs about 12 Stone, he then takes the common Doſe: One of nine Years takes three quarters of that; one of ſix the half, and one of three Years a quarter. Moreover, it having already been ſhewn, that the notable healthy Conſtitutions are but three, as alſo the notable Pulſes of each of theſe: Let then theſe Conſtitutions be as 2, 3, 4. That of the moſt fluid Blood as the firſt number, and ſo on; in that caſe, the Doſe of any Perſon will be found by multiplying the common Doſe for his Age into the Square of his Conſtitution and dividing by the Square of the middle Conſtitution. For inſtance, If ℥j. *Caſſiæ*, is the common Doſe, or the Doſe of the middle Conſtitution, ʒiij. ℈j. and *gr.* 13 $\frac{1}{3}$ is the Doſe of the firſt Conſtitution, and ʒxiv. *gr.* 13 $\frac{1}{3}$ that of the groſſeſt or laſt Conſtitution; and ſo proportionably for every Medicine in all the Ages, as appears by the Tables. This Method ſeems to anſwer ſo exactly, that there is not any thing neceſſary beſides, except when a Perſon is more Looſe or Coſtive than ordinary (which may be known from the Patient or otherways) it is to be reputed the ſame, as if he had taken an equivalent quantity of a Medicine proper to produce theſe effects. Any Phyſician, who has conſider'd this caſe in ſome People after Fluxing, will allow the juſtneſs of this Exception. As Vomiting Medicines have the ſame common Doſes with thoſe that Purge, they admit alſo of the like divided Doſes; which therefore may be found by the ſame Tables. Only, as People that are more Coſtive than ordinary require a proportionably greater Doſe of a Purging Medicine: So they require their Doſe of a Vomiting Medicine to be conſiderably leſs, as is very well known in hot Countries. But it muſt be obſerved, that in the Tables, *Age* ſtands inſtead of *Quantity of Blood*; becauſe they encreaſe pretty equally, and it makes the practice more eaſie to ſuch as are not accuſtom'd to Weights and Numbers. The more Skilful are deſir'd to obſerve, that the mean Ages, multiply'd into the mean Conſtitutions, give Doſes more nicely.

OEcon. anim. p. 51. Biblioth. Anat. p. 1124. Tom. 2.

The TABLES.

Some Inſtances ſhewing the Defects of the preſent Practice, and how mended by the foregoing Tables.

1. The Doſes of the foregoing Tables, ariſing from Calculation, agree perfectly well with the common Obſervation of the beſt Authors; tho' their Obſervation is very general and ill made, if we except the very firſt Steps. For inſtance, Authors of all Countries, *Engliſh, Dutch, German, Italian,* and *French,* reckon the Doſes after the ſame manner;

whereas, if they had been obſerv'd, they muſt have been different as are the Conſtitutions of Men in the different Countries.

2. The defect of their Obſervation is manifeſt by the diſproportion'd Doſes of ſome Medicines, their high Doſes being ſometimes double and ſometimes treble and more of their low Doſes, which is not conform to Nature; for, let the low Doſe be what it will, the high Doſe of one Medicine muſt always bear the ſame proportion to the high Doſe of another, as did their low Doſes, *viz.* even in Manna, they reckon it from ℥ij: commonly to ℥iij. and ℥iv. If it is ſaid that the firſt Doſe is the loweſt Doſe that is taken by a Man of a due Age, it is neither true in fact nor conform to their own way of reckoning: For inſtance, Rhubarb is ſaid to be taken from ʒj. to ℥ij. No body will ſay that this is the loweſt Doſe taken by a Man of a due Age as formerly; becauſe it is not in fact true, nor that the high natural Doſe is ℥ſs; for, as I ſaid before, If ℥j. of Manna and ʒj, of Rhubarb are the reſpective low Doſes, then ℥iv and ʒij. cannot be the reſpective high Doſes. As to what concerns ſome extraordinary Doſes given by themſelves, and far exceeding the ordinary Doſe, is eaſily accounted for by the Solution. There are many Examples of this Nature: Turbinth, *viz.* is commonly reckon'd among them from ʒj. to ʒij; yet *Marggravius*, and good Authors, have given it to ʒiv. So *Colocynthis* from *gr.* vi. to *gr.* xij. and *Fulgin. Fernel. Duncan.* ſay they have given it to ʒß.

3. Authors have been far from being exact; for they have only dos'd theſe Medicines for People of full Age; but have left the Doſes of the different Ages in ſilence; nor have they told us at what time a Man takes his higheſt Doſe, or how that alters in the Growth and Decline of Age, which is ſtill a very great difficulty for the moſt experienc'd Phyſicians to manage.

4. Their general Method is founded in a Miſtake; their loweſt Doſe being really the common Doſe taken by the generality of Men, which produces a multitude of Errours in the Practice. This is manifeſt in their doſing every Medicine.

5. The mentioned Caſe is more manifeſt by theſe Tables, and it is two to one but that a Phyſician over Purges or under Purges any Perſon in Health; and if more Caſes in Sickneſs are ſuppos'd, the odds will encreaſe proportionably. Experience confirms this exactly: For if the middle Doſe is given to one of the loweſt Conſtitution, and the middle Doſe is to purge 7 or 8 times; in that caſe, the Perſon of the loweſt Conſtitution is purg'd near twice as much as he ought to be; and if given to one of the higheſt Conſtitution, he is purg'd but half of what he ſhould be. But if the Doſe of the higheſt Conſtitution is given to one of the middle Conſtitution, he is purg'd twice as much as he ought to be; and if given to one of the loweſt, he is purg'd four times as much, or about thirty times, as we find true by daily Experience. But if the Quantity of Blood, the Age, or Sickneſs contribute to the

Errour, it may prove fatal. If this Conſideration were illuſtrated by a proper number of Examples, we ſhould find ſome hundreds of, otherways unavoidable, miſtakes now prevented by the Practice of theſe Tables.

Laſtly, We may eaſily account, by theſe Tables, for the Doſes of Children over purging ſome People of good Health, and of due Age; a *Phænomenon* ſo ſurprizing, that the ſmallneſs of the Doſe is commonly thought a good Excuſe for the Miſtake.

Dr. Freind's *Defence of his* Prælectiones Chymicæ. *n.* 331. *p.* 330.

II. Actorum Lipſienſium Editores, qui neque Experimenta à me prolata, neque modum quo ea ad naturæ Leges perpendere aggreſſus ſum ne attingunt quidem; principia ipſa, quæ jamdiu pro certiſſimis habita ſunt quibuſque innititur mea omnis rerum Chymicarum explicatio, convellere ſunt conati. Hocque primo impetu faciunt, poſthabita libri ipſius enarratione, ne quis ad legendum non præoccupatus accederet. Et certè minus æque ferenda eſt hæc eorum cavillatio, quoniam extra Provinciæ ſuæ fines evagati ſunt; id enim unicè præ ſe ferunt iſti Literatores & quaſi Indices Librorum, ut quid in quoque Scripto contineatur, compendiariâ quâdam operâ ſimpliciter fideliterque recenſeant, legentis interim Judicium relinquant integrum ac liberum. Pro fundamentis Theoriæ *Chymicæ* habui principia, ipſamque argumentandi Methodum, quam Mathematicorum Princeps in Philoſophiam intulit *Newtonus*: Qui quidem Vir, admirabili quo eſt ingenio, ad res Phyſicas promovendas certam patefecit viam, naturalemque Scientiam tanto rationum pondere ſtabilivit, tam incredibili rerum inventione locupletavit, ut ad eam illuſtrandam plura præſtiterit quàm omnes omnium gentium Philoſophi. Hoc itaque ſagaciſſimi Viri inſtitutum, quia *Editores* latere viſum eſt, paucis aperiam: Oſtendamque totum id, quodcunque eſt quod jam in hoc cognitionis genere exploratum atque perſpectum habemus, ex hâc ipſâ ratione ac viâ fluxiſſe. Porrò etiam Argumenta, quibus ad hanc Phyſicæ Doctrinam refutandam uſi ſunt, ex falſis, quas de hâc re imbiberint, Opinionibus promanâſſe planum faciam; pluresque iſtiuſmodi, quas adducunt, ratiunculas contra illa quæ ipſi amplectuntur principia, quàm contra *Newtoniana* proferri poſſe.

Carteſiani, iique fere omnes, qui ſe magiſtros Philoſophiæ *Mechanicæ* dici volunt, rationem hanc perpetuò tenuerunt, ut Hypotheſin aliquam ſumerent ſeu figmentum, quod nullibi niſi cogitatione fingentium exiſtit: Deinde, ut verbis neque perſpicuis neque definitis comminiſcerentur, quo demum modo omnia ad hujuſce Hypotheſeos normam efficiat natura. Aliam omnino *Newtonus* inſiſtit viam: Nihil ille fingit, nihil pro arbitrio ſuo aſſumit; id ſolum quod Experimento & Obſervatione notatum, ſenſibus omnium patet, pro rato habet: Ex his principiis certiſſimas Mathematicâ ἀκριβείᾳ elicit concluſiones, quas deinde ad alia Naturæ Phænomena explicanda feliciſſimè accommodat. Hanc inſiſtens viam elegantiſſimè demonſtravit, Planetas motu Elliptico

tico circa Solem verſari, areaſque temporibus uſque reſpondentes deſcribere: Satellites itidem ad eandem normam circa Planetas, quos ut comites perpetuo conſequuntur, volvi. Hinc extra dubium omne poſuit Planetas ad Solem, Satellites autem ad Planetas primarios ſe inclinare & tendere: Hanc autem inclinationem in ratione Diſtantiarum duplicatâ decreſcere: Ineſſe porrò immutabilem quibuſcunque corporibus vim, quâ itidem in ſeſe mutuò ferantur: Et inde fieri, ut Lunæ in Terram inflectio, idem planè valeat ac gravitatis vis, atque acceſſum receſſumque Maris efficiat. Inclinationem hanc ſive attractionem quidam, ſi ita lubet, *qualitatem occultam* nuncupent, & erit credo ſemper occulta: neque enim adhuc ex *Editoribus* quemquam extitiſſe video ita in penitiore Philoſophiâ perſpicacem, qui docere in ſe ſuſceperit, quo modo, quâ vi Mechanicâ Attractionem illam exerceat natura. Sed utcunque hæc Naturæ vis, ſi cauſam ſpectemus, occulta ſit, minimè tamen figmentum, ſive Hypotheſis (quod in eorum principia, ipſis etiam fatentibus, cadit) appellari poteſt; cum eam æquè revera exiſtere ac Solem aut Planetas, luculentiſſimis Argumentis confirmetur. Quòd ſi ſit hujuſmodi principium, quod in materiâ omni perpetuò inſidet, quid vetat quo minus id ad rem ſuam accommodent Philoſophi, explicentque nobis quo modo effectus plurimi, quos quotidianâ animadverſione notamus, vim inde ſuam atque Originem derivent. Pariter obſervatione diuturnâ clariſſimiſque Experimentis varium illum, quo radii Lucis refringi ſolent, modum exploravit idem *Newtonus*; hincque ita feliciter lucis colorumque naturam admirabilem aperuit, ut hanc Optices partem ante eum non niſi tenuiter admodum & nugatoriè pertractatam fuiſſe omnes ultrò agnoſcant. Hanc adeo rectiſſimam eſſe conſtat, quam Philoſophi in Scientiæ perveſtigatione tenere poſſunt, rationem, ut primùm multiplici experimento corporum naturas vireſque perquirant, deinde poſthabitâ omni cauſarum, unde eæ fluxerint, indagatione, Phænomena, quæ cujuſque virtutem ingenitam ſequuntur, enucleent atque exponant. Hâc ipſâ via ingreſſus Divinus ille *Archimedes* leges tum *Mechanicas* tum *Hydroſtaticas* exquiſivit, dum interim neque Gravitatis neque Liquoris cauſam aut ſtatueret aut inveſtigaret; ea ſolummodo quæ ſenſuum cognitione percipiuntur pro principiis habens utriuſque Scientiæ rationem pulcherrimè evolvit. Ita etiam *Galilæus*, quanquam nullam de Gravitatis cauſâ Hypotheſin commentus eſt, motûs tamen celeritatem, quam gravia corpora cadendo acquirunt, inveſtigavit, projectorum impetum & curſum, pendulorumque reciprocationes primus explicuit: Eaque Scientiæ fundamenta poſuit, quibus celeberrima Phyſicorum inventa hodiè innituntur. Quid? an non in Opticâ illuſtrandâ ampliſſimo cum fructu progreſſi ſunt Mathematici, duobus principiis, altero Refractionis, Reflectionis altero, conceſſis; utcunque alterutrius cauſa pauciſſimis adhuc innotuerit?

Si quid ponderis Editorum authoritas habeat, præclara hæc acutiſſimorum hominum inventa omnino repudianda ſunt, quia ſcilicet ex iis corporum virtutibus, quarum initia cauſæque prorſus incognita ſnnt, ducuntur; *nec ſine qualitate illâ occultâ veræ Philoſophiæ principia confundente, & in antiquum Chaos reducente, commode explicari poſſunt.* Video clariſ. Wolfium in Aerometriâ, gravitate Aeris, tanquam conceſſo principio, uſum eſſe; atque eo quidem multa Naturæ Phænomena haud abſurdè expediviſſe: Qui tamen gravitatis cauſam Mechanicam ratiocinatione ne attingit quidem; nec credo ullam unquam Hypotheſin ad cauſam hanc explicandam accommodatam fuiſſe, quam ipſe Wolfius à vero alieniſſimam eſſe non facillimè probare poſſit. An huic igitur objicient Editores, quod Scientiæ Phyſicæ *occultam qualitatem* invexerit? In hâc quidem Gravitate explicandâ, quam ſenſu percipimus, longiſſime omnium proceſſit *Newtonus*: Eam quippe à vi attractrice, quæ per omnem ſe undequaque materiam diſſeminat, oriri commonſtrat. Vim hanc Editores, pro ſuâ in rebus Philoſophicis authoritate, *figmenti* vocabulo appellant; ſed quo demum loquendi jure id, quod in rerum natura exiſtere oſtenditur, *figmentum* dici queat, ne intelligi quidem poteſt. Illam certe Attractionis Speciem quemadmodum in toto Planetarum orbe dominatur, luculentiſſimè expoſuit *Newtonus*; neque adhuc videre contigit, quid contra Viri perſpicaciſſimi demonſtrationes objectari poſſit. Alterum hoc Attractionis Genus, quæ in diſtantiæ ratione magis quàm duplicatâ decreſcit, & revera exiſtere, & vim ſuam in minutiſſimis corpuſculis acriter exercere, plura mihi præſto ſunt quæ probent Experimenta, quàm unquam ad demonſtrandam Aeris Gravitatem allaturus eſt Wolfius. Quorſum igitur Principia, quibus ratiocinatio hæc omnis nititur, in altero Argumento pro Commentitiis habere licet, in altero non item? Experientiâ comprobatum eſt, radios lucis quæ à Sole, ſtellis inerrantibus, vel etiam ab eo, quo utimur igne, dimanat, verſus oras ſolidorum corporum æqualiter allici; ea autem immutabilis naturæ lex eſt, ut ubicunque ſit Actio, ibi unà non poſſit non eſſe Reactio: Itaque verè & jure concluſuri videmur, Principium hoc quod Attractionis nomine vocamus, tum revera exiſtere, tum per univerſam omnino materiam diffundi. Quod licet in omni materia inhæreſcat, id tamen in minutiſſimis corpuſculis vim ſuam ad ſenſum magis pateſacere demonſtravit Vir in Phyſiologiâ acutiſſimus D. *Keillus*.

At aiunt, *Talibus ſemel admiſſis, apertâque fingendi licentiâ, mox erunt qui alias qualitates occultas, ſeu quas ipſi agnoſcunt abſolute inexplicabiles comminiſcentur, & paulatim ad vetera ignorantiæ aſyla redibunt. Si detur vis attrahendi, ſeu Sympathia, quidni pari jure vis repellendi, ſeu Antipathia? Ita facile etiam dabitur Antiperiſtaſis, dabuntur qualitates emiſſæ per modum ſpecierum cum ſuis Actu potentialibus, dabitur funiculus* Lini *Attractivus, dabitur in materia eadem Variatio* Exten*ſionis, non apparentis tantum, ſed etiam veræ.* Itane incœptant, *ſi detur vis Attrahendi*; cum eam dari Experientia ipſa apertiſſimè demonſtrat? Non eſt hoc opinionis commentum ad alia Phænomena explicanda excogitatum, ſed eſt per ſe conſtitutum à Naturâ Phænomenon; adeoque quanquam ſibi plaudant Editores, quod hujuſce Senntiæ ſautores ad abſurdum quid deduxerint, omnis tamen illa, de qua ſe ita fidenter jactant, huc tandem redit Argumentatio; nempe ſi unum aliquod principium, quod in rerum naturâ exiſtere obſervatione certâ compertum eſt, concedimus, ideo etiam oportet alia, quæ nuſquam extiterunt, approbare; uti verbi gratiâ, ſi Gravitatem agnoſcimus, quam corporibus quibuſcunque ineſſe certò animadvertimus, quanquam illius cauſam prorſus neſcimus, idcirco fabulas Philoſophorum omnes & commenta amplecti neceſſe eſt, quæ nec Experientiâ ullâ confirmari, nec ratione explicari queunt. Si hoc ſit Mathematicorum more ratiocinari, ſatius eſt profecto *ad vetera quævis ignorantiæ aſyla redire*, quam hanc argumentandi *licentiam aperire*. Sed vim Attractricem in eo maximè oppugnant, quod rationibus Mechanicis minime illuſtrari poſſit. An igitur volunt, ut nihil in rem Phyſicam introduci debeat, niſi cujus ratio & cauſa perſpecta ſit? An Editorum aliquis Elaterem aeris, quâ vi Mechanicâ conſtitutus ſit, unquam explicuit? Eum tamen & Philoſophi omnes ultrò concedunt, & ad multa Naturæ Phænomena enodanda fæliciſſimè accommodari unâ mente conſentiunt. Fabro, utique id libenter damus, ut Horologii artificium intelligat, quanquam interim gravitatis Elateriſque, ex quibus quidem pendet omnis rotarum converſio, rationem penitus ignoret: Hoc idem Phyſico denegabimus? Qui *vim* illam, quâ univerſa corpora aguntur, & ſuo quæque motu atque ordine diriguntur, inveſtigarit, qui hujuſce potentiæ motrices leges definire, eaſque ad præcipua Naturæ Phænomena explicanda adhibere poterit, tametſi cuinam cauſæ vis illa omnium gubernatrix ortum debet, plane ſe neſcire fateatur, illum de naturæ viribus & Machinatione nihil prorſus ſcire, nihil animo percipere dicemus? Quod ſi hoc Attractionis principium ad fontes uſque ſuos perſequendi ſtudio teneantur Editores, faciant quod lubet; hanc iis gloriam ultro relinquit *Newtonus*, ſatis præclarè ſecum agi ratus, ſi modo eorum offenſionem effugiat, quod involutum longeque difficillimum problema explicandum in ſe non ſuſceperit.

Non me latet quod Cl. *L.* quem quaſi Numen aliquod ſuſpiciunt Editores, in Specimine illo, quod vocabulâ eleganter ſonante nuncupat, *Dynamicum*, planiſſimè ſcripſerit, *Vim Activam ſeu niſum intimum corporum Naturam conſtituere.* Vis hæc ſive Niſus, ſi quid velit recte intelligo, idem eſt ac propenſio illa mutua, quàm corporibus quibuſcunque inſitam diximus; quámque multo ante pateſecerat *Newtonus*, quanquam eâ materiæ naturam contineri nuſpiam aſſeverarit. Si vera ſit *L.* ſententia, nobis æquo jure extenſionis ſoliditatiſque cauſa quærenda eſt, ac attrahentis hujuſce, quod omni materiæ ineſt, Principii ratio excutienda. Hoc autem poſito fundamento, effectus omnes quos in hâc univerſâ mundi machinâ contemplamur, ab ipſâ materiæ conſtitutione neceſſariam originem deducunt. At mihi quidem intima corporum natura ita parùm explorata eſt, ut longiſſime abſim, qui affirmem vim hanc æterno rerum fœdere illis intermiſceri, & eadem naturali colligatione, ac extenſionem ſoliditatemque inhæreſcere. Sane ita valde laborare videtur hæc Sententia, ut argumenta, quæ in contrarium afferre proclive eſſet, vix recenſeri nedum refelli poſſint. Quod ſi cum *L'o* ſentiant Editores, non video cur amplecti nolint principium, quod ille apprimè neceſſarium judicat, ut id in intimâ corporum naturâ conſtitutum eſſe pronunciet. Cum verò ex motu corporum conſtet, attractricem hanc potentiam revera exiſtere, ſi eam neque Materiæ neceſſario ingenitam, neque rationibus Mechanicis explicandam eſſe cenſeant, haud abſurdum aliquid credo aut Phyſico alienum facturi ſumus, ſi eam in voluntatem Dei reſolvimus: Legemque univerſam eſſe ſtatuimus, quâ omnis hæc mundi moles gubernatur & regitur, corporumque vario utcunque motu labentium convenientia atque concentus ſervatur: quanquam quidem hæc ipſa potentia, non minus quam naturæ conſtructio omnis, à divinâ voluntate unicè manaverit. Illi verò, qui nullam hujuſmodi legem agnoſcunt, ſed univerſum Phyſices negotium, non modò quoad proximas, ſed remotiſſimas etiam cauſas, ſuâpte naturâ & mechanicâ quâdam ratione geri volunt, ita ut nihil ſit quod non ab ipſâ materiæ vi immutabilique motûs conditione proficiſci putent, quid aliud agunt, niſi ut cum *Epicuro* notionem ex animo hominum evellant cuncta providentis atque moderantis Dei; argumentaque ſuppeditent, quæ in rem ſuam traducant impii? Quicquid verò de hâc Attractrice virtute ſtatuendum ſit, minime dubium eſt, rerum naturam ſine principio quodam actuoſo non poſſe conſiſtere: Quippe corpora, utcunque in motum ſemel excitata, ſi deinde iis ſuo more uti liceret, viciſſitudines ſuas certo tempore haud ita conſtanter conficerent. Hoc cum ita neceſſarium perceperit acutiſſimus *L.* pereleganter concluſit, quod *agere ſit Character Subſtantiarum.* Ubicunque autem vis hæc omnia ciens atque agitans motibus ſuis ſita ſit, in *occultam quandam qualitatem reſolvatur* neceſſe eſt; aliam enim illius cauſam, quam divini numinis voluntatem, fruſtra hactenus quæſivimus. Nonnulli autem, qui ſibi in rebus Mechanicis acutiùs cernere videntur, vim hanc in *Æthere* vel in Fluido quodam admodum ſubtili collocant; quos ſane interrogare velim, quid tandem ſit quod illum æthera agat, & in motione perenni continuatâque tueatur? Unde fit ut motus omnino contrarii ſe invicem non extinguant? Quid ſit porrò, quod motus hoſce eâ facultate inſtruat, ut ſuum ſinguli opus proprium ſibique aptum efficiant? Hæc omnia *occultâ qualitate*, quam in æthere ſitam eſſe volunt, oriri neceſſe eſt. Etenim ſi hanc Hypotheſin ad naturam revocamus, facilè patebit plures ab iis intromitti *occultas qualitates*, quàm ſunt quæ explicanda ſuſcipiunt Phænomena. Quanto rectiùs ille in Philoſophiâ ſuâ *Newtonus*? Qui principium non niſi unum, idque ſimplex maximè, & obſervatione confirmatum ſibi dari poſtulat, ——

Et ſpecioſa dehinc miracula promit.

Sed quantâ obſcuritate laboret, quamque infirmis rationibus fulta ſit tota illa æthereæ cujuſdam virtutis, aut ſubtilis fluidi Hypotheſis, nullum clarius, quàm ex iis quæ de hâc re diſſerunt *Editores*, peti poteſt argumentum. Aiunt quippe, *Hæc omnia ſine qualitate illâ occultâ attractrice, vera Philoſophiæ principia confundente, & in antiquum Chaos redigente,* *commode explicari poſſe, partim etiam à viris doctis explicata eſſe.* Hunc adeo ob finem, *ſtatuunt plurimas materiæ particulas Sphærâ quâdam magneticâ fluidi Subtilioris eſſe circundatas, cujus motu (ut in Magnetibus noſtris fieri videmus) attrahant ſe invicem, aut repellant, aut ad ſitum convenientem diſponant, quoties ſcilicet libertatem aliquam ſint nactæ.* Quid quæſo eſt *Sphæra quædam magnetica,* niſi aliquid admodum *occultum?* cui utique adſciſcitur quid adhuc occultius, ſcilicet *Magnetiſmus.* Unde fit, ut hæc materiæ ſubtilis Sphæra corpori, cui circundata eſt, perpetuò comitem ſe præbeat? Res quidem ipſa poſtulare videtur, ut corpus, cum ſemel motu impreſſo locum mutaverit, Sphæram hanc itidem ambientem poſt ſe relinquat: quippe ſi terra novo aliquo impetu acta alium prorſus curſum iniret, ex legibus Mechanicis ſatis liquet, quod non Atmoſphæra modò, ſed quicquid à Terrâ liberum ſolutumque eſſet, facto diſſidio, ſuâ ſe in ſede contineret. Quid igitur in hoc rerum ſtatu concludendum eſt? An quòd occultâ quâdam qualitate Atmoſphæra hæc motum corporis continuò ſequatur? An quòd vi etiam occultâ, altera materiæ ſubtilis Sphæra de novo gignatur? Uter ſit, libenter diſcere velim, quænam demum ea qualitas ſit, quæ Sphæram hanc Magneticam in motum cieat? cujus generis motus ille ſit, & quâ potiſſimùm ratione excitetur, qui efficit, ut *materiæ particulæ attrahant ſe invicem, aut repellant, aut ad ſitum convenientem diſponant?* Quot tandem *occultas qualitates* ad ſingula Phænomena explicanda accerſere coacti ſunt, dum unam illam & ſimplicem rejiciunt, quæ per univerſam naturæ fabricam ſe diffundit, & plurimis Phænomenis ſolvendis tam præclarè inſervit. Id verò in *Editoribus* ſatis mirari nequeo, quod qui contra vim attractricem ita acriter dimicant, & ſine eâ omnia commode explicari poſſe contendunt, eam tamen in hoc ſuum de rebus Phyſicis commentum ipſi transferant: Nec ab *attractionis Voculâ,* quæ quidem iis ita *eleganter ſonare* viſa eſt, *abſtineant,* ut *ignorantiam ſuam palliare* poſſint: Loquuntur enim de Sphærâ quâdam fluidâ, quæ *ATTRAHIT, repellit,* & ad *ſitum convenientem diſponit.* Cum nihil veri ſit in hâc Sphæræ ſubtilis fabulâ, cavendum certè fuit, ne deſideraretur iſta, quæ rem veriſimilem redderet, convenientia. Facillimum profectò eſſet, naturæ Phænomena omnia ad hunc modum illuſtrare; mirificè quippe rerum cauſas expedit *Sphæra Magnetica fluidumque ſubtile,* atque etiam maxime inter ſe pugnantia conciliat. Atque hoc quidem quod de *materia ſubtili* Vique *Magneticâ* excogitarunt figmento (dum agendi ratio ab inventoribus ferè intacta relinquitur) nullum præſentius *ignorantiæ aſylum;* etenim omnes illas occultas qualitates, quæ hactenus in Philoſophiam irrepſerunt, longè multumque ſuperat. Nemo certè non videt, quàm ficta hæc omnia atque commentitia ſint, cùm neque quale ſit hoc ſubtile fluidum, neque etiam ſi ullum omnino ſit, aut obſervatione animadverti, aut ratione colligi poſſit. Diſpiciat itaque Lector, annon ea quæ in veram, h. e. *Newtonianam* Phyſicen intentant argumentandi tela, in hanc ipſorum *infelicem Philoſophandi rationem* fortiùs retorqueri queant. Ea omnia quæ pro certis atque ratis jactanter ſatis venditant, vana prorſus ſunt & fabulis referta, nullâ obſervatione aut Experimento nitentia;

tia; quæ etiam ſi pro veris conceſſa fuerint, eo occultarum reconditiſſimarumque virtutum agmine ſtipantur, ut facilius multò ſit *Sympathiæ*, *Antipathiæ* & *Antiperiſtaſeos* naturam cogitatione complecti. Hujuſmodi nimirum Hypotheſes hoc vitio laborare ſemper comperi, ut obſcurius quid magiſque difficiles explicatus habeant, quam res ipſæ, quibus eæ explicandis accommodantur. In illâ *vorticum* Hypotheſi, quæ iis ante cæteras omnes arridet, rationem nullam afferunt, cur materia fluida curvam ſemitam affectet, ſeſeque circa centrum torqueat, cùm ea ſit corporum omnium natura, ut rectis lineis ferantur: unde tot vorticibus cautum eſt, ne in curſibus ſuis ſe invicem perturbent & impediant; unde per eos tranſeant Cometæ, motuque prorſus contrario ac ipſe vortex, verſentur; tantumque abſit, ut illius incitatiſſima converſio eos interpellet, ut in ſuis, quos circa Solem conficiunt, orbibus, ad eandem ac Planetæ normam dirigantur, ſeſeque verſus eum pari modo inflectant. Hâc adeo Vorticum Hypotheſi in eos ſe laqueos inducunt iſti *Pholoſophandi artifices*, à quibus nunquam expedire poſſunt: in in quâ tamen poſitum eſt omne hujuſmodi Philoſophiæ fundamentum. Quùm ad Phænomenon aliquod explicandum accedunt, ad illorum nutum præſto eſt ſubtilis materia, quæ modo motuque admodum ignoto atque inexplicabili rem quam velint efficiat. Num Philoſophiam magis ſapiunt hæc, quàm ſi quis dixerit id à *Sympathiâ*, *Antipathiâ*, *vel occultâ aliquâ qualitate proficiſci*? Num hæc Philoſophandi ratio, non æquè ac illa quam vellicant, in *Aſylum ignorantiæ* ceſſura eſt? Et ſi conſuetudini fictis hiſce fabulis indulgenti obſequimur, quidni cætera etiam, quæ ab hominibus ad comminiſcendum ingenioſis fingi poſſunt, amplectamur?

Quàm longè alia diſſimiliſque eſt vera Philoſophiæ inſtituendæ via! in quâ nihil ponitur, niſi quod in ipſâ rerum naturâ conſtitutum eſſe obſervatio evidentiſſima declarat; & quanquam principii, quo utimur, cauſa & origo deliteſcat, ex eo tamen multa, quæ quotidiano uſu animadvertimus, fluere & pendere poſſunt: Itaque ingenui eſt Philoſophi primò corporum virtutes experimentis elicere, deinde, ubi eæ diligenter exploratæ ſtabilitæque ſint, diſtinctè & perſpicuè commonſtrare, quinam illas effectus ſuâ ſponte conſequantur. Neque ulla credo tanti eſſe Adverſantium argumenta, quæ hanc veri inveſtigandi rationem evertant. Etenim ſi principia & poſtulata vim ſuam omnem in Experimentis poſitam obtineant; ſi propoſitionibus conceſſis & ritè præmiſſis, nihil contra Dialectices leges conficiatur, concluſio non poteſt non eſſe certiſſima: Ita ut quicquid hâc methodo evolutum explicatúmque habemus, rem Phyſicam inventis augere atque amplificare meritò cenſendum ſit. Igitur vim hanc Attractricem utcunque eam labefactare conentur *Editores*, firmam nos ſtabilemque tenere confidimus.

Eſt & alterum axioma, quod conſenſu ſuo non approbant *Editores*, viz. *Corporum momenta ſeu quantitates motuum oriri ex ratione quantitatis materiæ & celeritatis compoſitâ*; quæ qui ſic *æſtimant in communi errore verſantur*,

verſantur, uti paſſim in illorum Actis notatum eſt. In iis quidem video de hâc re unum aut alterum clariſſimi *L.* Commentariolum; qui tamen ita parum rationibus pugnat, ut nihil niſi fallaces concluſiunculas, nihil niſi in verbis captiones ſectetur: igitur ea quæ de hoc argumento protulit, unâ fere Mathematicorum voce atque ſententiâ improbantur: Quorum nonnulli hoc ipſum Axioma exquiſitè confirmârunt. Hos itaque adeat Lector; nam illa Diſceptatio, uti per ſe ſatis magna eſt, ita etiam huic inſtituto nimis aliena.

Of the Principles of Purgatives, by Monſ. Bolduc. *n.*278. *p.* 1099.

III. Monſieur *Bolduc* examined the Principles of Purgatives, and began with *Ipecacuanha,* which he ſaid he had endeavoured to ſweeten and qualifie, by trying to take away its too great Emetic Power. He aſſured, that how violent ſoever *Ipecacuanha* be, yet it is not ſo dangerous as *Scamony* or *Colloquintida,* which always leave Gripes and ſometimes Dyſenteries; whereas *Ipecacuanha* leaves only a gentle aſtriction after it. He ſaid next, that he having obſerv'd that the Emetick force of this Root conſiſts in its Reſinous parts, he had found out a way to take them from it, and to leave only the Saline parts; that he made uſe of Spirit of Wine to extract the one, and of Rain-water diſtilled to draw off the other; that he had afterwards given with very good Succeſs in Dyſenteries this *Ipecacuanha* ſo deſpoiled of its Reſinous parts. From *Ipecacuanha* he paſſed to Hellebore, which is another violent Emetick; which he diſtinguiſh'd into two ſorts, the black and the white. He ſaid, that ours was not different from that of the Antients, that the White caus'd mortal Convulſions, for which reaſon it was not uſed, and that he had never made any attempts with it. As for the black Hellebore it is to be obſerved, that that which comes by way of *England* is much weaker than that which grows upon the Mountains of *Switzerland*; which may well have been the reaſon that Phyſicians have neglected this Remedy. Then he related his operations on this Root. He ſaid, that having put it in a retort in a Reverbatory Fire, he at firſt drew off an acid Spirit, next an oily acid Spirit; thirdly a violent *alkali* Spirit came over mixt with Oyl of Tartar, and laſtly a fœtid Oyl. That from the *Caput mortuum* he had by a *Lixivium,* a fixt Salt, which fermented with Acids, ſuch as all other Plants give; beſides theſe Operations, he drew an Extract of this Root with Spirit of Wine, to get the Reſinous parts, and with diſtilled Rain Water for the Saline. He got but very few of the former, but a great deal of the other; ſo that he found that Spirit of Wine was uſeleſs in this caſe. Comparing then the Effects of theſe Purgatives, he ſaid, that the purely Reſinous purge little, and with much irritation; that the purely Saline purge only by Urine, but that both join'd together purge very well. That it is for this reaſon that Phyſicians make uſe of Salt of Tartar, to correct the bad Effects of Reſinous Purgatives; but if this Precaution were uſed, to make the Extract with aqueous

Diſſolvents, inſtead of the Sulphureous, there would be no need of the Corrective.

Obſervations on the Claſs of Sweet Taſtes, by Sir John Floyer. *n.* 279. *p.* 1160.

IV. All watry Plants ſhew their Flegm as well to the Taſt as in Diſtillation; and in all dry woody Taſts we obſerve the Earth, as well as we can by the Chymical Analyſis. By the Mucilage and Gummineſs, or oily Taſte, we diſtinguiſh the Oyl of Plants, as well as by Diſtillation. The Aromatical Smell ſhews us the Volatility of Oyl and Plants; and by the Fœtidneſs we alſo know that the Oyl and Salt are in a volatile State. By the Acrimony and Pungency we know there is a volatile Salt in Plants, and by the burning Taſte we find there is a Corroſive Salt in them. By a crude rough Acidity we obſerve the Tartar or eſſential Salt of Plants, but if the Acidity be of a Vinous Smell, we may obſerve 'tis of middle State of Digeſtion, and may be called a Vinous Tartar, and the Crude Tartar an Acerb Tartar, but if the Tartar had a pungent Smell, then 'tis a volatile Tartar or acid acrid Tartar. I will next deſcribe the principles obſervable in ſweet Taſtes, and their ſeveral Claſſes, but muſt firſt obſerve that ſweet Taſtes ſhew their Oyl by their ſlimy ſmoothneſs, and their Tartar is evident in their Extracts, as in the Juice of Liquoriſh.

1. The Graſs Sweets, as *granum caninum*, have much eſſential Salt and moderate Oyl, *Juncus equiſetum, arundo, typha, nymphæa,* are all of the Ruſh kind, ſweet and rough, and ſome of them have more Oyl, others more acid, and that the moſt crude have more Oyl than Tartar.

2. The Corn Sweets, as Barley, Rye, Wheat, Oats, Rice, Millet, have much Oyl and eſſential Salt, and a little volatile: So Bread yields Oyl and eſſential and volatile Salt. Note, that Fermentation or the Fire produces the volatile Salt, by exalting the Tartar into a volatile Salt; and the ſlimy mealineſs in Corn ſupplies the Oyl. *Tragopogon* and *Scorzonera* are referable to the Graſs, and contain much Oyl, and eſſential Salt.

3. The ſubacrid Sweets, as *Rampions, campanula, trachelium,* contain much Oyl and eſſential Salt, but the Acrimony in theſe Plants ſhews a volatile Salt not deſcribed by the Chymiſt.

4. The Ferns contain Oyl and eſſential Salt, as Polypody; but the acrid principle is not obſerved by the Chymiſt, nor the Fragrancy in Harts Tongue. *Oſmunda* and *Capillaries* have more Oyl than Salt, becauſe more mucilaginous and crude.

5. All the Leguminous ſlimy Sweets have more Oyl than Tartar, but all much of both, as Broom, *Ononis, Aquilegia, ſumaria,* Aſparagus, *ruſcus, thalyctrum, polygonatum Sena, galega, lathyrus, luteola, periclymenum, glychirriza, pſyllium,* much Oyl and volatile or eſſential Salt. Beans and Peas and Lentils have alſo a volatile Salt and *lens aquatica*. Note, that ſince there is no Acrimony in theſe Seeds, the volatile Salt is produced by the Fire. The aromatick Legumens, as *Melilotus* have an

exalted Oyl, and eſſential Salt. Periclymenum is deſcribed to have only eſſential Salt and Oyl, but ſince there is an aromatick Odor in the Flavour, and a great Acrimony, there muſt be ſome degree of volatile Salt in it. Fenagreke and Meadow Trefoyl have much Oyl and little Salt, and ſo has *Ophiogloſſum pinguicula & conſolida regalis.*

6. The ſweet Nuts, as Almonds, have much Oyl and eſſential Salt, but the bitter have more Salt than the ſweet Almonds, therefore 'tis probable that the Tartar abounds more in all bitters, and that Tartar is the effect of a higher Digeſtion, and the crudeſt Taſtes, as Stypticks, Sweets and Slimes have leaſt of it. So Cheſnuts and Beech-nuts have much Oyl and little Salt. Filberds are deſcribed to have eſſential Salt, and a little Volatile as well as Oyl; but it ſeems no probability that one may have it, and none in any of the other.

7. The ſweet acid or vinous Taſtes have much Oyl and eſſential Salt, as Prunes, Cherries, Strawberries, Raſberries. The Variety of the Taſtes of theſe Fruits ſhews the different Digeſtions and Mixtures, tho' the principles are the ſame. The ſweet viſcous Fruits, as Sebeſtens, have little eſſential Salt, and much Oyl.

8. The ſweet aromatic burning Taſtes contain a volatile Salt and Oyl, as *Schenanthus*, Ginger, Zed-Oary, Cubebs, Cardamums, *Vaynillas*, *Contrayerva*, *Calamus aromaticus*; but theſe following are miſtaken by the Chymiſts, who ſay that *Coſtus amarus, dulcis, cyperus, galanga*, Orris, have a volatile Oyl, but eſſential Salt only, for Orris is acrid and aromatic as well as the reſt, and therefore there is both Volatile Salt and Oyl in them, and alſo an eſſential Salt from their ſweetneſs.

9. The ſweet acrid Aromaticks of the Fænil Claſs have all a volatile Salt and a volatile Oyl, as Angelica, Lovage, Parſly, Meum, Dill. Note, Leaves and Roots of Fænil contain a volatile Oyl and eſſential Salt, the Seed a volatile Oyl and Salt; but ſince the Roots and Leaves have a pungent Taſte, there is alſo a volatile Salt in them, tho' the Chymiſts do not obſerve it. All the parts of Carraways have the odor of Punaiſes, except the Seed; from whence I may infer, that the fœtid Plants have the ſame principles as Aromatics, *viz.* a volatile Oyl and Salt; and this is confirmed by other Fœtids which have them, as Rue and *Aſſa fœtida*, and *Vulvaria*. *Peucedanum* is deſcribed of a bitter acid Taſte, with the odor of Pitch, and muſt have a volatile Salt, tho *L' Emery* deſcribes only its eſſential Salt and Oyl: So in Smallage he deſcribes only the eſſential Salt and Oyl, but its acrid Taſte manifeſts the volatile Salt.

10. The ſweet Gums, as Manna, Sarcocolla, contain much Oyl and eſſential Salt; tho Honey and Sugar have more eſſential Salt than Oyl; by which we may obſerve how much eſſential Salt is in all Sweets, and why they are apt to turn ſowre, and ferment; and from ſuch ſweet Gums all ſweet plants have their Acid and Oyl upon Diſtillation.

11. Citrulls, Melons, Gourds, Cucumbers, which are bitter Sweets, and very mucilaginous, contain much Oyl and little Salt. Since the whole Claſſes of Sweet Plants contain an Oyl and Eſſential Salt, ſome more ſome leſs of both, the Vertues of the ſeveral ſweet Taſtes can never be explained by the Chymical Principles, and no new Virtues by them is diſcovered; therefore all the advantage we have obtained by theſe Chymical Diſtillations is only to ſhew the nature of Sweetneſs in general, by diſcovering the principles contained in ſweet Plants, and this is a greater advantage to Natural Philoſophy than to Phyſick, to which the Taſting of Plants is more uſeful. By the Taſte we diſtinguiſh the Sweets into their ſeveral Claſſes, and we diſcern tempers and digeſtion, and mixture of their principles, and thence eaſily gueſs at their effects in Animals, and by the Taſte we diſtinguiſh the different ſtate of both the Oyl and Acid in Plants of different ſweet Taſtes; whereas the Chymiſts obſerve no difference of the Tartar Acid, whether it be Acerb, or Vinous, or Volatile; nor of the ſweet Oyls from the bitter and ſlimy. By he Taſte we diſtinguiſh the Acrid and the Acid Salts, which mix in diſtillation; and they are not well diſtiguiſhed by the Chymiſts. By the Taſte we diſcern when the Fire makes new products and mixtures, not naturally found in Plants; for in Corn, Beans, Peas, Bread, Fire produces a Volatile Salt not obſervable in them before: And a Volatile Salt is drawn from the lees of Wine by the Fire; and Leven yields alſo a Volatile Salt; whereas before in Corn only an Oyl and Acid were obſerved; and 'tis probable the Tartar is Volatilized, both by the Fermentation and the Fire. Coffee is a Bean by its Taſte and Cods, and acquires a Volatile Salt by roaſting; but *L' Emery* only mentions its Oyl and fixed Salt upon Diſtillation, but the fixed Salt is the effect of Fire, and its virtue depends on the Volatile.

No more Chymiſtry is neceſſary for diſcovery of the Phyſical Vertues of Plants, than to make Decoctions of them in fair Water, and to obſerve the Taſtes and other ſenſible qualities of thoſe Decoctions, and from them to take the natural Hints for the tryal of their Virtues on Animal Bodies. I will confine my ſelf to the Claſs of Sweet Taſtes, and give an account of ſeveral Decoctions made in all the ſpecies of ſweet Plants, and add ſome Taſtes I had not formerly deſcribed fully, and ſuch reaſons as induced me to place them in the Claſs of Sweet Plants. I own it is objected againſt Decoctions, that the Volatile parts expire, and the mucilage diſſolving in the Water obſcures the Taſte: I therefore do confeſs, that Plants are beſt taſted in their natural ſtate to diſcover all their Vertues, but theſe Decoctions do help to confirm our Taſtes, and diſcover the great variety of Medicines which may be made from Sweet Taſtes. Note, that all Decoctions muſt be taſted cold.

A Deſcription of the Taſtes and other ſenſible qualities in the Decoctions of Plants of the Sweet claſs.

1. The Graſs Sweets under which are contained all ſorts of Graſſes and Ruſhes, Reeds and Corn; I refer *Nymphea Alba* to the Ruſhes, both for its figure and taſte, which is crude and ſtyptic, with a bitteriſhneſs in the Seed; but the flowers are like Lillies of a ſweet ſmell, and mucilaginous, ſweet, ſtyptic in Taſte. I boyled Horſe-tayl a handful in a pint of Water, and I found the Decoction to taſte very ſtyptic with a bitteriſhneſs, and the Decoction looked like Small Beer, the Taſte being like Ruſhes: I concluded it to be of the ſame Claſs. The ſtyptic Virtue is uſeful for all Fluxes. *Plantago aquatica* boyled in Water gave a Cauſtic Acrid Taſte, joyned with a crude ruſh Taſte; the Decoction was greeniſh and pale. This Cauſtic Acrimony is uſeful in the Scurvy and Dropſy. It is *aranunculus*, by its *capitula echinata*, as well as by its Acrimony. *Nymphea lutea* has a ſweet Aſtringent luſtre in the Flowers, with an Acrid ſmell like Creſſes. A Reed is of a ſweet bitter and mucilaginous Taſte, it abſterges by the bitter without Acrimony, and cools by the Mucilage Subacrid ſtyptic, by which 'tis a Ruſh, *Sagittara* is ſweet, Subacrid, and ſtyptic, by which it is a Ruſh. *Alga Marina* is Subacrid and ſweet, and ſomething ſtyptic. *Fucus* is of a ſweet ſaltiſh Taſte. The Palm-Tree is ſweet and ſtyptic, and, becauſe of this Taſte and its *folia arundinacea*, I refer it to the ſweet ſtyptic; a ſweet Vinous Juice flows from its Bark being wounded, the Immature Fruit is very ſtyptic, but the mature ſweet, viſcid, vinous, and ſubaſtringent, proper for Fluxes. Wine, Vinegar and Sugar are made from the ſweet Juice. The Inſpiſſate juice of the Palm Tree is the *Terra Japonica*, whoſe ſubſtance is gummoſe, and of a bitteriſh ſtyptic Taſte, and alſo of a ſweet Taſte, and of a grateful odor. Boyl ℥j of *Terra Japonica* in ℔j of Water, and ſweeten it for a Cough or Looſeneſs. Leven is obſerved by *Ætius* to be cool by its acidity, to be hot being putrid, and alſo to have ſome Virtue from the Salt and Flower. He obſerves that Beer is hotter than Barley, and becauſe of its putrifaction 'tis of an ill juice; and he alſo obſerves, its Windineſs depends on the Air included in it; and the Wateriſh Beer and the Acid is cooling. All theſe Virtues were diſcovered by the ſenſible qualities of Leven and Beer, without Chymiſtry. And we may obſerve that what we call Fermentation in them, was called by *Ætius* a Putrifaction. We have rectified the notion, and given it a new name, but the Antients knew the nature and effects of Fermentation as well as the Moderns; and 'tis that which gives the different ſtates of the principles in Plants, and the ſeveral ſtates are beſt diſcovered by our Taſtes and Sences. I boyled *Gramen ſpica ſecalina*, and the Decoction was of a ſweet, mucilaginous and ſtyptic Taſte. The Roots of Graſs have ſomething of Acrimony and Aſtriction, but the Decoction taſtes

ſmooth and ſweetiſh. The green Leaves of Graſs are ſweet and ſtyptic. The Decoction of the Water was of a pale colour. The Seeds of Graſs are more diuretick and binding, I made a Decoction of them, which reſembled Water-gruel. *Paronychia foliis rutaceis* muſt be referred to the Graſſes, becauſe of its ſweetneſs.

2. Quere whether *Rapunculus* be not of the Sweet Claſs, becauſe the whole Plant is ſweet and, Milky, and for that both are given to Nurſes to increaſe Milk. The Roots of the rampions are ſomething ſtyptic and cooling, and proper againſt Inflammations. *Scorzonera* and *Tragopogon* are referable to the Graſſes, becauſe of their geniculate ſtalks, their Graſſy Leaves, ſweet Taſtes and Milky Juice, the Decoction was ſmooth and of a pale colour. *Trachelium* is alſo ſweet, milky, ſubacrid, and bitteriſh, of the ſame claſs.

3. The ſweet mucilaginous and crude *Lychnis*, the decoction of *Alſine*, is ſweetiſh and mucilaginous, the colour pale like Small Beer. I could obſerve no aſtringency in *Alſine*. I boyled ʒj in a pint of Water, it hath a cooling and moyſtening quality like common *Lychnis*. *Dioſcorides* ſays, Chickweed being bruiſed has the ſmell of a Cucumber. All the common *Lychnis*'s are bitter ſweets, and them I ſhall refer to the claſs of Bitters, tho their joynted Stalks and ſweetneſs may juſtly place them here, yet their chief Phyſical Virtue is from their Bitterneſs.

4. The Sweet Bitteriſh ſtyptic, and Subacrid Fern. I boyled ℥j of Fern Roots in ℔j of Water to half, the Taſte was very ſtiptic and Bitteriſh, the Colour Citron. The Mucilage and ſtypticity make Fern an excellent Vulnerary, and ſtyptic in all Fluxes: the young Buds rubbed in the Fingers ſwell ſomething like a Kernel, or the Laurel ſmell. *Dryopteris* is deſcribed Aſtringent and Sweet, Acid and Bitteriſh. *Hemionitis* has both Aſtringency and Bitteriſhneſs. *Adianthum album & nigrum* are Sweet and ſtyptic, and ſince Experience ſhews theſe to be good Pectorals, the other Ferns have the ſame Pectoral Vertue, as *Vulneraries*. *Herniaria* is deſcibed as a ſtyptic. In *Adianthum* there is ſomething odorate. *L' Emery*. Polypody is Bitteriſh, ſweet and Aſtringent, Nauſeous and Subacrid and ſlimy, it purges by this Taſte both Choler and Flegm. I boyled Harts Tongue ℥j in ℔j of Water to half; it was of the colour of Small Beer; the Taſte was Mucilaginous, ſweet, ſtyptic, with an Aromatick flavor of Raſberries or Orris, which is its Cordial Vertue, joyned with ſtypticity and Mucilage, by which 'tis proper for hot Hypochondriacs. The Roots of *Oſmunda* are of a Subacrid and Bitteriſh Taſte, beſides the Aſtringency, by which they open, but the Aſtringency much hinders that effect.

5. The ſweet, Acrid, Aromatics and Fœtids, the Roots have more ſweetneſs and Aromatic Acrimony than the Leaves, which are more crude; the Seeds have moſt Aromatic Acrimony. The Seeds of *Siler montanum* have an ungrateful ſmell like Cummin, and a Bitteriſh Taſte. *Peucedanum* is Bitter and ſweet. The green Leaves of Coriander are fœtid like *Punaiſes*. *Laſerpitium* is Acrid, Aromatic in ſmell, and of a

ſweet, Acrid, Aromatic Taſte. Sampire is of an Acrid Aromatic Taſte, and the ſmell of Smallage. Cumin is Acrid, Aromatic and Bitteriſh, and of a diſagreable ſmell. *Dracunculus Hortenſis* is ſweet like Anis. *Ferula* has an ungrateful Aromatic Odor. The Seed of *Meum* is bitter. *Cachrys* is Bitter, Acrid, Aromatic. *Cancalis* and *Daucus* agree in Virtue and Taſte. *Saxifragia pratenſis* has a great Root, ſweet, and Acrid; the Leaves are moſt Aromatic, and the Seeds; it reſembles the Taſte of Parſly. The Leaves of *Gingidium* are of a diſagreeable ſtrong ſmell, and the Roots bitter. *Tordylium* is ſweet, Aromatic, *gravi odore*. *Percipier* is like Chervil, and ſo is *Pecten Veneris*. I refer the Umbells to the Graſſes, becauſe of their Sweetneſs and Joynted ſtalks. Our Botaniſts have omitted the ſweet Taſte in ſome of them, which are bitter. Moſt of them are of an Aromatic ſmell and Taſte, but ſome are fœtid, viroſe and fervid in their Taſte. *Smyrnium* Root has the ſmell of Myrrh, with a Bitter Acrid Taſte, and it helps the Urine and the Menſes like it. The colour of the Specifick Juice is various; *Thyſſelinum* and *Seſeli pratenſe* have a Milk: The Ferula's are Milky, or have a Saffron colour: Their Juices make *Sagapenum* and *Galbanum*. The Juice of the Root of Peucedanum is reddiſh, and is called *Opoponax odore piceo vinoſo*. The Root of *Syphondilium* has a Saffron colour'd Juice, of a fœtid and bitteriſh Taſte. 'Tis obſerved that ſome ſeeds grow bitter by the fault of the ſoyl in which they grow. All of this claſs have a Volatile Oyl and Acrid ſalt, by which they are diuretic, carminative, and pectoral, alſo by their ſweetneſs, Emmenagogue, if fœtid. I boyled Parſley Roots ℥j in ℔j of Water to half, the Taſte was ſweet, and Acrid, Aromatic, the Colour pale. The Roots of *Meum* are Acrid, and ſmell ſtrong; too much of it offends the Head. *Libanotis* has the ſmell and Taſte of the ſeed of *Angelica*. *Sium* or *Apium paluſtre fol. oblongis* has an odor of Bitumen on all the Plant. *Thyſſelinum* is Bitter, Ingrate and Acrid. The Decoction of Angelica Roots are bitteriſh, Aromatic, Acrid, and of a yellowiſh Colour. *Imperatoria* Roots decocted ſmelled like Angelica, and taſted very Bitter and Acrid; of a green Colour.

6. The ſweet, Acrid, Terebinthinate and ſtyptic. *Calamus, Nardus, Cyperus,* agree in their Diuretic, Carminative and Emmenagogue Virtue, and their ſtypticity, *&c.* *Calamus* reſembles a Reed; it is an *Acorus*, the Taſte reſembles the Turpentine Plants, as well as its Cones. *Juncus odoratus* is a Ruſh, with the ſmell of a Roſe, when rubbed of a Burning, Acrid, Aromatic and Bitteriſh Taſte. *Cyperus* is of a pleaſant Odor, like *Lign. Aloes* whilſt it flowers, and of an Acrid, Aromatic, Bitteriſh, ſtyptic Taſte. *Nardus* reſembles the flavor of *Cyperus*; 'tis Bitteriſh, Aſtringent, Acrid and Aromatic. The Roots of *Cyperus* are uſed for *Nardus*. There is ſome fœtor in *Valerian*, *Aſurum*, *Serpentaria*, whoſe Roots have a little Sweetneſs, with a Terebinthinate Bitter Acrid, and all of them reſemble *Spikenard odoris gravitate*, and have the ſame Diuretic, Carminative, Emmenagogue

Vertue, and are proper for Malignant Fevers; and a little ſtyptic. The Red Valerian has a crude Taſte in the ſtalks and Leaves, the Flowers rubbed ſmell like Turpentine; the Leaves ſmell nauſeous, fœtid; the Roots agree with Spikenard. *Spica Celtica* and the Flowers of Valerian agree well. The Leaves of Wild Valerian boyled in Water yield a crude and fœtid Mucilage. The Tincture of the Roots of Valerian, extracted with Spirit of Wine and Sal Armoniac, fines it much. The Roots of Aſarum are Geniculate as Graſſes, and taſte very Acrid, Aromatic and Bitteriſh; 'tis called *Sylveſtre Nardum*, and for its ſimilitude referrible to it. The Antients attributed the ſame Vertue to *Aſarum* as to *Acorus*, but more intenſe and ſtrong. *Galen* obſerved that *Phu* was an Odorate Root, like *Nardum* in virtue. *Pliny* obſerved that Cyperus was a *Gladiolus bulboſa radice*, and like the Odor of *Nardum*. The Decoction of *Aſarum* was greeniſh, with a ſtrong ſmell of Spikenard, and Bitteriſh Acrid. The Decoction of Valerian Roots of the ſame Taſte, but pale colour, and ſmell of Spikenard, but weaker; they ſeem to be of the ſame Claſs.

7. The Sweet, Fervid, Acrid, Bitteriſh, and Aromatic like Orris in Taſte. The Roots of Orris are Geniculate like Graſs. The *Florentine* Roots are bitteriſh and ſweet, and of a burning Taſte. The Water Flag is burning and ſtyptic, but of no ſmell; The Flowers of Common *Orris* have an ill Smell, though the Roots be Aromatic. The whole Herb ſmells like Elder whilſt 'tis freſh bruiſed, but when dry 'tis Odorate; it gives a Rasberry Taſte to Drinks, and purges. The great Galangal is Aromatic, Acrid, Burning and Bitteriſh; the Roots alſo geniculate, odorate; in form like Cyperus. The Roots of Acorus are geniculate, acrid, burning, bitteriſh, and aromatic; it reſembles Orris both in Leaves and Roots. Ginger is acrid, burning and aromatic like Pepper; the Leaves are like *Iris Paluſtris*. *Zedoary* ſmells like Camphire, and is of a ſtrong Taſte, rather than ſweet, 'tis very bitter, and leſs acrid; but reſembles Ginger. Coſtus is very burning, acrid and aromatic, and bitteriſh; it agrees with the vertue of Orris, both in its diſcuſſing quality and deobſtructing. *Coſtus Dulcis* has a ſweet Taſte, and acrid; but the bitter has an ungrateful kind of Taſte; but there is but one ſpecies of *Coſtus*, the freſh is ſweet and white, the old is bitter and blackiſh. *Gladiolus* has a bulboſe Root, ſweet, and moderately acrid or burning; and the Leaves are like Orris. *Xyrs* is of an ungrateful fœtid odor, like Cimices; in form and figure like Orris. *Coſtus Arabicus* is acrid, bitter and aromatic in Taſte. The Roots of *Contrayerva* ſmell like Fig leaves, are like Orris Roots in figure; they being chewed taſte ſweet, aromatic and acrid, and have alſo an aſtringent Taſte. *Cardamomum* has the Stalk and Leaves of a Reed, the Seeds in cods of a burning, acrid, bitteriſh, aromatic Taſte. *Amomum* by its external figure and virtue agrees with *Cardamomum*; ſome reckon it like Acorus. *Grana Paradiſi* are of a fœtid and aromatic Taſte, betwixt Cardamums and Pepper, and agree with the ſame in vertue in Paralytic caſes. The Grains of

Paradice infuſed in Wine gives the odor of Quinces. *Salmon.* The Roots of our yellow Flag are large and reddiſh; our Countrymen ſcrape them and pound them, then mix them with Milk, and give them twice to a Dog, as an infallible Medicine againſt the bite of a Mad Dog. A perſon who took them told me they did not purge, but taſted very rough. This Decoction is of a reddiſh colour and rough.

8. The ſweet mucilaginous Pea-taſtes, or Legumens. I boyled *Vicia* ℥j in ℔j of Water to ℔ß, which taſted ſweet and crude ſtyptic, the colour was pale and greeniſh. I boyled *Lens Paluſtris* in the ſame manner, and found the Decoction of a turbid colour, and reddiſh like Mum; it is a Legumen by this intenſe colour, as well as by the Taſte, which was ſweetiſh, ſubacrid and ſtyptic. Note, that Snails and other Inſects are uſually mixt with it. I boyled *Lagopus* in the ſame quantity as the other; the Taſte of the Decoction was ſtyptic, crude and bituminous. The *Cortex Lentium* is ſtyptic and binding; the Seed of it mucilaginous and looſening; and this breeds an ill, flatulent and crude Juice, obſtructing the circulation of the Blood and Spirits, injurious to the Sight and Menſes; and by their flatulency producing turbulent Dreams. The Leaves of Liquoriſh feel gummoſe, the Roots are ſweet and ſubacid. Tamarinds are of an acid, acrid and ſweet mucilaginous Taſte; the Stones have the Figure of Lupins, the Leaves of Tamarinds are of an acid agreeable Taſte; and they muſt be referred to the Services and Berberries, tho by the Cods and Seeds I have erroneouſly claſs'd them with Caſſia or *Lupins*. *Fumaterry* is ſubacrid, bitteriſh, mucilaginous and legumen, tho the Flowers be not. *Radix Cava* has bitter ſweet Roots like Beans, and a mealy ſubacrid Taſte; the Leaves have a crude ſmell like Fumiterry, and of the ſame virtue. The Skin of the Bean is ſtyptick, but the fleſhy part bitteriſh leguminous. Coffee is bitteriſh and of the Bean kind, in which there is a volatile Salt, which raiſes the Spirits and produces Urine and the Menſes. *Orobus* is of an ingrateful bitteriſh Taſte; all the bitter Legumens open obſtructions and promote Urine, as Kidney Beans, *&c.* I boyled Broom ℥j in Water ℔j to ℔ß, the Taſte was moderately bitter, ſweet and ſtyptic, not nauſeous, and without any heat. I boyled of *Geniſta Spinoſa roots* ℥j in ℔j of Water to ℔ß, the Colour was like ſmall Beer, the Taſte mucilaginous, ſtyptic, without any great bitteriſhneſs or acrimony; the Root taſtes bitteriſh, mucilaginous, ſtyptic; it may be proper for the Stone by thoſe Taſtes, and for Fluxes. The Leaves of *Bruſcus* bruiſed have the ſmell of Broom, and its bitteriſhneſs, ſtypticity and ſweetneſs; and are ſubacrid, and by this Taſte a Legumen. *Crato* deſcribes Senna as if it had a *Viſcidum quid*, by which it gripes; and by the Taſte he diſcovered the Bitteriſhneſs and Aſtriction. *Matthiolus* obſerved its viſcous Taſte and Bitteriſhneſs, with a viroſe nauſeous odor, like *Cynogloſſe* or *Punica*. The ſiliquæ, as well as the ſweet Taſte and Flowers, prove it a Legumen. *Polygala* boyled with the Blue Flowers gave a Colour Blue like Violets, and the Taſte was

very mucilaginous, the Plant is aromatick, acrid. ℥j of Violatri Color boyled in ℔j of Water to half, made a Decoction of a greeniſh Colour, like Cowſlip Wine, and it taſted gratefully and mucilaginous; the Roots of *Polygala* are acrid and aromatic; it purges Bile. The Decoction of the Root of *Periclymenum* is ſtyptic, bitter, and of ſweet leguminous Taſte. Purple Trefoyle is mucilaginous and ſtyptic, by which it cools Feaveriſh Heats, and by its burning Acrimony it expels the putrid particles in Perechial Feavers and Fluxes. The Leaves of red Trefoyl rubbed ſmell acid, and the Flowers bruiſed ſmell like Woodbind. The Decoction of ℥j in ℔j of Water to half gave a greeniſh Colour, pale; and the Taſte was of a crude mucilaginous leguminous Taſte, with an acrimony, by which 'tis a great Diuretic and Ophthalmic. The Decoction of Purple Trefoyl has a turbid Colour, and mucilaginous Taſte. The Leaves of *Periclymenum* are acid and acrid; the Flowers more ſweet, the ſtyle is of a Bean Taſte, and the Twigs of the ſame. The *Stamina* have the Taſte of the Flower, and their Heads are very acrid.

Trifolium Fruticans is Bacciferous, with a blue Juice, and though it wants Cods 'tis a Legumen, as well as Periclymenum, which is bacciferous. The Flowers of Aſparagus are hexapetali; and of an Herbaceous Color; the Seed is a Pea, without a Cod, the Root is of a ſweet glutinous acrid Taſte; the Tops of it boyled reſemble Peas Pottage, and are evidently of a Leguminous Taſte. In the Legumens theſe are irregular, in Flowers or Cods, they are notwithſtanding certainly of that Claſs; by which we may infer, that the Taſte gives the moſt certain character of a Claſs. The *Herba Mimoſa* has an Herbaceous odor and a mucilaginous bitteriſh ingrateful Taſte; the Root is of a Taſte more grateful, without bitteriſhneſs, but of a violent ſmell like Garlick when firſt got, offending the Smell and Head, and is accounted a Poyſon. *Moriſon. Meſue* obſerves that *Pſyllium* has contrary Virtues; 'tis of a mucilaginous, acrid and nauſeous Taſte: The *Medulla* is hot and exulcerating; the *Cortex* is moyſtening and cooling. *Goſſipium* is a Plant like Linum, both in Leaves, Flower and Stalk. There is bitterneſs in *Linaria. Morſon. Linaria* is by its Flower of the leguminous Claſs, and probably Linum is of the ſame, tho the Flowers diſagree, being like it in Leaves. Linſeed has the ſame Virtue as Fenugreek, which is a Legumen. The Leaves of Flax are gummoſe and bitteriſh, ſubacrid; the Flowers are *Pentapetali*, which differ from the Legumens. I boyled ℥j of *Galega* in ℔j of Water to the half, the Color was pale, the Taſte nauſeous, bitter, ſweet and mucilaginous, and the Odor nauſeous and fœtid. The freſh Leaves of *Trifolium Aſphaltites* ſmell like Rue, wehn ripe like Bitumen. The Decoction of *Ononis* was mucilaginous, ſubacrid, the colour was greeniſh. The Leaves of *Conſolida Regalis* are of a crude Smell, the Seeds are in Cods and Taſte leguminous. The whole Plant is of a diſagreeable Taſte. *Glaſtum* and *Opiogloſſum* are of the ſame Virtue, and *Luteola* is like them; place them here or with the Creſſes.

Thalictrum muſt be referred to the Docks both by its Taſte and Virtue *Lignum Nephriticum* is ſubacrid and bitteriſh.

Of the inward uſe of Cantharides, *by Mr* Yonge, *n.* 280. *p.* 1210.

V. A Gentlewoman of 54 Years old, who for a long time had been tormented with frequent Fits of the Stone, and uſually brought off many, with the Gravel, *&c.* about a Year ſince grew Dropſical, of which being lately cured, ſhe fell into a total Suppreſſion of Urine, which many days baffled all Remedies. In this condition, about four in the Afternoon, the fifth Day of the Diſeaſe I gave her five Cantharides (without Heads, Wings or Legs) weighing four Grains and a half, and with as much Camphire and a little Conſerve, made them into 2 Pills or Bolus's. Next Morning, finding no effect good or bad, I repeated the Medicine; after which, about noon the Flood came, and continued above 48 Hours, bringing off in that time much more Urine, than could have been expected from her in the whole time of the Obſtruction. Some Gravel and *Sabulous* matter came away, but no Stones, nor did there any thing happen to the Stomach, Bladder or other Bowels, as uſual on the internal uſe of thoſe Inſects, but they operated ſo quietly, as if nothing but two Doſes of *Lapis Prunellæ* had been adminiſtred.

In ſeveral other Caſes I have often and ſucceſsfully given it, and without the *Dyſuria* and other painful accidents which attend the internal (oftentimes the external) uſe of this Remedy; altho' I mixt no Camphire, but I waſht it down with large Draughts of Poſſet, Ptyſan, Emulſions or Water-gruel; which in this Ladies Caſe I forbore to uſe, becauſe of her Dropſical Diſpoſition, and uſed only a Draught or two of middling Ale, impregnated with Broom, Juniper-Berries, Daucus Seeds, *&c.* The form in which I uſed to adminiſter this fiery Inſect, is that of a ſoft Pill or Bolus, compoſed of three *Cantharides* prepar. *Troch. e Myrrha* ℈ß *Sem. Amei* gr. vj R. *ob Cynosb.* q. ſ. This in ſtubborn Suppreſſions of the *Lochia* and *Menſtrua*, in difficult Child Birth, and Retention of the *Secundine*, does Wonders; what heat or pain it begets in the Neck of the Bladder, is much ſhort of what I have an hundred times ſeen (and ſometimes felt) to proceed from applying an Epiſpaſtick to the Back.

About twenty Years ſince, an enamoured Youth attempted to gain a coy Girl's Love, by giving her a Plumb-Cake, in which powder'd *Cantharides* were mixed; She eat part of it, and gave three others of the Family in which ſhe lived a ſhare: they were ſoon tormented with burning in the Stomach, bloody and ſcalding Urine, and great Pain in the Back; I cured them all in a ſhort time, by Powder of *Amoos* and *Lapis Prunellæ*, and Emulſions *aq. Sperm.* R. *&c.* How many *Cantharides* each devoured in the Bread, I could gueſs by a piece of it which remained, and ſuppoſe that viii or ix Grains fell to each ſhare.

VI. In

Effects of ſeveral ſorts of Poyſons on Animals, by W. Courten, *Eſq*; *n.* 335. *p.* 485.

VI. In *July*, *Anno* 1678. at *Montpelier*, we gave a Dog a piece of Bread ſteep'd in two Ounces of the Juice of *Dutch* Night-ſhade [*Solanum Batavicum*] expreſs'd from the green Plant, and mixt with Cheeſe. As far as we could perceive, he did not ſeem to receive any manifeſt Damage from it. The ſame Doſe of the Juice of the Leaves of *Hemlock* [*Cicuta*] had no more effect. We gave alſo the ſame Dog a pretty large Root of *Wolfs-Bane* [*Aconitum Pardalianches*] together with the Leaves and Flowers of the ſame Plant bruiſed and mixed with Fleſh; which did him no Hurt. Two Drachms of *White Hellebore* [*Helleborus albus*] very much diſorder'd him, and cauſed Reachings, Suffocations, Vomiting, and Voiding of Excrements. This Dog (as afterwards we often obſerved in others that had taken the like Corroſive Medicines) whether becauſe he was not able to endure the Pain, or by reaſon of any other Uneaſineſs, often ſcratched the Ground with his Feet: However he recover'd, and was well again. He ſwallowed alſo five Roots of *Meadow-Saffron* [*Colchicum Ephemerum*] dug freſh out of the Earth: With which he was violently tormented, but did not dye.

At laſt he took two Drachms of *Opium*, which caſt him into a deep ſleep; but after vomiting and voiding fœtid Excrements, he recover'd by degrees his former Briskneſs. So many, and thoſe ſo notorious Poiſons, could not kill this Dog.

Some Weeks after, when the ſame Dog had recover'd his former Vigour, we try'd on him the Force of a much ſtronger Poiſon. We cauſed him to be bit three or four times on the Belly, a little below the Navel, by an inraged Viper. There aroſe immediately little black Bladders, containing a liquid blackiſh ſort of Corruption; they were flaccid and tremulous, like the Gall Bladder when it is about half full; and a livid Colour by degrees ſpread over all the neighbouring Parts. The Venome propagated it ſelf with wonderful Quickneſs, and weakened all, but more eſpecially the animal Functions: For notwithſtanding the Diaphragme did ſtill perform its Office pretty ſtrongly, tho' with ſome diſorder, and the Heart continued beating, tho' faintly and irregular; yet they ſeemed to fare much better than the Brain, whoſe Strength was ſo weakened, that it could not perform the Functions of Senſe and Motion but very faintly; inſomuch, that the Dog lay without any Strength or Senſation, as if he had been ſeized with a Lethargy or Apoplexy: Which kind of Stupidity we alſo obſerv'd ſometimes in a greater, ſometimes in a leſs degree, in all other Dogs bit by a Viper. Being willing to ſave this Dog, (though we had found by many Experiments, that much ſlighter Wounds made by a Viper had occaſioned Death) we thought fit to have recourſe to ſeveral Remedies; and therefore cupped and ſcarified the part that was wounded, and applied Treacle [*Theriaca.*] After this we let him alone for about two Hours: But his Sleepineſs encreaſing more and more, and his vital and animal Functions ſinking, we were forced to have recourſe to another Method of Cure. Wherefore to diſpel his Sleepineſs, we forced into his Throat half a Drachm of Volatile Salt of Hartſhorn mixt in Broth; which we eaſily did, by reaſon of his Weakneſs. In a little time after, his Eyes, which before looked dead, began to revive, and he was able to ſtand on his Feet and walk. Whereupon we repeated the ſame Doſe of the Volatile Salt, by which he was freed from his Sleepineſs, and the Strength of his Heart recover'd; and notwithſtanding he remained weak for three days, yet he ſenſibly recover'd Strength, though he would eat nothing all that time: But he drank Water very plentifully and greedily; and on the ſecond day did not refuſe cold Broth. After the third day he began to eat ſolid Meats, and ſeemed out of danger; only ſome large foul Ulcers remain'd on that part of his Belly that was bit, of which he would ſcarce have died, had he not been killed by another Dog; which prevented us from ſeeing the Event of this Experiment. But to try more fully the force of the abovementioned Poiſon, it is neceſſary to make ſeveral Experiments of it: For tho the Bite of a Viper, if it be but ſlight, may kill ſome Dogs, yet in the Month following, a large ſtrong Dog, that was bit in the Tongue, which is a very dangerous part, recover'd without any Medicines. His Tongue indeed turned black, and ſwell'd ſo much, that it could ſcarce be contained in his Mouth: He was ſtupid, as is uſual from the Venome of a Viper, but not ſo much but that he could ſtand on his Feet. A few Hours after, his Sleepineſs decreaſed; and the next day he endeavour'd to lap Water, but the bigneſs of his Tongue prevented him. On the third day he threatned to bite any Body that diſturb'd him, and had recover'd ſo much Strength, as to be able to eſcape out of the place where he was kept: And two after, was ſeen in the Streets; but what became of him afterwards we could not learn.

On the 17th of *October* we gave a Dog fifteen Grains of the dryed Root of *Monks-Hood* [*Napellus*] powder'd, and mixt with Fleſh and Broth. He had no ſooner taken it, but he was ſeized with a difficulty of ſwallowing, or rather ſeemed as if he was like to be ſtrangled. He immediately grew faint and reſtleſs, and dug the Ground with his Feet; but ſoon deſiſted, by reaſon of a fainting Fit, as we imagined from the dull Colour of his Eyes, and a Weakneſs of all his Body. This Fainting was preſently ſucceeded by a violent Vomiting, in which he threw up the Fleſh that he had eaten, which was very little alter'd. His Fainting ſoon returning again, he laid himſelf on the Ground; but being ſeized with terrible Convulſions of the *Abdomen*, Diaphragme, and of almoſt the whole Body, he run from place to place, and vomited ſo great a quantity of frothy Matter, that he was like to have been ſtrangled. His vomiting encreaſed, with a kind of crying and ſobbing, like broken Sighs, as if he had endeavour'd to bark at thoſe that ſtood by. In this manner he was miſerably tormented for the ſpace of an Hour; at which time all his Symptoms remitted, and by degrees he recover'd. In the Summer before, we gave a little Dog a Drachm of the Root of *Monks-Hood* [*Napellus*:] He was ſoon after ſeized with the ſame Symptoms, but they were longer and more violent; and he in like manner recover'd. In both theſe Dogs we particularly took notice of theſe broken and interrupted Sighs, or kind of ſobbing; becauſe we did not obſerve the like to be occaſioned by any other Poiſon that we had made Trial of. An Ounce of the Leaves, Flowers, and Seed of the *Napellus* when green, being bruiſed and given to a Dog, ſcarce diſorder'd him any more than if he had eaten ſo much Graſs. About the ſame time we made tryal of the *Nux Vomica* on another Dog; not that we doubted of its being a Poiſon, but that we might ſee the effect of it on his Body when dead. The Dog accordingly dying in a ſhort time, we found his Stomach and ſmall Guts very red; and judged this Redneſs and Inflammation to be cauſed by the Corroſiveneſs of the Medicine.

On the 20th of *October*, we injected warm into the Jugular Vein of a ſtrong luſty Dog an Ounce of Emetic Wine [*Vinum Emeticum*:] For a quarter of an Hour after the Operation was over and he was let looſe, he continued pretty well, unleſs that he ſeemed ſomewhat dejected; but afterwards he began to grow ill, and an unuſual Agitation was manifeſt about the Diaphragme; this was followed by a continual Vomiting, and a little after by an Evacuation of ſome hard Excrements. By theſe Evacuations he ſeemed to be ſomewhat relieved, but ſoon grew uneaſy, moved from place to place, and vomited again. After this he laid himſelf down on the Ground pretty quietly; but his Vomiting returning again, diſturbed his Reſt, and abated his Strength, which grew weaker and weaker; for in the ſpace of an Hour he vomited 12 times or more, and ſometimes voided ſome liquid Excrements, but in ſmall quantity; having frequent Inclinations to go to Stool, but in vain, as in a *Teneſmus*. An Hour and half after the Operation, he being ſo weak that he could not ſtand, his Eyes dull, and looking as if he were half dead, we gave him ſome warm Broth thro' a Funnel. With this he was wonderfully refreſh'd immediately, and ſoon after could look about, ſtand on his Legs and walk; but by reaſon of his weakneſs, reel'd as if he had been drunk. We left him by himſelf in a warm Room, where he remained cold, and lay as if he had been dying; and in an Hour after, we forced him to take ſome more Broth, which revived him again: But in a little time, after ſome agitation of his Body, he vomited, made Urine very plentifully, howled miſerably, and dyed convulſed. Next Day in viewing his *Viſcera*, we found two things very obſervable (but neither of them occaſioned by the Liquor that was injected;) one of them was in the Heart, the other in the *Æſophagus*. In the Heart there were two *Polypus's*: That which poſſeſs'd the right Ventricle, ſtretched itſelf into the *Vena cava* and Pulmonary Artery; and that which was in the left Ventricle, ſent Branches into the adjoyning Veſſels, and was leſs than that in the Right Ventticle. The Subſtance of the *Polypus* was pretty firm, of a Fleſh colour, ſomewhat pellucid, and being cut thro' the middle was altogether of the ſame colour and conſiſtence as on the Surface. To the *Æſophagus* there grew a remarkable Gland, which was hard, callous and foul, and opened with a ſmall, round, fleſhy Orifice into the Inſide of the Stomach, where, upon preſſing it, a little Corruption came forth. Upon opening this Gland or Tubercle, we found in it a great many little Worms, wrapt and entangled together, and moiſten'd with a corrupted Matter. Some of theſe Worms were above 4 Inches long, others leſs. Afterwards we found the like Glands, full of Worms, in other Dogs, and in moſt we opened, but not ſo much corrupted as in this. We obſerv'd alſo the like foul Glands in the *Aorta deſcendens*, but in one only found a Worm like theſe, which was almoſt got out of it, thro' an Orifice, into the Cavity of the *Thorax*. After this we likewiſe obſerv'd more *Polypus*'s in Dogs.

On the 27th of *October* we injected warm into the Jugular Vein of a Dog a Drachm and half of *Sal Armoniac* diſſolved in a Ounce and half of Water. The Liquor had ſcarce arrived at the Heart, but the Dog preſently fell into deadly Convulſions over his whole Body: Wherefore we let him looſe, but he dy'd immediately. On the 18th of *November*, we cauſed a Whelp to be bit in the lower Lip by a Blind-Worm [*Cæcilia*] ſo that the Blood appear'd in the Wound. The Whelp dy'd indeed the ſame Day; but becauſe we had committed him to the care of another Perſon, we could not be certain whether he dy'd of the Wound or not; and what increaſed our Suſpicion, was, that there did not appear on the part that was bit any livid Colour. On the 12th of *December* we injected into the Jugular Vein of a Dog a Drachm of *Salt* of *Tartar* diſſolved in an Ounce of warm Water: He dy'd crying, and in Convulſions, almoſt immediately. On the 15th of *December* we found a *Polypus* in both the Ventricles of the Heart of a Dog, each *Polypus* ſtretching itſelf with a double Root into the Veſſels of the Ventricle it poſſeſſed. Afterwards we often obſerved the like *Polypus*'s in other Dogs.

On the 20th of *December* we injected warm into the Jugular Vein of a Dog an Ounce of Urine made by a Man faſting. The Dog was uneaſy during the Injection, and while the Liquor paſſed to the Heart; but was not ſeized with any Convulſions or other ill Symptoms; and being let looſe, eat Breed very greedily. The ſame Day we made a gentle Decoction of two Drachms of White Hellebore, well powder'd, in Spring Water, and evaporated it away to Nine Drachms and a half; and the next Day injected all the Decoction, ſtrongly preſſed out and turbid, into the Jugular Vein of a Dog. At firſt ſome few Drops only paſſed to the Heart, ſome concreted Blood obſtructing the Paſſage; but thoſe Drops very much affected the Dog, for he was ſeized immediately with convulſive Motions: But ſoon after, when the Liquor had removed what lay in its way, and had enter'd the Heart, it killed the Dog as ſuddenly as if he had been ſhot thro' the Heart with a Bullet; for having looſen'd him preſently, to ſee if any Life remain'd, he was quite dead and flaccid, and hung like a Fleece in the hand of the Perſon that held him.

On the 2d of *January* 1679. Vinegar was injected warm into the Jugular Vein of a Dog, without doing him any manifeſt harm. The ſame Day we cauſed a Whelp to be ſtung in the Tongue by ſeveral Scorpions; but the Wounds made by the Scorpions, by reaſon of their weakneſs, being but ſlight, and not penetrating deep, we made a ſmall Inciſion on the *Abdomen*, and drawing aſide the Skin, let the Scorpions make ſeveral Wounds on it; but without any effect, tho' we often forced the ſting into the Wounds, and preſſed the Bladder that is ſuppoſed to contain the Venome. In like manner a *Pidgeon*, being ſeveral times ſtung by a Scorpion, remained unhurt. *January* the 3d, two Drachms of Sugar diſſolved in an Ounce of Water, was injected into the Jugular Vein of a Dog: He received no harm from the injection, but continued well for the three Days after, that we kept him. On the 4th of *January*, a Drachm and half of *Spirit* of *Salt*, diluted in an Ounce and half of Water, and injected in the Jugular Vein of a Dog, killed him immediately. In the Right Ventricle of his Heart, we found the Blood partly grumous and concreted into harder Clots than ordinary, and partly frothy. In the ſame Dog that Gland that contains Worms, and is frequently found in the *Æſophagus*, opened with two Orifices into the Cavity of that Part, and in the *Sinus*'s of it there lay ſeveral ſmall Worms. *January* the 5th, we gave a Dog 12 ſmall Caterpillers of the Pine-tree [*Pityocampæ, vel Erucæ Pini*] weighing $\frac{1}{2}$ a Drachm, which we bruiſed alive, and mixed with Fleſh. The Dog, tho' he was but young, received no other hurt, than that now and then he ſeemed as if he endeavour'd to ſwallow ſomething, or was troubled with an Inclination to Vomit; from whence we judged the Stomach and *Æſophagus* to be only lightly affected: But theſe Symptoms vaniſhed in a few Hours, and the Dog continued Briſk, and greedy of Meat, all the reſt of the Day. The ſame Day we included a Rat in a large Glaſs with a Scorpion; but the Scorpion, being dull and benum'd with the extream coldneſs of the Weather, was able to wound the Rat but very weakly; with which however the Rat being provok'd, ſet upon the Scorpion, and knawed off and devour'd part of him, keeping his Eyes ſhut all the while, that he might not be hurt by his Claws or Sting. The ſame Fate happened to another Scorpion, which we added to the former; but the Rat notwithſtanding remained unhurt. *January* the 6th, we killed a Dog almoſt in a Moment, by injecting into his Jugular Vein an Ounce of *Spirit* of *Wine*, in which there was diſſolv'd a Drachm of *Camphire*. The ſame Day we injected warm into the Crural Vein of a Cat, 50 Grains of *Opium*, diſſolved in an Ounce of Water. The Cat preſently after the Injection ſeemed very much dejected, but did not cry; only made a low, interrupted, complaining Noiſe. After this followed Tremblings of the Limbs, Convulſive Motions of her Eyes, Ears, Lips, and almoſt of all parts of her Body, with violent Convulſions of her Breaſt: Sometimes ſhe would raiſe up her Head, and ſeem to look about her, but her Eyes were very dull and deadiſh; and tho' ſhe was let looſe, and had nothing tied about her Head or Neck, yet her Mouth was ſo filled with Foam or Froth, that ſhe was like to be ſtrangled. At laſt, her Convulſive Motions continuing, and being ſeized with a ſtretching of her Limbs, ſhe dy'd within a quarter of an Hour. Upon opening her Body, we did not find the Blood much altered from its Natural State.

February the 7th, we injected into the Crural Vein of a luſty ſtrong Dog a Drachm and half of *Opium*, diſſolved in an Ounce and half of Water. The Dog immediately ſhewed the great Pain he endured, by a violent ſtruggling of his whole Body, a loud Noiſe that he made, notwithſtanding his Jaws were tied, a great difficulty of Breathing and Palpitation of the Heart, with Convulſive Motions of almoſt all Parts of his Body: In a little time theſe remitted, and he was ſeized with a profound Sleep, as if he had been in a Lethargy or Apoplexy. Having let him looſe, he lay upon the Ground without moving or making any Noiſe, in ſo deep a Sleep, that he would not move with beating. About half an Hour after, if we beat him, he would move a little, but preſently lay down again. After an Hour, if we beat him, he would move a little more; and by degrees his Sleepineſs a little decreaſing, in an Hour and half or two Hours time, when he was beat he would make a Noiſe and walk a little, but ſeemed very heavy and ſtupified, and reeled as he went; but as ſoon as we left off beating him, as if he had ſoon forgot every thing that had paſt, he preſently laid himſelf down again and fell aſleep. Next day when we viewed the Place where he lay, we found a great quantity of fœtid Excrements, like a corrupted Blood, or the diluted *Opium* that he had taken: But ſtill his Drowſineſs continued, and tho' we beat him with Whips, that he ran crying about the Room, yet he preſently forgot it, and immediately fell aſleep again. In this ſleepy Condition he continued 3 Days, refuſing whatſoever was offer'd him to eat, or rather not minding that or any thing elſe. On the 4th Day we found him dead: But perhaps he would not have dy'd of the ſtupifying Quality of the *Opium*, if (conſidering the extream coldneſs of the Weather) we had put him in a warmer place, and had forced him to have taken ſome Broth. *February* the 8th, we found in the Bladder of a *Tortoiſe*, adhering to its Coat, a flat porous Stone, about twice as big as a Lentil. *February* the 9th, a Drachm and half of *Common Salt*, diſſolved in an Ounce and half of Water, was injected into the Jugular Vein of a Dog. After the Injection, he was thirſty, and drank Water greedily; but in other Reſpects he ſeemed to be pretty well, and the next Day was quite recover'd.

February the 20th 1679, we injected into the Crural Vein of a little Dog, half an Ounce of warm *Oil of Olives*, which we did with a great deal of difficulty, and very ſlowly, by reaſon of the ſmallneſs of the Vein and thickneſs of the Liquor. For half a quarter of an Hour that we were injecting the Liquor, the Dog did not ſeem to be uneaſy or out of order; but after that, he barked, cryed, looked dejected, and fell preſently into a deep Apoplexy; ſo that his Limbs were depriv'd of Senſe and Motion, and were flexible any way at pleaſure; his Reſpiration ſtill continuing very ſtrong, with a ſnorting and wheezing, and a thick watery Humour flowing in great quantity out of his Mouth, which was ſometimes mixt with Blood. He loſt all External Senſe: His Eyes tho' they continued open, were not ſenſible of any Objects that were put to them; and we touched and rubbed the *Cornea* (as ſenſible a part as it is) without any more ſign of his being ſenſible of it than if he had been dead. His Eye-lids notwithſtanding had a Convulſive Motion: His Hearing was quite loſt; and his Feeling, tho' at firſt he ſeem'd to have ſome ſmall Senſe of it when we touched his Wound, yet afterwards it was ſo dull, that we pinched his Claws and Fleſh with Pincers, and bored Holes thro' his Ears, without his moving or ſeeming to be the leaſt ſenſible of it. It is worth obſerving, that in the midſt of his Sleep, being ſometime ſeized with a Convulſive Motion of his Diaphragme and other Muſcles that help Reſpiration, he would bark ſtrongly as if he were awake, and in a little time would be quiet again: So that in leſs than a quarter of an Hour his Reſt would be diſturbed 3 or 4 times with this violent Barking. But conſidering this more attentively, we found that at the very time he barked, he was as void of Senſe as before; for we could neither make him Bark, nor leave off Barking, by either beating or pricking him; but in a little time he would leave off of himſelf, and return to it again ſome time after. Thus in three Hours after the Injection, ſpent in Sleeping and Barking, he dyed; and having opened his Body after he was dead, we found the *Bronchia* of the Lungs filled with a thick Froth. A few days after we injected a larger quantity, *viz.* an Ounce of *Oil of Olives* into the Jugular Vein of a Dog, which ſuffocated him the ſame Moment. Afterwards the ſame quantity of *Oil of Olives*, being injected into the Jugular Vein of a Dog, killed him in an Hours time. He was ſeized with a great Sleepineſs, Snorting and Wheezing, and a bloody Water run plentifully out of his Mouth. In this Dog, tho' he did not dye immediately, we did not obſerve the Barking as in the former: But in all that were ſuffocated by Oil, we found their Lungs filled with a very thick Froth. *February* the 27th, we injected 10 Drachms of highly rectified *Spirit of Wine* into the Crural Vein of a Dog. The Dog died in a very little time, very quietly, and as it were with pleaſure, licking his Jaws with his Tongue, and breathing quick, but eaſily, without barking, crying, or any Convulſive Motion. In the *Vena cava* and right Ventricle of the Heart, the Blood was concreted into a great many little hard Clots; which appeared yet more conſpicuous and harder in ſome Blood that flowed back from the Vein into the Syringe. In this Dog we found the Emulgent Artery of the Left Side to be double.

March the 2d, we injected three drachms of rectified Spirit of Wine into the Crural Vein of a ſmall Dog; which made him Apoplectick, and as it were half dead. In a little time he recovered from his Apoplexy, but grew giddy; and when he endeavour'd to go, reeled and fell down. Though his Strength encreaſed by degrees, yet his Drunkenneſs ſtill continued: His Eyes were red and fiery, and his Sight ſo dull, that he did not ſeem to take notice of any thing, and when he was beat would ſcarce move. However, in four Hours time he grew better, and would eat Bread when we gave it him. The next day he was brisker, and ſeemed paſt all danger. In diſſecting the ſame Dog ſome time after, we found in the ſmall Guts two Flat-Worms; one of them about 6 Spans long, and the other about 5. They had perforated the Gut; and one of them was got half out of it into the Cavity of the *Abdomen*. We found alſo in 2 Dogs a Worm of near a Foot in length, out of the Inteſtines, in the Cavity of the *Abdomen*, the Inteſtines being no ways perforated, but remaining ſound and whole. That we might be more certain of this, we ſeparated them from the Meſentery, and viewed them very carefully. But in both theſe Dogs the *Omentum* was of an ill Colour and putrified; whence we conjectured, that theſe Worms were bred from the Putrefaction of the *Omentum*. We injected into the Crural Vein of a Dog five Ounces of a ſtrong White Wine; which made him very drunk, and little different from what a leſs quantity of Spirit of Wine would have done: But in a few Hours his Drunkenneſs abated, and he recovered. In the ſame Month of *March* we injected into the Vein of a Dog, an Ounce of a ſtrong Decoction of Tobacco. He was ſeized immediately with ſtrange Convulſions of his whole Body. At firſt his Eyes looked wild and diſtorted, his Jaws trembled; and in a little time he died terribly convulſed. This Experiment we repeated ſeveral times after, and always with the ſame ſucceſs. Ten drops of diſtilled Oil of Sage, mixt with half a Drachm of Sugar, and diſſolved in an Ounce of Water, being injected into the Crural Vein of a Dog, did him no harm. In a caſtrated Dog we obſerved the Proceſſes of the *Peritoneum* and Spermatick Veſſels to be covered with Fat, and ſcarce to be ſeen; and that he did not ſmell ſo rank and ſtrong, as other Dogs that had not been caſtrated. A yellow-ſtreak'd *Lizard* [*Lacerta Chalcidica*] which had been kept all the Winter in Glaſs with Bran, being expoſed to the Sun to refreſh it, on the contrary died in a few Hours. We have alſo often found, that Scorpions expoſed to the hot Sun, eſpecially in the Summer, died in a ſhort time. A drachm of purified White Vitriol, injected into the Crural Vein of a Dog, killed him immediately. Fifteen Grains of Salt of Urine, diſſolved in an Ounce of Water, and injected into the Crural Vein of a Dog, caſt him into ſuchviolent Convulſions, that we were afraid he would dye under them. When he had recovered himſelf a little, we repeated the Injection with the ſame quantity; but the Dog got the better of it, though with a great deal of difficulty, and perfectly recovered.

April the 27th, we made a Decoction of two drachms of Senna in Water, and injected warm three Ounces of it into the Crural Vein of a very fat, and ſtrong Dog. He continued pretty quiet, without any ſign of pain or uneaſineſs, during the Operation; and when it was over we let him looſe, expecting the Event of it. He was melancholy and dejected, but eaſie and without any ſenſible Commotion, for the ſpace of an Hour. After that, his Reſpiration grew quicker, he had a murmuring Noiſe in his Belly, with violent Commotions of his *Abdomen*, Diaphragm, Stomach, and Inteſtines, and vomited plentifully a Bilioſe Matter. After his vomiting he grew faint, and in a little time his Vomiting returned again; ſo that in an Hour and half he vomited four times. His Strength and Appetite were very weak, and he would eat nothing for three days. But on the third day his Appetite, Strength, and former Briskneſs returned, and he recover'd.

Two Dogs, which had their Recurrent Nerves cut, loſt their Barking and Voice. But doubting whether the Wound or Scar might not affect and hurt the Motion of the Muſcles, we performed the ſame Operation on another Dog, but without cutting the Nerves; and when the Wound was healed, he barked freely as before. A Dog that had the Nerves of the *Par vagum* cut aſunder, preſently grew dejected and faint. He breath'd very freely, and with Sighs; for when he had drawn in his Breath leiſurely and inſenſibly, it came forth again immediately very forcibly and with a Sigh, as if it had been retained a long time in the Lungs. The Muſcles of the *Abdomen* and Diaphragm laboured hard, as if they were to ſupply the defect of the Lungs, which were grown almoſt uſeleſs by being denied an Influx of Spirits by the Pneumonick Nerves. The Dog refuſed all kind of Meat; ſometimes he vomited, or had an Inclination to vomit; and at laſt, in two days time, he dyed. Another Dog, that had the Nerves of the *Par vagum* only tied, lived 10 days. He vomited frequently and would not eat unleſs clandeſtinely: He breathed with Sighs, and was very faint. A Dog, that had the Trunk of the *Aorta deſcendens* tyed hard a little above the Diaphragm, immediately loſt the uſe of his hind Legs; for when he ſtood on his fore Legs, he would draw after him his hinder Legs, as if they had been dead: He grew weaker by degrees, and dyed in five Hours.

July the 12th, a Mole being ſtung in the Side by a *Scorpion*, dyed immediately convulſed. In this we obſerved, that the *Inteſtinum cæcum* is wanting in Moles.

The manner of making Styrax liquida, *com. by Mr.* Petiver. *n* 313. *p.* 44.

VII. *Roſa Mallas* grows upon the Iſland *Cobroſi*, at the upper end of the Red Sea, near *Cadeſs*, which is three days Journey from *Suez*: It is the Bark of a Tree (taken off every Year, and grows again) boiled in Salt Water, till it comes to a Conſiſtence like Birdlime, then ſeparated and put into a Cask, and brought to *Judda*, and ſo to *Mocha* in *June* and *July*, where it ſells from 60 to 120 Dollars *per* Barrel, according to its Goodneſs. The beſt is what is freeſt from Clay and Dirt, which is commonly mixt with it: and the way to try it is, to waſh it in Salt-Water, which will cleanſe it: The *Arabs* and *Turks* call it *Cotter Mija*. A Barrel is 420*l.*

Of the ſtrange Effects of the Indian *Varniſh, by Dr.* del Papa, *com. by Dr.* Sherrard. *n.* 274 *p.* 947

VIII. The uſing and handling of the *Indian* Varniſh (or Lackar) ſo far as is neceſſary to apply or lay it on ſubjects to be varniſht, having produced ſuch extraordinary effects on Seignior *Ignatio*, and more remarkably on his Maid-ſervant, *viz.* in great Swellings of their Heads and Eyes, and in their Arms, and indeed almoſt their whole Body, with an intolerable itching and inflammation, or Heat and Pimples, is ſo new and extraordinary a Phænomenon in Nature, as exacts the wonder, and preſſes the Curious to ſearch the Reaſon. The great difference between this and all things elſe that we know, and the Ingredients of which its made, being abſolutely unknown, renders it impoſſible to penetrate the Cauſe of the above-named Effects. Yet whoever would give ſome ſatisfaction to the Mind, by propoſing ſome probable Thought, may ſay that this Varniſh contains ſome Matter which when hot ſends into our Body a very ſubtile thin Vapour, which affects only the Skin, leaving the other parts of the Body untoucht; after the ſame manner, Cantharides, not only taken inwardly, but alſo outwardly applied to our Bodies, communicates a venemous quality of a particular Nature, affecting only the Kidneys, Bladder and Urinary Paſſes, cauſing there ſenſible Pains and Excoriations, not in the leaſt touching the other *Viſcera*. Some Phyſicians of Repute are of opinion that this particular diſagreement of *Cantharides* with the Urinary Ducts, comes from the Salt and Nitre contained in the Urine, which gives Life and Vigour to the Poyſon of the *Cantharides*, without which Salt the Poyſonous Quality of the *Cantharides* could have no Power: So after the ſame manner it may be ſaid that the noxious Fumes of the Varniſh becomes hurtful to the Skin, becauſe it mixes there with ſome Juice it meets with in the Skin itſelf, eſpecially in the miliary Glands, whereof the whole Skin is full. This is certain that this Varniſh exerts all its Malignity againſt the Skin, the *Viſcera* and Blood being untouch'd; beſides I obſerved, that the Maid (at the ſame time as her whole Skin almoſt was hard, inflamed, ſwell'd and full of Puſtules) had no Fever, no Pain in the Head, nor any Sickneſs or Illneſs; and as to what Diſorders ſhe or Signior *Ignatio* felt in their Eyes, this likewiſe was only a Swelling affecting the Eye-lids only, which may be reckon'd but as Skins: but 'tis poſſible the ill Effects of the Varniſh was more ſenſible and troubleſome in that part, becauſe the Skin there is thinner and more delicate than on the reſt of the Body. This Varniſh therefore is only an enemy to the Skin; and that this miſchief ſhould attend it, it is not neceſſary that the Varniſh ſhould be heated; for altho it is cold, it ſends forth this ill ſteam, which inſinuates itſelf into the Body, eſpecially when touched and handled.

I have ſeveral times ſpread a great deal of this Varniſh hot upon the naked skin of Poultry, and they never received any miſchief from it, either internal or external. I have cauſed other Fowl to ſwallow Crumbs of Bread ſopt in the Varniſh, and they ſeemed to like it very well. In others I have made ſeveral little pricks in their Breaſts till the Blood came out, and then anointed it all over with Varniſh, which, inſtead of hurting them, proved a Balſam to heal them. It is poſſible this Varniſh on the very thin skin of Fowl does not produce the ſame effects as on that of Men, becauſe they are very different from one another, in their ſtructures and quality of the humours contained in them. And to ſay ſomething of its ſubſtance; I have obſerved that this Varniſh is in a great part compoſed of a Gummy and unctuous matter, and ſince it is very light, ſwimming upon Brandy and Oyl, and unites neither with Water nor Spirit of Wine, nor any other Liquor but only with Oyl; and burns or takes fire; for I have dipt Cotton in it, which has burnt all away to aſhes, tho at firſt there was ſome difficulty to make it take fire; perchance ſome other matter not unctuous being mixt with it. And laſtly, ſince, being obſerved with the Microſcope, its compoſition ſhews like that of Oyl or Lard, or the like unctuous matter, it is very likely from all theſe, that it is compoſed of the Gum or Juice of ſome Reſinous Herb or Tree, or of the fat of ſome Animal, or Hogs Lard. And to make ſome gueſs; who knows but the Gall of ſome Creature may be mixt with it, to make it the more eaſily receive a ſmoothneſs and luſtre, as Limners uſe to put Gall into their Water-colours, to make them run and ſpread the better; and that the miſchief we find in touching and uſing it may proceed from hence?

I verily believe, there is no Mercury, of what ſort ſoever, in this Varniſh; not only becauſe it is very light (as was ſaid) but beſides, becauſe I have been very diligent in trying whether Gold would diſcover any ſign of Mercury, either in the Body of it, or the ſmoke, but could never find any; and moreover, Mercury produces very different effects in our Bodies, from thoſe before related of this Varniſh. Beſides, I have obſerved that the Varniſh, mixt with Spirit of Vitriol, or Juice of Limons or Vinegar, or Spirit of Wine, makes no ebullition nor change of colour; but it readily changes colour, when taken out of the Veſſel, it is expoſed to the air, becoming at firſt reddiſh, and afterwards almoſt quite black; the outward skin of it, which is next to the air, becoming very hard and black: This Skin is very thin, under which the reſt of the Varniſh remains ſoft and fluid, of the colour and conſiſtence of Honey; and as often as you take off this outward black hard skin, there will another be formed immediately like the former, and this as often as you pleaſe to repeat the experiment. So that the whole ſubſtance of the Varniſh will in time be changed into theſe hard and black skins. Finally, 'tis worth obſerving that this Varniſh has this known power; for having ſpread ſome of it on the naked Breaſt, of ſome Fowls, leaving it ſticking there 3 days, I afterwards found between the dried Varniſh and the Fleſh the place all feſter'd, and full of a yellowiſh *Serum* or Matter, but without any farther Miſchief to the Bodies of the Fowls themſelves. I have attempted the ſame thing on Dogs and Cats but without Succeſs, for theſe Animals with their Tongues and Claws, ſoon take all the Varniſh from their Bodies, and ſo have no hurt by it. Poſſibly in Horſes and Beaſts the Experiment may ſucceed better, if the Varniſh has this Corroſive or Cauſtick Quality on their Bodies, as it has on Poultry.

IX. *Accounts of Books omitted. viz.*

*n.*273. *p.* 918. 1. D. Dominici Sanguineti Appuli Diſſertationes Iatrophyſicæ. *Neapoli.* 1699.

*n.*283. *p.*1320. 2. A Mechanical Account of Poyſons, by Dr. *Mead.* 8vo. *Lond.*

*n.*306. *p* 2253. 3. Samuelis Dale Pharmacologiæ, ceu Manuductionis ad materiam Medicam Supplementum; Medicamenta Officinalia ſimplicia priore libro omiſſa complectens: Ut & notas Generum Characteriſticas, Specierum Synonyma, Differentias & Vires. Cum duplici Indice, generali altero Nominum & Synonymorum præcipuorum; altero Anglico Latino, in gratiam Tyronum. 12mo. *Lond.* 1705.

n. 320. *p.*319. 4. Prælectiones Chymicæ Oxoniæ habitæ; a *Johanne Freind*, M. D. Æd. Chriſt. Alumn.

F I N I S.

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